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Introduction to neutrosophic soft ideal topological spaces

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Abstract. In this paper, we give the definition of neutrosophic soft Cl^*-closure, which is more general than neutrosophic soft closure of neutrosophic soft set, with the help of neutrosophic soft ideal and neutrosophic soft point. Then we investigate some basic properties of this new concept. Moreover, we define the notion of τ^* and give the concept of neutrosophic soft-$\mathfrak{I}-$compactness. Besides, we examine the relationship between these concepts and give the relations with previously given concepts.

Keywords: neutrosophic set; soft set; neutrosophic soft topology; neutrosophic soft ideal; neutrosophic soft-$\mathfrak{I}$-compact

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1. **Introduction**

Due to the importance of mathematically expressing uncertain concepts, which can't be defined by classical logic, researchers offer new theories everyday. Some of the most important well known theories are of probability, statistic, fuzzy sets, soft sets and neutrosophic sets.

Researchers in economics, medicine, engineering, business and etc are struggling with the complexity of undeterminate data modelling. Classical methods can't always be successful, because of the unambiguities that arise in these areas can take various forms. Probability theory, fuzzy sets, soft sets and neutrosophic sets are of famous and useful approaches in uncertainty identification.

In 1965, Zadeh [12] studied and improved fuzzy set theory, which plays a curcial role in covering the concept of uncertainty. Fuzzy set (briefly FS) is specified by membership function that defined on the closed interval $[0, 1]$. FS sets have been very popular in almost all branches of science. Later Atanassov [1] in 1986 defined intuitionistic fuzzy sets as a generalization of FS. Then Smarandache [11], defined neutrosophic sets in 2005 as a generalization of IFS (briefly, intuitionistic fuzzy set), where we have the degree of membership, the degree of indeterminacy

and the degree of non-membership of each element in X. In [7] Molodtsov initiated the concept of soft sets as a new mathematical tool for dealing with uncertainties. Hence, this theory has successful applications in various fields. Then P.K. Maji [6] gave the definition of neutrosophic soft set (briefly NSS) by using the NS set and SS sets. Bera [2] investigated the notion of neutrosophic soft topological spaces.(NSTS) T.Y. Öztürk et al. [8] redefined some operations on NSS and NSTS. They gave the definition of neutrosophic soft point (NSP).
In this study, we defined the concept of NS-ideal and the NS local function by using NSP which is given by Gündüz et al. [4]. Furthermore we investigate some basic properties of NS local function. Moreover we give the concept of NS $\star$-topology which is finer than NS topology. We also define and study the concept of nNS $\mathfrak{I}$-compactness via NSI. We present the relationship between the concepts of NS-compactness and NS-$\mathfrak{I}$-compactness.

2. **Preliminaries**

Definition 2.1. [11] A neutrosophic set N on the universe set W is described as

$$N = \{< \varpi, T_N(\varpi), I_N(\varpi), F_N(\varpi) >: \varpi \in W\}$$

where $T, I, F : W \to]^-0, 1^+[$ and $^-0 \leq T_N(\varpi) + I_N(\varpi) + F_N(\varpi) \leq 3^+$.

Maji [6] first defined the concept of the NSS, then Deli and Broumi [3] modified this definition as follow:

Definition 2.2. [3] A NSS $\widetilde{F}_P$ over W is a set defined by a set valued function $\widetilde{F} : P \to P(W)$, where $\widetilde{F}$ is called the approximate function of the NSS $\widetilde{F}_P$ and it can be written as a set of ordered pairs:

$$\widetilde{F}_P = \{(p, < \varpi, T_{\widetilde{F}(p)}(\varpi), I_{\widetilde{F}(p)}(\varpi), F_{\widetilde{F}(p)}(\varpi) >: \varpi \in W) : p \in P\}$$

where $T_{\widetilde{F}(p)}(\varpi), I_{\widetilde{F}(p)}(\varpi), F_{\widetilde{F}(p)}(w) \in [0, 1]$ called the truth-membership, indeterminancy-membership and falsity-membership function of $\widetilde{F}(p)$ respectively. Since the supremum of each T, F, I is 1, the equality $0 \leq T_{\widetilde{F}(p)}(\varpi) + I_{\widetilde{F}(p)}(\varpi) + F_{\widetilde{F}(p)}(\varpi) \leq 3$ is obtained. Let $NSS(W, P)$ be the family of all neutrosophic soft set over W.

Definition 2.3. [2] Let $\widetilde{F}_P \in NSS(W, P)$. The complement of $\widetilde{F}_P$ is denoted by $(\widetilde{F}_P)^c$ and is described as:

$$\widetilde{F}^c_P = (\widetilde{F}_p)^c = \{(p, < \varpi, F_{\widetilde{F}(p)}(\varpi), 1 - I_{\widetilde{F}(p)}(\varpi), T_{\widetilde{F}(p)}(\varpi) >: \varpi \in W) : p \in P\}.$$

It is obviuous that $((\widetilde{F}_P)^c)^c = (\widetilde{F}_P)$

Definition 2.4. [8] Let $\widetilde{F}_P \in NSS(W, P)$.

(a) $\widetilde{F}_P$ is said to be a null NSS if $T_{\widetilde{F}(p)}(\varpi) = 0$, $I_{\widetilde{F}(p)}(\varpi) = 0$, $F_{\widetilde{F}(p)}(\varpi) = 1$ $\forall \varpi \in W$, $\forall p \in P$. It is denoted by $0_{(W,P)}$.

(b) $\widetilde{F}_P$ is said to be an absolute NSS if $T_{\widetilde{F}(p)}(\varpi)$= 1, $I_{\widetilde{F}(p)}(\varpi)$=1, $F_{\widetilde{F}(p)}(\varpi)$=0 $\forall\varpi \in W$, $\forall p \in P$. It is denoted by $1_{(W,P)}$.
It is obviuous that $(1_{(W,P)})^c$=$0_{(W,P)}$.

Definition 2.5. [6] Let $\widetilde{F}_P$, $\widetilde{G}_P \in NSS(W,P)$. $\widetilde{F}_P$ is said to be NS subset of $\widetilde{G}_P$ if $T_{\widetilde{F}(p)}(\varpi)$ $\leq T_{\widetilde{G}(p)}(\varpi)$, $I_{\widetilde{F}(p)}(\varpi)\leq I_{\widetilde{G}(\varpi)}(p)$ and $F_{\widetilde{F}(p)}(\varpi)\geq F_{\widetilde{G}_A}(\varpi)$, $\forall p \in P$, $\forall\varpi \in W$. It is denoted by $\widetilde{F}_P \sqsubseteq \widetilde{G}_P$. $\widetilde{F}_P$ is said to be neutrosophic soft equal to $\widetilde{G}_P$ if $\widetilde{F}_P$ is NS subset of $\widetilde{G}_P$ and $\widetilde{G}_P$ is NS subset of $\widetilde{F}_P$. It is denoted by $\widetilde{F}_P$=$\widetilde{G}_P$

Definition 2.6. [8] Let $\widetilde{F}_P$, $\widetilde{G}_P \in NSS(W,P)$. Then their union is denoted by $\widetilde{F}_P \sqcup \widetilde{G}_P = \widetilde{H}_P$ and is described as

$$\widetilde{H}_P = \{(p, <\varpi, T_{\widetilde{H}(p)}(\varpi), I_{\widetilde{H}(p)}(\varpi), F_{\widetilde{H}(p)}(\varpi) >: \varpi \in W) : p \in P\}$$

where $T_{\widetilde{H}(p)}(\varpi) = max\{T_{\widetilde{F}(p}(\varpi), T_{\widetilde{G}(p)}(\varpi)\}$, $I_{\widetilde{H}(p)}(\varpi) = max\{I_{\widetilde{F}(p)}(\varpi), I_{\widetilde{G}(p)}(\varpi)\}$ and $F_{\widetilde{H}(p)}(\varpi)$ =$min\{F_{\widetilde{F}(p)}(\varpi), F_{\widetilde{G}(p)}(\varpi)\}$.

Definition 2.7. [8] Let $\widetilde{F}_P$, $\widetilde{G}_P \in NSS(W,P)$. Then their intersection is denoted by $\widetilde{F}_P \sqcap \widetilde{G}_P = \widetilde{H}_P$ and is described as

$$\widetilde{H}_P = \{(p, <\varpi, T_{\widetilde{H}(p)}(\varpi), I_{\widetilde{H}(p)}(\varpi), F_{\widetilde{H}(p)}(\varpi) >: \varpi \in W) : p \in P\}$$

where $T_{\widetilde{H}(p)}(\varpi) = min\{T_{\widetilde{F}(p)}(\varpi), T_{\widetilde{G}(p)}(\varpi)\}$, $I_{\widetilde{H}(p)}(\varpi) = min\{I_{\widetilde{F}(p)}(\varpi), I_{\widetilde{G}(p)}(\varpi)\}$ and $F_{\widetilde{H}(p)}(\varpi) = max\{F_{\widetilde{F}(p)}(\varpi), F_{\widetilde{G}(p)}(\varpi)\}$.

Definition 2.8. [4] The NSS $\varpi^a_{(\omega,u,\varrho)}$ is said to be neutrosophic soft point $(briefly, NSP(W,P))$ for every $\varpi \in W$, 0< ω, u, ϱ< 1, p $\in$ P, and is described as follows:

$$\varpi^p_{(\omega,u,\varrho)}(p')(v) = \begin{cases} (\omega,u,\varrho) & if\ p' = p\ and\ \varpi = v \\ (0,0,1), & if p' \neq p\ or\ \varpi \neq v. \end{cases} \tag{1}$$

A NSP $\varpi^a_{(\omega,u,\varrho)}$ belongs to NSS $\widetilde{F}_P$ over W if $\omega \leq T_{\widetilde{F}(p)}(\varpi)$, $u \leq I_{\widetilde{F}(p)}(\varpi)$, $\varrho \geq F_{\widetilde{F}(p)}(\varpi)$.

Definition 2.9. [8] Let $\tau \subset NSS(W,P)$. Then τ is said to be NS topology on W if :

(NST1) $0_{(W,P)}$, $1_{(W,P)}\in \tau$
(NST2) the intersection of any two NSS in τ belongs to τ;
(NST3) the union of any number of NSS in τ belongs to τ.
Then the triple (W,τ,P) is called a neutrosophic soft topological space (NSTS) over W. The members of τ are said to be $\tau-$ neutrosophic soft open set or simply, NS open (NSO) sets in X. A NSS over W is said to be neutrosophic soft closed (NSC) in W if its complement belongs to τ.

Theorem 2.10. *[10] Let (W, τ, P) be NSTS and $\beta \subset \tau$. Then the family β is NS basis of τ iff there exists a NSS $\widetilde{B}_P \in \beta$ such that $\varpi^p_{(\omega,u,\varrho)} \in \widetilde{B}_P \sqsubseteq \widetilde{F}_P$ for each $\widetilde{F}_P \in \tau$ and $\varpi^p_{(\omega,u,\varrho)} \in \widetilde{F}_P$.*

Definition 2.11. [5] Let (W, τ, P) and (V, τ', R) be two NSTS. $(f, \vartheta) : (W, \tau, P) \rightarrow (V, \tau, B)$ be a soft mapping and $\widetilde{F}_A$ be a NSS over W. Then the image of $\widetilde{F}_P$ under the mapping (f, ϑ) denoted by $(f, \vartheta)(\widetilde{F}_P)$ is NSS over V defined by for each p $\in$ P,

$$(f, \vartheta)(\widetilde{F}_P) = ((f, \vartheta)(T), (f, \vartheta)(I), (f, \vartheta)(F)),$$

Definition 2.12. [5] Let (U, τ, P) and (V, τ', R) be two NSTS. $(f, \vartheta) : (W, \tau, P) \rightarrow (V, \tau', R)$ be a soft mapping and

$$\widetilde{H}_R = \{(r, < v, K_{\widetilde{H}(r)}(v), L_{\widetilde{H}(r)}(v), M_{\widetilde{H}(r)}(v) >: v \in V) : r \in R\}$$

be a NSS over V. Then the inverse image of $\widetilde{H}_R$ under the mapping (f, ϑ) denoted by $(f, \vartheta)^{-1}(\widetilde{H}_R)$ is NSS over W described as for each p $\in$ P,

$$(f, \vartheta)^{-1}(\widetilde{H}_R) = ((f, \vartheta)^{-1}(K), (f, \vartheta)^{-1}(L), (f, \vartheta)^{-1}(M)),\ (f, \vartheta)^{-1}(K)(p)(\varpi) = K(\vartheta(p))(f(\varpi))$$

$$(f, \vartheta)^{-1}(L)(p)(\varpi) = L(\vartheta(p))(f(\varpi))$$

$$(f, \vartheta)^{-1}(P)(p)(\varpi) = M(\vartheta(p)(f(\varpi))$$

Definition 2.13. [5] Let (W, τ, P) and (V, τ', R) be two NSTS. $(f, \vartheta) : (W, \tau, P) \rightarrow (V, \tau', R)$ is a neutrosophic soft continuous function iff for each NSO $\widetilde{F}_R$ over V, $(f, \vartheta)^{-1}(\widetilde{F}_R)$ is NSO over W.

Definition 2.14. [5] Let (W, τ, P) and (V, τ', R) be two NSTS. $(f, \vartheta) : (W, \tau, P) \rightarrow (V, \tau', R)$ is NS open iff for each NSO $\widetilde{F}_P$ over W, $(f, \vartheta)(\widetilde{F}_P)$ is NSO over V.

Definition 2.15. [9] Let (W, τ, P) be a NSTS. If every NSO cover of $1_{(W,P)}$ has a finite subcover then (W, τ, P) is called NS-compact space. A space (W, τ, P) is said to be NS-compact if every NSO cover $\{(\widetilde{G_\lambda})_A : \lambda \in \Lambda\}$ of $\widetilde{1}_{(U,A)}$ there exists a finite subset Λ_0 of Λ such that $1_{(W,P)} = \bigsqcup\{(\widetilde{G_\lambda})_A : \lambda \in \Lambda_0\}$.

3. **Neutrosophic Soft Ideal Topological Spaces**

In this section, we define NS local function of a NSS with respect to $\mathfrak{I}$ and NST over W. By using this definition, we give the concept of NS Cl^*-closure which is a generalization of the concept of NS-closure of a NSS.

Definition 3.1. A neutrosophic soft ideal $\mathfrak{I}$ (NSI) is a non-empty collection of NSS over W which satisfies the following conditions :

(a) $\widetilde{F}_P \in \mathfrak{I}$ and $\widetilde{G}_P \sqsubseteq \widetilde{F}_P$ implies $\widetilde{G}_P \in \mathfrak{I}$

(b) $\widetilde{F}_P \in \Im$, $\widetilde{G}_P \in \Im$ implies $\widetilde{F}_P \sqcup \widetilde{G}_P \in \Im$

Example 3.2. Let $m^p_{(\omega,u,\varrho)} \in NSP(W,P)$. Then each of the following families are neutrosophic soft ideal over W.

(a) $\Im=\{0_{(W,P)}\}$

(b) $\Im$=NSS(W,P)

(c) $\Im=\{\widetilde{F}_P \in NSS(W,P) : m^p_{(\omega,u,\varrho)} \not\in \widetilde{F}_P\}$

Theorem 3.3. *Let $\Im$ be a NSI over W and $\Im = \{(\widetilde{F_k})_P : (\widetilde{F_k})_P \in NSS(W,P),\ k \in K\}$ where $(\widetilde{F_k})_P = \{(p, <\varpi, T_{(\widetilde{F_k})(p)}(\varpi), I_{(\widetilde{F_k})(p)}(\varpi), F_{(\widetilde{F_k})(p)}(\varpi) >: \varpi \in W) : p \in P\}$ for each $k \in K$. Then*

$$\Im_1 = \{T_{(\widetilde{F_k})(p)} : (\widetilde{F_k})_P \in \Im \quad p \in P\}$$
$$\Im_2 = \{I_{(\widetilde{F_k})(p)} : (\widetilde{F_k})_P \in \Im \ p \in P\}$$

are fuzzy soft ideals over W.

Proof. It is obvious. □

Remark 3.4. Let $\Im$ be a NSI over W and $\Im = \{(\widetilde{F_k})(p) : (\widetilde{F_k})_P \in NSS(W,P),\ k \in K\}$ where $(\widetilde{F_k})_P = \{(p, <\varpi, T_{(\widetilde{F_k})(p)}(\varpi), I_{(\widetilde{F_k})(p)}(\varpi), F_{(\widetilde{F_k})(p)}(\varpi) >: \varpi \in W) : p \in P\}$ for each k $\in$ K. Then

$$\Im_3 = \{F_{(\widetilde{F_k})(p)} : (\widetilde{F_k})_P \in \Im, p \in P\}$$

couldn't be a fuzzy soft ideal over W.

Example 3.5. Let $W = \{\varpi_1, \varpi_2, \varpi_3\}$, $P = \{p_1, p_2, p_3\}$ and NSI
$\Im = \{\widetilde{G}_P \in NSS(W,P) : \widetilde{G}_P \sqsubseteq \widetilde{F}_P\}$, where NSS $\widetilde{F}_P$, over W defined as follows:

$$\widetilde{F}_A = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{ccc} p_1 & p_2 & p_3 \end{array} \\ \begin{pmatrix} (0.1,0.5,0.3) & (0.1,0.2,0.3) & (0.5,0.6,1) \\ (0.4,0.4,0.7) & (0.4,0.4,0.7) & (0.4,0.3,0.7) \\ (0.4,0.2,0.1) & (0.4,0.6,0.1) & (0.3,0.2,1) \end{pmatrix} \end{array}$$

Then $\Im_1 = \{T_{(\widetilde{H_k})(p)}(\varpi) : (\widetilde{H_k})_P \in \Im\} = \{T_{\widetilde{G}(p)} : T_{\widetilde{G}(p)} \leq T_{\widetilde{F}(p)}, p \in P\}$ where
$T_{\widetilde{F}(p_1)}=\{(\frac{\varpi_1}{0.1}, \frac{\varpi_2}{0.4}, \frac{\varpi_3}{0.4})\}$, $T_{\widetilde{F}(p_2)}=\{(\frac{\varpi_1}{0.1}, \frac{\varpi_2}{0.4}, \frac{\varpi_3}{0.4})\}$ and $T_{\widetilde{F}(p_3)}=\{(\frac{\varpi_1}{0.5}, \frac{\varpi_2}{0.4}, \frac{\varpi_3}{0.3})\}$.
Then $\Im_1$ is a fuzzy soft ideal over W.
But $\Im_3 = \{F_{(\widetilde{H_k})_P}(\varpi) : (\widetilde{H_k})_\varpi \in \Im, p \in P\} = \{F_{\widetilde{G}_P} : F_{\widetilde{F}_P} \leq F_{\widetilde{G}_P}\}$ where
$F_{\widetilde{F}(p_1)}=\{(\frac{\varpi_1}{0.3}, \frac{\varpi_2}{0.7}, \frac{\varpi_3}{0.1})\}$, $F_{\widetilde{F}(p_2)}=\{(\frac{\varpi_1}{0.3}, \frac{\varpi_2}{0.7}, \frac{\varpi_3}{0.1})\}$ and $F_{\widetilde{F}(\varpi_3)}=\{(\frac{\varpi_1}{1}, \frac{\varpi_2}{0.7}, \frac{\varpi_3}{1})\}$.
Then $\Im_3$ is not a fuzzy soft ideal over W since $0_W \not\in \Im_3$

Definition 3.6. Let (W, τ, P) be a NST and $\mathfrak{I}$ be a NSI over W. Then

$$(\widetilde{F}_P)^*(\mathfrak{I}, \tau) = \widetilde{F}_A^* = \bigsqcup\{\varpi^a_{(\omega,u,\varrho)} : \widetilde{O}_P \sqcap \widetilde{F}_P \notin \mathfrak{I} \ for \ every \ \widetilde{O}_P \in \mathcal{U}(\varpi^p_{(\omega,u,\varrho)})\}$$

is called the neutrosophic soft local function of $\widetilde{F}_A$ with respect to $\mathfrak{I}$ and τ, where $\mathcal{U}(\varpi^p_{(\omega,u,\varrho)})$=$\{\widetilde{O}_P \in \tau : \varpi^p_{(\omega,u,\varrho)} \in \widetilde{O}_P\}$.

Proposition 3.7. *Let (W, τ, P) be a NSTS, $\mathfrak{I}$ and $\mathfrak{I}_1$ be NSI over W with the same set of parameters A. For $\widetilde{F}_P, \widetilde{G}_P \in NSS(W, P)$,*

(a) $\widetilde{F}_P \sqsubseteq \widetilde{G}_P$ implies that $\widetilde{F}_P^* \sqsubseteq \widetilde{G}_P^*$.
(b) $\widetilde{F}_P^* \sqsubseteq Cl(\widetilde{F}_P)$.
(c) $\widetilde{F}_P^*$ is neutrosophic soft closed.
(d) $(\widetilde{F}_P^*)^* \sqsubseteq \widetilde{F}_P^*$.
(e) $(\widetilde{F}_P \sqcup \widetilde{G}_P)^* = \widetilde{F}_P^* \sqcup \widetilde{G}_P^*$
(f) $(\widetilde{F}_P \sqcup \widetilde{F}_P^*) = \widetilde{F}_P^*$.
(g) $\mathfrak{I} \subset \mathfrak{I}_1$ implies that $\widetilde{F}_P^*(\mathfrak{I}_1, \tau) \sqsubseteq \widetilde{F}_P^*(\mathfrak{I}, \tau)$.

Proof.

(a) Let $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{G}_P^*$. Then there exists a subset $\widetilde{O}_P \in \tau$ containing $\varpi^p_{(\omega,u,\varrho)}$ such that $\widetilde{O}_P \sqcap \widetilde{G}_P$ $\in \mathfrak{I}$. By hypothesis, $\widetilde{O}_P \sqcap \widetilde{F}_P \sqsubseteq \widetilde{O}_P \sqcap \widetilde{G}_P$ so that $\widetilde{O}_P \sqcap \widetilde{F}_P \in \mathfrak{I}$ and $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{F}_P^*$.
(b) Let $\varpi^p_{(\omega,u,\varrho)} \notin Cl(\widetilde{F}_P)$. Then there exists a subset $\widetilde{O}_P \in \tau$ containing $\varpi^p_{(\omega,u,\varrho)}$ such that $\widetilde{O}_P \sqcap \widetilde{F}_P = 0_{(W,P)} \in \mathfrak{I}$. Then $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{F}_P^*$.
(c) Let $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{F}_P^*$. Then there exists a subset $\widetilde{O}_P \in \tau$ containing $\varpi^p_{(\omega,u,\varrho)}$ such that $\widetilde{O}_P \sqcap \widetilde{F}_P$ $\in \mathfrak{I}$. By definition of $\widetilde{F}_P^*$, we obtain $\widetilde{O}_P \sqcap$ $\widetilde{F}_P^* = 0_{(W,P)}$ and $\varpi^a_{(\omega,u,\varrho)} \notin Cl(\widetilde{F}_P^*)$. So we get $\widetilde{F}_P^*$= $Cl(\widetilde{F}_P^*)$ i.e. $\widetilde{F}_P$ is NSC.
(d) It is obvious from (b) and (c).
(e) We obtain that $(\widetilde{F}_P \sqcup \widetilde{G}_P)^* \sqsupseteq \widetilde{F}_P^* \sqcup \widetilde{G}_P^*$ by (a). Let $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{F}_P^* \sqcup$ $\widetilde{G}_P^*$. Then, there exist subsets $\widetilde{O_1}_P, \widetilde{O_2}_P \in \tau$ containing $\varpi^p_{(\omega,u,\varrho)}$ such that $\widetilde{O_1}_P \sqcap \widetilde{F}_P \in \mathfrak{I}$ and $\widetilde{O_2}_P \sqcap \widetilde{G}_P \in \mathfrak{I}$. By definition of ideal, $\widetilde{O}_P \sqcap$ $(\widetilde{F}_P \sqcup \widetilde{G}_P) \in \mathfrak{I}$ where $\widetilde{O}_P$=$\widetilde{O_1}_P \sqcap$ $\widetilde{O_2}_P$ Then, we get $\varpi^p_{(\omega,u,\varrho)} \notin$ $(\widetilde{F}_P$ $\sqcup \widetilde{G}_P)^*$ Hence $(\widetilde{F}_P \sqcup \widetilde{G}_P)^* \sqsubseteq \widetilde{F}_P^* \sqcup \widetilde{G}_P^*$
(f) It is obvious from (d) and (e).
(g) Let $\mathfrak{I} \subset \mathfrak{I}_1$ and $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{F}_P^*(\mathfrak{I}, \tau)$. Then there exists a subset $\widetilde{O}_P \in \tau$ containing $\varpi^p_{(\omega,u,\varrho)}$ such that $\widetilde{O}_P \sqcap \widetilde{F}_P \in \mathfrak{I}$. So $\widetilde{O}_P \sqcap \widetilde{F}_P \in \mathfrak{I}_1$ and we obtain $\varpi^p_{(\omega,u,\varrho)} \notin \widetilde{F}_P^*(\mathfrak{I}_1, \tau)$. Hence $\widetilde{F}_P^*(\mathfrak{I}_1, \tau)$ $\sqsubset \widetilde{F}_P^*(\mathfrak{I}, \tau)$. □

Example 3.8. Let $W = \{\varpi_1, \varpi_2, \varpi_3\}$, $P = \{p_1, p_2\}$ and $\tau = \{1_{(\varpi,P)}, 0_{(\varpi,P)}, \widetilde{F}_P, \widetilde{G}_P, \widetilde{H}_P, \widetilde{Z}_P\}$, $\Im = \{\widetilde{K}_P : \widetilde{K}_P \sqsubseteq \widetilde{L}_P\}$, where NSS $\widetilde{F}_P$, $\widetilde{G}_P$, $\widetilde{H}_P$, $\widetilde{Z}_P$ and $\widetilde{L}_P$ over W defined as follows:

$$\widetilde{F}_P = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.6, 0.5, 0.3) & (0.5, 0.6, 1) \\ (0.4, 0.3, 0.7) & (0.4, 0.3, 0.7) \\ (0.9, 0.2, 0.1) & (0.3, 0.2, 1) \end{pmatrix} \end{array}$$

$$\widetilde{G}_P = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.4, 0.7, 0.2) & (0.5, 0.3, 0.9) \\ (0.5, 0.3, 0.2) & (0.4, 0.3, 0.6) \\ (0.4, 0.2, 0.1) & (0.3, 0.2, 0.1) \end{pmatrix} \end{array}$$

$$\widetilde{H}_P = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.6, 0.7, 0.2) & (0.5, 0.6, 0.9) \\ (0.5, 0.3, 0.2) & (0.4, 0.3, 0.6) \\ (0.9, 0.2, 0.1) & (0.3, 0.2, 0.1) \end{pmatrix} \end{array}$$

$$\widetilde{Z}_P = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.4, 0.5, 0.3) & (0.5, 0.3, 1) \\ (0.4, 0.3, 0.7) & (0.4, 0.3, 0.7) \\ (0.4, 0.2, 0.1) & (0.3, 0.2, 1) \end{pmatrix} \end{array}$$

$$\widetilde{L}_P = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.4, 0.3, 0.7) & (0.5, 0.4, 0.8) \\ (0.6, 0.3, 0.5) & (0.7, 0.4, 0.3) \\ (0.5, 0.4, 0.4) & (0.5, 0.4, 0.1) \end{pmatrix} \end{array}$$

Then $\widetilde{L}_P^* \neq Cl(\widetilde{L}_P)$.

Example 3.9. Let us consider Example 3.8. and take $\widetilde{M}_P$ as follows

$$\widetilde{M}_P = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.6, 0.4, 0.1) & (0.4, 1, 0) \\ (0.7, 0.3, 0.4) & (0.6, 0.4, 0.2) \\ (0.6, 0.7, 0.1) & (0.7, 0.4, 0.1) \end{pmatrix} \end{array}$$

Then $\widetilde{L}_P \not\sqsubseteq \widetilde{M}_P$ but $\widetilde{L}_P^* \sqsubset \widetilde{M}_P^*$

Let (W, τ, P) be a NSTS and $\Im$ be a NSI over W.If $\widetilde{F}_A$ is NSS in W, then $Cl^*(\widetilde{F}_P) = \widetilde{F}_P \sqcup \widetilde{F}_P^*$.

Proposition 3.10. *Let (W, τ, P) be a NSTS and $\Im$ be a NSI over W with the same set of parameters P. Let $\widetilde{F}_P$, $\widetilde{G}_P$ are neutrosophic soft sets in (W, τ, P), then*

(a) $Cl^*(\widetilde{1}_{(W,P)}) = \widetilde{1}_{(W,P)}$, $Cl^*(\widetilde{0}_{(W,P)}) = \widetilde{0}_{(W,P)}$
(b) $\widetilde{F}_P \sqsubseteq \widetilde{G}_P$ *implies that* $Cl^*(\widetilde{F}_P) \sqsubseteq Cl^*(\widetilde{G}_P)$.
(c) $\widetilde{F}_P \sqsubseteq Cl^*(\widetilde{F}_P)$.
(d) $Cl^*(Cl^*(\widetilde{F}_P)) = Cl^*(\widetilde{F}_P)$.
(e) $Cl^*(\widetilde{F}_P \sqcup \widetilde{G}_P) = Cl^*(\widetilde{F}_P) \sqcup Cl^*(\widetilde{G}_P)$

Proof.

(a) It is obvious.
(b) By Proposition 3.7 (a), we get $\widetilde{F}^*_P \sqsubseteq \widetilde{G}^*_P$. Then $Cl^*(\widetilde{F}_P) = \widetilde{F}_P \sqcup \widetilde{F}^*_P \sqsubseteq \widetilde{G}_P \sqcup \widetilde{G}^*_P = Cl^*(\widetilde{G}_P)$. Then we have $Cl^*(\widetilde{F}_P) \sqsubseteq Cl^*(\widetilde{G}_P)$.
(c) By Proposition 3.7 (f), we get $Cl^*(Cl^*(\widetilde{F}_P)) = Cl^*(\widetilde{F}_P \cup \widetilde{F}^*_P) = (\widetilde{F}_P \sqcup \widetilde{F}^*_P) \sqcup \widetilde{F}^*_P = Cl^*(\widetilde{F}_P)$. Then we get $Cl^*(Cl^*(\widetilde{F}_P)) = Cl^*(\widetilde{F}_P)$.
(d) $Cl^*(\widetilde{F}_P \sqcup \widetilde{G}_P) = (\widetilde{F}_P \sqcup \widetilde{G}_P) \sqcup (\widetilde{F}_P \sqcup \widetilde{G}_P)^* = (\widetilde{F}_P \sqcup \widetilde{G}_P) \sqcup (\widetilde{F}^*_P \sqcup \widetilde{G}^*_P)$ from Proposition 3.7 (e). Then we get $Cl^*(\widetilde{F}_P \sqcup \widetilde{G}_P) = Cl^*(\widetilde{F}_P) \sqcup Cl^*(\widetilde{G}_P)$. □

Theorem 3.11. *Let* (W, τ, P) *be a NSTS and* $\mathfrak{I}$ *be a NSI over* W*.Then*

$$\tau^*(\mathfrak{I}, \tau) = \tau^* = \{\widetilde{F}_P \in NSS(W, P) : Cl^*((\widetilde{F}_P)^c) = \widetilde{F}^c_p\}$$

is a NS topology over W*. Each member of* $\tau^*(\mathfrak{I}, \tau)$ *is said to be NS-$*$-open.*

Proof. It is obvious from Proposition 3.10. □

Theorem 3.12. *Let* (W, τ, P) *be a NSTS and* $\mathfrak{I}$ *be a NSI over* W*. Then,* $\tau \subset \tau^*(\mathfrak{I}, \tau)$

Proof. Let $\widetilde{O}_P \in \tau$. Then $\widetilde{O}^c_P$ is neutrosophic soft $*$-closed iff $\widetilde{O}^c_P = Cl^*(\widetilde{O}^c_P)$. Then by Proposition 3.7. (c), we obtain $(\widetilde{O}^c_P)^* \sqsubseteq \widetilde{O}^c_P$ and we get $\widetilde{O}^c_P = \widetilde{O}^c_P \sqcup (\widetilde{O}^c_P)^* = Cl^*(\widetilde{O}^c_P)$. So $\widetilde{O}^c_P$ is neutrosophic soft $*$-closed i.e $\widetilde{O}_P \in \tau^\star$. □

Corollary 3.13. *Let* (W, τ, P) *be a NSTS and* $\mathfrak{I}$ *be a NSI. If* $\widetilde{F}_P$ *is a NSS, then* $\widetilde{F}_P$ *is neutrosophic soft -$*$-closed iff* $\widetilde{F}^*_P \sqsubset \widetilde{F}_P$*.*

Theorem 3.14. *Let* (W, τ, P) *be a NSTS and* $\mathfrak{I}$ *be NSI over* W*. Then*

$$\beta^*(\mathfrak{I}, \tau) = \beta^* = \{\widetilde{F}_P - \widetilde{G}_P : \widetilde{F}_P \in \tau \text{ and } \widetilde{G}_P \in \mathfrak{I}\}$$

is a neutrosophic soft basis of neutrosophic soft topology τ^**.*

Proof. Let $\varpi^p_{(\omega,u,\varrho)} \in \widetilde{O}_P$ and $\widetilde{O}_P \in \tau^*$. Then $Cl^*(\widetilde{O}^c_P) = \widetilde{O}^c_P$ and $\varpi^p_{(\omega,u,\varrho)} \notin Cl^*(\widetilde{O}^c_P)$. By definition of $Cl^*((\widetilde{O}_P)^c)$, $\varpi^p_{(\omega,u,\varrho)} \notin ((\widetilde{O}_P)^c)^*$. Then there exists a subset $\widetilde{F}_P \in \tau$ containing

$\varpi^p_{(\omega,u,\varrho)}$ such that $\widetilde{I}_P = \widetilde{F}_P \sqcap (\widetilde{O}_P)^c \in \mathfrak{I}$. Hence we obtain $\widetilde{F}_P$ - $\widetilde{I}_P \sqsubset \widetilde{O}_P$. By Theorem 2.10, $\beta^*(\mathfrak{I},\tau)$ is a NS basis of NSTS τ^* . □

Example 3.15. Let us consider Example 3.8 and take $\widetilde{O}_P$ as follows:

$$\widetilde{O}_A = \begin{array}{c} \\ \varpi_1 \\ \varpi_2 \\ \varpi_3 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_2 \end{array} \\ \begin{pmatrix} (0.6, 0.5, 0.4) & (0.5, 0.6, 1) \\ (0.4, 0.3, 0.7) & (0.3, 0.3, 0.7) \\ (0.4, 0.2, 0.5) & (0.1, 0.2, 1) \end{pmatrix} \end{array}$$

Then $\widetilde{O}_P \in \tau^*$ but it is not NSO in (W, τ, P)

Proposition 3.16. *Let (W, τ, P) be a NSTS and $\mathfrak{I}$ be a NSI over W. Then, the followings are equivalent:*

(a) $Cl(\widetilde{F}_P) \sqsubseteq Cl^*(\widetilde{F}_P)$ for each $\widetilde{F}_P \in NSS(W, P)$.
(b) Every NS $*$-closed set in U is NSC.
(c) $\tau = \tau^*$.

Proof. $(a) \Rightarrow (b)$: It is obvious. $(b) \Rightarrow (a)$: Let $\widetilde{F}_P \in NSS(W, P)$. Since $Cl^*(\widetilde{F}_P)$ is neutrosophic soft $*$-closed and by condition (b), we get $Cl^*(\widetilde{F}_P)$ is NSC. Then we obtain $Cl(\widetilde{F}_P) \sqsubseteq Cl^*(\widetilde{F}_P)$ by Proposition 3.7 (d).
$(b) \Rightarrow (c)$: Let $\widetilde{F}_P \in \tau^*$. Then $\widetilde{F}^c_P = Cl^*(\widetilde{F}^c_P)$ and so $\widetilde{F}^c_P$ is NS $*$-closed. By condition (b), we get $\widetilde{F}^c_P$ is NSC. Then, we obtain $\tau = \tau^*$ by Theorem 3.12.
$(c) \Rightarrow (b)$: It is obvious. □

Proposition 3.17. *Let (W, τ, P) be a NSTS and $\mathfrak{I}$ be a NSI over W. Then, the followings are equivalent:*

(a) $Cl^*(\widetilde{F}_P) \sqsubseteq Cl(\widetilde{F}_P)$ for each $\widetilde{F}_P \in NSS(W, P)$.
(b) Every NSC set in U is NS $*$-closed.
(c) $\tau \subset \tau^\star$.

Proof. It is obvious. □

4. Neutrosophic Soft $\mathfrak{I}$-Compact Spaces

In this section, we introduce the concept of NS-$\mathfrak{I}$-compactness on a set W and investigate some basic properties.

Definition 4.1. A space $(W, \tau, \mathfrak{I}, P)$ is said to be neutrosophic soft-$\mathfrak{I}$-compact (NSIC) if for every NSO-cover $\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda\}$ of $\widetilde{1}_{(W,P)}$ there exists a finite subset Λ_0 of Λ such that $1_{(W,P)} - \bigsqcup\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda_0\} \in \mathfrak{I}$.

A NSS $\widetilde{F}_P$ of a space $(X, \tau, \mathfrak{I}, P)$ is called NS- $\mathfrak{I}$-compact set in $(W, \tau, \mathfrak{I}, P)$ if for every NSO-cover $\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda\}$ of $\widetilde{F}_P$ there exists a finite subset Λ_0 of Λ such that $\widetilde{F}_P - \bigsqcup\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda_0\} \in \mathfrak{I}$.

Example 4.2. Let $W = IN$ be a universe, $P = \{p_1, p_2\}$ a parametric set and N_G be a neutrosophic set on W defined as follows:

$$N_G = \begin{cases} < \varpi, (1,1,0) > & if\ \varpi \in G \\ < \varpi, (0,0,1), > & if\ \varpi \notin G. \end{cases} \tag{2}$$

Let β=$\{(\widetilde{F_G})_A : 1 \in G, G \subset W\}$ be a NS base for τ where $(\widetilde{F_G})_P$ be a NSS defined as $\widetilde{F_G}(p_1)$ =$\widetilde{F_G}(p_2)$= $\{N_G : 1 \in G, G \subset W\}$. Let $\mathfrak{I}$=$\{(\widetilde{I_G})_P : 1 \notin G, G \subset W\} \cup \{0_{(W,P)}\}$ be a NSI where $(\widetilde{I_G})_P$ is a NSS defined as $\widetilde{I_G}(p_1)$ =$\widetilde{I_G}(p_2)$= $\{N_G : 1 \notin G, G \subset W\}$. Then $(W, \tau, \mathfrak{I}, P)$ is NSIC.

Proposition 4.3. *If (W, τ, P) is neutrosophic soft-compact, then $(U, \tau, A, \mathfrak{I})$ is NSIC.*

Proof. It is obvious since $0_{(W,P)} \in \mathfrak{I}$. □

Example 4.4. Let us consider Example 4.2. Then $(W, \tau, \mathfrak{I}, P)$ is neutrosophic soft-$\mathfrak{I}$-compact but it is not neutrosophic soft-compact.

Theorem 4.5. *Let (W, τ, P) be a NSTS and $\mathfrak{I}$ be a NSI over W. $(W, \tau, P, \mathfrak{I})$ is NSIC if and only if for every family $\{(\widetilde{F_\lambda})_P : \lambda \in \Lambda\}$ of NSC over W for which $\sqcap\{(\widetilde{F_\lambda})_P : \lambda \in \Lambda\} = 0_{(W,P)}$, there exists a finite subset Λ_0 of Λ such that $\sqcap\{(\widetilde{F_\lambda})_P : \lambda \in \Lambda_0\} \in \mathfrak{I}$.*

Proof. Let $(W, \tau^*(\mathfrak{I}, \tau), P)$ be NSIC and let $\{(\widetilde{F_\lambda})_P : \lambda \in \Lambda\}$ be NSC over W for which $\sqcap$ $\{(\widetilde{F_\lambda})_P : \lambda \in \Lambda\}$ =$0_{(W,P)}$. Then $\{1_{(W,P)} - (\widetilde{F_\lambda})_P : \lambda \in \Lambda\}$ is the NSO over W such that $\sqcup$ $\{1_{(W,P)} - (\widetilde{F_\lambda})_P : \lambda \in \Lambda\}$ =$1_{(W,P)}$. By hypothesis and Theorem 3.12, there exists a finite subset Λ_0 of Λ such that $1_{(W,P)} - \sqcup \{1_{(W,P)} - (\widetilde{F_\lambda})_P : \lambda \in \Lambda_0\}$= $(\sqcup\{1_{(W,P)} - (\widetilde{F_\lambda})_P : \lambda \in \Lambda_0\})^c$=$\sqcap\{(\widetilde{F_\lambda})_P : \lambda \in \Lambda_0\}$$\in \mathfrak{I}$. Thus, $(W, \tau, P, \mathfrak{I})$ is NSIC. □

Corollary 4.6. *[9] (W, τ, P) is NSC iff for every family of NSC sets with empty intersection in (W, τ, P) has a finite subfamily with empty intersection.*

Proof. It is obvious by previous theorem taking $\mathfrak{I} = \{0_{(W,P)}\}$. □

Theorem 4.7. *Let (W, τ, P) be a NSTS and $\mathfrak{I}$ be NSI over W.If $(W, \tau^*(\mathfrak{I}, \tau), P)$ is NSC, then $(W, \tau, P, \mathfrak{I})$ is NSIC.*

Proof. Let $(W, \tau^*(\mathfrak{I}, \tau), P)$ be NSC and $\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda\}$ be a NSO-cover of W. By Theorem 3.12, $\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda\}$ is a NS-$*$-open cover of W. Assuming that $(W, \tau^*(\mathfrak{I}, \tau), P)$ be NSC, there exists a finite subset Λ_0 of Λ such that $1_{(W,P)}=\bigsqcup\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda_0\}$. Then $1_{(W,P)}-\bigsqcup\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda_0\} \in \mathfrak{I}$. Thus, $(W, \tau, P, \mathfrak{I})$ is NSIC. □

Theorem 4.8. *If $(W, \tau, P, \mathfrak{I})$ is NSIC and $\mathfrak{I}$ is a NSI on W with $\mathfrak{I} \subset \mathfrak{J}$, then $(W, \tau, P, \mathfrak{J})$ is NSIC.*

Proof. Let $(W, \tau, P, \mathfrak{I})$ is NSIC, $\mathfrak{J}$ is a NSI on W with $\mathfrak{I} \subset \mathfrak{J}$. Let $\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda\}$ be a NSO-cover of U. Since $(W, \tau, \mathfrak{J}, P)$ is NSIC, there exists a finite subset Λ_0 of Λ such that $1_{(W,P)}$-$\bigsqcup\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda_0\} \in \mathfrak{I}$. Since $\mathfrak{I} \subset \mathfrak{J}$, $1_{(W,P)}-\bigsqcup\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda_0\} \in \mathfrak{J}$. Thus, $(W, \tau, P, \mathfrak{J})$ is NSIC. □

Proposition 4.9. *Let (W, τ, P) and (V, τ', R) be two NST, $(f, \vartheta) : (W, \tau, P) \to (V, \tau', R)$ be a soft mapping and $\mathfrak{I}$ a NSI on W. Then, $(f, \vartheta)(\mathfrak{I}) = \{(f, \vartheta)(\widetilde{F}_P) : \widetilde{F}_P \in \mathfrak{I}\}$ is a NSI on V.*

Theorem 4.10. *Let $f : (W, \tau, P, \mathfrak{I}) \to (V, \tau', R, \mathfrak{J})$ be a neutrosophic soft continuous surjection and $(f, \vartheta)(\mathfrak{I}) \subseteq \mathfrak{J}$. If $(W, \tau, P, \mathfrak{I})$ is neutrosophic soft- $\mathfrak{I}$-compact, then $(V, \tau', R, \mathfrak{J})$ is NSIC.*

Proof. Let $\{(\widetilde{G_\lambda})_R : \lambda \in \Lambda\}$ be a NSO-cover of V. Since f is neutrosophic soft continuous , $\{(f, \vartheta)^{-1}((\widetilde{G_\lambda})_R) : \lambda \in \Lambda\}$ is a neutrosophic soft open cover of W. Assuming that $(W, \tau, P, \mathfrak{I})$ is NSIC, there exists a finite subset Λ_0 of Λ such that $1_{(W,P)}-\bigsqcup\{(f, \vartheta)^{-1}((\widetilde{G_\lambda})_R) : \lambda \in \Lambda_0\} \in \mathfrak{I}$. This implies that, $(f, \vartheta)(1_{(W,P)}-\bigsqcup\{f^{-1}((\widetilde{G_\lambda})_R) : \lambda \in \Lambda_0\}) \in (f, \vartheta)(\mathfrak{I})$. Then $1_{(V,R)}-\bigsqcup\{(\widetilde{G_\lambda})_R : \lambda \in \Lambda_0\} \in \mathfrak{J}$. So, $(V, \tau', R, \mathfrak{J})$ is NSIC. □

Proposition 4.11. *Let (W, τ, P) and (V, τ, R) be two NST. $(f, \vartheta) : (W, \tau, P) \to (V, \tau, R)$ be an injective soft mapping and $\mathfrak{J}$ a NSI on V. Then, $(f, \vartheta)^{-1}(\mathfrak{J})$ is a NSI on W.*

Proof. It is obvious. □

Theorem 4.12. *Let $f : (W, \tau, P) \to (V, \tau', R, \mathfrak{J})$ be a neutrosophic soft open bijection. If $(V, \tau', R, \mathfrak{J})$ is neutrosophic soft-$\mathfrak{J}$-compact, then $(W, \tau, P, (f, \vartheta)^{-1}(\mathfrak{J}))$ is neutrosophic soft-$(f, \vartheta)^{-1}(\mathfrak{J})$-compact.*

Proof. Let $\{(\widetilde{G_\lambda})_P : \lambda \in \Lambda\}$ be a NSO-cover of U. Since f is neutrosophic soft open bijection, $\{(f,\vartheta)((\widetilde{G_\lambda})_P) : \lambda \in \Lambda\}$ is a neutrosophic soft τ'-open cover of V. Assuming that $(V, \tau', R, \mathfrak{J})$ is NSIC, there exists a finite subset Λ_0 of Λ such that $1_{(V,R)} - \bigsqcup\{(f,\vartheta)((\widetilde{G_\lambda})_P) : \lambda \in \Lambda_0\} \in \mathfrak{J}$. This implies that, $(f,\vartheta)^{-1}(1_{(V,R)} - \bigsqcup\{(f,\vartheta)((\widetilde{G_\lambda})_P) : \lambda \in \Lambda_0\}) \in (f,\vartheta)^{-1}(\mathfrak{J})$. Then $1_{(W,P)} - \bigsqcup\{(\widetilde{G_\lambda})_R : \lambda \in \Lambda_0\} \in (f,\vartheta)^{-1}(\mathfrak{J})$. So, $(W, \tau, P, (f,\vartheta)^{-1}(\mathfrak{J}))$ is neutrosophic soft-$(f,\vartheta)^{-1}(\mathfrak{J})$-compact. □

5. **Conclusions**

Smarandance introduced neutrosophic set as an extension of intuitionistic fuzzy set. Maji gave the concept of neutrosophic soft set and neutrosophic soft set theory has a lot of applications in different areas. Then Gündüz et all redefined basic notions of neutrosophic soft set and soft point. In this paper, we gave the definition of NS-Cl^*-closure, which is more general than neutrosophic soft closure of NSS, with the help of NSI and NSTS. Also we examined basic properties of this new concept. Moreover, we define the concept of NS-$*$-topology and NSIC. Besides, we studied the relationship between these concepts and investigate relations with those concepts. We plan to study the concept of connected in neutrosophic soft ideal topological space.

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Abbreviations:

FS fuzzy set

IFS intuitionistic fuzzy set

NS neutrosophic set

SS soft set

NSS neutrosophic soft set

NSP neutrosophic soft point

NSTS neutrosophic soft topological space

NSO neutrosophic soft open

NSC neutrosophic soft closed

NSI neutrosophic soft ideal

NSC neutrosophic soft compact

NSIC neutrosophic soft I-compact

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