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On Extension of Metricspace to Neutro Hypermetricspace

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Abstract. Polvan introduced the impression of strong neutro metricspaces. He demonstrtated this impression depending on the neutro precepts from the neutro function one of the impressions of Smarandache. In this study, we introduce an extension of neutro metricspaces as neutro hypermetricspaces and investigated their qualities. We show that neutro hypermetricspaces are an extension of metricspaces and analyzed the relations between metricspaces and neutro hypermetricspaces. The basic impressions such as open positive-ball subsets and open negative-ball subsets are defined and are presented by some finite and finite neutro hypermetricspaces. Based on the open positive-ball subsets and open negative-ball subsets, we demonstrate the impressions of neutro negative-open sets and neutro positive-open sets, respectively.

Keywords: (Neutro) hypermetricspace, neutro-negative-open set, neutro-positive-open set, unified set.

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1. Introduction

M. K. EL Gayyar, introduced the notion of neutrosophic topological structures. He presented it in two different types of impressions of neutrosophic topological space, closure interior [6]. Florentin Smarandache [8] et al. introduced an impression of generalized neutrosophic bipolar vague sets (N.B.V.S) and vague topology in topological spaces. Neutrosophic vague theory is a pragmatic manner to practice fragmentary and erratic information. Indeed, they interpolated the perception of an N.B.V.S as a combination of N. S, B. S, and V. S. Some researchers have presented the N. S in neutro metricspace and other neutro algebras such as [1–5, 7–12]. We interpolate the new thought of hypermetricspace as a generalization of metricspaces and present their qualities. In the real world, there is no exact size and most sizes are approximate. For example, we say the length of a wall that has a fracture and is somewhat angled is between 100 and 102 meters, or we say the weight of a package is between 200 and 201 kg. These approximations indicate that the sizes are not exact and fall within a range. These real issues motivate us to interpolate supermeters. In addition, sometimes we can't measure some objects and we say that this size is not known to us. Sometimes we cannot take the distance of an object from itself to zero, because an object fills a volume in three-dimensional space and we have a distance on this object itself. These issues make us interpolate the cloud meter to have a more realistic model of the world around us. In this study, we extended the metricspace to neutro hypermetricspace and considered the relations between of their precepts. We have shown that the open sets in neutro hypermetricspaces are different, for instance, we show that there exist two types of neutro-open sets negative-open set and positive-open set. In a final show, the union of neutro-open sets is not a neutro set necessarily.

2. Preliminaries

Here we address the needs of our work.

Definition 2.1. [9] Assume $\nu : \mathcal{Q}^2 \to \mathbb{R}$, with $\mathcal{Q} \neq \emptyset$. Then, $(\mathcal{Q}, \nu)$ is called a *neutro metricspace*, if obtain

$(NM\text{-}1)$ $(o, o' \in \mathcal{Q}$ so $o\nu o' \geq 0)$, $\big(o_1, o_2 \in \mathcal{Q}$ so $o_1\nu o_2 < 0$ or inconclusive $\big)$;

$(NM\text{-}2)$ $(o \in \mathcal{Q}$ so $o\nu o = 0)$, $\big(o' \in \mathcal{Q}$ such that, $o'\nu o' \neq 0$ or inconclusive $\big)$;

$(NM\text{-}3)$ $(o, o', o'' \in \mathcal{Q}$, such that $o\nu o'' \leq o\nu o' + o'\nu o'')$, $\big(o_1, o_2, o_3 \in \mathcal{Q}$, such that $o_1\nu o_3 > o_1\nu o_2 + o_2\nu o_3$ or inconclusive $\big)$;

$(NM\text{-}4)$ $(o, o' \in \mathcal{Q}$, such that $o\nu o' = o'\nu o)$, $\big(o_1, o_2 \in \mathcal{Q}$, such that $o_1\nu o_2 \neq o_2\nu o_1$ or inconclusive $\big)$.

Definition 2.2. [7] Assume $\nu : \mathcal{Q}^2 \to \mathbb{R}^{\geq 0}$, with $\mathcal{Q} \neq \emptyset$. Then, $(\mathcal{Q}, \nu)$ is called a *strong neutro*, if

$(NM\text{-}1)$ $\big(o \in \mathcal{Q}$ so $o\nu o = 0)$ and $\big(o' \in \mathcal{Q}$ so $o'\nu o' \neq 0$ or inconclusive $\big)$;

$(NM\text{-}2)$ $\big(o, o', o'' \in \mathcal{Q}$, such that $o\nu o'' \leq o\nu o' + o'\nu o''$ and $\big(o_1, o_2, o_3 \in \mathcal{Q}$, such that $0_1\nu o_3 > o_1\nu o_2 + o_2\nu o_3$ or inconclusive $\big)$;

$(NM\text{-}3)$ $\big(o, o' \in \mathcal{Q}$, such that $o\nu o' = o'\nu o$ and $\big(o_1, o_2 \in \mathcal{Q}$, such that $o_1\nu o_2 \neq o_2\nu o_1$ or inconclusive $\big)$.

3. Hypermetricspace

In this section, we interpolate the notation of hypermetricspace as an extension of metricspaces and present their qualities.

Definition 3.1. Presume $\vartheta : M \times M \to P^*(\mathbb{R})$ be a map and $M \neq \emptyset$. Then (M, ϑ) is a *hypermetricspace*, if

$(Hm\text{-}1)$ obtain $r \geq 0$, s.t $[0, r] \Subset \vartheta(x, y)$;

$(Hm\text{-}2)$ $\vartheta(x,x) = \{0\}$;

$(Hm\text{-}3)$ $\vartheta(x,z) \Subset \vartheta(x,y) + \vartheta(y,z)$;

$(Hm\text{-}4)$ $\vartheta(x,y) = \vartheta(y,x)$;

which for any $\mathcal{A}, \mathcal{B} \in P^*(\mathbb{R}), \mathcal{A} \Subset \mathcal{B}$ iff for any $a \in \mathcal{B}$ obtain $b \in \mathcal{B}$ so $b - a \geq 0 (a \leq b)$ and ϑ is called a *hypermeter* on M.

For each set $\{A_i\}_{i=1}^{n}$ of subset of $\mathbb{R}$, recall, $\sum_{i=1}^{n} A_i = \{a_1 + a_2 + a_3 + \ldots + a_n \mid a_i \in A_i\}$. Then get the following results.

Theorem 3.2. *Presume $(\mathcal{Q}, \vartheta)$ be a hypermetricspace. Then for each $x, y \in \mathcal{Q}, A, B \in P^*(\mathbb{R})$;*

(i) $\{0\} \Subset \vartheta(x,y)$,

(ii) $\vartheta(x,y) \Subset \vartheta(x,y)$,

(iii) *if* $A \subseteq B$, *then* $A \Subset B$,

(iv) $A \Subset A + B$.

Proof. (i) For each $x, y \in \mathcal{Q}, [0,0] \subseteq \vartheta(x,y)$, $\{0\} \subseteq \vartheta(x,y)$. Hence $0 \in \vartheta(x,y)$ and so $\{0\} \Subset \vartheta(x,y)$.

(ii) Presume $x, y \in \mathcal{Q}$, since for each $a \in \vartheta(x,y), a \leq a$, we get that $\vartheta(x,y) \Subset \vartheta(x,y)$.

(iii) For each $x \in A, x \in B$ and $x \leq x$, we get that $A \Subset B$.

(iv) Since $A \subseteq A + B$, by item $(iii), A \Subset A + B$. □

Corollary 3.3. *Presume $(\mathcal{Q}, \vartheta)$ be a hypermetricspace. Then for each $x, y, z \in \mathcal{Q}, A_i \in P^*(\mathbb{R}), 1 \leq i \leq n$;*

(i) $A_i \Subset A_i$,

(ii) $A_i \Subset \sum_{i=1}^{n} A_i$,

(iii) $\vartheta(x,y) \Subset \vartheta(x,y) + \vartheta(y,z)$.

Example 3.4. (i) Any metricspace $(\mathcal{Q}, \vartheta)$ is a hypermetricspace.

$(Hm\text{-}1)$ For each $x, y \in \mathcal{Q}$, since $0 \leq \vartheta(x,y)$, for $r = 0$, we get that $[0,r] \Subset \vartheta(x,y)$. Clearly $(Hm\text{-}2)$ and $(Hm\text{-}4)$ are valid.

$(Hm\text{-}3)$ For each $x, y, z \in \mathcal{Q}$, since $\vartheta(x,z) \leq \vartheta(x,y) + \vartheta(y,z)$, based Theorem 3.2, $\vartheta(x,z) \Subset \vartheta(x,y) + \vartheta(y,z)$.

(ii) Presume $\mathcal{Q} = \{a,b,c,d\}$. Then $(\mathcal{Q}, d)$ is a hypermetricspace as Table 2.

ϑ	a	b	c	d
a	$\{0\}$	$[0,1]$	$[0,2]$	$[0,3]$
b	$[0,1]$	$\{0\}$	$[0,2]$	$[0,3]$
c	$[0,2]$	$[0,2]$	$\{0\}$	$[0,3]$
d	$[0,3]$	$[0,3]$	$[0,3]$	$\{0\}$

TABLE 1. Hypermetricspace$(\mathcal{Q},\vartheta)$.

For each intervals $In(a,b), In(c,d)$, recall that $In(a,b)+In(c,d)=In(a+c,b+d)$, which $In(w,y)=[w,y]$ and get the following results.

Theorem 3.5. *Any non–avoid subset of $\mathbb{R}$ can be a hypermetricspace.*

Proof. Presume $\emptyset \neq A \subseteq \mathbb{R}$ be arbitrary. For each $x,y\in A$, define $\vartheta:\mathbb{R}\times\mathbb{R}\to P^*(\mathbb{R})$ by $\vartheta(x,y)=[0,|\max\{x,y\}-\min\{x,y\}|]$. For each $(x_1,y_1),(x_2,y_2)\in A^2$, since $(x_1,y_1)=(x_2,y_2)$ implies that $(0,|\max\{x_1,y_1\}-\min\{x_1,y_1\}|)=(0,|\max\{x_2,y_2\}-\min\{x_2,y_2\}|)$, we get that $\vartheta(x_1,y_1)=\vartheta(x_2,y_2)$ and so ϑ is well-defined. Moreover,

$(Hm\text{-}1)$ Presume $x,y\in A$. Then for $r=\dfrac{|\max\{x,y\}-\min\{x,y\}|}{2}$, we get that $[0,r]\Subset[|x|,|y|]$.

$(Hm\text{-}2)$ Presume $x\in A$. Computations show that $\vartheta(x,x)=[0,|\max\{x,x\}-\min\{x,x\}|]=[0,0]=\{0\}$.

$(Hm\text{-}3)$ Presume $x,y,z\in A$. Then

$$\begin{aligned}
&\vartheta(x,z)=[0,|\max\{x,z\}-\min\{x,z\}|]\\
&\Subset[0,|\max\{x,y\}-\min\{x,y\}|+|\max\{y,z\}-\min\{y,z\}|]\\
&\Subset[0,|\max\{x,y\}-\min\{x,y\}|]+[0,|\max\{y,z\}-\min\{y,z\}|]=\vartheta(x,y)+\vartheta(y,z).
\end{aligned}$$

$(Hm\text{-}4)$ Presume $x,y\in A$. Then

$$\vartheta(x,y)=[0,|\max\{x,y\}-\min\{x,y\}|]=[0,|\max\{y,x\}-\min\{y,x\}|]=\vartheta(y,x).$$

It follows that (A,ϑ) is a hypermetricspace. □

Presume $\mathcal{Q}$ be a non-avoid set. We say that $\mathcal{Q}$ is a reproduced hypermetricspace, if obtain a hypermeter ϑ on $\mathcal{Q}$, such that $(\mathcal{Q},\vartheta)$ is a hypermetricspace. In addition, we denote $\mathcal{H}m(\mathcal{Q})=\{\vartheta\mid\vartheta \text{ is a hypertmeter on } \mathcal{Q}\}$, as the set of all hypermeters on $\mathcal{Q}$ and will get the following result.

Theorem 3.6. *For each non-avoid set $\mathcal{Q}$,*

(i) $\mathcal{Q}$ is a reproduced hypermetricspace.

(ii) $|\mathcal{H}m(\mathcal{Q})|=2^{\aleph}(\aleph=card(\mathbb{R}))$.

Proof. Presume $\mathcal{Q}$ be a non-avoid set.

$(i),(ii)$ For each $x,y \in \mathcal{Q}$, and for each $r \in \mathbb{R}$, define $\vartheta_r(x,y) = \begin{cases} \{0\} & if x = y, \\ [0,r] & if x \neq y, \end{cases}$. It is clear that $(\mathcal{Q},\vartheta_r)$ is a hypermetricspace. Hence $|\mathcal{H}m(\mathcal{Q})| = 2^{\aleph}(\aleph = card(\mathbb{R}))$. □

Presume $(\mathcal{Q},\vartheta)$ be a hypermetricspace and $\epsilon \in \mathbb{R}^{>0}$. So $B_\epsilon(x) = \{y \in \mathcal{Q} \mid Sup(\vartheta(x,y)) < \epsilon\}$, is called an open ball of radius ϵ and center $x \in \mathcal{Q}$.

Example 3.7. Consider the hypermetricspace $(\mathcal{Q},\vartheta)$ in Example 3.4. For each $0 < \epsilon < 1$, and each $x \in \mathcal{Q}, B_\epsilon(x) = \{x\}, B_1(a) = \{a\}, B_2(a) = \{a,b\}, B_3(a) = \{a,b,c\}, B_4(\epsilon) = \{a,b,c,d\}$ and for each $\epsilon \geq 4, B_\epsilon(a) = \mathcal{Q}$.

Theorem 3.8. *Presume $(\mathcal{Q},\vartheta)$ be a hypermetricspace and $\epsilon \in \mathbb{R}^{>0}$.*

(i) $Inf\{\vartheta(x,y) \mid x,y \in \mathcal{Q}\} = 0$.
(ii) $x \in B_\epsilon(x)$.
(iii) $\mathcal{Q} = \bigcup\limits_{x\in\mathcal{Q}} B_\epsilon(x)$.
(iv) *If* $Sup\{\vartheta(x,y) \mid x,y \in \mathcal{Q}\} = \epsilon$, *then for each* $\epsilon' > \epsilon, B_\epsilon(x) = B_{\epsilon'}(x)$.
(v) *There exists* $\epsilon \in \mathbb{R}^{>0}$ *so* $B_\epsilon(x) = \mathcal{Q}$.

Proof. (i) For each $x,y \in \mathcal{Q}$ and each $r \in \mathbb{R}^{\geq 0}, [0,r] \subseteq \vartheta(x,y)$, implies that $[0,r] \Subset \vartheta(x,y)$. It follows that obtain $z \in \vartheta(x,y)$ so $z \leq 0$. It follows that $(-\infty,0]$ is the set of all lower bounds of $\vartheta(x,y)$ and so $Inf\{\vartheta(x,y) \mid x,y \in \mathcal{Q}\} = 0$.

(ii) Since $\epsilon > 0$ and obtain $r \in \mathbb{R}^{\geq 0}, [0,r] \Subset \vartheta(x,y)$, for each $x \in \mathcal{Q}, B_\epsilon(x) = \{y \in \mathcal{Q} \mid Sup(\vartheta(x,y)) < \epsilon\} \neq \emptyset$. In addition, by Theorem 3.2, $\{0\} \Subset \vartheta(x,x) \leq Sup(\vartheta(x,y)) < \epsilon$, includes that $x \in B_\epsilon(x)$.

(iii) For each $x \in \mathcal{Q}$, by item $(i), \{x\} \subseteq B_\epsilon(x)$ and so $\mathcal{Q} = \bigcup\limits_{x\in\mathcal{Q}} \{x\} \subseteq B_\epsilon(x)$. Hence $\mathcal{Q} = \bigcup\limits_{x\in\mathcal{Q}} B_\epsilon(x)$.

(iv) Presume $Sup\{\vartheta(x,y) \mid x,y \in \mathcal{Q}\} = \epsilon$. Then for each $x,y \in \mathcal{Q}, \vartheta(x,y) \leq Sup(\vartheta(x,y)) < \epsilon$ and so $B_\epsilon(x) = \{y \in \mathcal{Q} \mid Sup(\vartheta(x,y)) < \epsilon\} = \mathcal{Q}$. So for each $\epsilon' > \epsilon, B_\epsilon(x) = \{y \in \mathcal{Q} \mid Sup(\vartheta(x,y)) < \epsilon'\} = \mathcal{Q}$.

(v) By item (iv), consider $\epsilon = Sup\{\vartheta(x,y) \mid x,y \in \mathcal{Q}\}$, then $B_\epsilon(x) = \{y \in \mathcal{Q} \mid Sup(\vartheta(x,y)) < \epsilon\} = \mathcal{Q}$. □

3.1. **On neutro hypermetricspace**

In this subsection, we interpolate the new impression of neutro hypermetricspace as an extension of neutro metricspaces and in general as an extension of metricspaces. We analyze the qualities of neutro hypermetricspaces with respect to joint of sumsets and presented the relation of neutro hypermetricspaces and neutro metricspaces.

Definition 3.9. Presume $\vartheta : M \times M \to P^*(\mathbb{R})$ be a map and $M \neq \emptyset$.The structure (M, ϑ) is called a *neutro hypermetricspace*, if

(Hm-1) ($\exists\ r \geq 0, x, y \in M$, so $[0, r] \Subset \vartheta(x, y)$) and ($\exists\ s \leq 0, z, w \in M$ so $[s, 0] \Subset \vartheta(z, w)$ or inconclusive);

(Hm-2) ($\exists\ x \in \mathcal{Q}, such\ that\ \vartheta(x, x) = \{0\}$) and ($\exists\ y \in \mathcal{Q}, such\ that\ \vartheta(y, y) \neq \{0\}$ or inconclusive);

(Hm-3) ($\exists\ x, y \in \mathcal{Q} such\ that\ \vartheta(x, z) \Subset \vartheta(x, y) + \vartheta(y, z)$) and ($\exists\ z, w, u \in \mathcal{Q}, such\ that\ \vartheta(z, w) \succ \vartheta(z, u) + \vartheta(u, w)$ or inconclusive);

(Hm-4) ($\exists\ x, y \in \mathcal{Q}, such\ that\ \vartheta(x, y) = \vartheta(y, x)$) and ($\exists\ w, z \in \mathcal{Q}, such\ that\ \vartheta(z, w) \neq \vartheta(w, z)$ or inconclusive);

which for each $A, B \subseteq \mathbb{R}, A \Subset B$ iff for each $a \in A$ obtain $b \in B$ so$b - a \geq 0(a \leq b)$ and $A \succ B$ obtain $a \in A$ so for each $b \in B$ $a > b$. We will call ϑ as a *neutro hypermeter* on $\mathcal{Q}$.

Example 3.10. Presume $\mathcal{Q} = \{a, b, c, d, e\}$. Then $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace as Table 2.

ϑ	a	b	c	d	e
a	$\{0\}$	$[0,1]$	$[-2,0]$	$[0,3]$	$[-3,0]$
b	$[0,1]$	$\{0\}$	$[0,2]$	$[0,3]$	$[0,3]$
c	$[0,2]$	$[0,2]$	$\{0\}$	$[0,3]$	$[0,3]$
d	$[0,3]$	$[0,3]$	$[0,3]$	$\{-2,-3\}$	$[0,3]$
e	$[0,3]$	$[0,3]$	$[0,3]$	$[0,3]$	$\{4,5\}$

TABLE 2. Neutro hypermetricspace $(\mathcal{Q}, \vartheta)$.

We see that $\vartheta(a, a) = \{0\}, \vartheta(d, d) \neq \{0\}, \vartheta(a, d) = \vartheta(d, a), \vartheta(a, b) \Subset \vartheta(a, d) + \vartheta(d, b)$ and $\vartheta(e, e) \succ \vartheta(e, a) + \vartheta(a, e)$. Hence $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace.

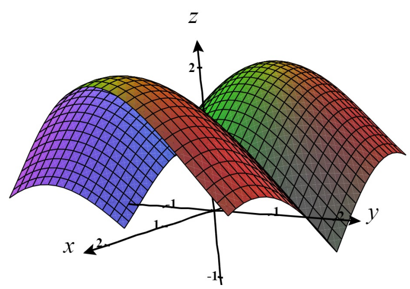
Presume $n \in \mathbb{N}$ and $f : \mathbb{R}^n \to \mathbb{R}$ be odd bounded continuous map. Then define $\vartheta : \mathbb{R}^n \times \mathbb{R}^n \to P^*(\mathbb{R}^n)$ by $\vartheta((\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}), (\overline{s_1}, \overline{s_2}, \ldots, \overline{s_n})) = Range(f(\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}) + f(\overline{s_1}, \overline{s_2}, \ldots, \overline{s_n}))$ and $\vartheta((\pi, \pi, \ldots, \pi), (2\pi, 2\pi, \ldots, 2\pi)) \neq \vartheta((2\pi, 2\pi, \ldots, 2\pi), (\pi, \pi, \ldots, \pi))$.

Theorem 3.11. *Presume $n \in \mathbb{N}$. Then $(\mathbb{R}^n, \vartheta)$ is a hypermetricspace.*

Proof. Since f is bounded, obtain $m \in \mathbb{R}$, so for each $(\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}) \in \mathbb{R}^n, m \leq f(\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}) \leq M$. Presume $(\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}), (\overline{s_1}, \overline{s_2}, \ldots, \overline{s_n}) \in \mathbb{R}^n$ be arbitrary. Then define $\vartheta((\overline{r_1}, \overline{r_2}, \ldots, \overline{r_n}), (\overline{s_1}, \overline{s_2}, \ldots, \overline{s_n})) = \sum_{i=1}^{n} Range(f(x_i))$. Since the map f is odd continuous, obtain $(\overline{t_1}, \overline{t_2}, \ldots, \overline{t_n}) \in \mathbb{R}^n$ and $(\overline{w_1}, \overline{w_2}, \ldots, \overline{w_n}) \in \mathbb{R}^n$ so$f(\overline{t_1}, \overline{t_2}, \ldots, \overline{t_n}) \leq 0$ and $f(\overline{w_1}, \overline{w_2}, \ldots, \overline{w_n}) \geq 0$. Since f is bounded, obtain $(\overline{t_1}, \overline{t_2}, \ldots, \overline{t_n}) \in \mathbb{R}^n$ and $(\overline{w_1}, \overline{w_2}, \ldots, \overline{w_n}) \in \mathbb{R}^n$ so that $Range(f(\overline{t_1}, \overline{t_2}, \ldots, \overline{t_n}) + f((\overline{w_1}, \overline{w_2}, \ldots, \overline{w_n})) = \{0\}$, obtain $(\overline{z_1}, \overline{z_2}, \ldots, \overline{z_n}) \in \mathbb{R}^n$ and $(\overline{u_1}, \overline{u_2}, \ldots, \overline{u_n}) \in \mathbb{R}^n$ so that $Range(f(\overline{z_1}, \overline{z_2}, \ldots, \overline{z_n}) + f((\overline{u_1}, \overline{u_2}, \ldots, \overline{u_n}) = [0, 2M]$ and obtain $(\overline{v_1}, \overline{v_2}, \ldots, \overline{v_n}) \in \mathbb{R}^n$ and $(\overline{q_1}, \overline{q_2}, \ldots, \overline{q_n}) \in \mathbb{R}^n$ so that $Range(f(\overline{v_1}, \overline{v_2}, \ldots, \overline{v_n}) + f(g(\overline{q_1}, \overline{q_2}, \ldots, \overline{q_n}) = [-2m, 0]$. Hence $(\mathbb{R}^n, \vartheta)$ is a hypermetricspace. □

Example 3.12. Define $\vartheta : \mathbb{R} \times \mathbb{R} \to P^*(\mathbb{R})$, by $\vartheta(x, y) = Rang(|Sinx| + Cosy)$ as Figure 1.

FIGURE 1. $|sin(x)| + cos(y)$



Using Figure 1, for each $\pi/2 \leq x \leq 3\pi/4$ and $3\pi/4 \leq y \leq \pi, [-1, 0] \Subset \vartheta(x, y)$, for each $0 \leq x, y \leq \pi/2, [0, 1] \Subset \vartheta(x, y), \vartheta(\pi/3, \pi/6) = \vartheta(\pi/6, \pi/3), \vartheta(\pi/2, \pi) \neq \vartheta(\pi, \pi/2), \vartheta(3\pi/4, 3\pi/4) = 0, \vartheta(\pi/4, \pi/4) \neq 0$, and $\vartheta(\sqrt{2}, \sqrt{3}) \neq \vartheta(\sqrt{3}, \sqrt{2})$. In addition, for each x, y, which $\vartheta(y, y) \geq 0$, we get that $\vartheta(x, z) \Subset \vartheta(x, y) + \vartheta(y, z)$ and for each x, y, which $\vartheta(y, y) \leq< 0$, we get that $\vartheta(x, z) \succ \vartheta(x, y) + \vartheta(y, z)$. Hence $(\mathbb{R}, \vartheta)$ is a hypermetricspace.

Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace, $\epsilon \in \mathbb{R}^{>0}$ and $\delta \in \mathbb{R}^{<0}$. Then $B_\epsilon(x) = \{y \in \mathcal{Q} \mid sup(\vartheta(x, y)) < \epsilon\}$, is called an open positive-ball of radius ϵ and center $x \in \mathcal{Q}$ and $B_\delta(x) = \{y \in \mathcal{Q} \mid inf(\vartheta(x, y)) > \delta\}$, means by open negative-ball of radius δ and center $x \in \mathcal{Q}$.

Example 3.13. (i) Consider the neutro hypermetricspace $(\mathcal{Q}, \vartheta)$ in Example 3.10. Then $B_1(a) = \{a, c, e\}, B_2(a) = \{a, b, c, e\}, B_3(a) = \{a, b, c, e\}, B_4(a) = \mathcal{Q}, B_{-1}(a) = \{a, b, d\}, B_{-2}(a) = \{a, b, d\}, B_{-3}(a) = \{a, b, c, d\}$ and $B_{-4}(a) = \mathcal{Q}$.

(ii) Consider the neutro hypermetricspace $(\mathcal{Q}, \vartheta)$ in Example 3.12. Then for each $x \in \mathbb{R}$,

$B_2(x) = \emptyset, B_{-2}(x) = \mathbb{R}, B_1(x) = \{y \in \mathbb{R} \mid |sinx| + cos(y) > 1\} = \{y \in \mathbb{R} \mid |sinx| > 1 - cos(y)\}$.

Theorem 3.14. *Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace, $\epsilon \in \mathbb{R}^{>0}$ and $\delta \in \mathbb{R}^{<0}$. Then $B_\epsilon(x) = \{y \in \mathcal{Q} \mid \vartheta(x, y) \prec [0, \epsilon]\}$.*

Proof. (i) Presume $y \in B_\epsilon(x)$. Then $\vartheta(x, y) \Subset [0, \epsilon]$ and so for each $z \in \vartheta(x, y)$, obtain $a \in [0, \epsilon]$ so $z \leq a < \epsilon$. Hence $sup(\vartheta(x, y)) < \epsilon$ and so $y \in \{y \in \mathcal{Q} \mid sup(\vartheta(x, y)) < \epsilon\}$. Presume $y \in \{y \in \mathcal{Q} \mid sup(\vartheta(x, y)) < \epsilon\}$. Then $sup(\vartheta(x, y)) < \epsilon$ and so $\vartheta(x, y) < sup(\vartheta(x, y)) < \epsilon$. It follows that $\vartheta(x, y) < \epsilon$ and so $y \in B_\epsilon(x)$. Hence $B_\epsilon(x) = \{y \in \mathcal{Q} \mid sup(\vartheta(x, y)) < \epsilon\}$. □

Theorem 3.15. *Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace, $\epsilon \in \mathbb{R}^{>0}$ and $\delta \in \mathbb{R}^{<0}$.*

(i) *For each $x \in \mathcal{Q}, B_\epsilon \neq \emptyset$ and $B_\delta \neq \emptyset$.*

(ii) *If $Inf\{\vartheta(x, y) \mid x, y \in \mathcal{Q}\} = \delta$, then for each $\delta', \delta'' < \delta, B_{\delta'}(x) = B_{\delta''}(x)$.*

(iii) *If $Sup\{\vartheta(x, y) \mid x, y \in \mathcal{Q}\} = \epsilon$, then for each $\epsilon', \epsilon'' > \epsilon, B_{\epsilon''}(x) = B_{\epsilon'}(x)$.*

(iv) *There exist $\epsilon \in \mathbb{R}^{>0}, \delta \in \mathbb{R}^{<0}$ so$B_\epsilon(x) \cup B_\delta(x) = \mathcal{Q}$.*

Proof. (i) Presume $x \in \mathcal{Q}$. Since $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace, for each $y \in \mathcal{Q}$, obtain, $r > 0$, so $[0, r] \Subset \vartheta(x, y)$ and obtain $s \leq 0$, so $[s, 0] \Subset \vartheta(x, y)$ or indeterminate. It follows that $B_\epsilon(x) = \{y \in \mathcal{Q} \mid sup(\vartheta(x, y)) < \epsilon\} \neq \emptyset$ and $B_\delta(x) = \{y \in \mathcal{Q} \mid inf(\vartheta(x, y)) > \delta\} \neq \emptyset$.

(ii) Since $Inf\{\vartheta(x, y) \mid x, y \in \mathcal{Q}\} = \delta$, for each $x, y \in \mathcal{Q}, \delta \leq \vartheta(x, y)$. Now for each $\delta' < \delta$, we get that for each $x, y \in \mathcal{Q}, \delta' < \vartheta(x, y)$ and so $B_{\delta'}(x) = \{y \in \mathcal{Q} \mid inf(\vartheta(x, y)) > \delta'\} = \mathcal{Q}$. It follows that for each $\delta', \delta'' < \delta, B_{\delta'}(x) = B_{\delta''}(x)$.

(iii) Presume $Sup\{\vartheta(x, y) \mid x, y \in \mathcal{Q}\} = \epsilon$. Then for each $x, y \in \mathcal{Q}, \vartheta(x, y) \leq \epsilon$. Now for each $\epsilon < \epsilon'$ and for each $x, y \in \mathcal{Q}, \vartheta(x, y) < \epsilon'$. Thus $B_{\epsilon'}(x) = \{y \in \mathcal{Q} \mid sup(\vartheta) < \epsilon'\} = \mathcal{Q}$ and so for each $\epsilon', \epsilon'' > \epsilon, B_{\epsilon''}(x) = B_{\epsilon'}(x)$.

(iv) It is clear by items $(ii), (iii)$. □

Corollary 3.16. *Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace, $x \in \mathcal{Q}, \epsilon, \epsilon' \in \mathbb{R}^{>0}$ and $\delta, \delta' \in \mathbb{R}^{<0}$.*

(i) *If $\epsilon < \epsilon'$, then $B_\epsilon(x) \subseteq B_{\epsilon'}(x)$.*

(ii) *If $\delta < \delta'$, then $B_{\delta'}(x) \subseteq B_\delta(x)$.*

Definition 3.17. Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace and $U \subseteq \mathcal{Q}$. Then U is called a neutro positive-open set, if obtain $x \in U$ and $\epsilon \in \mathbb{R}^{>0}$ so $B_\epsilon(x) \subseteq U$ and obtain $y \in U, \epsilon' \in \mathbb{R}^{>0}$ such that $B_{\epsilon'}(x) \nsubseteq U$. Moreover, U is called a neutro negative-open set, if obtain $x \in U$ and $\delta \in \mathbb{R}^{<0}$ so $B_\delta(x) \subseteq U$ and obtain $y \in Y, \delta' \in \mathbb{R}^{>0}$ such that $B_{\delta'}(x) \nsubseteq U$. We say that U is a neutro-open set, if it is a neutro positive-open set and a neutro negative-open set.

Example 3.18. (i) Presume $\mathcal{Q} = \{a, b, c, d\}$. Then $(\mathcal{Q}, \vartheta)$ is a netro hypermetricspace as Table 3.

ϑ	a	b	c	d
a	$\{0\}$	$[-1, 1]$	$[-2, 2]$	$[-3, 3]$
b	$[0, 1]$	$\{0\}$	$[0, 2]$	$[0, 3]$
c	$[0, 2]$	$[0, 2]$	$\{0\}$	$[0, 3]$
d	$[-3, 3]$	$[0, 3]$	$[0, 3]$	$\{5, 6\}$

Table 3. Neutro hypermetricspace $(\mathcal{Q}, \vartheta)$.

We see that $\vartheta(a, a) = \{0\}, \vartheta(d, d) \neq \{0\}, \vartheta(a, d) = \vartheta(d, a), \vartheta(a, b) \Subset \vartheta(a, c) + \vartheta(c, b)$ and $\vartheta(d, d) \succ \vartheta(d, a) + \vartheta(a, d)$. Hence $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace. Then $U = \{a, b\}$ is a neutro-open set, since $B_{\frac{1}{2}}(a) \subseteq U, B_3(a) \nsubseteq U$, we get U is a neutro positive-open set. In addition, $B_{\frac{-1}{2}}(a) \subseteq U, B_{-3}(a) \nsubseteq U$, we get U is a neutro negative-open set. Hence, U is a neutro-open set.

(ii) Consider the neutro hypermetricspace $(\mathcal{Q}, \vartheta)$ in Example 3.10 and $U = \{d\}$. Then U is not a neutro negative-open set, since for each $\delta \in \mathbb{R}^{<0}, B_\delta(d) = \mathcal{Q} \nsubseteq U$. Hence U is not a neutro-open set.

Theorem 3.19. *Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace. Then*

(i) *$\emptyset$ is a neutro-open set.*

(ii) *$\mathcal{Q}$ is not a neutro-open set.*

Proof. (i) Immediate by definition.

(ii) Since for each $x \in \mathcal{Q}, \epsilon \in \mathbb{R}^{>0}$, and each $\delta \in \mathbb{R}^{<0}$, have $B_\epsilon(x) \subseteq \mathcal{Q}$ and $B_\delta(x) \subseteq \mathcal{Q}$, we get that $\mathcal{Q}$ is not a neutro-open set. □

Theorem 3.20. *Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace and $\emptyset \neq U \subseteq \mathcal{Q}$. If $\{U_i\}_{i \in I}$ is a set of neutro-open sets, then $\bigcap_{i \in I} U_i$ is a neutro-open set.*

Proof. Presume $\{U_i\}_{i \in I}$ be a set of neutro-open sets of $\mathcal{Q}$. If $\bigcap_{i \in I} U_i = \emptyset$, then by item (i), $\bigcap_{i \in I} U_i$ is a neutro-open set. Suppose that $\bigcap_{i \in I} U_i \neq \emptyset$ and $u, u' \in \bigcap_{i \in I} U_i$. It implies that for each $i \in I, u, u' \in U_i$. Since for each $i \in I, U_i$ is a neutro positive-open set, obtain $\epsilon^i \in \mathbb{R}^{>0}$ so $B_{\epsilon^i}(u) \subseteq U_i$ and obtain $u' \in U_i, \epsilon'^i \in \mathbb{R}^{>0}$ such that $B_{\epsilon'^i}(u') \nsubseteq U_i$. Now, consider $\epsilon = \min\{\epsilon^i \mid i \in I\}$, then based Corollary 3.16, for each $i \in I, B_\epsilon(u) \subseteq B_{\epsilon^i}(u)$. Thus $B_\epsilon(u) \subseteq \bigcap_{i \in I} B_{\epsilon^i}(u) \subseteq \bigcap_{i \in I} U_i$. In addition, consider $\epsilon' = \max\{\epsilon^i \mid i \in I\}$, then based Corollary

3.16, for each $i \in I, B_{\epsilon'^i}(u) \subseteq B_{\epsilon'}(u)$. Thus $B_{\epsilon'}(u) \nsubseteq \bigcap_{i\in I} B_{\epsilon'^i}(u) \nsubseteq \bigcap_{i\in I} U_i$. It follows that $\{U_i\}_{i\in I}$ is a neutro positive-open set. In a similar way, each one can see that $\{U_i\}_{i\in I}$ is a neutro negative-open set and so $\{U_i\}_{i\in I}$ is a neutro-open set in $\mathcal{Q}$. □

Example 3.21. Presume $\mathcal{Q} = \{a, b, c, d\}$. Then $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace as Table 4.

ϑ	a	b	c	d
a	$\{0\}$	$[-1, 1]$	$[-2, 5]$	$[1, 3]$
b	$[-1, 1]$	$\{0\}$	$[-3, 2]$	$[-4, 3]$
c	$[-2, 5]$	$[-5, -4]$	$\{0\}$	$[-5, 3]$
d	$[-3, 3]$	$[-3, 3]$	$[-4, 3]$	$\{0, -1, -2\}$

Table 4. Neutro hypermetricspace $(\mathcal{Q}, \vartheta)$.

We see that $\vartheta(a, a) = \{0\}, \vartheta(d, d) \neq \{0\}, \vartheta(a, d) = \vartheta(d, a), \vartheta(a, b) \Subset \vartheta(a, c) + \vartheta(c, b)$ and $\vartheta(d, b) \succ \vartheta(d, c) + \vartheta(c, b)$. Hence $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace. Then $U_1 = \{a\}(B_1(a) \subseteq U_1$ and $B_2(a) \nsubseteq U_1), U_2 = \{b\}(B_1(b) \subseteq U_2$ and $B_2(b) \nsubseteq U_2), U_3 = \{c\}(B_1(c) \subseteq U_3$ and $B_5(c) \nsubseteq U_3)$ and $U_4 = \{d\}(B_1(d) \subseteq U_4$ and $B_4(d) \nsubseteq U_4)$ are neutro-open sets, while $\mathcal{Q} = U_1 \cup U_2 \cup U_3 \cup U_4$ is not a neutro-open set.

Theorem 3.22. *Presume $(\mathcal{Q}, \vartheta)$ be a neutro hypermetricspace. Then obtain $\epsilon \in \mathbb{R}^{>0}$ and $\delta \in \mathbb{R}^{<0}$ and $x \in \mathcal{Q}$, such that*

(i) *$B_\epsilon(x)$ is a neutro-open set, provided $B_\epsilon(x) \neq \mathcal{Q}$;*
(ii) *$B_\delta(x)$ is a neutro-open set, provided $B_\delta(x) \neq \mathcal{Q}$.*

Proof. (i) Since $(\mathcal{Q}, \vartheta)$ is a neutro hypermetricspace, obtain $x, y, z \in \mathcal{Q}$ so$\vartheta(x, z) \Subset \vartheta(x, y) + \vartheta(y, z)$. Presume $y \in B_\epsilon(x)$. Then $sup(\vartheta(x, y)) < \epsilon$. Consider $\rho = \epsilon - sup(\vartheta(x, y))$ and $z \in B_\rho(y)$. Thus $sup(\vartheta(x, z)) < sup(\vartheta(x, y)) + sup(\vartheta(y, z)) < sup(\vartheta(x, y)) + \epsilon - sup(\vartheta(x, y)) = \epsilon$. Hence $sup(\vartheta(x, z)) < \epsilon$ and so obtain $\rho \in \mathbb{R}^{>0}$ so$B_\rho(y) \subseteq B_\epsilon(x)$. Also obtain $x, y, z \in \mathcal{Q}$ so $\vartheta(x, z) \succ \vartheta(x, y) + \vartheta(y, z)$. Consider $\rho = \epsilon + sup(\vartheta(x, y))$ and $z \in B_\rho(y)$, then $B_\rho(y) \nsubseteq B_\epsilon(x)$. Therefore, $B_\epsilon(x)$ is a neutro-open set.

(ii) It is similar to the item (i). □

4. Conclusions

In the real world there is no exact size and most sizes are approximate. For example, we say the length of a wall that has a fracture and is somewhat angled is between 100 and 102 meters, or we say the weight of a package is between 200 and 201 kg. These approximations indicate that the sizes are not exact and fall within a range. These real issues motivate us to interpolate supermeters. In addition, sometimes it is not possible for us to measure some objects and we say that this size is not known to us. Sometimes we cannot take the distance of an object from itself to zero, because an object fills a volume in three-dimensional space and we have a distance on this object itself. These issues make us interpolate the cloud meter to have a more realistic modeling of the world around us. Hence, we connect the notations of metricspaces and neutro hypermetricspaces. All the basic impressions such as neutro open balls and neutro open sets are interpolated and investigated. We intend to amplify these impressions in the next works in others neutro structures.

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