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Implementation of Circle-Breaking Algorithm on Fermatean Neutrosophic Graph to discover Shortest Path

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Abstract: In many scientific domains, there is a growing interest in the shortest path problem. Traffic routes that can be precisely defined become arbitrary due to the damage that natural catastrophes inflict on roads and bridges. The truth membership, indeterminacy membership, and falsity membership of the component elements make up a neutrosophic set. Their axis of symmetry is indeterminacy membership, and it has a symmetric form. The neutrosophic number is a better way to express the edge distance in uncertain circumstances. With an edge distance stated using Fermatean neutrosophic numbers (**FrNN**), the study aims to solve the shortest path problem of the Fermatean neutrosophic graph. Additionally, the edge distance will be resolved based on the score and precise functions derived from the FrNN. In order to solve the shortest path problem and determine the shortest distance, the application of a circle-breaking algorithm is suggested.

Keywords: circle-breaking algorithm; neutrosophic graph; shortest path problem; Fermatean neutrosophic numbers.

1. Introduction

In many sectors, the Shortest Path Problem (SPP) is one of the most important and well-known problems, and it is the heart of network instabilities. Fuzzy numbers are useful in conventional problems where the node distance is supposed to be known. In these situations, the optimal solution can be obtained in an unknown setting. Calculating the minimum cost of a path for each vertex is known as single source SPP. Especially when trying to find the shortest way, finding the path with the fewest bends is crucial and will yield the greatest outcomes.

Senapati et al. came up with the novel concept of the Fermatean fuzzy set [21, 22]. One of its constraints is that the total of the membership and non-membership grades' third powers cannot be equal to one. More spatial severity in the membership and non-membership grades allows Fermatean Fuzzy Sets (FFSs) to assist with unpredictability of data more successfully. Later on, they use FFSs to figure out a few techniques. A thorough examination of FFSs and its uses is given in reference. Thamizhendhi et al. [25] defined Fermatean Fuzzy Hyper-Graphs (FFHGs) and gave them many interpretations and characteristics. Operators on single-valued Neutrosophic graphs are investigated in the source material [6] by Talea et al. Furthermore, activities on Neutrosophic ambiguous graphs are clarified in [7].

The Bellman algorithm was suggested by Tan et al. [7]. Broumi used the original Bellman algorithm to find the shortest path from the start point to the end point. Broumi et al. [5] found the solution for an Isolated single valued neutrosophic graph. Tan [24] applied an improved dynamic programming algorithm to the SPP of a trapezoidal fuzzy medium intelligence graph, beginning the search from the end point, and the NN was not accurate during the operation process. Scoring function and accuracy function for the pentagonal NN's were applied to the SPP by Chakraborty [9].

The circle-breaking algorithm functions more effortlessly than the Dijkstra approach in order to tackle the least spanning tree problem of undirected graphs, Guan [15] invented the circle-breaking technique in 1975. Beginning with the original graph, the circle-breaking technique iteratively removes the longest edge within the closed circle until it yields a minimal spanning tree. Consequently, one edge of the longer of the two paths that create the closed circle can be constantly eliminated when this technique is applied to the directed neutrosophic graph, allowing any two points to be connected while disconnecting the considerably longer path. Numerous clarifications of neutroshopic sets and graphs are provided in [18,19,20] by various researchers. Ultimately, the algorithm resolves the neutrosophic graph's SPP. Lehua et al [17] explored the Shortest Path Trapezoidal Fuzzy Neutrosophic Graph with Circle-Breaking Algorithm. Amala et al [1], presented Modified Circle Breaking Algorithm to solve a Shortest Path Problem. Very recently, an innovative optimal technique entitled Dhouib-Matrix-SPP (DM-SPP) is invented by Dhouib in [11]. In addition, the DM-SPP is enhanced for cognitive mobile robot (using four and eight movement directions) by Dhouib in [12, 13, 14]. Zeng et al proposed an Islanding Algorithm of Distribution System with Distributed Generations based on Circle-breaking Algorithm [28].

In unidentified scenarios, it is customary to determine the edge distance utilizing the neutrosophic number. The investigation seeks to solve the shortest path difficulties of the Fermatean neutrosophic graph with an edge distance provided by Fermatean neutrosophic numbers (FrNN). For weighted, connected, and undirected graphs, the lowest cost spanning tree problem can be solved with the help of the circle-breaking algorithm. The edge distance will be resolved based on the score and precise functions derived from the FrNN. Furthermore, the SPP for directed graphs will be expanded, and the graphs will be examined in search of a closed circle that connects the beginning and ending vertices. Until there are no more circles in the picture, the previous procedures are repeated after removing the final edge of the longer path. The ability to handle issues occurring in Fermatean neutrosophic graphs is what makes the proposed method new. The results of this study will be used to develop new frameworks and algorithms that will determine the optimal path for a given network in a variety of fixed contexts and neutrosophic environments in the future. The aforementioned method has relevance to all kinds of neutrosophic structures. This algorithmic rule can be implemented to machine learning, shipping, computerized systems, research facilities, manufacturing facilities, and other locales where shortest path explanations are crucial. The datasets evolved and/or scrutinized during this study are not exposed to the community at large.

The layout of this article is arranged systematically as follows: Introduction about the research work is given in section1.Section 2 provides some basic concepts of Fermatean neutrosophic numbers and graphs. In section 3, Circle-Breaking Algorithm is provided. In Section 4, a numerical example is deliberated. The conclusion of this research work is summarized in the last Section 5.

2. *Preliminaries:*

Definition: 2.1 [18]

Let X be a non-empty set (universe). The concept of Fermatean neutrosophic sets (FN sets) $\mathcal{N}$ on $\mathfrak{U}$ is an object that can be represented mathematically by A on x is of the following format: $A = \{(x, ó_P(x), ύ_P(x), ώ_P(x))/x \in X\}$, Where $ó_P(x), ύ_P(x), ώ_P(x) \in [0,1]$, $0 \le (ó_P(x))^3 + (ύ_P(x))^3 \le 1$ and $0 \le (ώ_P(x))^3 \le 1$.
Then $0 \le (ó_P(x))^3 + (ύ_P(x))^3 + (ώ_P(x))^3 \le 2$, $for\ all\ x\ in\ X$ $ó_P(x)$ is the degree of membership, $ύ_P(x)$ is the degree of indeterminacy and $ώ_P(x)$ is the degree of non-membership. Here $ó_P(x)\, and\ ύ_P(x)$ are dependent components and $ώ_P(x)$ is an independent component.

Definition: 2.2 [18]

Let X be a nonempty set and I the unit interval [0, 1]. A Fermatean neutrosophic sets K and L of the form $K = \{(x, ó_P(x), ύ_P(x), ώ_P(x))/x \in X\}$ and $L = \{(x, ó_Q(x), ύ_Q(x), ώ_Q(x))/x \in X\}$. Then

1) $K^c = \{(x, ó_P(x), 1 - ύ_P(x), ώ_P(x)) : x \in X\}$

2) $K \cup L = \{(x, \max(ó_P(x), ó_Q(x)), \min(ύ_P(x), ύ_Q(x)), \min(ώ_P(x), ώ_Q(x)) : x \in X\}$

3) $K \cap L = \{(x, \min(ó_P(x), ó_Q(x)), \max(ύ_P(x), ύ_Q(x)), \max(ώ_P(x), ώ_Q(x)) : x \in X\}$

Definition: 2.3 [18]

Let $P = (ó_P, ύ_P, ώ_P)$ and $Q = (ó_Q, ύ_Q, ώ_Q)$ be any two Fermatean neutrosophic numbers on a vertices on any graph G then the edge length from Q to P is defined as

$$ó_Q(u,v) = \min\{ó_P(u), ó_P(v)\}$$

$$ύ_Q(u,v) = \min\{ύ_P(u), ύ_P(v)\}$$

$$ώ_Q(u,v) = \min\{ώ_P(u), ώ_P(v)\}, \text{ if } ó_Q(u,v), ύ_Q(u,v), ώ_Q(u,v) \in [0,1]$$

Definition: 2.4 [18]

Let the vertices $\acute{o}_s(u,v)$, $\acute{v}_s(u,v)$, $\acute{\omega}_s(u,v)$ be Fermatean neutrosophic number then the score function is defined as

$$S = \frac{\acute{o}s(u,v)+\acute{v}s(u,v),+1-\acute{\omega}s(u,v)}{3} \quad (1)$$

Definition: 2.5 [20]

Let G = (V, E) be a graph which is an ordered pair a set of vertices (nodes or points) and a set of edges (links or lines), which an edge is associated with two distinct vertices.

Definition: 2.6 [20]

Any fuzzy relation μ: $S \times S \to [0,1]$ can be regarded as defining a weighted graph, or fuzzy graph, where the arc $(x, y) \in S \times S$, for all x, y in S has weight $\mu(X, Y) \in [0,1]$.

Definition: 2.7 [20]

A Neutrosophic graph is of the form $G^* = (V, \sigma, \mu)$ where $\sigma = (T_1, I_1, F_1)$ & $\mu = (T_2, I_2, F_2)$

(i) $V = \{v_1, v_2, v_3, \ldots v_n\}$ such that $T_1 : V \longrightarrow [0,1]$, $I_1 : V \longrightarrow [0,1]$ and $F_1 : V \to [0,1]$ denote the degree of truth-membership function, indeterminacy–membership function and falsity membership function of the vertex $v_1 \in V$ respectively and $0 \le T_i(v) + I_i(v) + (v) \le 3$, $\forall\, vi \in V$ (i = 1,2 , 3, … n).

(ii) $T_2 : V \times V \longrightarrow [0,1]$, $I_2 : V \times V \longrightarrow [0,1]$ and $F_2: V \times V \to [0,1]$ where $T_2(v_i, v_j)$, $I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function , indeterminacy –membership function and falsity-membership function of the edge (v_i, v_j) respectively such that for every edge (vi, vj),

$$T_2(\ , vj) \le \min\{T_1(v_i), T_1(v_j)\},$$

$$I_2(\ , vj) \le \min\{I_1(v_i), I_1(v_j)\},$$

$$F_2(\ , vj) \le \max\{F_1(v_i), F_1(v_j)\},$$

and $T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \le 3$

Definition: 2.8 [20]

A Fermatean fuzzy Graph (FFG) on a universal set X is a pair $\mathbb{G} = (\mathcal{P}, \mathcal{Q})$ where $\mathcal{P}$ is Fermatean fuzzy set on X and $\mathcal{Q}$ is a Fermatean fuzzy relation on X such that :

$\{_q(u, v) \le \min\{T_{\mathcal{P}}(u), T_{\mathcal{P}}(v)\}$

$\{_q(u, v) \ge \max\{F_{\mathcal{P}}(u), F_{\mathcal{P}}(v)\}$ and

$0 \le T_Q^3(u, v) + F_Q^3(u, v) \le 1$ *for all* $u, v \in X$, where , $T_{\mathcal{Q}}: X \times X \to [0,1]$, $F_{\mathcal{Q}}: X \times X \to [0,1]$ indicates degree of membership and degree of non-membership of $\mathcal{Q}$, correspondingly. Here $\mathcal{P}$ is the Fermatean fuzzy vertex set of $\mathbb{G}$ and $\mathcal{Q}$ is the Fermatean fuzzy edge set of $\mathbb{G}$.

Definition: 2.9 [20]

Let X be a universal set. A mapping $\mathcal{P} = (T_{\mathcal{P}},, F_{\mathcal{P}}) : X \times X \to [0,1]$ is called a Fermatean Neutrosophic relation on X such that $T_{\mathcal{P}}(u, v)$, $I_{\mathcal{P}}(u,v)$, $F_{\mathcal{P}}(u,v) \in [0,1]$ for all $u, v \in X$.

Definition: 2.10 [20]

Let $\mathcal{P} = (T_{\mathcal{P}}, I_{\mathcal{P}}, F_{\mathcal{P}})$ and $\mathcal{Q} = (T_{\mathcal{Q}}, I_{\mathcal{Q}}, F_{\mathcal{Q}})$ be Fermatean Neutrosophic sets on X if $\mathcal{Q}$ is Fermatean Neutrosophic relation on X, then $\mathcal{Q}$ is called a Fermatean Neutrosophic relation on $\mathcal{P}$ if { $T_{\mathcal{Q}}(u,v) \le$

min{$T_{\mathcal{P}}(u)$, $T_{\mathcal{P}}(v)$} $I_{\mathcal{Q}}(u,v) \geq$ max{$I_{\mathcal{P}}(u)$,$I_{\mathcal{P}}(v)$} $F_{\mathcal{Q}}(u,v) \geq$ max{$F_{\mathcal{P}}(u)$, $F_{\mathcal{P}}(v)$} if $T_{\mathcal{P}}(u, v)$, $I_{\mathcal{P}}(u, v)$, $F_{\mathcal{P}}(u,v) \in$ [0,1] for all $u, v \in X$.

Definition: 2.11 [20]

A Fermatean neutrosophic graph on a universal set X is a pair $\mathbb{G} = (\mathcal{P}, \mathcal{Q})$ where $\mathcal{P}$ is Fermatean Neutrosophic set on X and $\mathcal{Q}$ is a Fermatean Neutrosophic relation on X such that:

$$\begin{cases} T_Q(u,v) \leq \min\{T_p(u), T_p(v)\} \\ I_Q(u,v) \geq \max\{I_p(u), I_p(v)\} \\ F_Q(u,v) \geq \max\{F_p(u), F_p(v)\} \end{cases}$$

and $0 \leq T_{\mathcal{Q}}^3\ (u, v) + T_{\mathcal{Q}}^3\ (u, v) + T_{\mathcal{Q}}^3\ (u, v) \leq 2$, for all $u, v \in X$

where , $T_{\mathcal{Q}}$: x × x → [0,1], $I_{\mathcal{Q}}$: x × x → [0,1] and $F_{\mathcal{Q}}$:x × x → [0,1] indicates degree of membership, degree of indeterminacy-membership and degree of non-membership of Q, correspondingly. Here, P is the Fermatean Neutrosophic vertex set of G and Q is the Fermatean Neutrosophic edge set of G.

Definition: 2.12 [19]

Let (η, ξ) ∈ N, L = $\langle (\eta, \xi);\ T^{\tau}, I^{\tau}, F^{\tau} \rangle$, is a Fermatean neutrosophic normal number (FNNN), where its degree of truth, indeterminacy and falsity membership are defined, $T_L^T = T_L^T e^{-\left(\frac{x-\eta}{\xi}\right)^3}$, $T_L^I = T_L^I e^{-\left(\frac{x-\eta}{\xi}\right)^3}$, $T_L^F = 1-(1-T_L^F)e^{-\left(\frac{x-\eta}{\xi}\right)^3}$,x∈$X$ respectively, where x is a non empty set and $T^{\tau}, I^{\tau}, F^{\tau} \in [0,1]$ and $0 \leq T_L^T(x)^3 + T_L^I(x)^3 + T_L^F(x)^3 \leq 2$.

If $L_1 = \langle (\eta_1, \xi_1);\ T_1^T, T_1^I, T_1^F \rangle$, $L_2 = \langle (\eta_2, \xi_2);\ T_2^T, T_2^I, T_2^F \rangle$ be any two FNNNs and ∧ be a positive integer, then

1. $L_1 \oplus L_2 = \ ((\eta_1 + \eta_2, \xi_1 + \xi_2);\ \sqrt[3\wedge]{(T_1^T)^{3\wedge} + (T_2^T)^{3\wedge} - (T_1^T)^{3\wedge}(T_2^T)^{3\wedge}}$,

$$\sqrt[\wedge]{(T_1^I)^{\wedge} + (T_2^I)^{\wedge} - (T_1^I)^{\wedge}(T_2^I)^{\wedge}}, (T_1^F . T_2^F) \quad (2)$$

2. $L_1 \otimes L_2 = (((\eta_1 . \eta_2, \xi_1 . \xi_2); (T_1^T, T_2^T), \sqrt[\wedge]{(T_1^I)^{\wedge} + (T_2^I)^{\wedge} - (T_1^I)^{\wedge}(T_2^I)^{\wedge}}$,

$$\sqrt[3\wedge]{(T_1^F)^{3\wedge} + (T_2^F)^{3\wedge} - (T_1^F)^{3\wedge}(T_2^F)^{3\wedge}}\ ) \quad (3)$$

3. $\wedge . L_1 = ((\wedge.\eta_1, \wedge.\xi_1);\ \sqrt[3\wedge]{1-(1-(T_1^T)^{3\wedge})^{\wedge}}\ , (T_1^I)^{\wedge}, (T_1^F)^{\wedge})$ (4)

4. $L_1^{\wedge} = ((\eta_1^{\wedge} . \xi_1^{\wedge}); (T_1^T)^{\wedge}(T_2^T)^{\wedge}, \sqrt[3\wedge]{1-(1-(T_1^F)^{3\wedge})^{\wedge}})$ (5)

Definition: 2.13 [19]

Let $L_1 = \langle (\eta_1, \xi_1);\ T_1^T, T_1^I, T_1^F \rangle$, $L_2 = \langle (\eta_2, \xi_2);\ T_2^T, T_2^I, T_2^F \rangle$ be any two FNNNs be the any two FrNN s. Then,

(i) The Euclidean distance between L_1 and L_2 is defined as $D_E(L_1, L_2)$, where

$$D_E(L_1, L_2) = \frac{1}{3}\sqrt[3]{\left(\frac{1+(T_1^T)^3+(T_1^I)^3-(T_1^F)^3}{3}\eta_1 - \frac{1+(T_2^T)^3+(T_2^I)^3-(T_2^F)^3}{3}\eta_2\right)^3 + \frac{1}{3}\left(\frac{1+(T_1^T)^3+(T_1^I)^3-(T_1^F)^3}{3}\xi_1 - \frac{1+(T_2^T)^3+(T_2^I)^3-(T_2^F)^3}{3}\xi_2\right)^3} \tag{6}$$

(ii) The Hamming distance between L_1 and L_2 is defined as $D_E(L_1, L_2)$, where

$$D_H(L_1, L_2) = \frac{1}{3}\sqrt[3]{\left|\frac{1+(T_1^T)^3+(T_1^I)^3-(T_1^F)^3}{3}\eta_1 - \frac{1+(T_2^T)^3+(T_2^I)^3-(T_2^F)^3}{3}\eta_2\right| + \frac{1}{3}\left|\frac{1+(T_1^T)^3+(T_1^I)^3-(T_1^F)^3}{3}\xi_1 - \frac{1+(T_2^T)^3+(T_2^I)^3-(T_2^F)^3}{3}\xi_2\right|} \tag{7}$$

3. Procedure for Solving SPP with Circle-Breaking Algorithm on Fermatean Neutrosophic Graph.

With the circle-breaking algorithm, the lowest cost spanning tree issue for weighted, linked, and undirected graphs can be tackled (see Figure 1). In addition, it will extend the SPP for directed graphs and explore the graphs for a closed circle that binds the starting and terminating vertices. After eliminating the last edge of the longer path, the preceding steps are repeated until there are no more circles in the figure.

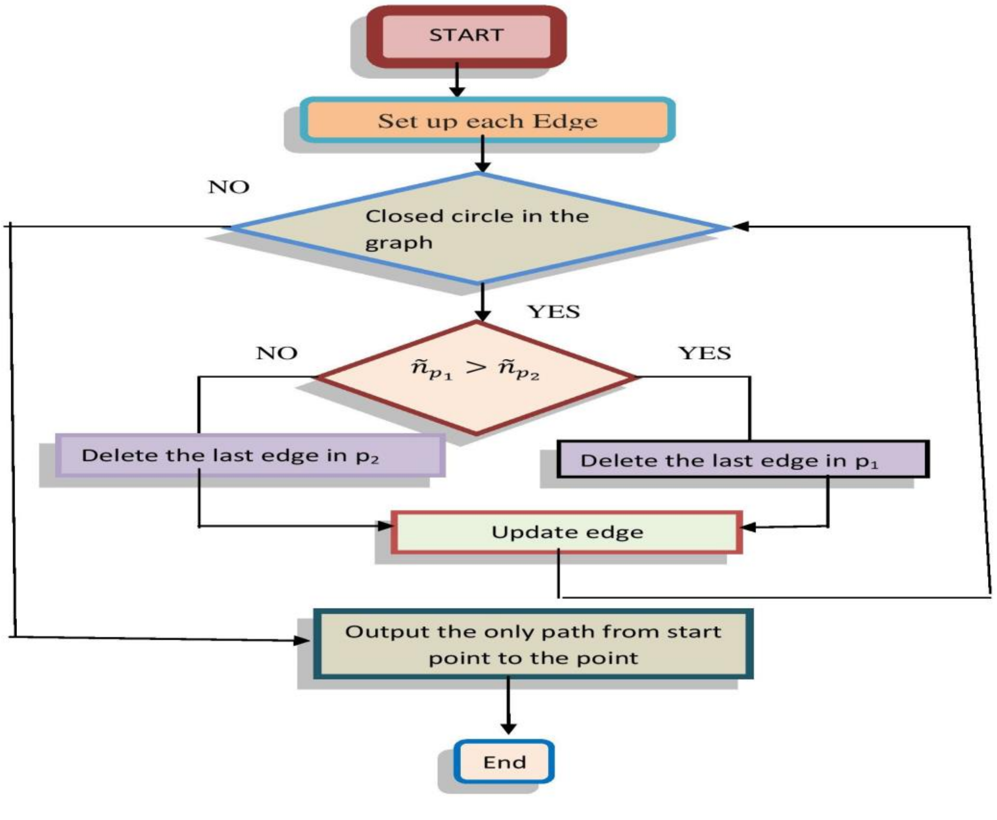

Figure 1: Procedure for Circle-Breaking Algorithm

The particular steps are outlined below,

Step 1: In the Fermatean neutrosophic graph, define a closed circle at random.

Then, determine the two pathways, p_1 and p_2, that encircle the closed circle.

These paths have a common starting node, N_0, and a common ending node, N_1.

Step 2: E v ery path's edges add up. The two pathways are then

represented by the trapezoidal fuzzy numbers $\tilde{n}_{p_1}$ and $\tilde{n}_{p_2}$.

Step 3: Determine the value of the scoring function.

Step 4: Employing the ranking function, correlate the sizes of S($\tilde{n}_{p_1}$)1 and S($\tilde{n}_{p_2}$), then

locate and eliminate.

4. Numerical Example:

Consider the Fermatean neutrosophic shortest path problem, with six nodes (see Figure 2).

Table 1. Details of edge information in terms of Fermatean neutrosophic numbers (FrNN).

Vertices	Fermatean neutrosophic number
(1,2)	<(0.2,0.4);0.3,0.7,0.5 >
(1,3)	<(0.1,0.3);0.5,0.3.0.8>
(2,4)	<(0.2,0.3);0.4,0.8,0.6>
(2,5)	<(0.3,0.4);0.6,0.5,0.7>
(3,4)	<(0.1,0.4);0.7,0.6,0.8>
(3,5)	<(0.2,0.3);0.9,0.6,0.5>
(4,6)	<(0.3,0.5);0.8,0.4,0.6>
(5,6)	<(0.1,0.2); 0.4,0.3,0.5>

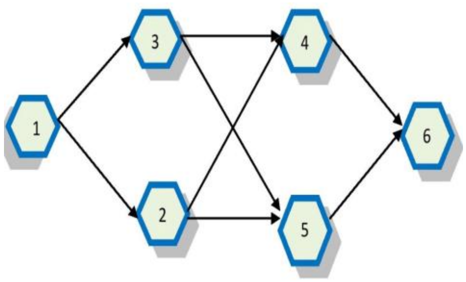

Figure 2: Fermatean neutrosophic graph.

The shortest path from ① to ⑥ is solved based on the circle-breaking algorithm. A circle is randomly selected in the figure, the larger of the two paths surrounding the circle is determined, and the last edge of the circle is deleted. This process is repeated until no other circle can be found. Finally, the only path remaining from ① to ⑥ is the shortest path.

Step 1: Circle ①②④③① in Figure and paths

P_1= {(1,3),(3,4)} and P_2= {(1,2),(2,4)} that make up the closed circle, as indicated by the thick lines in Figure 3, are found.

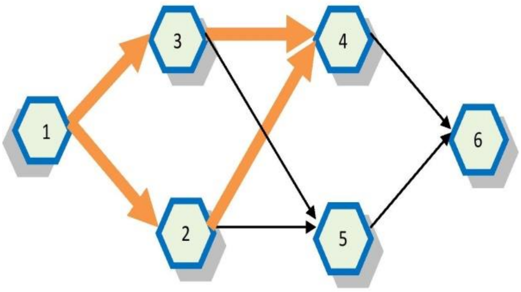

Figure 3: Closed circle ①②④③①

Step 2: According to Equation (2), all Fermatean neutrosophic edges of each path are summed, and the TrFNN of the two paths are obtained by determining and $\tilde{n}_{p_1}$and $\tilde{n}_{p_2}$.

$$\tilde{n}_{p_1} = \tilde{n}_{(1,2)} \oplus \tilde{n}_{(2,4)}$$

= <(0.2,0.4);0.3,0.7,0.5 > $\oplus$ <(0.2,0.3);0.4,0.8,0.6>

$$=((\ 0.2 + 0.2, 0.4 + 0.3);\ \sqrt[3\wedge]{(0.3)^{3\wedge} + (0.4)^{3\wedge} - (0.3)^{3\wedge}(0.4)^{3\wedge}}\ ,$$

$$\sqrt[\wedge]{(0.7)^{\wedge} + (0.8)^{\wedge} - (0.7)^{\wedge}(0.8)^{\wedge}}, (0.5.0.6))$$

$$= ((0.4,0.7){:}\ \sqrt[3]{(0.3)^3 + (0.4)^3 - (0.3)^3(0.4)^3}\ ,$$

$$\sqrt[1]{(0.7)^1 + (0.8)^1 - (0.7)^1(0.8)^1}, (0.5.0.6))$$

$$= ((0.4,\ 0.7){:}\ \sqrt[3]{0.027 + 0.064 - 0.027 * 0.064}\ ,\ \ \sqrt[1]{0.7 + 0.8 - 0.7 * 0.8}, (0.5.0.6))$$

$$= ((0.4,\ 0.7){:}\ \sqrt[3]{0.027 + 0.064 - 0.001728}\ ,\ \sqrt[1]{0.7 + 0.8 - 0.56}, (0.5.0.6))$$

$$= ((0.4,\ 0.7) : \sqrt[3]{0.089272},\ \sqrt[1]{0.94}, (0.5.0.6))$$

= ((0.4, 0.7) :0.45,0.97,0.3)

$\tilde{n}_{p_2} = \tilde{n}_{(1,3)} \oplus \tilde{n}_{(3,4)}$

= <(0.1,0.3);0.5,0.3.0.8> $\oplus$<(0.1,0.4);0.7,0.6,0.8>

$((0.1 + 0.1, 0.3 + 0.4);\ \sqrt[3\wedge]{(0.5)^{3\wedge} + (0.7)^{3\wedge} - (0.5)^{3\wedge}(0.7)^{3\wedge}}\ ,$

$\sqrt[\wedge]{(0.3)^{\wedge} + (0.6)^{\wedge} - (0.3)^{\wedge}(0.6)^{\wedge}}, (0.8.0.8))$

= ((0.2,0.7): $\sqrt[3]{(0.5)^3 + (0.7)^3 - (0.5)^3(0.7)^3}$,

$\sqrt[\wedge]{(0.3)^1 + (0.6)^1 - (0.3)^1(0.6)^1}, (0.8.0.8))$

= ((0.2,0.7): $\sqrt[3]{0.125 + 0.343 - 0.125 * 0.343}$, $\sqrt[1]{0.3 + 0.6 - 0.3 * 0.6}, (0.8.0.8))$

= ((0.2, 0.7): $\sqrt[3]{0.125 + 0.343 - 0.042875}$. $\sqrt[1]{0.3 + 0.6 - 0.18}, (0.8.0.8))$

= ((0.2, 0.7) : $\sqrt[3]{0.425125}$, $\sqrt[1]{0.72}, (0.8.0.8))$

= ((0.2, 0.7) : $0.75\ , 0.85, 0.64)$

Step 3: The Score function of is given by S = $\frac{T + I + 1 - F}{3}$

$\tilde{n}_{p_1}$= ((0.4, 0.7) : 0.45,0.97,0.3)

$S_{p_1} = \frac{0.45 + 0.97 + 1 - 0.3}{3} = \frac{2.12}{3} = 0.71$

$\tilde{n}_{p_2}$= ((0.2, 0.7) : $0.75\ , 0.85, 0.64)$

$S_{p_2} = \frac{0.75 + 0.85 + 1 - 0.64}{3} = \frac{1.96}{3} = 0.653$

Step 4: Because $S_{p_1} > S_{p_2}$, $\tilde{n}_{p_1} > \tilde{n}_{p_2}$according to Definition 7. Thus, by deleting the last edge, (3, 4), in p1, the Fermatean neutrosophic graph in Figure. 4 can be obtained.

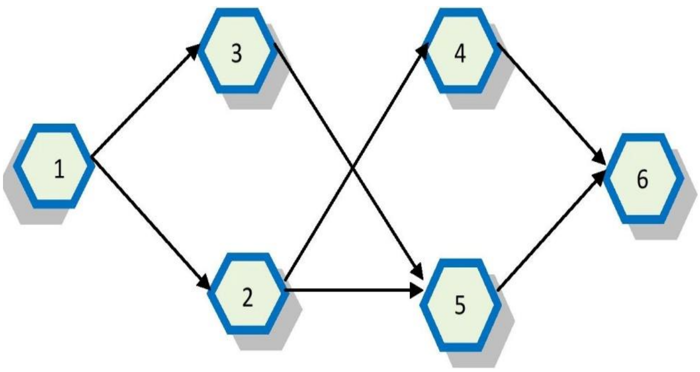

Figure 4: Removal of edge (3, 4).

Step 5: Closed circle ①②⑤③① in Figure 4 is selected, and paths P_3 = {(1,3),(3,5)} and P_4 ={(1,2),(2,5)} that make up the closed circle, as indicated by the thick lines in Figure 5, are obtained. Once this task is completed, we return to Step 2.

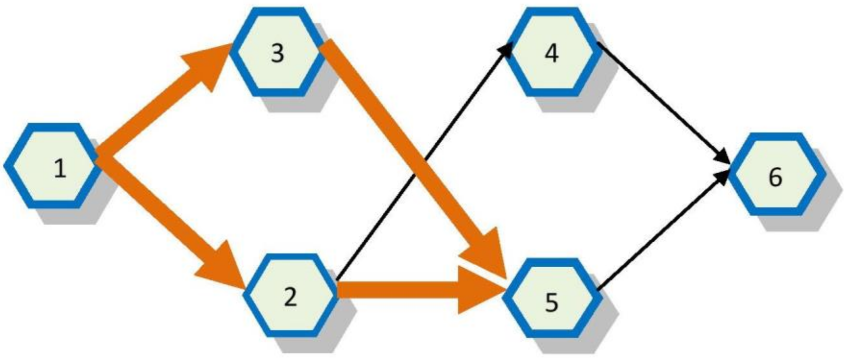

Figure 5: Closed circle ①②⑤③①

Step 6: According to Equation (2), all neutrosophic edges of each path are summed, and the FrNN of the two paths are obtained by determining $\tilde{n}_{p_3}$ and $\tilde{n}_{p_4}$.

$\tilde{n}_{p_3} = \tilde{n}_{(1,3)} \oplus \tilde{n}_{(3,5)}$

= <(0.1,0.3);0.5,0.3.0.8> $\oplus$ <(0.2,0.3);0.9,0.6,0.5>

$= ((0.1 + 0.2, 0.3 + 0.3);\ \sqrt[3\wedge]{(0.5)^{3\wedge} + (0.9)^{3\wedge} - (0.5)^{3\wedge}(0.9)^{3\wedge}}\ ,$

$\sqrt[\wedge]{(0.3)^{\wedge} + (0.6)^{\wedge} - (0.3)^{\wedge}(0.6)^{\wedge}}, (0.8.0.5))$

$=((0.3,0.6)\text{:}\ \sqrt[3]{(0.5)^3 + (0.9)^3 - (0.5)^3(0.9)^3}\ ,$

$\sqrt[\wedge]{(0.3)^1 + (0.6)^1 - (0.3)^1(0.6)^1}, (0.8.0.5))$

= ((0.3,0.6): $\sqrt[3]{0.125+0.729-0.125*0.729}$, $\sqrt[1]{0.3+0.6-0.3*0.6}, (0.8.0.5))$

= ((0.3,0.6): $\sqrt[3]{0.125+0.729-0.091125}$, $\sqrt[1]{0.3+0.6-0.18}, (0.8.0.5))$

= ((0.3,0.6) : $\sqrt[3]{0.7629}$, $\sqrt[1]{0.72}, (0.8.0.5))$ = ((0.3,0.6): $0.914\ , 0.85, 0.40)$

$\tilde{\eta}_{p_4} = \tilde{\eta}_{(1,2)} \oplus \tilde{\eta}_{(2,5)}$

= <(0.2,0.4);0.3,0.7,0.5 >$\oplus$ <(0.3,0.4);0.6,0.5,0.7>

$((0.2+0.3, 0.4+0.4);\ \sqrt[3\wedge]{(0.3)^{3\wedge}+(0.6)^{3\wedge}-(0.3)^{3\wedge}(0.6)^{3\wedge}}$,

$\sqrt[\wedge]{(0.7)^{\wedge}+(0.5)^{\wedge}-(0.7)^{\wedge}(0.5)^{\wedge}}, (0.5.0.7))$

= ((0.5,0.8): $\sqrt[3]{(0.3)^3+(0.6)^3-(0.3)^3(0.6)^3}$,

$\sqrt[\wedge]{(0.7)^1+(0.5)^1-(0.7)^1(0.5)^1}, (0.5.0.7))$

= ((0.5,0.8): $\sqrt[3]{0.027+0.216-0.027*0.216}$, $\sqrt[1]{0.7+0.5-0.7*0.5}, (0.5.0.7))$

= ((0.5,0.8): $\sqrt[3]{0.027+0.216-0.027*0.216}$, $\sqrt[1]{0.7+0.5-0.7*0.5}, (0.5.0.7))$

= ((0.5,0.8) : $\sqrt[3]{0.237}$, $\sqrt[1]{0.85}, (0.5.0.7))$ = ((0.5, 0.8) : $0.62, 0.92, 0.35)$

Step 7: The Score function of is given by S = $\frac{T+I+1-F}{3}$

$\tilde{\eta}_{p_3}$= ((0.3,0.6): $0.91\ , 0.85, 0.40)$, $S_{p_3} = \frac{0.91+0.85+1-0.40}{3} = \frac{2.36}{3}$ = 0.79

$\tilde{\eta}_{p_4}$= ((0.5, 0.8) : $0.62, 0.92, 0.35)$. $S_{p_4} = \frac{0.62+0.92+1-0.35}{3} = \frac{2.19}{3}$ = 0.73

Step 8: Because $\tilde{\eta}_{p_3} > \tilde{\eta}_{p_4}$, $S_{p_3} > S_{p_4}$ can be obtained according to Definition 7. Thus, by deleting the last edge, (3, 5), in p3, the Fermatean neutrosophic graph depicted in Figure 6 can be obtained.

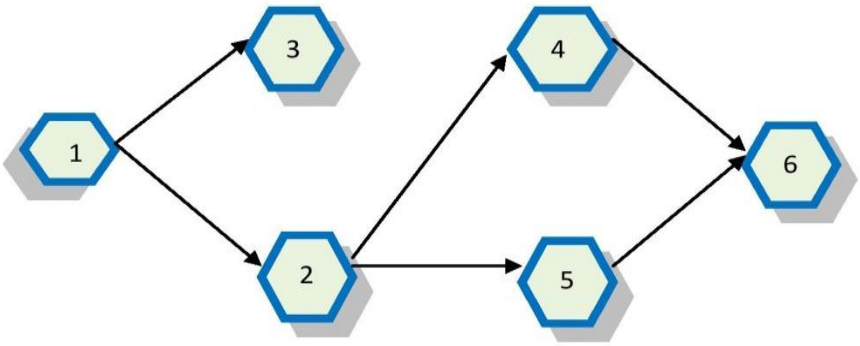

Figure 6: Removal of edge (3, 5)

Step 9: Another closed circle ②④⑥⑤② in Figure 6 is identified, such that paths P_5 ={(2,4),(4,6)} and P_6={(2,5),(5,6)} make up a closed circle, as indicated by the thick lines in Figure 7. Once this task is completed, we return to Step 2.

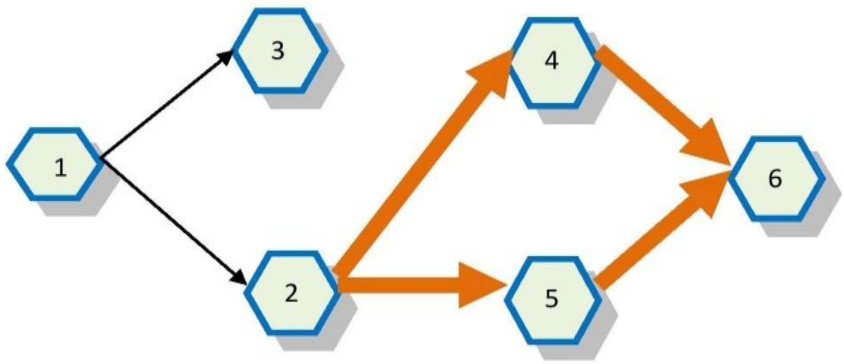

Figure 7: Closed circle ②④⑥⑤②.

Step 10: According to Equation (5), all Fermatean neutrosophic edges of each path are summed, and the FrNN of the two paths are obtained by computing $\tilde{\eta}_{P_5}$ and $\tilde{\eta}_{P_6}$.

$$\tilde{\eta}_{P_5} = \tilde{\eta}_{(2,4)} \oplus \tilde{\eta}_{(4,6)}$$

<(0.2,0.3);0.4,0.8,0.6>$\oplus$ <(0.3,0.5);0.8,0.4,0.6>

$$= \left(\left(0.2 + 0.3, 0.3 + 0.5\right); \sqrt[3^\wedge]{(0.4)^{3\wedge} + (0.8)^{3\wedge} - (0.4)^{3\wedge}(0.8)^{3\wedge}}\ ,\right.$$

$$\left. \sqrt[\wedge]{(0.8)^\wedge + (0.4)^\wedge - (0.8)^\wedge(0.4)^\wedge}, (0.6.0.6)\right)$$

= ((0.5,0.8): $\sqrt[3]{(0.4)^3 + (0.8)^3 - (0.4)^3(0.8)^3}$,

$$\sqrt{(0.8)^1 + (0.4)^1 - (0.8)^1(0.4)^1}, (0.6.0.6))$$

= ((0.5,0.8): $\sqrt[3]{0.064 + 0.512 - 0.064 * 0.512}$, $\sqrt[1]{0.8 + 0.4 - 0.8 * 0.4}$, (0.6.0.6)

= ((0.5, 0.8): $\sqrt[3]{0.064 + 0.512 - 0.0328}$, $\sqrt[1]{0.8 + 0.4 - 0.32}$, (0.6.0.6))

$= ((0.5, 0.8) : \sqrt[3]{0.543},\ \sqrt[1]{0.88}, (0.6.0.6)) \quad = \quad ((0.5, 0.8) : 0.82, 0.94, 0.36)$

$\tilde{\eta}_{P_6} = \tilde{\eta}_{(2,5)} \oplus \tilde{\eta}_{(5,6)}$

$= \langle(0.3,0.4);0.6,0.5,0.7\rangle \oplus \langle(0.1,0.2); 0.4,0.3,0.5\rangle$

$((0.3 + 0.1, 0.4 + 0.2);\ \sqrt[3\wedge]{(0.6)^{3\wedge} + (0.4)^{3\wedge} - (0.6)^{3\wedge}(0.4)^{3\wedge}}\ ,$

$\sqrt[\wedge]{(0.5)^{\wedge} + (0.3)^{\wedge} - (0.5)^{\wedge}(0.3)^{\wedge}}, (0.7.0.5))$

$=((0.4,0.6): \sqrt[3]{(0.6)^3 + (0.4)^3 - (0.6)^3(0.4)^3}\ ,$

$\sqrt[\wedge]{(0.5)^1 + (0.3)^1 - (0.5)^1(0.3)^1}, (0.7.0.5))$

$= ((0.4,0.6): \sqrt[3]{0.216 + 0.064 - 0.216 * 0.064}\ ,$

$\sqrt[1]{0.5 + 0.3 - 0.5 * 0.3}, (0.7.0.5))$

$= ((0.4, 0.6): \sqrt[3]{0.216 + 0.064 - 0.014}\ , \sqrt[1]{0.5 + 0.3 - 0.15}, (0.7.0.5))$

$= ((0.4, 0.6) : \sqrt[3]{0.266},\ \sqrt[1]{0.65}, (0.7.0.5)),$

$= \quad ((0.4, 0.6) : 0.64\ , 0.81, 0.35)$

The Score function of is given by $S = \frac{T + I + 1 - F}{3}$

$\tilde{\eta}_{P_5} = \quad ((0.5, 0.8) : 0.82, 0.94, 0.36)$

$S_{P_5} = \frac{0.82 + 0.94 + 1 - 0.36}{3} = \frac{2.4}{3} = 0.8$

$\tilde{\eta}_{P_6} = \quad ((0.4, 0.6) : 0.64\ , 0.81, 0.35)$

$S_{P_6} = \frac{0.64 + 0.81 + 1 - 0.35}{3} = \frac{2.1}{3} = 0.7$

Step 11: Because $S(\tilde{\eta}_{P_5}) > S(\tilde{\eta}_{P_6})$, according to Definition 7, we can obtain $\tilde{\eta}_{P_5} > \tilde{\eta}_{P_6}$. The last edge, (4, 6), in P_5 is deleted, and the Fermatean neutrosophic graph is obtained, as depicted in Figure 8.

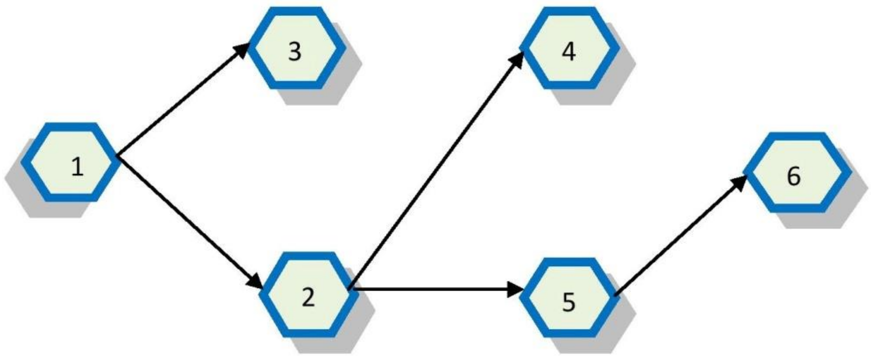

Figure 8: Removal of edge (4, 6)

Step 12: In the Fermatean neutrosophic graph illustrated in Figure 8, it is no longer possible to find a closed circle and the circle-breaking algorithm ends. As indicated by the dotted line in Figure 9, only one path exists from starting node 1 to ending node 6, which is the shortest path sought.

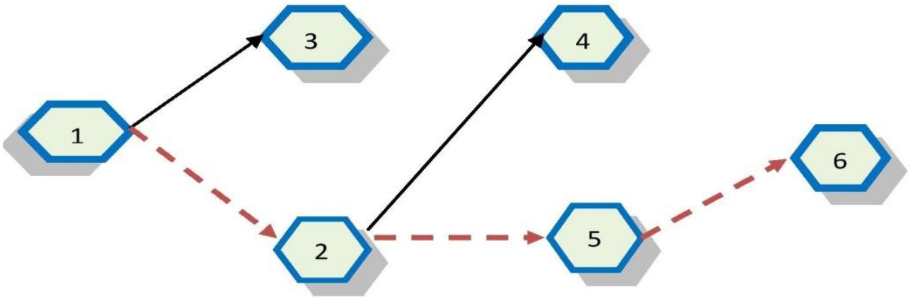

Figure 9: Shortest Path

5. Conclusion:

Circle-breaking algorithm is applied in this paper to fix the SPP of a Fermatean neutrosophic graph, and we utilized an example to confirm the algorithm's viability. The suggested framework may be expanded upon and enhanced in the future to enable it to figure out the distance or shortest path between two locations on a map that depicts various kinds of networks. This paper concludes that it has painstakingly developed a sound theoretical framework, pointing out the shortcomings of the existing methods and emphasizing the need for Fermatean neutrosophic numbers. Trigonometric similarities and more distance measurements can be presented in the future, and many real-world decision-making issues can be resolved.

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References

1. Amala S., Richard, A.Rajkumar, D.Nagarajan, Broumi Said, Shortest Path Problem on

Neutrosophic Environment using Modified Circle Breaking Algorithm, *International Journal of Neutrosophic Science (IJNS) ,Vol. 18, No. 04, PP. 385-394, 2022*.

2. Atanassov K., Intuitionistic fuzzy sets, *Fuzzy Sets and Systems 20: 87-96, 1986*.
3. Aydın, S., A Fuzzy MCDM Method Based on New Fermatean Fuzzy Theories. In *International Journal of Information Technology & Decision Making (pp. 1–22). World Scientific Pub Co Pte Lt. https://doi.org/10.1142/s021962202150019x .2021.*
4. Barraza, R., Sepúlveda, J. M., & Derpich, I., Application of Fermatean fuzzy matrix in codesign of urban projects, *Procedia Computer Science, Vol. 199, pp. 463–47.2022.*
5. Broumi S., Bakali A., Talea M., Smarandache F., Isolated single valued neutrosophic graphs. *Neutrosophic Sets Systems, 11, pp.74–78, 2016b.*
6. Broumi S., Talea M., Bakali A., Smarandache F, Single valued neutrosophic graphs. *J New Theory, 10, pp.86–101, (2016a).*
7. Broumi, S., Dey, A., Talea, M., Bakali, A., Smarandache, F.Nagarajan, D. Lathamaheswari, M.Kumar, R., Shortest path problem using Bellman algorithm under neutrosophic environment. *Complex Intell. Syst. 5*, 409–416, 2019.
8. Broumi, S., krishna P., Fermatean Neutrosophic Matrices and Their Basic Operations. *Neutrosophic Sets and Systems 58, 1,35, 2023. https://digitalrepository.unm.edu*
9. Chakraborty, A., Application of Pentagonal Neutrosophic Number in Shortest Path Problem. *Int. J. Neutrosophic Sci*, *3*, 21–28, 2020.
10. Deng, Z., Wang, J., Evidential Fermatean fuzzy multicriteria decision-making based on Fermatean fuzzy entropy. *In International Journal of Intelligent Systems, Vol. 36, Issue 10, pp. 5866–5886, 2021.Wiley.* https://doi.org/10.1002/int.22534.
11. Dhouib, S., An optimal method for the Shortest Path Problem: The Dhouib-Matrix-SPP (DM-SPP*), Results in Control and Optimization, Vol. 12, ID: 100269,* 2023, *DOI: 10.1016/j.rico.2023.100269.*
12. Dhouib, S., Shortest Path Planning via the Rapid Dhouib-Matrix-SPP (DM-SPP) Method for the Autonomous Mobile Robot. *Results in Control and Optimization, Vol. 13*, ID: 100299, 2023, DOI: 10.1016/j.rico.2023.100299.
13. Dhouib, S., Faster than Dijkstra and A* Methods for the Mobile Robot Path Planning Problem Using Four Movement Directions: The Dhouib-Matrix-SPP-4. *Advances in Transdisciplinary Engineering, Mechatronics and Automation Technology*, Vol. 46, pp. 284-290, 2024, DOI: 10.3233/ATDE231116.
14. Dhouib, S., Intelligent Path Planning for Cognitive Mobile Robot Based on Dhouib-Matrix-SPP Method. *Cognitive Robotics, Vol. 4:62-73, 2024 DOI: 10.1016/j.cogr.2024.02.001.*
15. Guan, M.G., Algorithm of breaking circle for minimum tree. *J. Math. Pract. Theory*, *4*, 40–43, 1975.
16. Gul, M., Lo, H.-W., & Yucesan, M., Fermatean fuzzy TOPSIS-based approach for occupational risk assessment in manufacturing. *In Complex & Intelligent Systems ,Vol. 7, Issue 5, pp. 2635–2653,2021.*
17. Lehua, Y., Dongmei, L., Ruipu, T., Shortest Path Solution of Trapezoidal Fuzzy

Neutrosophic Graph Based on Circle-Breaking Algorithm,*Symmetry*, 12,1-20, doi:10.3390/sym12081360, 2020.

18. Raut P.K., Behera. S.P, Broumi. S., Mishra, D., Calculation of shortest path on Fermatean Neutrosophic Networks, *Neutrosophic Sets and Systems*, Vol. 57, 2023.
19. Palanikumar, M., Arulmozhi, K., Santanu, A., MADM Approach For Fermatean Neutrosophic Normal Aggregation Operator Framework, arXiv:2206.03700v1 [math.GM] 8 Jun 2022.
20. Broumi, S., Sundareswaran, R., Shanmugapriya, M., Bakali, A., Talea, M., Theory and Applications of Fermatean Neutrosophic Graphs, *Neutrosophic Sets and Systems, Vol. 50, 2022*
21. Senapati, T., Yager, R., Fermatean Fuzzy Sets, *Journal of Ambient Intelligence and Humanized Computing , 11, pp. 663-674, 2019.*
22. Senapati, T., Yager, R., Some New Operations Over Fermatean Fuzzy Numbers and Application of Fermatean Fuzzy WPM in Multiple Criteria Decision Making. *In Informatica , Vol. 30, Issue 2, pp. 391–412, 2019.*
23. Smarandache, F., A unifying field in logics neutrosophy: neutrosophic probability, set and logic, *American Research Press, Rehoboth 1998.*
24. Tan, R.P., Zhang, W.D., Broumi, S., Solving methods for the shortest path problem based on trapezoidal fuzzy neutrosophic numbers. *Control Decis.*, *34*, 851–860,2019.
25. Thamizhendhi, G., Kiruthica, C., Suresh, S., Fermatean fuzzy hypergraph,*Journal of Hunan University Natural Sciences, Vol. 48. No. 12, pp.2333-2340, 2011.*
26. Yang, Z., Garg, H., Li, X., Differential Calculus of Fermatean Fuzzy Functions: Continuities, Derivatives, and Differentials. *In International Journal of Computational Intelligence Systems ,Vol. 14, Issue 1, p. 282, 2020. Springer Science and Business Media LLC. https://doi.org/10.2991/ijcis.d.201215.001.*
27. Zadeh, L.A, Fuzzy sets, *Information and Control*, 8(5): 338- 353; 1965.
28. Zeng, J.B., Zhang, W., Li, X.Y., Islanding Algorithm of Distribution System with Distributed Generations based on Circle-breaking Algorithm. *Power Sys. Eng*, *35*, 15–19, 2019.

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