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Exploring Neutrosophic Over Supra Exterior Modal Topological Structure: Theory and Application in Healthcare Decision-Making

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Abstract. Neutrosophic environments have gained significant relevance across diverse disciplines. Neutrosophic Over Supra Exterior Modal Topological Structure is introduced, demonstrating the generation of two distinct structures through traditional Neutrosophic Over supra topological operations, denoted as $\boxplus$ (closure) and $\boxdot$ (interior), along with novel Neutrosophic Over Supra Exterior Modal Operations, expressed as $\circledast$ (modal closure) and $\odot$ (modal interior). This exploration delves into the essential properties and characteristics of these structures, illuminating their behavior and implications. To illustrate the practical relevance, a numerical example is presented, demonstrating the identification of disease types using a score function. The study conclusively shows that the Neutrosophic Over Supra Exterior Modal Topological Structure effectively classifies diseases such as tuberculosis (bacterial), malaria (parasitic), swine flu (viral), and ringworm (fungal). This dual approach ensures a holistic understanding of the topic, appealing to both theoretical and practitioners seeking practical solutions in the neutrosophic over environment.

1. Introduction

In the tapestry of everyday life, uncertainty emerges prominently in various scenarios, such as rolling a die or tossing a coin onto an uneven surface. These moments underscore the prevalence of uncertainty. Zadeh, [25] credited in 1965 for the origination of fuzzy sets, introduced the concept of membership degrees, laying the foundation not only for a theory of possibility [26] but also inspiring Bellman and colleagues to explore decision-making in contexts influenced by fuzziness [4]. Atanassov later extended Zadeh's work by introducing intuitionist fuzzy sets, emphasizing both degrees of membership non-membership [3].

Smaracheand [21] is credited with the discovery exploration of novel trends applications within neutrosophic theory in 1999. Bustince Burillo, in 1995 [5] investigated the correlation of intuitionist fuzzy sets, particularly in scenarios involving interval values. Christianto postulated three potential utilities of neutrosophic sets [7]. Molodtsov [17], in 1999, brought attention to primary results concerning soft sets.Maji in 2003 develped soft set theory [15]. Neutrosophic sets have found practical utility in medical contexts, with correlation measures employed to discern intricate connections between variables.

In 2021, Radha et al. [19] introduced the concept of neutrosophic Pythagorean sets and their heightened correlation. Ye, in 2013 [24] presented an alternative facet of correlation. Wang et al. delved into single-valued neutrosophic sets [23], while Mallick Pramanik, in 2020 [16] discussed pentapartitioned neutrosophic sets. Jansi et al., [13] in 2019, explored neutrosophic Pythagorean sets, featuring both dependent and independent components. Smaracheand introduced the innovative notion of neutrosophic sets with over, under, and off limits in 2016 [22]. Many researchers like Chinnadurai,Al-Hamido etc are intruducing different ways of neutrosophic logic [1, 2, 6, 8, 12, 18, 20]. Additionally, RN Devi and Yamini Parthiban introduced a decision-making process using neutrosophic Pythagorean soft sets, utilizing a measure of correlation [9] in 2023.

Neutrosophic sets play a crucial role in addressing uncertainties that classical sets and fuzzy sets may struggle to manage. Unlike classical sets, which define membership in a binary manner, and fuzzy sets, which introduce partial membership degrees, neutrosophic sets accommodate indeterminacy where elements can simultaneously belong, partially belong, and not belong to a set due to incomplete or conflicting information (Smarandache, 1999). This capability to represent indeterminate and conflicting information makes neutrosophic sets particularly powerful in modeling and analyzing complex systems where precise boundaries and membership degrees are ambiguous or evolving.

Existing methods like fuzzy logic and traditional neutrosophic sets inadequately handle complex uncertainties and struggle to integrate with topological structures. The Neutrosophic Over Supra Exterior Modal Topological Structure addresses these gaps by incorporating modal operations, enhancing uncertainty management, and bridging the integration gap with operations respecting topological properties and neutrosophic elements. This framework extends applicability to fields such as IT and healthcare, accommodating complex real-world data and decision-making scenarios. Additionally, it includes practical demonstrations, validating its effectiveness in real-world applications.

The Neutrosophic Over Supra Exterior Modal Topological Structure addresses limitations in current neutrosophic frameworks by integrating modal operations with topological structures, effectively handling over and under limits. This integration enhances the modeling of complex relationships and spatial dependencies, crucial for applications in healthcare decision-making, information systems, and multi-criteria decision analysis.

In this manuscript an introduction to the Neutrosophic Over Supra Exterior Modal Topological Structure is given. It is not only introduces the theoretical underpinnings of this innovative framework but also provides a tangible demonstration of its effectiveness in healthcare decision-making. This dual approach ensures a holistic understanding of the topic, appealing to both theoretical and practitioners seeking practical solutions in the neutrosophic over environment using a score function.

2. Preliminary

Definition 2.1. Let $\mathcal{H}$ be an non empty set $\mathcal{J}$ is said to be an NS.Then

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph(\mathsf{h}), \eth(\mathsf{h}), \Upsilon(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0,1]$ $0 \le \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \le 3$. Here $\aleph(\mathsf{h}), \eth(\mathsf{h})$ and $\Upsilon(\mathsf{h})$ are degree of true membership,degree of indeterminacy and degree of falsity.

Definition 2.2. Let $\mathcal{J}$ be an NS in $\mathcal{H}$.If $\mathcal{J}$ is said to be an $\mathcal{N}^{\mathfrak{o}}$ in an non-empty set $\mathcal{H}$ then it has at-least one neutrosophic component is > 1 and no other component are < 0 is defined as,

$$\mathcal{J} = \{\langle \mathsf{h}, \aleph(\mathsf{h}), \eth(\mathsf{h}), \Upsilon(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

Where $\aleph, \eth, \Upsilon : \mathcal{H} \to [0, \Omega]$, $0 \le \aleph(\mathsf{h}) + \eth(\mathsf{h}) + \Upsilon(\mathsf{h}) \le 3$ and Ω is said to be over-limit of $\mathcal{N}^{\mathfrak{o}}$.

Note: $\rho(\mathcal{H}^{\natural})$ is a set of all Neutrosophic Over Set($\mathcal{N}^{\mathfrak{o}}$) in $\mathcal{H}$

Definition 2.3. The $\mathcal{N}^{\mathfrak{o}}$ $\rhd = \{\langle \mathsf{h}, 0, 0, \Omega\rangle : \mathsf{h} \in \mathcal{H}\}$ is said to be a Null $\mathcal{N}^{\mathfrak{o}}$ and $\blacktriangleright = \{e, \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} : e \in \mathcal{E}\}$ is said to be an universal $\mathcal{N}^{\mathfrak{o}}$.

Definition 2.4. Let τ_{NT} is a collection of subsets of $\mathcal{H}$.Then τ_{NT} is known as neutrosophic topology if it is satisfies the conditions

(i) $\rhd, \blacktriangleright \in \tau_{NT}$.

(ii) The union of an arbitrary collection τ_{NT} is in τ_{NT}.

(iii) The finite intersection of subsets τ_{NT} is in τ_{NT}.

Then $(\mathcal{H}, \tau_{NT})$ is called neutrosophic topological structure(NTS).An element of τ_{NT} is called an neutrosophic open set and the complement of τ_{NT} is called an neutrosophic closed set.

Definition 2.5. Let $\tau_{\mathcal{N}^{\mathfrak{o}}}$ is a collection of subsets of $\mathcal{H}$.Then $\tau_{\mathcal{N}^{\mathfrak{o}}}$ is known as neutrosophic over topology if it is satisfies the conditions (i) $\rhd, \blacktriangleright \in \tau_{\mathcal{N}^{\mathfrak{o}}}$.

(ii) The union of an arbitrary collection $\tau_{\mathcal{N}^{\mathfrak{o}}}$ is in $\tau_{\mathcal{N}^{\mathfrak{o}}}$.

(iii) The finite intersection of subsets $\tau_{\mathcal{N}^{\mathfrak{o}}}$ is in $\tau_{\mathcal{N}^{\mathfrak{o}}}$.

Then $(\mathcal{H}, \tau_{\mathcal{N}^{\mathsf{o}}})$ is called neutrosophic over topological structure($\mathcal{N}^{\mathsf{o}}$-topological structure).An element of $\tau_{\mathcal{N}^{\mathsf{o}}}$ is called an neutrosophic over open set and the complement of $\tau_{\mathcal{N}^{\mathsf{o}}}$ is called an neutrosophic over closed set.

Definition 2.6. Let $\mathcal{J}$ and $\mathcal{W}$ be two $\mathcal{N}^{\mathsf{o}}$, Then the union,intersection, compliment and subset are defined by

(i)$\mathcal{J} \mho \mathcal{W} = \{\langle \mathsf{h}, \max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), \max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), \min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}$

(ii)$\mathcal{J} \Omega \mathcal{W} = \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}$

(iii)$\mathcal{J}^{☾} = \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$

(iv)$\mathcal{J} \subset \mathcal{W}$, then $\aleph_{\mathcal{J}}(\mathsf{h}) \leq \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{J}}(\mathsf{h}) \leq \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h}) \geq \Upsilon_{\mathcal{W}}(\mathsf{h})$

In other words, $\mathcal{W}$ is an super set of $\mathcal{J}$

(v) Let $\mathcal{J} \subset \mathcal{W}$ and $\mathcal{W} \subset \mathcal{J}$ then $\mathcal{J} = \mathcal{W}$

Proposition 2.7. *Let $\mathcal{J}$ be an $\mathcal{N}^{\mathsf{o}}$ on $\mathcal{H}$. Then*

(i). $\triangleright^{☾} = \blacktriangleright$

(ii). $\blacktriangleright^{☾} = \triangleright$

(iii). $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

Proof. 1. $\triangleright^{☾} = \blacktriangleright$

$$\triangleright = \{\langle \mathsf{h}, 0, 0, \Omega\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\triangleright^{☾} = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \blacktriangleright$$

$$\Longrightarrow \triangleright^{☾} = \blacktriangleright$$

2. $\blacktriangleright^{☾} = \triangleright$

$$\blacktriangleright = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\blacktriangleright^{☾} = \{\langle \mathsf{h}, \Omega, \Omega, 0\rangle : \mathsf{h} \in \mathcal{H}\} = \triangleright$$

$$\Longrightarrow \blacktriangleright^{☾} = \triangleright$$

3. $(\mathcal{J}^{☾})^{☾} = \mathcal{J}$

$$\mathcal{J}^{☾} = \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$(\mathcal{J}^{☾})^{☾} = \{\langle \mathsf{h}, \aleph_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\} = \mathcal{J}$$

$$\Longrightarrow (\mathcal{J}^{☾})^{☾} = \mathcal{J}$$

□

Proposition 2.8. *Let $\mathcal{J}$ and $\mathcal{W}$ be an $\mathcal{N}^{o}$ on $\mathcal{H}$. Then*

(i). $\mathcal{J} \Cup \mathcal{J} = \mathcal{J} \Cap \mathcal{J} = \mathcal{J}$

(ii). $\mathcal{J} \Cup \mathcal{W} = \mathcal{W} \Cup \mathcal{J}$

(iii). $\mathcal{J} \Cap \mathcal{W} = \mathcal{W} \Cap \mathcal{J}$

(iv). $\mathcal{J} \Cup \triangleright = \mathcal{J}\ \ \mathcal{J} \Cup \blacktriangleright = \blacktriangleright$

(v). $\mathcal{J} \Cap \triangleright = \triangleright\ \ \mathcal{J} \Cap \blacktriangleright = \mathcal{J}$

Proof. The proof is obvious from the definition. □

Theorem 2.9. *Let $\mathcal{J}$ and $\mathcal{W} \in \mathcal{N}^{o}$. Then*

(i). $(\mathcal{J} \Cup \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \Cup \mathcal{W}^{\complement}$

(ii). $(\mathcal{J} \Cap \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \Cap \mathcal{W}^{\complement}$

Proof. (i).By the union definition,

$$\mathcal{J} \Cup \mathcal{W} = \{\langle \mathsf{h}, max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$(\mathcal{J} \Cup \mathcal{W})^{\complement} = \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\} \tag{1}$$

By the definition of compliment

$$\mathcal{J}^{\complement} = \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\mathcal{W}^{\complement} = \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\mathcal{J}^{\complement} \Cup \mathcal{W}^{\complement} = \{\langle \mathsf{h}, min(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), max(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), max(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\} \tag{2}$$

From (1) and (2) we get,

$$(\mathcal{J} \Cup \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \Cup \mathcal{W}^{\complement}$$

(ii). By the union definition we know that,

$$\mathcal{J} \Cap \mathcal{W} = \{\langle \mathsf{h}, min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{J}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$(\mathcal{J} \Cap \mathcal{W})^{\complement} = \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\} \tag{3}$$

By the definition of compliment

$$\mathcal{J}^{\complement} = \{\langle \mathsf{h}, \Upsilon_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{J}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\mathcal{W}^{\complement} = \{\langle \mathsf{h}, \Upsilon_{\mathcal{W}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$$

$$\mathcal{J}^{\complement} \Cap \mathcal{W}^{\complement} = \{\langle \mathsf{h}, max(\Upsilon_{\mathcal{J}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})), min(\Omega - \eth_{\mathcal{J}}(\mathsf{h}), \Omega - \eth_{\mathcal{W}}(\mathsf{h})), min(\aleph_{\mathcal{J}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}))\rangle : \mathsf{h} \in \mathcal{H}\} \tag{4}$$

From (3) and (4) we get,

$(\mathcal{J} \Omega \mathcal{W})^{\complement} = \mathcal{J}^{\complement} \Omega \mathcal{W}^{\complement}$ □

3. Neutrosophic Over Supra Exterior Modal Topological Structure

Definition 3.1. Let $\tau_{\mathcal{N}^{o}}$ is a collection of subsets of $\mathcal{H}$.Then $\tau_{\mathcal{N}^{o}}$ is known as neutrosophic over supra topology if it is satisfies the conditions

(i) $\triangleright, \blacktriangleright \in \tau_{\mathcal{N}^{o}}$.

(ii) The union of an arbitrary collection $\tau_{\mathcal{N}^{o}}$ is in $\tau_{\mathcal{N}^{o}}$.

Then $(\mathcal{H}, \tau_{\mathcal{N}^{o}})$ is called neutrosophic over supra topological structure($\mathcal{N}^{o}$-supra topological structure). An element of $\tau_{\mathcal{N}^{o}}$ is called an neutrosophic over supra open set and the complement of $\tau_{\mathcal{N}^{o}}$ is called an neutrosophic over supra closed set.

Definition 3.2. An $\mathcal{N}^{o}$ $\mathcal{T} \in \mathcal{H}$, then neutrosophic over supra topological interior and closure are $\boxdot(\mathcal{T})$ and $\boxplus(\mathcal{T})$ is defined as:

$\boxdot(\mathcal{T}) = \mho\{\mathfrak{N} : \mathfrak{N} \subseteq \mathcal{H} \quad \text{and} \quad \mathfrak{N} \in \tau_{\mathcal{N}^{o}}\}$

$\boxplus(\mathcal{T}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \quad \text{and} \quad \mathcal{O}^{\complement} \in \tau_{\mathcal{N}^{o}}\}$.

Proposition 3.3. *Let $(\mathcal{H}, \tau_{\mathcal{N}^{o}})$ be a $\mathcal{N}^{o}$-supra topological structure $\mathcal{R}$ is a subset of $\mathcal{H}$, then*

(i) $\boxdot(\mathcal{R})$ is the largest $\mathcal{N}^{o}$-supra open set contained in $\mathcal{R}$.

(ii) $\boxplus(\mathcal{R})$ is the smallest $\mathcal{N}^{o}$-supra closed set containing $\mathcal{R}$.

Proof. (i) By the definition of interior, $\boxdot(\mathcal{R})$.Let $\mathcal{N}$ be an open set such that $\mathfrak{N} \subset \mathcal{R}$. $\because \mathfrak{N}$ is open and $\mathcal{N} \subset \mathcal{R}$,then

$\mathfrak{N} \subset \boxdot(\mathcal{R}) \implies \boxdot(\mathcal{R})$ is the largest open set contained in $\mathcal{R}$.

(ii) By the closure definition,

$\boxplus(\mathcal{R}) = \Omega\{\mathcal{O} : \mathcal{H} \subseteq \mathcal{O} \qquad \mathcal{O}^{\complement} \in \tau_{\mathcal{N}^{o}}\}$

$\boxplus(\mathcal{R})$ is the smallest closed set containing $\mathcal{R}$. □

Definition 3.4. Let $\mathcal{T}$ be a $\mathcal{N}^{o}$ over $\mathcal{H}$.Then two stard Neutrosophic Over Supra Modal Operations are expressed as $\circledast$ and $\odot$ is defined as

$\circledast\, \mathcal{T} = \{\langle \mathsf{h}, min(\aleph_{\mathcal{T}}(\mathsf{h}), \Omega), \eth_{\mathcal{T}}(\mathsf{h}), \Omega - \sqrt{|\aleph_{\mathcal{T}}(\mathsf{h}) - \eth_{\mathcal{T}}(\mathsf{h})|}\rangle : \mathsf{h} \in \mathcal{H}\}$

$\odot\mathcal{T} = \{\langle \mathsf{h}, max(\aleph_{\mathcal{T}}(\mathsf{h}), \Omega), \sqrt{(\aleph_{\mathcal{T}}(\mathsf{h}))(\Upsilon_{\mathcal{T}}(\mathsf{h}))}, |\sqrt{\Omega} - \Upsilon_{\mathcal{T}}(\mathsf{h})|\rangle : \mathsf{h} \in \mathcal{H}\}$

Example 3.5. Let $\mathcal{T} = \{\langle \mathsf{h}, 1.3, 0.8, 0.7\rangle : \mathsf{h} \in \mathcal{H}\}$ and $\Omega = 1.5$ be a $\mathcal{N}_{o}$. Then,

$\circledast\, \mathcal{T} = \{\langle \mathsf{h}, 1.3, 0.8, 0.5\rangle : \mathsf{h} \in \mathcal{H}\}$

$\odot\mathcal{T} = \{\langle \mathsf{h}, 1.3, 0.9, 0.4\rangle : \mathsf{h} \in \mathcal{H}\}$

Note: For any $\mathcal{N}_{o}$ $\mathcal{T}$,$\circledast\, \mathcal{T} \subseteq \mathcal{T} \subseteq \odot\mathcal{T}$

Definition 3.6. Let neutrosophic over supra topological structure $(\mathcal{H}, \tau_{\mathcal{N}^{o}})$ then neutrosophic over supra exterior $\mathcal{N}^{o}$ $\mathcal{T}$ is denoted by $\mathfrak{ext}(\mathcal{T})$ $\mathfrak{ext}_{\circledast}(\mathcal{T}) = (\circledast\,(\mathcal{T}))^{☾}$ or $\mathfrak{ext}_{\odot}(\mathcal{T}) = \odot(\mathcal{T}^{☾})$

Definition 3.7. An $\mathcal{N}^{o}$ $\mathcal{T}$ $\mathcal{W} \in \mathfrak{P}(\mathcal{H}^{♮})$. Then $\langle \mathfrak{P}(\mathcal{H}^{♮}), \boxplus, \mho, \Omega, \mathfrak{ext}_{\circledast} \rangle$ is said to be an neutrosophic over supra exterior modal topological closure if it satisfies the following condition:

$(i) \boxplus(\mathcal{T} \mho \mathcal{W}) = \boxplus(\mathcal{T}) \mho \boxplus(\mathcal{W})$,

$(ii) \mathcal{T} \subseteq \boxplus(\mathcal{T})$

$(iii) \boxplus(\mathfrak{K}) = \mathfrak{K}$

$(iv) \boxplus(\boxplus(\mathcal{T})) = \boxplus(\mathfrak{T})$

$(v) \mathfrak{ext}_{\circledast}(\mathcal{T} \Omega \mathcal{W}) = \mathfrak{ext}_{\circledast}(\mathcal{T}) \Omega \mathfrak{ext}_{\circledast}(\mathcal{W})$

$(vi) \mathfrak{ext}_{\circledast}(\mathcal{T}) \subseteq \mathcal{T}$

$(vii) \mathfrak{ext}_{\circledast}(\mathfrak{K}) = \mathfrak{K}$

$(viii) \mathfrak{ext}_{\circledast}(\mathfrak{ext}_{\circledast}(\mathcal{T})) = \mathfrak{ext}_{\circledast}(\mathcal{T})$

$(ix) \mathfrak{ext}_{\circledast}(\boxplus(\mathcal{T})) = \boxplus(\mathfrak{ext}_{\circledast}(\mathcal{T}))$

Theorem 3.8. *If $\mathcal{T}$ $\mathcal{W}$ be two $\mathcal{N}^{o}$. Then $\langle \mathfrak{P}(\mathcal{H}^{♮}), \boxplus, \mho, \Omega, \mathfrak{ext}_{\circledast} \rangle$ is a neutrosophic over supra modal topological closure.*

Proof. Let $\mathcal{T}$ $\mathcal{W}$ be two $\mathcal{N}_{\mathfrak{D}\mathfrak{G}}$

$\mathcal{T}, \mathcal{W} \in \mathcal{P}(\mathcal{H}^{♮})$ then,

$(i) \boxplus(\mathcal{T} \mho \mathcal{W}) = \boxplus(\{\langle \mathsf{h}, \aleph_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{T}}(\mathsf{h}), \Upsilon_{\mathcal{T}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}\} \mho \{\langle \mathsf{h}, \aleph_{\mathcal{W}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}\})$

$= \boxplus(\{\langle \mathsf{h}, max(\aleph_{\mathcal{T}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), max(\eth_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), min(\Upsilon_{\mathcal{T}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h})) \rangle : \mathsf{h} \in \mathcal{H}\})$

$= \{\langle \mathsf{h}, \sup_{\mathsf{g} \in \mathcal{H}} max(\aleph_{\mathcal{T}}(\mathsf{g}), \aleph_{\mathcal{W}}(\mathsf{g})), \sup_{\mathsf{g} \in \mathcal{H}} max(\eth_{\mathcal{T}}(\mathsf{g}), \eth_{\mathcal{W}}(\mathsf{g})),$
$\inf_{\mathsf{g} \in \mathcal{H}} min(\Upsilon_{\mathcal{T}}(\mathsf{g}), \Upsilon_{\mathcal{W}}(\mathsf{g})) \rangle : \mathsf{h} \in \mathcal{H}\}$

$= \{\langle \mathsf{h}, max(\sup_{\mathsf{g} \in \mathcal{H}} \aleph_{\mathcal{T}}(\mathsf{g}), \sup_{\mathsf{g} \in \mathcal{H}} \aleph_{\mathcal{W}}(\mathsf{g})), max(\sup_{\mathsf{g} \in \mathcal{H}} \eth_{\mathcal{T}}(\mathsf{g}), \sup_{\mathsf{g} \in \mathcal{H}} \eth_{\mathcal{W}}(\mathsf{g})),$
$min(\inf_{\mathsf{g} \in \mathcal{H}} \Upsilon_{\mathcal{T}}(\mathsf{g}), \inf_{\mathsf{g} \in \mathcal{H}} \Upsilon_{\mathcal{W}}(\mathsf{g})) \rangle : \mathsf{h} \in \mathcal{H}\}$
$= \boxplus(\mathcal{T}) \mho \boxplus(\mathcal{W})$

$(ii) \mathcal{T} = \{\langle \mathsf{h}, \aleph_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{T}}(\mathsf{h}), \Upsilon_{\mathcal{T}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}\} \subseteq \{\langle \mathsf{h}, \sup_{\mathsf{g} \in \mathcal{H}} \aleph_{\mathcal{T}}(\mathsf{g}), \sup_{\mathsf{g} \in \mathcal{H}} \eth_{\mathcal{T}}(\mathsf{g}),$
$\inf_{\mathsf{g} \in \mathcal{H}} \Upsilon_{\mathcal{T}}(\mathsf{g}) \rangle : \mathsf{h} \in \mathcal{H}\} = \boxplus(\mathcal{T})$

$(iii) \boxplus(\mathfrak{K}) = \boxplus(\{\langle \mathsf{h}, 0, 0, \Omega \rangle : \mathsf{h} \in \mathcal{H}\}) = \{\langle \mathsf{h}, \sup_{\mathsf{g} \in \mathcal{H}} 0, \sup_{\mathsf{g} \in \mathcal{H}} 0, \inf_{\mathsf{g} \in \mathcal{H}} \Omega \rangle : \mathsf{h} \in \mathcal{H}\}$
$= \{\mathsf{h}, 0, 0, \Omega : \mathsf{h} \in \mathcal{H}\} = \mathfrak{K}$

$(iv) \boxplus(\boxplus(\mathcal{T})) = \boxplus(\boxplus(\{\langle \mathsf{h}, \aleph_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{T}}(\mathsf{h}), \Upsilon_{\mathcal{T}}(\mathsf{h}) \rangle : \mathsf{h} \in \mathcal{H}\}))$
$= \boxplus(\{\langle \mathsf{h}, \sup_{\mathsf{g} \in \mathcal{H}} \aleph_{\mathcal{T}}(\mathsf{g}), \sup_{\mathsf{g} \in \mathcal{H}} \eth_{\mathcal{T}}(\mathsf{g}), \inf_{\mathsf{g} \in \mathcal{H}} \Upsilon_{\mathcal{T}}(\mathsf{g}) \rangle : \mathsf{h} \in \mathcal{H}\})$

$= \{\langle \mathsf{h}, \sup_{\mathsf{g}\in\mathcal{H}} \aleph_{\mathcal{T}}(\mathsf{g}), \sup_{\mathsf{g}\in\mathcal{H}} \eth_{\mathcal{T}}(\mathsf{g}), \inf_{\mathsf{g}\in\mathcal{H}} \Upsilon_{\mathcal{T}}(\mathsf{g})\rangle : \mathsf{h} \in \mathcal{H}\} = \boxplus(\mathcal{T})$

$$
\begin{aligned}
(v)\mathfrak{ext}_{\circledast}(\mathcal{T}\Omega\mathcal{W}) &= \mathfrak{ext}_{\circledast}(\{\mathsf{h}, \langle min(\aleph_{\mathcal{T}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), max(\Upsilon_{\mathcal{T}}(\mathsf{h}), \Upsilon_{\mathcal{W}}(\mathsf{h}))\rangle\}) \\
&= \{\mathsf{h}, \langle min(min(\aleph_{\mathcal{T}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}), \Omega), min(\eth_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), \\
&\quad \Omega - \sqrt{min(\aleph_{\mathcal{T}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})) - min(\eth_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})))}\rangle\} \\
&= \{\mathsf{h}, \langle min(\aleph_{\mathcal{T}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h})), min(\eth_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{W}}(\mathsf{h})), \\
&\quad \Omega - \sqrt{max(\aleph_{\mathcal{T}}(\mathsf{h}) - \eth_{\mathcal{T}}(\mathsf{h}), \aleph_{\mathcal{W}}(\mathsf{h}) - \eth_{\mathcal{W}}(\mathsf{h})))}\rangle\} \\
&= \mathfrak{ext}_{\circledast}(\mathcal{T})\Omega\mathfrak{ext}_{\circledast}(\mathcal{W})
\end{aligned}
$$

The proof of $(vi) - (ix)$ is obvious from the Definition(3.10,3.11 and 3.12). □

Definition 3.9. An $\mathcal{N}^{o}$ $\mathcal{T}$ $\mathcal{W} \in \mathfrak{P}(\mathcal{H}^{\natural})$. Then $\langle \mathfrak{P}(\mathcal{H}^{\natural}), \boxdot, \Omega, \mho, \mathfrak{ext}_{\odot}\rangle$ is said to be an neutrosophic over supra exterior modal topological interior if it satisfies the following condition:

$(i) \boxdot(\mathcal{T}\Omega\mathcal{W}) = \boxdot(\mathcal{T})\Omega\boxdot(\mathcal{W})$,

$(ii) \boxdot(\mathcal{T}) \subseteq \mathcal{T}$

$(iii) \boxdot(\mathfrak{K}) = \mathfrak{K}$

$(iv) \boxdot(\boxdot(\mathcal{T})) = \boxdot(\mathfrak{T})$

$(v)\mathfrak{ext}_{\odot}(\mathcal{T}\mho\mathcal{W}) = \mathfrak{ext}_{\odot}(\mathcal{T})\mho\mathfrak{ext}_{\odot}(\mathcal{W})$

$(vi)\mathcal{T} \subseteq \mathfrak{ext}_{\odot}(\mathcal{T})$

$(vii)\mathfrak{ext}_{\odot}(\mathfrak{K}) = \mathfrak{K}$

$(viii)\mathfrak{ext}_{\odot}(\mathfrak{ext}_{\odot}(\mathcal{T})) = \mathfrak{ext}_{\odot}(\mathcal{T})$

$(ix)\mathfrak{ext}_{\odot}(\boxdot(\mathcal{T})) = \boxdot(\mathfrak{ext}_{\odot}(\mathcal{T}))$

Theorem 3.10. *If $\mathcal{T}$ $\mathcal{W}$ be two $\mathcal{N}^{o}$.Then $\langle \mathfrak{P}(\mathcal{H}^{\natural}), \boxdot, \Omega, \mho, \mathfrak{ext}_{\odot}\rangle$ is a neutrosophic over exterior modal topological interior.*

Proof. The proof is obvious. □

Definition 3.11. Let two $\mathcal{N}^{o}$ as $\mathcal{T} = \{\langle \mathsf{h}, \aleph_{\mathcal{T}}(\mathsf{h}), \eth_{\mathcal{T}}(\mathsf{h}), \Upsilon_{\mathcal{T}}(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$ $\mathcal{W} = \{\langle \mathsf{g}, \aleph_{\mathcal{W}}(\mathsf{g}), \eth_{\mathcal{W}}(\mathsf{g}), \Upsilon_{\mathcal{W}}(\mathsf{g})\rangle. \mathsf{g} \in \mathcal{H}\}$ $\mathfrak{f} : \mathcal{H} \to \mathcal{H}$ be a function,

(i)$\mathfrak{f}(\mathcal{T}) = \{\langle \mathsf{g}, \mathfrak{f}(\aleph_{\mathcal{T}})(\mathsf{g}), \mathfrak{f}(\eth_{\mathcal{T}})(\mathsf{g}), (1 - \mathfrak{f}(1 - \Upsilon_{\mathcal{T}}))(\mathsf{g})\rangle : \mathsf{g} \in \mathcal{H}\}$ is a $\mathcal{N}^{o}$ on $\mathcal{H}$ called the image of $\mathcal{T}$ under $\mathfrak{f}$

(ii) .$\mathfrak{f}^{-1}(\mathcal{W}) = \{\langle x, \mathfrak{f}^{-1}(\aleph_{\mathcal{W}})(\mathsf{h}), \mathfrak{f}^{-1}(\eth_{\mathcal{W}})(\mathsf{h}), \mathfrak{f}^{-1}(\Upsilon_{\mathcal{W}})(\mathsf{h})\rangle : \mathsf{h} \in \mathcal{H}\}$ is a $\mathcal{N}^{o}$ on $\mathcal{H}$ is called the pre-image of $\mathcal{W}$ under $\mathfrak{f}$, where

$$\mathfrak{f}(\aleph_{\mathcal{T}})(\mathsf{g}) = \begin{cases} \sup_{\mathsf{h}\in\mathfrak{f}^{-1}}(\mathsf{g})\aleph_{\mathcal{T}}(\mathsf{h}), & \text{if } \mathfrak{f}^{-1}(\mathsf{g}) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\mathfrak{f}(\eth_{\mathcal{T}})(\mathsf{g}) = \begin{cases} \sup_{\mathsf{h}\in\mathfrak{f}^{-1}}(\mathsf{g})\eth_{\mathcal{T}}(\mathsf{h}), & \text{if } \mathfrak{f}^{-1}(\mathsf{g}) \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$(1-\mathfrak{f}(1-\Upsilon_{\mathcal{T}}))(\mathsf{g}) = \begin{cases} \inf_{\mathsf{h}\in\mathfrak{f}^{-1}}(\mathsf{g})\Upsilon_{\mathcal{T}}(\mathsf{h}), & \text{if only if } \mathfrak{f}^{-1}(\mathsf{g}) \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

Let us introduce the $\mathfrak{f}_{-}(\Upsilon_{\mathcal{T}})$ for $(1-\mathfrak{f}(1-\Upsilon_{\mathcal{T}}))$

Corollary 3.12. *Let $\{\mathcal{T}\}_{i\in\alpha}^{\infty}, \mathcal{T}\ \mathcal{W}$ be $\mathcal{N}^{\mathfrak{o}}$ on $\mathcal{H}$, then the following are true*

(i) $(\Omega_{i\in\alpha}\mathcal{T}_i)^{☾} = \mho_{i\in\alpha}\mathcal{T}_i^{☾}, (\mho_{i\in\alpha}\mathcal{T}_i)^{☾} = \Omega_{i\in\alpha}\mathcal{T}_i^{☾}$

(ii) $(\mathcal{T}^{☾})^{☾} = \mathcal{T} \cdot \mathcal{W}^{☾} \subseteq \mathcal{T}^{☾}$, *if* $\mathcal{W} \subseteq \mathcal{T}$

Proof. (i) $(\Omega_{i\in\alpha}\mathcal{T}_i)^{☾} = \{\langle \mathsf{h}, |1-\inf_{i\in\alpha}\{\aleph_{\mathcal{T}}(\mathsf{h})\}|, |1-\inf_{i\in\alpha}\{\eth_{\mathcal{T}}(\mathsf{h})\}|, |\ 1-\sup_{i\in\alpha}\{\Upsilon_{\mathcal{T}}(\mathsf{h})\ |\}\rangle : \mathsf{h} \in \mathcal{H}\}$

$= \{\langle x, \sup_{i\in\alpha}(|1-\aleph_{\mathcal{T}i}(x)|), \sup_{i\in\alpha}(|1-\eth_{\mathcal{T}i}(x)|).., \inf .. \in \alpha(|1-\Upsilon_{\mathcal{T}i}(x)|)\rangle : x \in \mathcal{H}\}$

$= \mho_{i\in\alpha}\mathcal{T}_i^{☾}$.

Similarly, $(\mho_{i\in\alpha}\mathcal{T}_i)^{☾} = \Omega_{i\in\alpha}\mathcal{T}_i^{☾}$ is obvious

(ii) we can prove it from (i) □

Definition 3.13. Let single valued neutrosophic over score function is denoted by $\mathfrak{S}_{\mathfrak{F}}$ defined by $\mathfrak{S}_{\mathfrak{F}} = \frac{1}{3m}[\sum_{i=1}^{m}[|2+\aleph_i-\eth_i-\Upsilon_i|]]$

4. Algorithm

Step 1:Collection of data

Consider $\mathfrak{m}$ attributes $\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, \ldots, \mathcal{A}_{\mathfrak{m}}$,$\mathfrak{n}$ attributes $\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3, \ldots, \mathfrak{E}_{\mathfrak{n}}$ and $\mathfrak{p}$ attributes $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3, \ldots, \mathfrak{H}_{\mathfrak{p}}$

$(\mathfrak{n} \leq \mathfrak{p})$ for multi-attributes decision making problem(MADMP).

$\mathcal{A}_{\mathsf{i}}\backslash\mathfrak{E}_{\mathsf{j}}$	$\mathfrak{E}_1$	$\mathfrak{E}_2$	.	.	.	$\mathfrak{E}_n$
$\mathcal{A}_1$	$\mathfrak{a}_{11}$	$\mathfrak{a}_{12}$	.	.	.	$\mathfrak{a}_{1n}$
$\mathcal{A}_2$	$\mathfrak{a}_{21}$	$\mathfrak{a}_{22}$	.	.	.	$\mathfrak{a}_{2n}$
$\mathcal{A}_3$	$\mathfrak{a}_{31}$	$\mathfrak{a}_{32}$	.	.	.	$\mathfrak{a}_{3n}$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$\mathcal{A}_m$	$\mathfrak{a}_{m1}$	$\mathfrak{a}_{m2}$	.	.	.	$\mathfrak{a}_{mn}$

$\mathfrak{H}_{\mathfrak{p}}\backslash\mathcal{A}_{\mathsf{i}}$	$\mathcal{A}_1$	$\mathcal{A}_2$	.	.	.	$\mathcal{A}_m$
$\mathfrak{H}_1$	$\mathfrak{d}_{11}$	$\mathfrak{d}_{12}$	.	.	.	$\mathfrak{d}_{1m}$
$\mathfrak{H}_2$	$\mathfrak{d}_{21}$	$\mathfrak{d}_{22}$	.	.	.	$\mathfrak{d}_{2m}$
$\mathfrak{H}_3$	$\mathfrak{d}_{31}$	$\mathfrak{d}_{32}$	.	.	.	$\mathfrak{d}_{3m}$
.	.	.	.	.	.	.
.	.	.	.	.	.	.
$\mathfrak{H}_p$	$\mathfrak{d}_{p1}$	$\mathfrak{d}_{p2}$	.	.	.	$\mathfrak{d}_{pm}$

Step 2: Construction of neutrosophic over supra exterior topology for $(\mathfrak{E}_{\mathsf{j}})$ $(\mathfrak{H}_{\mathfrak{k}})$**:**

(i)$\tau_{\mathsf{j}}^{\varsigma} = \mathcal{A}\mho\mathcal{B}$, Where $\mathcal{A} = \{\triangleright, \blacktriangleright, \mathfrak{a}_{1\mathsf{j}}, \mathfrak{a}_{2\mathsf{j}}, \mathfrak{a}_{3\mathsf{j}}, \ldots, \mathfrak{a}_{\mathfrak{m}\mathsf{j}}\}$ and $\mathcal{B} = \{\mathfrak{a}_{1\mathsf{j}} \cup \mathfrak{a}_{2\mathsf{j}}, \mathfrak{a}_{1\mathsf{j}} \cup \mathfrak{a}_{3\mathsf{j}}, \ldots, \mathfrak{a}_{\mathfrak{m}-1\mathsf{j}} \cup \mathfrak{a}_{\mathfrak{m}\mathsf{j}}\}$

(ii)$\nu^{\varsigma}_{\mathfrak{k}} = \mathcal{C}\mho\mathcal{D}$, Where $\mathcal{C} = \{\triangleright, \blacktriangleright, \eth_{\mathfrak{k}1}, \eth_{\mathfrak{k}2}, \eth_{\mathfrak{k}3}, \ldots, \eth_{\mathfrak{k}\mathfrak{m}}\}$ and $\mathcal{D} = \{\eth_{\mathfrak{k}1}\cup\eth_{\mathfrak{k}2}, \eth_{\mathfrak{k}1}\cup\eth_{\mathfrak{k}3}, \ldots, \eth_{\mathfrak{k}\mathfrak{m}-1}\cup \eth_{\mathfrak{k}\mathfrak{m}}\}$

Step 3:Find single valued neutrosophic over score function:

Single valued neutrosophic over score functions($\mathfrak{S}_{\mathfrak{F}}$) of $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathfrak{E}_{\mathfrak{j}}$ $\mathfrak{H}_{\mathfrak{k}}$

(i)$\mathfrak{S}_{\mathfrak{F}}(\mathfrak{E}_{\mathfrak{j}})= \begin{cases} \mathfrak{S}_{\mathfrak{F}}(\mathcal{A}) & if \mathfrak{S}_{\mathfrak{F}}(\mathcal{B}) = 0 \\ \frac{1}{2}[\mathfrak{S}_{\mathfrak{F}}(\mathcal{A}) + \mathfrak{S}_{\mathfrak{F}}(\mathcal{B})] & otherwise \end{cases}$

where, (1) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{A})= \frac{1}{3(\mathfrak{m}+2)}\big(\sum_{\mathfrak{i}=1}^{\mathfrak{m}+2}[2 + \aleph_i - \eth_i - \Upsilon_i]\big)$

(2) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{B})= \frac{1}{3\mathfrak{q}}\big(\sum_{\mathfrak{i}=1}^{\mathfrak{q}}[2 + \aleph_i - \eth_i - \Upsilon_i]\big)$

$\forall$ $\mathfrak{j}$ denotes the number of elements in $\mathcal{B}$ $\mathfrak{j} = 1, 2, \ldots, n$

(ii) $\mathfrak{S}_{\mathfrak{F}}(\mathfrak{H}_{\mathfrak{k}})= \begin{cases} \mathfrak{S}_{\mathfrak{F}}(\mathcal{C}) & if \mathfrak{S}_{\mathfrak{F}}(\mathcal{D}) = 0 \\ \frac{1}{2}[\mathfrak{S}_{\mathfrak{F}}(\mathcal{C}) + \mathfrak{S}_{\mathfrak{F}}(\mathcal{D})] & otherwise \end{cases}$

where, (1) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{C})= \frac{1}{3(\mathfrak{m}+2)}\big(\sum_{\mathfrak{i}=1}^{\mathfrak{m}+2}[2 + \aleph_i - \eth_i - \Upsilon_i]\big)$

(2) $\mathfrak{S}_{\mathfrak{F}}(\mathcal{D})= \frac{1}{3\mathfrak{r}}\big(\sum_{\mathfrak{i}=1}^{\mathfrak{r}}[2 + \aleph_i - \eth_i - \Upsilon_i]\big)$

$\forall$ $\mathfrak{r}$ denotes the number of elements in $\mathcal{D}$ $\mathfrak{k} = 1, 2, \ldots, \mathfrak{p}$

Step 4:Final decision

Establish the single-valued neutrosophic over score values for decisions $\mathfrak{E}_1 \leq \mathfrak{E}_2 \leq \cdots \leq \mathfrak{E}_{\mathfrak{n}}$ and also for the attributes $\mathfrak{H}_1 \leq \mathfrak{H}_2 \leq \cdots \leq \mathfrak{H}_{\mathfrak{p}}$ .Choose the attribute $\mathfrak{H}_{\mathfrak{p}}$ for $\mathfrak{E}_1$,$\mathfrak{H}_{\mathfrak{p}-1}$ for $\mathfrak{E}_2$ etc,. If $\mathfrak{p} > \mathfrak{n}$ then leave out $\mathfrak{H}_{\mathfrak{k}}$, where $\mathfrak{k} = 1, 2, \ldots, \mathfrak{n} - \mathfrak{p}$.

5. Numerical Illustration

In contemporary times, there is a discernible surge in the prevalence of infectious diseases, primarily attributed to the proliferation of viruses, bacteria, fungi, and other pathogens. The task of systematically categorizing diverse sets of symptoms under a singular disease classification presents a formidable challenge within the domain of medical diagnosis. Within this context, the previously mentioned strategy is employed to underscore its efficacy and pertinence.

As infectious diseases persist in their evolution diversification, healthcare professionals grapple with the intricate task of precisely diagnosing these conditions. This involves discerning the specific causative agents, considering the myriad symptoms exhibited by affected individuals, establishing a comprehensive understing of the disease's pathogenesis. The process of classifying a myriad of symptoms under a single disease umbrella necessitates a methodical well-considered approach.

In the context of our discussion, a disease identification problem serves as a practical demonstration of the aforementioned strategy's efficiency and relevance. This illustrative example highlights how a systematic and holistic approach to diagnosis can assist in tackling the burgeoning challenges posed by infectious diseases, ultimately leading to more efficacious treatments improved public health outcomes.

Step 1:Collection of data

The information acquired from consultations with medical professionals, covering four classifications of microorganisms viruses, bacteria, fungi, and parasites recognized for inducing infectious diseases, along with the corresponding symptoms such as rhinorrhea, decreased appetite, diarrhea, alopecia, and anemia, has been methodically structured into a tabular format. The aim is to ascertain the pathogens establish the diseases they induce, which include common cold, stomach flu, E.coli infection, ringworm, and hookworm infestation.

TABLE 1. Symptoms for microorganism

Symptoms\Microorganism	Parasite($\mathfrak{E}_1$)	Fungi($\mathfrak{E}_2$)	Bacteria($\mathfrak{E}_3$)	Virus($\mathfrak{E}_4$)
Runny nose($\mathcal{A}_1$)	(1.35,0.9,0.1)	(1.34,0.3,0.25)	(1.19,0.53,1.01)	(1.26,0.3,0.11)
Loss of appetite($\mathcal{A}_2$)	(1.22,0.3,0.6)	(1.21,0.2,0.3)	(1.07,0.53,1)	(1.2,0.14,0.22)
Diarrhea ($\mathcal{A}_3$)	(1.17,0.2,0.3)	(0.52,0.1,1.18)	(1.13,0.21,0.6)	(0.4,0.2,1.16)
Hair loss ($\mathcal{A}_4$)	(1.35,0.15,0.5)	(1.3,0.3,0.7)	(1.09,0.1,0.1)	(1.25,0.2,0.5)
Anemia ($\mathcal{A}_5$)	(0.6,0.3,1.14)	(1.34,0.2,0.6)	(1.02,0.17,0.8)	(1.21,0.3,0.6)

TABLE 2. Symptoms for the diseases

Diseases\Symptoms	Runnynose($\mathcal{A}_1$)	Loss of appetite($\mathcal{A}_2$)	Diarrhea ($\mathcal{A}_3$)	Hair loss ($\mathcal{A}_4$)	Anemia($\mathcal{A}_5$)
Tuberculosis($\mathfrak{H}_1$)	(1.31,0.53,0.04)	(1.32,0.61,0.3)	(1.24,0.73,0.1)	(1.2,0.4,0.1)	(1.32,0.4,0.1)
Common cold ($\mathfrak{H}_3$)	(1.1,0.1,0.9)	(1.22,0.3,0.8)	(1.12,0.3,0.8)	(1.13,0.4,0.6)	(1.09,0,0.1)
Ring worm ($\mathfrak{H}_5$)	(1.09,0,0.1)	(0,0.2,1.1)	(1.07,0.2,0.3)	(1.2,0.3,0.1)	(1.02,0,0.8)
Swin flu ($\mathfrak{H}_4$)	(1.12,0.5,0.8)	(1.1,0.4,0.9)	(1.22,0.5,0.8)	(1.2,0.3,0.8)	(1.32,0.4,0.8)
Malaria ($\mathfrak{H}_2$)	(1.29,0.1,0.1)	(1.23,0.6,0.7)	(1.23,0.6,0.7)	(1.13,0.1,0.7)	(1.15,0.3,0.5)

Step 2: Construction of neutrosophic over supra exterior topology for ($\mathfrak{E}_{\mathrm{j}}$) ($\mathfrak{H}_{\mathfrak{k}}$):

TABLE 3. $\mathfrak{E}_{\mathrm{j}}$

	$\mathcal{A}$ and $\mathcal{B}$
τ_1^{ς}	$\mathcal{A} = \{(1.5, 1.5, 0), (0, 0, 1.5), (1.35, 0.9, 0.1), (1.22, 0.3, 0.6), (1.17, 0.2, 0.3)(1.35, 0.15, 0.5), (0.6, 0.3, 1.14)\}$ and $\mathcal{B} = \{(1.35, 0.9, 0.1)\}$
τ_2^{ς}	$\mathcal{A} = \{(1.5, 1.5, 0), (0, 0, 1.5), (1.34, 0.3, 0.25), (1.21, 0.2, 0.3), (0.52, 0.1, 1.18), (1.3, 0.3, 0.7), (1.34, 0.2, 0.6)\}$ and $\mathcal{B} = \{(1.34, 0.3, 0.25)\}$
τ_3^{ς}	$\mathcal{A} = \{(1.5, 1.5, 0), (0, 0, 1.5), (1.19, 0.53, 1.01), (1.07, 0.53, 1), (1.13, 0.21, 0.6), (1.09, 0.1, 0.1), (1.02, 0.17, 0.8)\}$ and $\mathcal{B} = \{(1.19, 0.53, 1.01)\}$
τ_4^{ς}	$\mathcal{A} = \{(1.5, 1.5, 0), (0, 0, 1.5), (1.26, 0.3, 0.11), (1.2, 0.14, 0.22), (0.4, 0.2, 1.16), (1.25, 0.2, 0.5), (1.21, 0.3, 0.6)\}$ and $\mathcal{B} = \{(1.26, 0.3, 0.11)\}$

TABLE 4. $\mathfrak{H}_{\mathfrak{k}}$

	$\mathcal{C}$ and $\mathcal{D}$
ν_1^{ς}	$\mathcal{C} = \{(1.5,1.5,0),(0,0,1.5),(1.31,0.53,0.04),(1.1,0.1,0.9),(1.09,0,0.1),(1.12,0.5,0.8),(1.29,0.1,0.1)\}$ and $\mathcal{D} = \{(1.31,0.53,0.04)\}$
ν_2^{ς}	$\mathcal{C} = \{(1.5,1.5,0),(0,0,1.5),(1.32,0.61,0.3),(1.22,0.3,0.8),(0,0.2,1.1),(1.1,0.4,0.9),(1.23,0.6,0.7)\}$ and $\mathcal{D} = \{(1.32,0.61,0.3)\}$
ν_3^{ς}	$\mathcal{C} = \{(1.5,1.5,0),(0,0,1.5),(1.24,0.73,0.1),(1.12,0.3,0.8),(1.07,0.2,0.3),(1.22,0.5,0.8),(1.23,0.6,0.7)\}$ and $\mathcal{D} = \{(1.24,0.73,0.1)\}$
ν_4^{ς}	$\mathcal{C} = \{(1.5,1.5,0),(0,0,1.5),(1.2,0.4,0.1),(1.13,0.4,0.6),(1.2,0.3,0.1),(1.2,0.3,0.8),(1.13,0.1,0.7)\}$ and $\mathcal{D} = \{(1.2,0.4,0.1)\}$
ν_5^{ς}	$\mathcal{C} = \{(1.5,1.5,0),(0,0,1.5),(1.32,0.4,0.1),(1.09,0,0.1),(1.02,0,0.8),(1.32,0.4,0.8),(1.15,0.3,0.5)\}$ and $\mathcal{D} = \{(1.32,0.4,0.1)\}$

Step 3:Find single valued neutrosophic over score function:

TABLE 5. $\mathfrak{S}_{\mathfrak{F}}\mathfrak{E}_{\mathrm{j}}$

$\mathcal{A}$	$\mathcal{B}$	$\mathfrak{E}_{\mathrm{j}}$
0.6523	0.7833	$\mathfrak{E}_1 = 0.7178$
0.6647	0.9300	$\mathfrak{E}_2 = 0.7973$
0.6166	0.5500	$\mathfrak{E}_3 = 0.5833$
0.6309	0.9500	$\mathfrak{E}_4 = 0.7904$

TABLE 6. $\mathfrak{S}_{\mathfrak{F}}\mathfrak{H}_{\mathfrak{k}}$

$\mathcal{C}$	$\mathcal{D}$	$\mathfrak{H}_{\mathfrak{k}}$
0.7257	0.1826	$\mathfrak{H}_1 = 0.4541$
0.5457	0.1606	$\mathfrak{H}_2 = 0.3531$
0.6357	0.1606	$\mathfrak{H}_3 = 0.3981$
0.6933	0.1800	$\mathfrak{H}_4 = 0.4366$
0.7142	0.1880	$\mathfrak{H}_5 = 0.4511$

Step 4:Final decision

Arrange $\mathfrak{S}_{\mathfrak{F}}$ for the attributes $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3, \mathfrak{H}_4, \mathfrak{H}_5$ and alternatives $\mathfrak{E}_1, \mathfrak{E}_2, \mathfrak{E}_3, \mathfrak{E}_4$ in progression is result to $\mathfrak{H}_2 < \mathfrak{H}_3 < \mathfrak{H}_4 < \mathfrak{H}_5 < \mathfrak{H}_1$ and $\mathfrak{E}_3 < \mathfrak{E}_1 < \mathfrak{E}_4 < \mathfrak{E}_2$.

6. Conclusion

This manuscript introduces the innovative concept of Neutrosophic Over Supra Exterior Modal Topological Structure and outlines its foundational properties. Where model closure ($\circledast$) and model interior ($\odot$) operations in the Neutrosophic Over Supra Exterior Modal Topological Structures framework provide tools for managing and analyzing sets with complex membership degrees and modalities. They ensure comprehensive coverage of set elements (closure) and extraction of core characteristics (interior), respectively, thereby enhancing the framework's capability to handle nuanced uncertainty and ambiguity in practical applications. It concludes by definitively classifying several diseases: tuberculosis as bacterial, malaria as parasitic, swine flu as viral, and ringworm as fungal. Moreover, $\mathcal{N}^{\mathbf{o}}$ is shown to be indispensable across various domains such as information technology, decision-making support systems, electronic databases, disease detection, and multi-criteria cognitive problem-solving. Its versatility and applicability underscore its vital role in enhancing understanding and decision-making in complex systems and uncertain environments.

7. Future Research

For future research, expanding the Neutrosophic Over Supra Exterior Modal Topological Structure to incorporate additional modalities and refine existing operations could further enhance its robustness and applicability. Empirical validation across diverse domains and scenarios would provide concrete evidence of its effectiveness and reliability. Furthermore, developing computational tools and algorithms tailored to implement this framework in practical settings could facilitate its adoption and integration into decision support systems and real-time applications. Collaborative efforts across disciplines could explore new applications and synergies, unlocking novel insights and capabilities offered by neutrosophic theory and modal topological structures.

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