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Evaluating the nuclear power plant safety system under neutrosophic environment

Debapriya Mondal[1], Totan Garai[2], Gopal Chandra Roy [3] and Shariful Alam[1*]

[1] Department of Mathematics, IIEST, Shibpur, Howrah-711103, India.

[2]Department of Mathematics, Syamsundar College, Purba Bardhaman, WB, India.

[3]Department of Mining Engineering, IIEST, Shibpur, Howrah-711103, India.

[*]Correspondence: salam@math.iiests.ac.in; Tel.: +919874452995

Abstract. The safety system of nuclear power plants is very crucial for protecting the environment and the living world. In this article, we have addressed the nuclear power plant safety system in a neutrosophic environment. Here, we have mathematically presented a nuclear power plant safety system in terms of neutrosophic differential equations. We have thoroughly examined and discussed both analytically and numerically the solution of the nuclear power plant safety model considering the underlying model parameter as single-valued triangular neutrosophic number. We have focused to find out probability of the nuclear power plant's safety system working normally and shutdown in safe mode when the model parameters are imprecise in nature. It has been observed that the normal operation and shutdown probability of nuclear power plant safety systems under uncertain conditions are predicted with robust accuracy.

Keywords: Neutrosophic set; Single valued neutrosophic set; Triangular single valued neutrosophic number; Neutrosophic differential equation; Nuclear power plant safety model

1. Introduction

Safety measures in Nuclear Power Plants (NPP) have become essential for reducing risks to human health and the critical environmental hazards. Good precautionary safety strategies can help safeguard all the workers involved in NPPs from the incidence of catastrophic accidents and significantly protect the environment from the risks of unintentional radiation exposure. Recently, several research studies have been carried out on the safety issues of NPPs. Jyotish et. al. [1] used reachability graphs to analyse the failure mode of different components of the safety system involved in the NPPs and evaluated the system through mathematical modelling. Hyvärinen et. al. [2] argued that comprehensive understanding of the safety of nuclear reactors is very much essential and incorporated defense-in-depth knowledge to improve the

NPP safety systems. Bao et. al. [3] has suggested an improved framework to assess the risk of digital control systems of a nuclear power plant by integrating three typical risk stages, namely 'qualitative hazard analysis', 'quantitative reliability' and 'consequence analyses'. Zhang et. al. [4] recommended multi-criterion and multi-context evaluations to enhance the safety of nuclear power plants. So and Kim [5] shows that the 'Chernoff-bound' can be used to estimate the higher level of the failure probability in a NPP safety system incorporating the concept of passive safety system. Wu et. al. [6] developed safety archetype identification and used behaviour modelling to get ride off human dependability in NPP operations. Marton et. al. [7] pointed out that the ageing impact on the reliability of system equipment has to be appraised to guarantee the safety standards of an NPP. They presented a risk impact evaluation model incorporating several operation safety factors like ageing & obsolescence of system equipment, self-assessment of maintenance plans, surveillance requirements, wear-out phase of the equipment lifetime, etc., where the model parameters are estimated through long-term historical data. Ahn et. al. [8] have designed and improved operator support system used in NPPs through artificial intelligence algorithm that validates operational system activity and alert human errors to ward off the adverse effect of the integrity of power plant. Ayodeji et. al. [9] proposed deep learning for assessing the dependability, explainability, and research potential of nuclear power reactors. Singh and Singh [10] addressed on the dependability measurement of NPP control and instrumentation systems. Abdel-Basset et. al. [11] described an effective risk assessment and mitigation strategy for solar power plants based on real data. Abdel-Basset et. al. [12] investigated the assessment of sustainable flue gas treatment systems for the iron and steel industry using a novel multi-stage technique based on spherical fuzzy MCDM. Abdel-Basset et. al. [13] presented a conceptual framework for assessing the optimal sustainability of electric car charging stations for green energy in smart cities. Mondal et. al. [14] examined the design and assessment of a picture fuzzy mining safety system using picture fuzzy differential equations. Although a substantial number of research works have been carried out on the various aspects of safety issues of nuclear power plats, but the majority these studies was performed in the crisp environment.

The safety system of an NPP is a complex phenomenon [15] and many factors like human operations error, human-machine interaction, assessment of prospective risk, and evaluation of reliability & engineering-resilience are involved, which are inherently imprecise in nature. Thus, it is very much realistic to analyze the NPP model system in an imprecise environment. To handle various types of uncertainty and indistinctness, the theories of impreciseness have been geared up in which the structure of neutrosophic numbers is the latest addition. In the year 1998 Smarandache [16] first demonstrate the impression of the neutrosophic set as a compatible mixture of three independent membership functions: truthness, falsity and doubtfulness. Indeed, neutrosophic set was a genuine and sensible extension of intuitionistic set introduced by Atanassov [17]. Purba et. al. [18] discussed about the fuzzy probability-based event tree analysis, which is used in the NPP probabilistic safety assessment. Moreover, Purba [19] incorporated fuzzy based probability to assess reliability of NPP safety model system. Sleem and Elhenawy [20] investigated the energy efficiency and material cost reductions realised by the evolution of solar panels used in photovoltaic systems using the neutrosophic model. Sallam and Mohamed [21] highlighted the neutrosophic MCDM approach for analysing onshore wind for power generation and ecological sustainability. Mondal et. al. [22] addressed safety measures of oil and gas sector by using an FDE system. On the other hand, Melliani and Chadli [23] introduced intuitionistic fuzzy differential equations (IFDE) by use of the derivatives of degrees of membership and non-membership functions. Amma et. al. [24] developed diagonally implicit block backward differential formulae to provide a numerical solution to the IFDE problem. Man et. al. [25] provided a clear procedure for resolving a fuzzy heat equation that is based on IFDE. Melliani et. al. [26] utilized the homotrophy analysis approach to provide an approximate solution to the IFDE using the linear differential operator. But all these work did not consider refusal terms which is common in real life problems. Smarandache use the word “indeterminacy” to capture the refusal terms in the concept neutrosophic numbers and subsequently, in order to simulate indeterminacy, NDE was created. Sumathi and Priya [27] presented a novel viewpoint on the NDE. Sumathi and Sweety [28] discussed a novel method for solving differential equations using the trapezoidal neutrosophic number. Son et. al. [29] discussed an equation with a linear quadratic regulator that is controlled by granular neutrosophic fractional differential equations. Alamin et. al. [30] discussed solution and interpretation of neutrosophic homogeneous difference equations. Parikh and Shani [31] researched on the Sumudu Transform in order to solve the NS initial conditions of the second ODE. Mondal et. al. [32] developed a model outlining the use of the NDE in mining safety. Parikh et. al. [33] investigated using an analytical and numerical approach in an NS context to solve a first-order initial value issue. Acharya et. al. [34] investigated an NDE method for simulating glucose transport in the circulation using neutrosophic sets. Garai et. al. [35] discussed softmax function based neutrosophic aggregation operators and application in multi-attribute decision making problem.

In reality, in many situations, it was observed that the collected data are insufficient and transmit some misinformation. In such situations, incorporating neutrosophic numbers offers better results. Although some studies have been done on NPP safety model system in fuzzy environments, but have not noticed any work in this regard in the neutrosophic environments. In this article, we have mathematically described the nuclear power plant safety model system(NPPSMS) [36], in terms of neutrosophic differential equations (NDE). We have thoroughly examined and discussed both analytically and numerically the solution of the model system considering the underlying model parameter as single-valued triangular neutrosophic number and focused to find out probability shutting down the NPP in safe mode.

1.1. *Motivation*

The safety system of NPP is very much essential and crucial as it has a high risk impact both on the biotic and abiotic elements of environment. Many uncertain information & factors are involved in the NPP system. Naturally, several ambiguities and uncertainties are associated with the underlying parameters involved in the mathematical model of NPP safety system. Now with the introduction of the neutrosophic set by Smarandache [16], we have noticed that a neutrosophic data set can handle imprecise data more efficiently where doubtfulness is inherently and independently embedded into the data set. Moreover, as per our knowledge, no study of the NPP model system has been carried out in the NS environment. Thus, we mathematically formulate the NPP system in terms of neutrosophic differential equations and analyze it both analytically and numerically.

1.2. *Novelty*

Despite the fact that some patterns are followed, fresh attention and new efforts have been made by way of ourselves, which is stated as below:

• NPP safety issues has been first time addressed in the neutrosophic set environment. Here, we have mathematically presented NPP safety system in terms of neutrosophic differential equations.

• Both analytical and numerical solutions of the NPP safety model system have been obtained in crisp as well as neutrosophic environment using classical method, zadeh's extension principle, and generalised hukuhara differentiability(GH_{ii}) and results are compared.

1.3. *Structure of this paper*

The remaining portions of this article are ornamented as follows: Section-2 gives some preliminaries concept and definition. Section-3 contains first order linear homogeneous neutrosophic differential equation. Section-4 contains description of NPPSMS. Section-5 contains analytical solution of the NPPSMS model system. Section-6 contains numerical solution of the NPPSMS and consequently, conclusions are discussed in Section-7.

Table 1. List of abbreviation with description

Abbreviation	Description
CS	crisp set
FS	fuzzy set
IFS	intuitonistic fuzzy set
CSNs	crisp number
NS	neutrosophic set
SVNS	single valued neutrosophic set
TSVNNs	triangular single valued neutrosophic number
ODE	Ordinary differential equation
FDE	fuzzy differential equation
IFDE	intutionistic fuzzy differential equation
NDE	neutrosophic differential equation
IC	initial condition
NPPSMS	nuclear power plant safety model system

1.4. *List of abbreviation*

2. **Preliminaries**

2.1. ***Definition of NS*** *[32]*

Let $\mathfrak{V}$ represent an universal set. A NS $\tilde{\mathbb{A}}^{NS}$ of $\mathfrak{V}$ be defined by $\tilde{\mathbb{A}}^{NS}=\langle(\hat{\mathfrak{u}};\ \mu_T(\hat{\mathfrak{u}}),\ \nu_I(\hat{\mathfrak{u}}),\ \sigma_F(\hat{\mathfrak{u}})) :\ \hat{\mathfrak{u}} \in \mathfrak{V}\rangle$ where $\mu_T(\hat{\mathfrak{u}}),\ \nu_I(\hat{\mathfrak{u}}),\ \sigma_F(\hat{\mathfrak{u}})$ be outlined as the truth membership, indeterminacy membership, falsity membership grade of $\hat{\mathfrak{u}}$ in $\tilde{\mathbb{A}}^{NS}$ which are real standard or non-standard subsets of $^{-}]0,1[^{+}$ & $\mu_T(\hat{\mathfrak{u}}) + \nu_I(\hat{\mathfrak{u}}) + \sigma_F(\hat{\mathfrak{u}}) \leqslant 3^{+}$.

2.2. ***Definition of SVNS*** *[32]*

Let $\mathfrak{V}$ be a Universal set. A SVNS $\tilde{\mathbb{A}}^{NS}$ of $\mathfrak{V}$ be defined by $\tilde{\mathbb{A}}^{NS}=\langle(\hat{\mathfrak{u}};\ \mu_T(\hat{\mathfrak{u}}),\ \nu_I(\hat{\mathfrak{u}}),\ \sigma_F(\hat{\mathfrak{u}})) :\ \hat{\mathfrak{u}} \in \mathfrak{V}\rangle$ where $\mu_T(\hat{\mathfrak{u}}),\ \nu_I(\hat{\mathfrak{u}}),\ \sigma_F(\hat{\mathfrak{u}})$ be outlined as the truth membership, indeterminacy membership, falsity membership grade of $\hat{\mathfrak{u}}$ in $\tilde{\mathbb{A}}^{NS}$ which are subset of $[0,1]$ & $\mu_T(\hat{\mathfrak{u}}) + \nu_I(\hat{\mathfrak{u}}) + \sigma_F(\hat{\mathfrak{u}}) \leqslant 3$.

2.3. ***Definition of TSVNNs*** *[32]*

A TSVNNs is denoted by $\tilde{\mathbb{A}}^{NS}=\langle\hat{\mathfrak{a}}_1,\ \hat{\mathfrak{a}}_2,\ \hat{\mathfrak{a}}_3;\ f_\mu,\ f_\nu,\ f_\sigma\rangle$ whose truth, indeterminacy and falsity membership functions are defined by

$$\mu_T(\hat{\mathfrak{u}})=\begin{cases}(\frac{\hat{\mathfrak{u}}-\hat{\mathfrak{a}}_1}{\hat{\mathfrak{a}}_2-\hat{\mathfrak{a}}_1})f_\mu & \text{when } \hat{\mathfrak{a}}_1 \leqslant \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_2\\ f_\mu & \text{when } \hat{\mathfrak{u}} = \hat{\mathfrak{a}}_2\\ (\frac{\hat{\mathfrak{a}}_3-\hat{\mathfrak{u}}}{\hat{\mathfrak{a}}_3-\hat{\mathfrak{a}}_2})f_\mu & \text{when } \hat{\mathfrak{a}}_2 \leqslant \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_3\\ 0 & \text{when } \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_1 \text{ or } \hat{\mathfrak{u}} \geqslant \hat{\mathfrak{a}}_3\end{cases}$$

$$\nu_I(\hat{\mathfrak{u}}) =\begin{cases}\frac{\hat{\mathfrak{a}}_2-\hat{\mathfrak{u}})+(\hat{\mathfrak{u}}-\hat{\mathfrak{a}}_1)f_\nu}{\hat{\mathfrak{a}}_2-\hat{\mathfrak{a}}_1} & \text{when } \hat{\mathfrak{a}}_1 \leqslant \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_2\\ f_\nu & \text{when } \hat{\mathfrak{u}} = \hat{\mathfrak{a}}_2\\ \frac{(\hat{\mathfrak{u}}-\hat{\mathfrak{a}}_2)+(\hat{\mathfrak{a}}_3-\hat{\mathfrak{u}})f_\nu}{\hat{\mathfrak{a}}_3-\hat{\mathfrak{a}}_2} & \text{when } \hat{\mathfrak{a}}_2 \leqslant \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_3\\ 1 & \text{when } \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_1 \text{ or } \hat{\mathfrak{u}} \geqslant \hat{\mathfrak{a}}_3\end{cases}$$

$$\sigma_F(\hat{\mathfrak{u}}) =\begin{cases}\frac{(\hat{\mathfrak{a}}_2-\hat{\mathfrak{u}})+(\hat{\mathfrak{u}}-\hat{\mathfrak{a}}_1)f_\sigma}{\hat{\mathfrak{a}}_2-\hat{\mathfrak{a}}_1} & \text{when } \hat{\mathfrak{a}}_1 \leqslant \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_2\\ f_\sigma & \text{when } \hat{\mathfrak{u}} = \hat{\mathfrak{a}}_2\\ \frac{(\hat{\mathfrak{u}}-\hat{\mathfrak{a}}_2)+(\hat{\mathfrak{a}}_3-\hat{\mathfrak{u}})f_\sigma}{\hat{\mathfrak{a}}_3-\hat{\mathfrak{a}}_2} & \text{when } \hat{\mathfrak{a}}_2 \leqslant \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_3\\ 1 & \text{when } \hat{\mathfrak{u}} \leqslant \hat{\mathfrak{a}}_1 \text{ or } \hat{\mathfrak{u}} \geqslant \hat{\mathfrak{a}}_3\end{cases}$$

where $\mu_T(\hat{\mathfrak{u}}) + \nu_I(\hat{\mathfrak{u}}) + \sigma_F(\hat{\mathfrak{u}}) \leqslant 3$ & $w_\mu \in (0, 1]$, $w_\nu,\ w_\sigma \in [0, 1)$.

2.4. ***Cut Set*** *[32]*

Let, $\tilde{\mathbb{A}}^{NS}$ be any SVNS, then $(r,\ \beta,\ \gamma)$-cut of SVNS is denoted by $\mathbb{A}(r,\ \beta,\ \gamma)$ and it is defined by $\mathbb{A}(r,\ \beta,\ \gamma)$=$\langle \hat{\mathfrak{u}} \in \mathfrak{V} :\ \mu_T(\mathfrak{u}) \geqslant r,\ \nu_I(\mathfrak{u}) \leqslant \beta,\ \sigma_F(\mathfrak{u}) \leqslant \gamma;\ 0 < r \leqslant 1, 0 \leqslant \beta < 1,\ 0 \leqslant \gamma < 1\rangle$.
Now the basic arithmetic operation on TSVNNs can be written as:
Consider two TSVNNs, $\tilde{\mathbb{A}}^{NS}$=$\langle \hat{\mathfrak{a}}_1,\ \hat{\mathfrak{a}}_2,\ \hat{\mathfrak{a}}_3;\ f_\mu,\ f_\nu,\ f_\sigma\rangle$; $\tilde{\mathbb{B}}^{NS}$=$\langle \hat{\mathfrak{b}}_1,\ \hat{\mathfrak{b}}_2,\ \hat{\mathfrak{b}}_3;\ g_\mu,\ g_\nu,\ g_\sigma\rangle$, the following operation are:

- **Addition:**

$\tilde{\mathbb{A}}^{NS}$+$\tilde{\mathbb{B}}^{NS}$ $= \langle[(\hat{\mathfrak{a}}_1 + \hat{\mathfrak{b}}_1,\ \hat{\mathfrak{a}}_2 + \hat{\mathfrak{b}}_2,\ \hat{\mathfrak{a}}_3 + \hat{\mathfrak{b}}_3);\ f_\mu \wedge g_\mu,\ f_\nu \vee g_\nu,\ f_\sigma \vee g_\sigma]\rangle$

- **Substraction:**

$\tilde{\mathbb{A}}^{NS}$-$\tilde{\mathbb{B}}^{NS}$ $= \langle[(\hat{\mathfrak{a}}_1 - \hat{\mathfrak{b}}_3,\ \hat{\mathfrak{a}}_2 - \hat{\mathfrak{b}}_2,\ \hat{\mathfrak{a}}_3 - \hat{\mathfrak{b}}_1);\ f_\mu \wedge g_\mu,\ f_\nu \vee g_\nu,\ f_\sigma \vee g_\sigma]\rangle$

- **Multiplication:**

$\tilde{\mathbb{A}}^{NS}$.$\tilde{\mathbb{B}}^{NS}$ $= \langle[(\hat{\mathfrak{a}}_1\hat{\mathfrak{b}}_1,\ \hat{\mathfrak{a}}_2\hat{\mathfrak{b}}_2,\ \hat{\mathfrak{a}}_3\hat{\mathfrak{b}}_3);\ f_\mu \wedge g_\mu,\ f_\nu \vee g_\nu,\ f_\sigma \vee g_\sigma]\rangle$

- **Division:**

$$\frac{\tilde{\mathbb{A}}^{NS}}{\tilde{\mathbb{B}}^{NS}} = \Big\langle\Big[\Big(\frac{\hat{\mathfrak{a}}_1}{\hat{\mathfrak{b}}_3},\ \frac{\hat{\mathfrak{a}}_2}{\hat{\mathfrak{b}}_2},\ \frac{\hat{\mathfrak{a}}_3}{\hat{\mathfrak{b}}_1}\Big);\ f_\mu \wedge g_\mu,\ f_\nu \vee g_\nu,\ f_\sigma \vee g_\sigma\Big]\Big\rangle$$

Where $\wedge = Min, \vee = Max$

3. **First Order linear homogeneous neutrosophic differential equation: [32]**

Let us consider an ODE

$$\frac{d\mathcal{Y}}{d\mathfrak{t}} = \mathcal{L}\mathcal{Y}, \mathfrak{t} \in [0, \infty) \tag{1}$$

with IC $\mathcal{Y}(\mathfrak{t}_0)$=$\mathcal{Y}_0$. Now the above ODE will be treated as NDE when at-least one of the parameter $\mathcal{L}$ or initial condition $\mathcal{Y}_0$ are considered as SVNNs.

Now, the classical technique [37], Zadeh's extension principle [38], generalised Hukuhara differentiability(GH_{ii}) [39], and other approaches can each be used to solve the NDE mentioned above.

3.1. Definition of strong solution

Let $\tilde{\mathcal{Y}}^{NS}(\mathfrak{t})$ be the classical solution (as described in [37]) and its cut be $\mathcal{Y}(\mathfrak{t},\ r,\ \beta,\ \gamma)$= $\langle[\mathcal{Y}_1(\mathfrak{t},\ r), \mathcal{Y}_2(\mathfrak{t},\ r)],\ [\mathcal{Y}'_1(\mathfrak{t},\ \beta), \mathcal{Y}'_2(\mathfrak{t},\ \beta)],\ [\mathcal{Y}''_1(\mathfrak{t},\ \gamma),\ \mathcal{Y}''_2(\mathfrak{t},\ \gamma)]\rangle$.

The solution is strong if

(i) $\dfrac{d\mathcal{Y}_1(\mathfrak{t},\ r)}{dr} > 0,\ \dfrac{d\mathcal{Y}_2(\mathfrak{t},\ r)}{dr} < 0\ \forall\ r \in (0,\ 1],\ \mathcal{Y}_1(\mathfrak{t},\ 1) \leqslant \mathcal{Y}_2(\mathfrak{t},\ 1)$

(ii) $\dfrac{d\mathcal{Y}'_1(\mathfrak{t},\ \beta)}{d\beta} < 0,\ \dfrac{d\mathcal{Y}'_2(\mathfrak{t},\ \beta)}{d\beta} > 0\ \forall\ \beta \in [0, 1),\ \mathcal{Y}'_1(\mathfrak{t}, 0) \leqslant \mathcal{Y}'_2(\mathfrak{t},\ 0)$

(iii) $\dfrac{d\mathcal{Y}''_1(\mathfrak{t},\ \gamma)}{d\gamma} < 0,\ \dfrac{d\mathcal{Y}''_2(\mathfrak{t},\ \gamma)}{d\gamma} > 0\ \forall\ \gamma \in [0, 1),\ \mathcal{Y}''_1(\mathfrak{t},\ 0) \leqslant \mathcal{Y}''_2(\mathfrak{t},\ 0)$

A weak solution would result from anything less.

4. Description of NPPSMS

The design, production, operation, and maintenance of several types of nuclear power power-producing cost billions of dollars per year. Around 436 commercial nuclear reactors were operational in 2011 and produced 16% of the world's electricity. NPPSMS frequently have issues with dependability, safety, human factors, and human error. For instance, human error was responsible for almost 27% of commercial NPPSMS breakdowns in the US between 1990 and 1994.

Here, we proposed to study on NPPSMS. [36] in terms of a NDE. Following are our key areas of focus in this paper:

- Establish a model for NPPSMS.
- Determine the model's solution in a CS environment.
- Determine the model's solution in the NS environment.

4.1. *Mathematical formulation of NPPSMS*

4.2. *Acceptation*

(I) Every event exists independently of the others.

(II) In the finite time period $\delta\mathfrak{t}$, the probability of changing from one system state to another is given by the expression $\Psi\delta\mathfrak{t}$, where Ψ is the rate of change between system states.

(III) A single transition from one system state to another taking place more than once in the finite time interval $\delta\mathfrak{t}$ is extremely unlikely, that is $(\Psi\delta\mathfrak{t})(\Psi\delta\mathfrak{t}) \rightarrow 0$.

4.3. *Notation*

$\mathfrak{t}$	time interval.
$\delta\mathfrak{t}$	finite time interval.
λ_s	is the continuous failure rate of the NPPSMS.
$\mathcal{P}_0(\mathfrak{t})$	is the probability that the NPPSMS is working normally.
$\mathcal{P}_1(\mathfrak{t})$	is the probability that the NPPSMS will collapse.
$\mathcal{P}_0(\mathfrak{t}+\delta\mathfrak{t})$	is the probability that the NPPSMS will be in the off-state at time $\mathfrak{t}+\delta\mathfrak{t}$.
$\mathcal{P}_1(\mathfrak{t}+\delta\mathfrak{t})$	is the probability that the NPPSMS will collapse at any given time $\mathfrak{t}+\delta\mathfrak{t}$.
$j=0$	represents the normal position.
$j=1$	is the collapse state.
$\mathcal{P}_j(\mathfrak{t})$	denotes the probability of being in position j at time interval $\mathfrak{t}$.
$\lambda_s\delta\mathfrak{t}$	is the probability that NPPSMS will collapse within the time period $\delta\mathfrak{t}$.
(1 - $\lambda_s\delta\mathfrak{t}$)	is the probability that there won't be a collapse in the time frame $\delta\mathfrak{t}$.

Assume that a system utilised by the NPPSMS may either be in a functioning condition or a collapsing state. The system's constant failure rate is λ_s. The diagram of the system state space is shown in Fig-1. The statuses of the NPPSMS are indicated by the numbers 0 and 1 in the boxes. State 0 denotes the NPPSMS in normal operation, whereas State 1 denotes NPPSMS shutdown. Let λ_s indicate the constant failure rate from state 0 to state 1. The meaning of $\mathfrak{t}$, $\delta\mathfrak{t}$, $\mathcal{P}_0(\mathfrak{t}+\delta\mathfrak{t})$, $\mathcal{P}_1(\mathfrak{t}+\delta\mathfrak{t})$, $\mathcal{P}_0(\mathfrak{t})$, $\mathcal{P}_1(\mathfrak{t})$, $\lambda_s\delta\mathfrak{t}$ and (1 - $\lambda_s\delta\mathfrak{t}$) already discussed in section-4.3. Suppose that the NPPSMS is operated under the conditions of normal operation, independent occurrence of system failure, and constant rate of system failure.

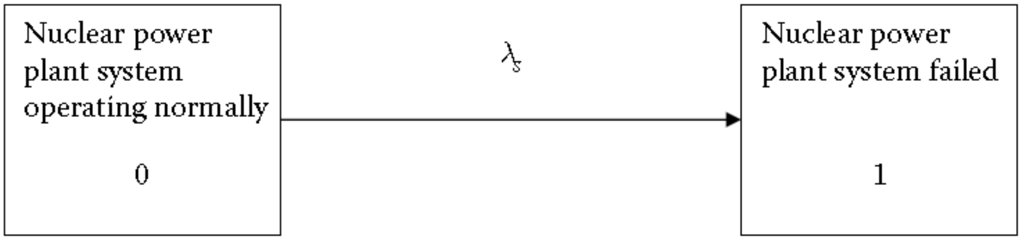


FIGURE 1. Nuclear power plant system state space diagram

Now using the notation of section-4.2, it is clear that at the initial time, $\mathcal{P}_0(0)$=1 and $\mathcal{P}_1(0)$=0. Now, the first two equations may be constructed using the fundamentals of probability theory:

$$\mathcal{P}_0(t+\delta t) = \mathcal{P}_0(t)(1-\lambda_s\delta t) \tag{2}$$

$$\mathcal{P}_1(t+\delta t) = \mathcal{P}_0(t)(\lambda_s\delta t) + \mathcal{P}_1(t)(1-o\delta t) \tag{3}$$

Now, from equation (2) we get

$$\mathcal{P}_0(t+\delta t) - \mathcal{P}_0(t) = -\lambda_s\mathcal{P}_0(t)\delta t \tag{4}$$

Now using definition of differentiability from equation (4), we get

$$\lim_{\delta t\to 0}\frac{\mathcal{P}_0(t+\delta t)-\mathcal{P}_0(t)}{\delta t} = -\lambda_s\mathcal{P}_0(t)$$

$$\therefore \frac{d\mathcal{P}_0(t)}{dt} = -\lambda_s\mathcal{P}_0(t) \tag{5}$$

Similarly from equation (3), we get

$$\frac{d\mathcal{P}_1(t)}{dt} = \lambda_s\mathcal{P}_0(t) \tag{6}$$

So, the required NPPSMS model system as follows

$$\therefore \frac{d\mathcal{P}_0(t)}{dt} = -\lambda_s\mathcal{P}_0(t) \tag{7}$$

$$\frac{d\mathcal{P}_1(t)}{dt} = \lambda_s\mathcal{P}_0(t) \tag{8}$$

with IC: $\mathcal{P}_0(0) = 1$ & $\mathcal{P}_1(0) = 0$.

5. **Analytical solution of the NPPSMS model system**

There are two approaches to solving equations (7) and (8): in a CS environment and in an imprecise one. To solve imprecise environments, NS environments are employed. The details of everything are provided below.

Crisp Solution of the NPPSMS model system: Taking the input value λ_s as crisp number, we get following solutions:

$$P_0(\mathtt{t}) = e^{-\lambda_s \mathtt{t}} \tag{9}$$

$$P_1(\mathtt{t}) = 1 - e^{-\lambda_s \mathtt{t}} \tag{10}$$

NS Solution of the NPPSMS model system:

Taking the input value $\tilde{\lambda_s}^{NS}$ as triangular neutrosophic number, let, $\mathcal{P}_0\tilde{(\mathtt{t})}^{NS}$, $\mathcal{P}_1\tilde{(\mathtt{t})}^{NS}$ be the NS solution of the model system whose $r,\ \beta,\ \gamma$-cut values are as follows:

$\mathcal{P}_0(\mathtt{t},\ r,\ \beta,\ \gamma)$=$\langle[\mathcal{P}_{01}(\mathtt{t},\ r), \mathcal{P}_{02}(\mathtt{t},\ r)],\ [\mathcal{P}'_{01}(\mathtt{t},\ \beta),\ \mathcal{P}'_{02}(\mathtt{t},\ \beta)],\ [\mathcal{P}''_{01}(\mathtt{t},\ \gamma),\ \mathcal{P}''_{02}(\mathtt{t},\ \gamma)]\rangle$

$\mathcal{P}_1(\mathtt{t},\ r,\ \beta,\ \gamma)$=$\langle[\mathcal{P}_{11}(\mathtt{t},\ r),\ \mathcal{P}_{12}(\mathtt{t},\ r)],\ [\mathcal{P}'_{11}(\mathtt{t},\ \beta),\ \mathcal{P}'_{12}(\mathtt{t},\ \beta)],\ [\mathcal{P}''_{11}(\mathtt{t},\ \gamma),\ \mathcal{P}''_{12}(\mathtt{t},\ \gamma)]\rangle$

Consequently, following significant computation, we may write,

$$\mathcal{P}_{01}(\mathtt{t},\ r) = \frac{e^{\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ 1 - \sqrt{\frac{\mathbb{A}_2(r)}{\mathbb{A}_1(r)}}\right) + \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ 1 + \sqrt{\frac{\mathbb{A}_2(r)}{\mathbb{A}_1(r)}}\right) \tag{11}$$

$$\mathcal{P}_{02}(\mathtt{t},\ r) = \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}_1(r)}{\mathbb{A}_2(r)}} + 1\right) + \frac{e^{\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ 1 - \sqrt{\frac{\mathbb{A}_1(r)}{\mathbb{A}_2(r)}}\right) \tag{12}$$

$$\mathcal{P}'_{01}(\mathtt{t}, \beta) = \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}'_2(\beta)}{\mathbb{A}'_1(\beta)}} + 1\right) - \frac{e^{\mathtt{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}'_2(\beta)}{\mathbb{A}'_1(\beta)}} - 1\right) \tag{13}$$

$$\mathcal{P}'_{02}(\mathtt{t},\ \beta) = \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}'_1(\beta)}{\mathbb{A}'_2(\beta)}} + 1\right) + \frac{e^{\mathtt{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left.\ 1 - \sqrt{\frac{\mathbb{A}'_1(\beta)}{\mathbb{A}'_2(\beta)}}\right) \tag{14}$$

$$\mathcal{P}''_{01}(\mathtt{t},\ \gamma) = \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}''_2(\gamma)}{\mathbb{A}''_1(\gamma)}} + 1\right) - \frac{e^{\mathtt{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}''_2(\gamma)}{\mathbb{A}''_1(\gamma)}} - 1\right) \tag{15}$$

$$\mathcal{P}''_{02}(\mathtt{t},\ \gamma) = \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}''_1(\gamma)}{\mathbb{A}''_2(\gamma)}} + 1\right) + \frac{e^{\mathtt{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left.\ 1 - \sqrt{\frac{\mathbb{A}''_1(\gamma)}{\mathbb{A}''_2(\gamma)}}\right) \tag{16}$$

$$\mathcal{P}_{11}(\mathtt{t},\ r) = 1 - \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}_1(r)}{\mathbb{A}_2(r)}} + 1\right) - \frac{e^{\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ 1 - \sqrt{\frac{\mathbb{A}_1(r)}{\mathbb{A}_2(r)}}\right) \tag{17}$$

$$\mathcal{P}_{12}(\mathtt{t},\ r) = 1 - \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}_2(r)}{\mathbb{A}_1(r)}} + 1\right) + \frac{e^{\mathtt{t}\sqrt{\mathbb{A}_1(r)\mathbb{A}_2(r)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}_2(r)}{\mathbb{A}_1(r)}} - 1\right) \tag{18}$$

$$\mathcal{P}'_{11}(\mathtt{t},\ \beta) = 1 - \frac{e^{-\mathtt{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left.\ \sqrt{\frac{\mathbb{A}'_1(\beta)}{\mathbb{A}'_2(\beta)}} + 1\right) - \frac{e^{\mathtt{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left.\ 1 - \sqrt{\frac{\mathbb{A}'_1(\beta)}{\mathbb{A}'_2(\beta)}}\right) \tag{19}$$

$$\mathcal{P}'_{12}(\mathfrak{t},\ \beta) = 1 - \frac{e^{-\mathfrak{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left(\sqrt{\frac{\mathbb{A}'_2(\beta)}{\mathbb{A}'_1(\beta)}} + 1\right) + \frac{e^{\mathfrak{t}\sqrt{\mathbb{A}'_1(\beta)\mathbb{A}'_2(\beta)}}}{2}\left(\sqrt{\frac{\mathbb{A}'_2(\beta)}{\mathbb{A}'_1(\beta)}} - 1\right) \tag{20}$$

$$\mathcal{P}''_{11}(\mathfrak{t},\ \gamma) = 1 - \frac{e^{-\mathfrak{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left(\sqrt{\frac{\mathbb{A}''_1(\gamma)}{\mathbb{A}''_2(\gamma)}} + 1\right) - \frac{e^{\mathfrak{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left(1 - \sqrt{\frac{\mathbb{A}''_1(\gamma)}{\mathbb{A}''_2(\gamma)}}\right) \tag{21}$$

$$\mathcal{P}''_{12}(\mathfrak{t},\ \gamma) = 1 - \frac{e^{-\mathfrak{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left(\sqrt{\frac{\mathbb{A}''_2(\gamma)}{\mathbb{A}''_1(\gamma)}} + 1\right) + \frac{e^{\mathfrak{t}\sqrt{\mathbb{A}''_1(\gamma)\mathbb{A}''_2(\gamma)}}}{2}\left(\sqrt{\frac{\mathbb{A}''_2(\gamma)}{\mathbb{A}''_1(\gamma)}} - 1\right) \tag{22}$$

Where $\langle[\mathbb{A}_1(r),\ \mathbb{A}_2(r)],\ [\mathbb{A}'_1(\beta),\ \mathbb{A}'_2(\beta)],\ [\mathbb{A}''_1(\gamma),\ \mathbb{A}''_2(\gamma)]\rangle$ is the cut set of $\tilde{\lambda_S}^{NS}$. Solution is strong or weak if it satisfies the condition of NDE.

6. Numerical solution of the NPPSMS

Assume that 0.0002 failures occur per hour on a system that is utilised in an NPPSMS. Determine how likely it is that the NPPSMS will function correctly and fail throughout an 800-hour mission in (i) the CS environment and (ii) the NS environment.

6.1. *Crisp Solution*

When we take respective input values λ_s=0.0002; $\mathfrak{t}$=800-h, we get the following output results $\mathcal{P}_0(800)$=0.852144 and $\mathcal{P}_1(800)$=0.147856.

6.2. *NS Solution*

We numerically solve the model system in the NS environment. Here, we take, $\tilde{\lambda_s}^{NS}=$ $\langle 0.0001,\ 0.0002,\ 0.0003;\ 0.9,\ 0.6,\ 0.5\rangle$, and $\mathfrak{t}$=800-h. We get the numerical values of $\langle r, \beta, \gamma\rangle$-cut of probability of NPPSMS working normally $[\ \mathcal{P}_0(\mathfrak{t},\ r,\ \beta,\ \gamma)]$ and probability of NPPSMS failed $[\mathcal{P}_1(\mathfrak{t},\ r,\ \beta,\ \gamma)]$ for $r \in (0,\ 1]$ and $\beta,\ \gamma \in [0,\ 1)$ that are shown in Table-2. From Table-2 it is observed that when r increases, the values of $\mathcal{P}_{01}(\mathfrak{t},\ r)$ and $\mathcal{P}_{11}(\mathfrak{t},\ r)$ are increasing but the values of $\mathcal{P}_{02}(\mathfrak{t},\ r)$ and $\mathcal{P}_{12}(\mathfrak{t},\ r)$ are decreasing. If both the values of β and γ increase then the values of $\mathcal{P}'_{02}(\mathfrak{t},\ \beta)$, $\mathcal{P}'_{12}(\mathfrak{t},\ \beta)$, $\mathcal{P}''_{02}(\mathfrak{t},\ \gamma)$, $\mathcal{P}''_{12}(\mathfrak{t},\ \gamma)$ increase whereas values of $\mathcal{P}'_{01}(\mathfrak{t},\ \beta)$ and $\mathcal{P}'_{11}(\mathfrak{t},\ \beta)$; $\mathcal{P}''_{01}(\mathfrak{t},\ \gamma)$ and $\mathcal{P}''_{11}(\mathfrak{t},\ \gamma)$ decrease. We developed the solution's graph, which is depicted in Figure-2. Figure-2(a) present the graph of truth, indeterminacy and falsity membership function of the nuclear power plant system when it is working normally mode and Figure-2(b) present the graph of truth, indeterminacy and falsity membership function of the nuclear power plant system when it is failed mode. So the triangular single valued neutrosophic value of the probability of NPPSMS working normally in 800 hours time interval is $\mathcal{P}_0(\tilde{800})^{NS}$=$\langle 0.768847,\ 0.852144,\ 0.929359;\ 0.9,\ 0.6,\ 0.5\rangle$, where truth $(\mu_{\mathcal{P}_0}(\mathfrak{u}))$, indeterminacy $(\eta_{\mathcal{P}_0}(\mathfrak{u}))$ and falsity $(\nu_{\mathcal{P}_0}(\mathfrak{u}))$ membership functions are respectively as follows:

$$\mu_{\mathcal{P}_0}(\mathfrak{u}) = \begin{cases} (\frac{\mathfrak{u}-0.768847}{0.083297}).0.9 & \text{when } 0.768847 \leqslant \mathfrak{u} \leqslant 0.852144 \\ 0.9 & \text{when } \mathfrak{u} = 0.852144 \\ (\frac{0.929359-\mathfrak{u}}{0.077215}).0.9 & \text{when } 0.852144 \leqslant \mathfrak{u} \leqslant 0.929359 \\ 0 & \text{when } \mathfrak{u} \leqslant 0.768847 \text{ or } \mathfrak{u} \geqslant 0.929359 \end{cases}$$

$$\eta_{\mathcal{P}_0}(\mathfrak{u}) = \begin{cases} \frac{0.390836-0.4.\mathfrak{u}}{0.083297} & \text{when } 0.768847 \leqslant \mathfrak{u} \leqslant 0.852144 \\ 0.6 & \text{when } \mathfrak{u} = 0.852144 \\ \frac{0.4\mathfrak{u}-0.294529}{0.077215} & \text{when } 0.852144 \leqslant \mathfrak{u} \leqslant 0.929359 \\ 1 & \text{when } \mathfrak{u} \leqslant 0.768847 \text{ or } \mathfrak{u} \geqslant 0.929359 \end{cases}$$

$$\nu_{\mathcal{P}_0}(\mathfrak{u}) = \begin{cases} \frac{0.467721-0.5.\mathfrak{u}}{0.083297} & \text{when } 0.768847 \leqslant \mathfrak{u} \leqslant 0.852144 \\ 0.5 & \text{when } \mathfrak{u} = 0.852144 \\ \frac{0.5\mathfrak{u}-0.387465}{0.077215} & \text{when } 0.852144 \leqslant \mathfrak{u} \leqslant 0.929359 \\ 1 & \text{when } \mathfrak{u} \leqslant 0.768847 \text{ or } \mathfrak{u} \geqslant 0.929359 \end{cases}$$

where $\mu_{\mathcal{P}_0}(\mathfrak{u}) + \eta_{\mathcal{P}_0}(\mathfrak{u}) + \nu_{\mathcal{P}_0}(\mathfrak{u}) \leqslant 3$

Also the triangular single valued neutrosophic value of the probability of nuclear power plant system failed in 800 hours time interval is $\mathcal{P}_1(\tilde{800})^{NS}$=$\langle 0.070641,\ 0.147856,\ 0.231153;\ 0.9,\ 0.6,\ 0.5\rangle$, where truth($\mu_{\mathcal{P}_1}(\mathfrak{u})$), indeterminacy($\eta_{\mathcal{P}_1}(\mathfrak{u})$) and falsity($\nu_{\mathcal{P}_1}(\mathfrak{u})$) membership functions are respectively as follows:

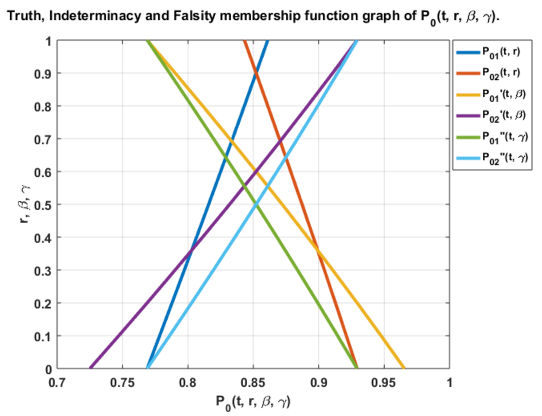
$$\mu_{\mathcal{P}_1}(\mathfrak{u}) = \begin{cases} (\frac{\mathfrak{u}-0.070641}{0.077215}).0.9 & \text{when } 0.070641 \leqslant \mathfrak{u} \leqslant 0.147856 \\ 0.9 & \text{when } \mathfrak{u} = 0.147856 \\ (\frac{0.231153-\mathfrak{u}}{0.083297}).0.9 & \text{when } 0.147856 \leqslant \mathfrak{u} \leqslant 0.231153 \\ 0 & \text{when } \mathfrak{u} \leqslant 0.070641 \text{ or } \mathfrak{u} \geqslant 0.231153 \end{cases}$$

$$\eta_{\mathcal{P}_1}(\mathfrak{u}) = \begin{cases} \frac{0.105471-0.4.\mathfrak{u}}{0.077215} & \text{when } 0.070641 \leqslant \mathfrak{u} \leqslant 0.147856 \\ 0.6 & \text{when } \mathfrak{u} = 0.147856 \\ \frac{0.4\mathfrak{u}-0.009164}{0.083297} & \text{when } 0.147856 \leqslant \mathfrak{u} \leqslant 0.231152 \\ 1 & \text{when } \mathfrak{u} \leqslant 0.0.070641 \text{ or } \mathfrak{u} \geqslant 0.231153 \end{cases}$$

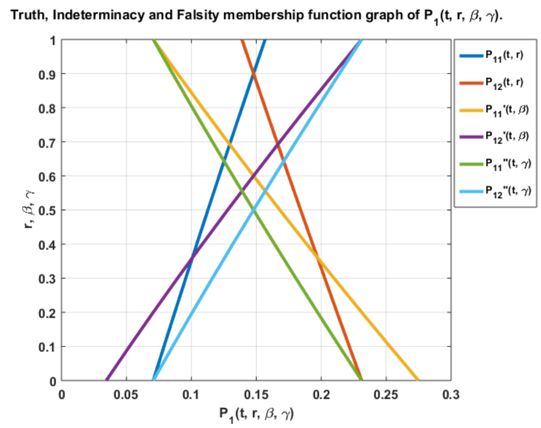
$$\nu_{\mathcal{P}_1}(\mathfrak{u}) = \begin{cases} \frac{0.112536-0.5.\mathfrak{u}}{0.077215} & \text{when } 0.070641 \leqslant \mathfrak{u} \leqslant 0.147856 \\ 0.5 & \text{when } \mathfrak{u} = 0.147856 \\ \frac{0.5\mathfrak{u}-0.032280}{0.083297} & \text{when } 0.147856 \leqslant \mathfrak{u} \leqslant 0.231152 \\ 1 & \text{when } \mathfrak{u} \leqslant 0.0.070641 \text{ or } \mathfrak{u} \geqslant 0.231153 \end{cases}$$

where $\mu_{\mathcal{P}_1}(\mathfrak{u}) + \eta_{\mathcal{P}_1}(\mathfrak{u}) + \nu_{\mathcal{P}_1}(\mathfrak{u}) \leqslant 3$.

From the table values and graph, we see that $\mathcal{P}_{01}(\mathfrak{t},\ r)$, $\mathcal{P}_{11}(\mathfrak{t},\ r)$ are increasing function and

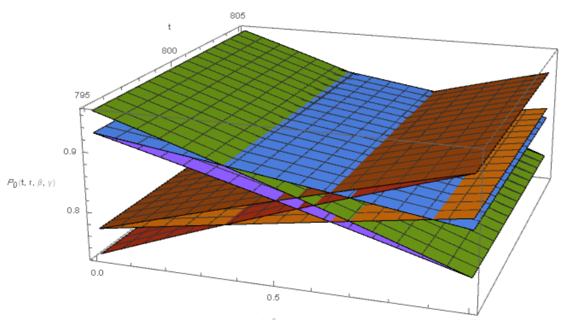


(a) Depicts the truth, indeterminacy and falsity membership function graph of the probability of nuclear power plant system working normally at time (t=800).

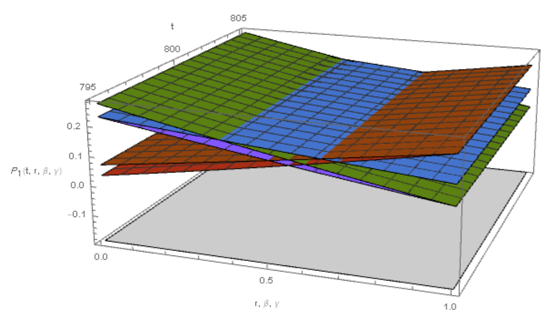


(b) Depicts the truth, indeterminacy and falsity membership function graph of the probability of nuclear power plant system failed at time (t=800).

FIGURE 2. Depicts the truth, indeterminacy and falsity membership function graph of the probability of nuclear power plant system at time t=800 when parameter λ_s is TSVNNs.



(a) Depicts the 3D plot of the the truth, indeterminacy and falsity membership function graph of the probability of nuclear power plant system working normally at time interval between 795 hrs to 805 hrs.



(b) Depicts 3D plot of the truth, indeterminacy and falsity membership function graph of the probability of nuclear power plant system failed at time interval between 795 hrs to 805 hrs.

FIGURE 3. Depicts the 3D plot of the truth, indeterminacy and falsity membership function graph of the probability of nuclear power plant system at time interval between 795 hrs to 805 hrs when parameter λ_s is TSVNNs.

$\mathcal{P}_{02}(\mathfrak{t},\ r)$, $\mathcal{P}_{12}(\mathfrak{t},\ r)$ are decreasing function, whereas $\mathcal{P}'_{01}(\mathfrak{t},\ \beta)$, $\mathcal{P}''_{01}(\mathfrak{t},\ \gamma)$, $\mathcal{P}'_{11}(\mathfrak{t},\ \beta)$, $\mathcal{P}''_{11}(\mathfrak{t},\ \gamma)$

TABLE 2. Values of $\mathcal{P}_0(\mathfrak{t},\ r,\ \beta,\ \gamma)$ and $\mathcal{P}_1(\mathfrak{t},\ r,\ \beta,\ \gamma)$

when $\tilde{\lambda_s}^{NS}$ is a TSVNNs.

$r,\ \beta,\ \gamma$	$\mathcal{P}_{01}(\mathfrak{t},\ r)$	$\mathcal{P}_{02}(\mathfrak{t},\ r)$	$\mathcal{P}'_{01}(\mathfrak{t},\ \beta)$	$\mathcal{P}'_{02}(\mathfrak{t},\ \beta)$	$\mathcal{P}''_{01}(\mathfrak{t},\ \gamma)$	$\mathcal{P}''_{02}(\mathfrak{t},\ \gamma)$	$\mathcal{P}_{11}(\mathfrak{t},\ r)$	$\mathcal{P}_{12}(\mathfrak{t},\ r)$	$\mathcal{P}'_{11}(\mathfrak{t},\ \beta)$	$\mathcal{P}'_{12}(\mathfrak{t},\ \beta)$	$\mathcal{P}''_{11}(\mathfrak{t},\ \gamma)$	$\mathcal{P}''_{12}(\mathfrak{t},\ \gamma)$
0	0.768847	0.929359	0.965531	0.725082	0.929359	0.768847	0.070641	0.231153	0.274918	0.034470	0.231153	0.070641
0.1	0.778386	0.921096	0.947654	0.747134	0.914427	0.785968	0.078904	0.221614	0.252866	0.052346	0.210432	0.085573
0.2	0.787856	0.912752	0.929359	0.768847	0.899235	0.802862	0.087248	0.212144	0.231153	0.070641	0.197138	0.100765
0.3	0.797256	0.904328	0.910653	0.790213	0.883788	0.819526	0.095672	0.202744	0.209787	0.089347	0.180474	0.116212
0.4	0.806585	0.895825	0.891543	0.811223	0.868090	0.835954	0.104175	0.193415	0.188777	0.018260	0.164046	0.131910
0.5	0.815843	0.887243	0.872038	0.831870	0.852144	0.852144	0.112757	0.184157	0.168130	0.108457	0.147856	0.147856
0.6	0.825028	0.878583	0.852144	0.852144	0.835954	0.868090	0.121417	0.174972	0.147856	0.147856	0.131910	0.164046
0.7	0.834141	0.869846	0.831870	0.872038	0.819526	0.883788	0.130154	0.165859	0.127962	0.168130	0.116212	0.180474
0.8	0.843179	0.861033	0.811223	0.891543	0.802862	0.899235	0.138967	0.156821	0.108457	0.188777	0.100765	0.197138
0.9	0.852144	0.852144	0.790213	0.910653	0.785968	0.914427	0.147856	0.147856	0.089347	0.209787	0.08573	0.214032
1.0	0.861033	0.843179	0.768847	0.929359	0.768847	0.929359	0.156821	0.138967	0.070641	0.231153	0.070641	0.231153

are decreasing functions and $\mathcal{P}'_{02}(\mathfrak{t},\ \beta)$, $\mathcal{P}''_{02}(\mathfrak{t},\ \gamma)$, $\mathcal{P}'_{12}(\mathfrak{t},\ \beta)$, $\mathcal{P}''_{12}(\mathfrak{t},\ \gamma)$ are increasing functions. Hence, the solutions are strong solution.

6.3. *Comparison study*

TABLE 3. Comparison analysis

Method	$\mathcal{P}_0(\mathfrak{t})$	$\mathcal{P}_1(\mathfrak{t})$
Zadeh's extension principle	Strong solution	Strong solution
Generalized Hukuhara differentiability(GH_{ii})	Weak solution	Weak solution
Classical method	Strong solution	Strong solution

After completing the lengthy calculation task and consulting Table-3, We found that the solutions will be of a strong variety of our NPPSMS is solved by employing Zadeh's extension principle approach. Once again, the result will be weak if the model is solved using the Generalized Hukuhara Differentiability(GH_{ii}) method. We used the classical technique to solve our problem, resulting in a strong solution. Now it turns out that we are getting robust solutions with both Zadeh's extension principle and classical methods. But, $\langle r,\ \beta,\ \gamma\rangle$-cut solution cannot be obtained directly while we are using Zadeh's extension principle; instead, we must depend on crisp solution. On the other hand, we may obtain the $\langle r,\ \beta,\ \gamma\rangle$-cut solution without depending on the crisp solution when we use the classical method. Thus, the solution produced using the classical approach appears to be quite good. Comparing the numerical results from these two procedures reveals that the classical method's approach is the most acceptable. Based on all these findings, the classical technique is the most acceptable and error-free.

We will now perform a comparative analysis between the results obtained in crisp and neutrosophic environments. In a crisp environment, for λ_s=0.0002; $\mathfrak{t}$=800-h, the probability value of normal operation of a NPPSMS is 0.852144, and the probability value of failure is 0.147856. We take the value of parameter λ_s as a single-valued triangular neutrosophic number instead of a crisp number. So the maximum tolerance is assumed to be 50% on both sides while shifting the value of the parameter from a crisp environment to a neutrosophic environment. Here, the maximum truth membership value is assumed to be 0.9, with the minimum indeterminacy and falsity membership values being 0.6 and 0.5, respectively. Now in the neutrosophic environment, for the values of $\tilde{\lambda_s}^{NS}$=$\langle$0.0001, 0.0002, 0.0003; 0.9, 0.6, 0.5$\rangle$, and t=800-h., the single valued triangular neutrosophic value of the probability of normal operation of the NPPSMS is $\mathcal{P}_0(\tilde{800})^{NS}$=$\langle$0.768847, 0.852144, 0.929359; 0.9, 0.6, 0.5$\rangle$, and the single valued triangular neutrospheric value of the probability of failure is $\mathcal{P}_1(\tilde{800})^{NS}$=$\langle$0.070641, 0.147856, 0.231153; 0.9, 0.6, 0.5$\rangle$. Here, the values in the neutrosophic environment are much more flexible compared to the crisp environment. Also, the value of the probability of normal operation of a NPPSMS in a neutrosophic environment indicates that the maximum permissible tolerances on its left and right sides are 9.77% and 9.06%, respectively; the maximum truth membership value is 0.9; and the minimum indeterminacy and falsity membership values are 0.6 and 0.5, respectively. Again, the failure probability value of the NPPSMS indicates that the maximum allowable tolerances on its left and right sides are 52.22% and 56.34%, the maximum truth membership value is 0.9, and the minimum indeterminacy and falsity membership values are 0.6 and 0.5, respectively. These conclusive findings show that the neutrosophic environment is more flexible, realistic, and acceptable than the crisp environment.

7. Conclusion

In this article we have discussed the definition of NS, SVNS, TSVNNs, and $\langle r, \beta, \gamma\rangle$-cut set of SVNS. We have also explored the idea of a first-order linear homogeneous neutrosophic differential equation and discussed its robustness in view of strong and weak nature of it's solutions. Then, we argued and pointed out the need of analyzing NPP safety system in NS-environment and mathematically formulate an NPPSMS model system in terms of a single-valued triangular neutrosophic differential equation. We have used TSVNNs to analyse and quantitatively discuss the solutions of the NPPSMS above model system with appropriate emphasis on the model parameter rates of failure λ_s. We have obtained the solutions of the model system both in crisp and NS environment. Robust solutions of the NPPSMS model system have been found in both crisp and NS environments. Moreover, solutions are compared by applying there distinct methods namely Zadeh's extension principle, Generalized Hukuhara differentiability (GH_{ii}) and Classical method. It is observed that solutions obtained by Classical method are reliable with less error. However, we have noticed one limitation in our model system. The initial conditions of the model cannot be treated as single-valued triangular neutrosophic numbers. If the initial conditions are assumed to be single-valued triangular neutrosophic numbers, then it contradicts the theory of basic probability. A special advantage of our nuclear power plant safety model is that it can provide robust solutions in any imprecise environment as it is composed of only one parameter.

As an future extension of this work, neutrosophic differential equation can be utilized incorporating other powerful fuzzy environments like trapezoidal neutrosophic numbers or trapezoidal single-valued neutrosophic numbers to capture and handle many real-world problems where impreciseness are inherently involved. In the future, the outcomes of our research may be used to patient safety systems, medical safety systems, human error safety systems, power generation safety systems, compressor safety systems, engineering maintenance safety systems, and so on.

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