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Deneutrosophication of Neutrosophic Bézier Surface Approximation Model

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Abstract: The deneutrosophication process is a process transforming from neutrosophic values to crisp output values. It is the final step for the operations within a neutrosophic set and system. Neutrosophic set theories are a generalization of intuitionistic fuzzy and fuzzy set theories, focusing on truth, indeterminacy, and falsity memberships independently. However, it isn't easy to generate a geometrical model such as a Bézier surface by using neutrosophic set theory through the deneutrosophication process. Therefore, this paper used an average of triangular footprint method in the deneutrosophication process to construct the neutrosophic Bézier surface (NBS) models by using approximation methods. Before generating the model, the neutrosophic control net (NCN) must first be introduced using the deneutrosophication process. After that, the NCN will be blended with the Bernstein basis function to generate the NBS approximation model. Next, some numerical examples of NBS will be provided. Finally, the deneutrosophication of NBS approximation models will be visualized, and its algorithm will be shown.

Keywords: Deneutrosophication process; Bézier Surface; Approximation Method; Average of Triangular Footprints

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1. Introduction

Numerous theories, such as probability theory, intuitionistic fuzzy set theory, fuzzy set theory, rough set theory, and many more, have been developed to address uncertainties. The notion of the neutrosophic set (NS), which involves an independent indeterminacy membership, was first suggested by Smarandache [1,2] in 1999. Various new set theories, such as fuzzy sets [3], intuitionistic fuzzy sets [4], and classic sets, are generalized by this function. Three different functions are utilized to quantify truth-membership (T), indeterminacy-membership (I), and false-membership (F) in an NS. These degrees operate independently of one another.

Deneutrosophication is determining actual output from neutrosophic data [5]. In 2005, Smarandache et al. [6] proposed a deneutrosophication approach utilizing synthesis and the center of gravity method. The synthesis technique described in [6] assigns an NS to FSs with different crisp values. Chakraborty et al. (2018) proposed deneutrosophication techniques for triangular neutrosophic numbers by using the removal area method and then applying it to imprecise project evaluation and route selection problems [10]. Meanwhile, in 2019, they also used the removal area method to study a deneutrosophication strategy for pentagonal neutrosophic numbers in the minimal spanning tree (MST) problem [7]. According to Azzah Awang et al. (2019), the deneutrosophication equation used in [8] provides the neat truth grade rather than the actual event output. Said Broumi et al. (2019) discovered a deneutrosophicated value for a trapezoidal fuzzy number (TFN) utilizing a scoring function based on the TFN's center of gravity in the study [9]. However, there is another method for converting neutrosophic values to crisp output: an average of triangular footprint method introduced by Zakaria [15,16] for the fuzzification method and expanded for intuitionistic set by Zulkifly [29]. Therefore, as the targeted gap is based on the previous studies mentioned, this paper will contribute to introducing the deneutrosophication process based on neutrosophic geometric modeling and the average of the triangular footprint method.

Neutrosophic sets can be utilized to deal with challenges in curve and surface construction for computer-aided graphic designs (CAGD). This concept is required for building a smoother curve or surface model. NS and CAGD are two techniques that can be used to solve the ambiguity of control or data points. Wahab [11-13] developed a method for fuzzifying control points to create Bezier and B-spline curves. The study of fuzzy set theory with geometric modeling has been expanding in [14-19]. Besides that, some studies used the neutrosophic set theory with geometric modeling [20-25,28,30-32]. Additionally, some researchers use neutrosophic for other applications such as in decision-making for medical supplies, ranking cloud service, quantum communication, and virtual manufacturing environments by using type-2 neutrosophic set theory [33-36]. Nevertheless, no research has been done on how to visualize the modeling of neutrosophic geometry when presented with the deneutrosophication process.

The following is a summary of this work. Background information about the topic is given in the first part. A few neutrosophic set theories and their fundamental characteristics are covered in Section 2. The creation of a neutrosophic control net (NCN) via the deneutrosophication process will be covered in Section 3. The deneutrosophication process for the neutrosophic Bézier Surface (NBS) approximation is then seen and described in Section 4 using a numerical example of NCN. At the end of Section 4, an algorithm demonstrating the deneutrosophication procedure for NBS will be shown. Section 5 concludes with the discussion and conclusions.

2. Basic Properties

The fuzzy set only deals with membership, whereas the intuitionistic set deals with membership, non-membership, and uncertainty degrees and are dependent on each other. However, the NS treats the truth, indeterminacy, and falsity degrees independently [1]. An NS has three membership functions with the addition of the parameter "indeterminacy" to the NS specification [2]: a membership function, represented by the letter *T*; an indeterminacy membership function, denoted by the letter *I*; and a non-membership function, appointed by the letter *F*. This section will define neutrosophic set theory, neutrosophic relation notion, neutrosophic point, the (α,β,γ)-cut operations as the fundamentals and the triangular neutrosophic number. Therefore, the definitions are as follows.

Definition 1. [1] Let X be the universal set, with elements in X represented as x. The NS is an element in the structure as follows:

$$\hat{A} = \{\left\langle x : T_{\hat{A}(x)}, I_{\hat{A}(x)}, F_{\hat{A}(x)} \right\rangle | \ x \in X\} \tag{1}$$

where $T, I, F : X \to]^{-}0,1^{+}[$ represented as the degree of truth, indeterminacy, and falsity memberships respectively of the element $x \in X$ to the set $\hat{A}$ with the condition;

$$0^{-} \le T_{\hat{A}}(x) + I_{\hat{A}}(x) + F_{\hat{A}}(x) \le 3^{+} \tag{2}$$

There is no limit to the amount of $T_{\hat{A}}(x), I_{\hat{A}}(x)$ and $F_{\hat{A}}(x)$.

NS takes the value from real standard or non-standard subsets of $]^{-}0,1^{+}[$. But in technical applications the real value of the interval $[0,1]$ will be used since $]^{-}0,1^{+}[$ difficult to apply in real data such as scientific and engineering problems. Thus, the membership values used as follows:

$$\hat{A}=\{\langle x:T_{\hat{A}(x)},I_{\hat{A}(x)},F_{\hat{A}(x)}\rangle|\ x\in X\}\text{ and }T_{\hat{A}}(x),I_{\hat{A}}(x),F_{\hat{A}}(x)\in[0,1]$$

There is no limitation on the summation of $T_{\hat{A}}(x), I_{\hat{A}}(x)$ and $F_{\hat{A}}(x)$. But comply with the following conditions:

$$0\le T_{\hat{A}}(x)+I_{\hat{A}}(x)+F_{\hat{A}}(x)\le 3 \tag{3}$$

Definition 2. [20,21] Suppose $\hat{A}=\left\{\langle x:T_{\hat{A}(x)},I_{\hat{A}(x)},F_{\hat{A}(x)}\rangle|\ x\in\hat{A}\right\}$ and $\hat{B}=\left\{\langle y:T_{\hat{B}(y)},I_{\hat{B}(y)},F_{\hat{B}(y)}\rangle|\ y\in\hat{B}\right\}$ will be the neutrosophic elements. Thus, $R=\left\{\langle (x,y):T_{(x,y)},I_{(x,y)},F_{(x,y)}\rangle|\ x\in\hat{A},y\in\hat{B}\right\}$ is a neutrosophic relation (NR) on $\hat{A}$ and $\hat{B}$.

Definition 3 [20,21] Neutrosophic set of $\hat{A}$ in space X is neutrosophic point (NP) and $\hat{A}=\{\hat{A}_i\}$ where $i=0,\ldots,n$ is a set of NPs where $T_{\hat{A}}:X\to[0,1]$ As a true degree, $I_{\hat{A}}:X\to[0,1]$ as indeterminacy membership and $F_{\hat{A}}:X\to[0,1]$ as false membership with

$$T_{\hat{A}}(\hat{A})=\begin{cases}0 & \text{if } \hat{A}_i\notin\hat{A}\\ a\in(0,1) & \text{if } \hat{A}_i\hat{\in}\hat{A}\\ 1 & \text{if } \hat{A}_i\in\hat{A}\end{cases}$$
$$I_{\hat{A}}(\hat{A})=\begin{cases}0 & \text{if } \hat{A}_i\notin\hat{A}\\ b\in(0,1) & \text{if } \hat{A}_i\hat{\in}\hat{A}\\ 1 & \text{if } \hat{A}_i\in\hat{A}\end{cases} \tag{4}$$
$$F_{\hat{A}}(\hat{A})=\begin{cases}0 & \text{if } \hat{A}_i\notin\hat{A}\\ c\in(0,1) & \text{if } \hat{A}_i\hat{\in}\hat{A}\\ 1 & \text{if } \hat{A}_i\in\hat{A}\end{cases}$$

Definition 4 [26] The $(\alpha,\beta,\gamma)-$cut of NS is denoted as $C_{(\alpha,\beta,\gamma)}$ where $\alpha,\beta,\gamma\in[0,1]$ and are fixed numbers, such that $0\le\alpha+\beta+\gamma\le 3$ is defined as $C_{(\alpha,\beta,\gamma)}=\left\{\begin{matrix}\langle T_{\hat{A}}(x),I_{\hat{A}}(x),F_{\hat{A}}(x)\rangle:x\in X,\\ T_{\hat{A}}(x)\ge\alpha,I_{\hat{A}}(x)\le\beta,F_{\hat{A}}(x)\le\gamma\end{matrix}\right\}$

Definition 5 [26] A NS $\hat{A}$ is mentioned in the whole conversation of real numbers $\mathbb{R}$ and it will be neutrosophic numbers if it follows the following properties:

i. $\hat{A}$ is normal if exist $x_0\in\mathbb{R}$ such that $T_{\hat{A}}(x_0)=1$ while $I_{\hat{A}}(x_0)=F_{\hat{A}}(x_0)=0$.

ii. $\hat{A}$ is convex set for truth function $T_{\hat{A}}(x)$ where $T_A(\mu x_1+(1-\mu)x_2)\ge\min(T_{\hat{A}}(x_1),T_{\hat{A}}(x_2))$, $\forall x_1,x_2\in\mathbb{R},\mu\in[0,1]$.

iii. $\hat{A}$ is concave set for indeterminacy $I_{\hat{A}}(x)$ and false function $F_{\hat{A}}(x)$ where $I_{\hat{A}}(\mu x_1+(1-\mu)x_2)\ge\max(I_{\hat{A}}(x_1),I_{\hat{A}}(x_2))$, $\forall x_1,x_2\in\mathbb{R},\mu\in[0,1]$ and $F_{\hat{A}}(\mu x_1+(1-\mu)x_2)\ge\max(F_{\hat{A}}(x_1),F_{\hat{A}}(x_2))$, $\forall x_1,x_2\in\mathbb{R},\mu\in[0,1]$.

Definition 6 [10] Suppose $w_{\hat{A}}, u_{\hat{A}}, y_{\hat{A}} \in [0,1]$. A triangular neutrosophic number for $\hat{A} = \langle (a_1, b_1, c_1); w_{\hat{A}}, u_{\hat{A}}, y_{\hat{A}} \rangle$ is a special NS on the real number set, since there are three memberships which are truth, indeterminacy, and falsity membership functions, the following functions are defined:

$$T_{\hat{A}}(x) = \begin{cases} \dfrac{(x-a)w_{\hat{A}}}{(b-a)w_{\hat{A}}} & (a \le x \le b) \\ w_{\hat{A}} & (x = b_1) \\ \dfrac{(c-x)w_{\hat{A}}}{(c-b)} & (b \le x \le c) \\ 0 & otherwise \end{cases}$$

$$I_{\hat{A}}(x) = \begin{cases} \dfrac{\left(b-x+u_{\hat{A}}(x-a)\right)}{(b-a)} & (a \le x \le b) \\ \mathrm{u}_{\hat{A}} & (x = b) \\ \dfrac{\left(x-b+u_{\hat{A}}(c-x)\right)}{(c-b)} & (b \le x \le c) \\ 1 & otherwise \end{cases} \tag{5}$$

$$F_{\hat{A}}(x) = \begin{cases} \dfrac{\left(b-x+y_{\hat{A}}(x-a)\right)}{(b-a)} & (a \le x \le b) \\ \mathrm{y}_{\hat{A}} & (x = b) \\ \dfrac{\left(x-b+y_{\hat{A}}(c-x)\right)}{(c-b)} & (b \le x \le c) \\ 1 & otherwise \end{cases}$$

Definition 6 can be visualized as in Figure 1. Figure 1 shows the triangular neutrosophic number for a set $\hat{A} = \langle (a_1, b_1, c_1); w_{\hat{A}}, u_{\hat{A}}, y_{\hat{A}} \rangle$ where $T_{\hat{A}}, I_{\hat{A}}, F_{\hat{A}} \in [0,1]$ represented as truth, indeterminacy, and falsity membership.

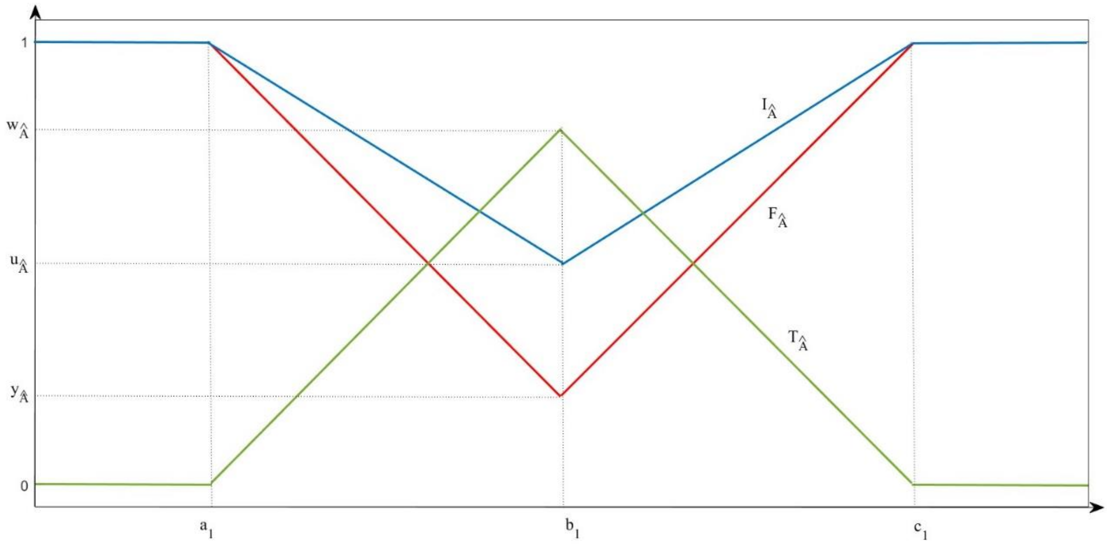

Figure 1 Triangular neutrosophic numbers for a set $\hat{A} = \langle (a,b,c); w_{\hat{A}}, u_{\hat{A}}, y_{\hat{A}} \rangle$

3. Deneutrosophication of Neutrosophic Control Net (NCN)

This section discussed the neutrosophic point relation (NPR), as well as the deneutrosophication of neutrosophic control net (NCN) for each $\langle \alpha, \beta, \gamma \rangle - \text{cut}$ are introduced by using the average of triangular footprint method by [15,16,29]. NPR is built on the concept of NR as in Definition 2 and NP as in Definition 3, which was addressed in the previous section. If P, Q is a collection of Euclidean universal space points and $P, Q \in \mathbf{R}^2$ then NPR is defined as follows:

Definition 7 [22] Let X, Y collection of universal space points with a non-empty set and $P, Q, I \in \mathbf{R} \times \mathbf{R} \times \mathbf{R}$, then NPR is defined as

$$\hat{R} = \left\{ \left\langle \left(p_i, q_j \right), T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j) \right\rangle \middle| T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j) \in I \right\} \tag{6}$$

where $\left(p_i, q_j \right)$ is an ordered pair of coordinates and $\left(p_i, q_j \right) \in P \times Q$ while $T_R(p_i, q_j), I_R(p_i, q_j), F_R(p_i, q_j)$ are the truth membership, indeterminacy membership, and false membership that follow the condition of the neutrosophic set which is $0 \le T_{\hat{A}}(x) + I_{\hat{A}}(x) + F_{\hat{A}}(x) \le 3$.

The control point is crucial in the design, control, and manufacture of smooth curves [22]. The NCP is defined in this part by first applying the notion of fuzzy control point from Wahab et al. [12] in the following:

Definition 8 [22] Let $\hat{R}$ be an NPR, then $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ denoted as NCPs for membership truth, indeterminacy where $i = 1, \ldots, n+1$ and the position vector of $n+1$ as control polygon vertices.

$$\begin{aligned} \hat{P}_i^T &= \left\{ \hat{p}_1^T, \hat{p}_2^T, \ldots, \hat{p}_{n+1}^T \right\} \\ \hat{P}_i^I &= \left\{ \hat{p}_1^I, \hat{p}_2^I, \ldots, \hat{p}_{n+1}^I \right\} \\ \hat{P}_i^F &= \left\{ \hat{p}_1^F, \hat{p}_2^F, \ldots, \hat{p}_{n+1}^F \right\} \end{aligned} \tag{7}$$

This study is primarily focused on surface modeling. Thus, it is crucial to introduce the concept of a neutrosophic control net. In surface modeling, the control net is obtained by combining the relations of each control point [27]. In contrast, the curve model only requires control points [27]. Thus, the neutrosophic control net (NCN) can be described as follows.

Definition 9 [25,23,28] Let $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ as NCPs for each membership, and $\hat{P}_{i,j}^{\;T,I,F}$ represents as NCN for truth, indeterminacy, and falsity memberships where $i=1,...,n+1$ and $j=1,...,m+1$ and the position vector of $n+1$ and $m+1$ as control polygon vertices.

$$
\begin{aligned}
\hat{P}_{i,j}^{\;T} &= \begin{bmatrix} \hat{P}_{1,1}^{\;T} & \hat{P}_{1,2}^{\;T} & \cdots & \hat{P}_{1,m+1}^{\;T} \\ \hat{P}_{2,1}^{\;T} & \hat{P}_{2,2}^{\;T} & \cdots & \hat{P}_{2,m+1}^{\;T} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n+1,1}^{\;T} & \hat{P}_{n+1,2}^{\;T} & \cdots & \hat{P}_{n+1,m+1}^{\;T} \end{bmatrix} \\
\hat{P}_{i,j}^{\;I} &= \begin{bmatrix} \hat{P}_{1,1}^{\;I} & \hat{P}_{1,2}^{\;I} & \cdots & \hat{P}_{1,m+1}^{\;I} \\ \hat{P}_{2,1}^{\;I} & \hat{P}_{2,2}^{\;I} & \cdots & \hat{P}_{2,m+1}^{\;I} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n+1,1}^{\;I} & \hat{P}_{n+1,2}^{\;I} & \cdots & \hat{P}_{n+1,m+1}^{\;I} \end{bmatrix} \\
\hat{P}_{i,j}^{\;F} &= \begin{bmatrix} \hat{P}_{1,1}^{\;F} & \hat{P}_{1,2}^{\;F} & \cdots & \hat{P}_{1,m+1}^{\;F} \\ \hat{P}_{2,1}^{\;F} & \hat{P}_{2,2}^{\;F} & \cdots & \hat{P}_{2,m+1}^{\;F} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n+1,1}^{\;F} & \hat{P}_{n+1,2}^{\;F} & \cdots & \hat{P}_{n+1,m+1}^{\;F} \end{bmatrix}
\end{aligned}
\tag{8}
$$

As mentioned in the introduction part, deneutrosophication is the process of determining actual output from neutrosophic data [5]. Therefore, $\hat{P}_{i,j}^{T,I,F}$ denoted as the neutrosophic value will transform to crisp output represented as $P_{i,j}^{T,I,F}$ for their respective memberships. Thus, Definition 10 describes the deneutrosophication process for each $\langle \alpha,\beta,\gamma \rangle -\text{cut}$ in triangular forms. NCNs are deneutrosophicated based on $\langle \alpha,\beta,\gamma \rangle$ values when $\alpha,\beta,\gamma \in (0,1]$. Figure 2 and Definition 10 show the deneutrosophication of NCNs by using an average triangular footprint method. The average triangular footprint method is a division process that uses the footprint of left, mean, and right values of a triangular control point [29]. Therefore, the deneutrosophication of NCN is as follows:

Definition 10 Let $\hat{P}_{i,j}^{T,I,F}$ be the set of NCNs where $i=1,...,n+1$ and $j=1,...,m+1$. Then, ${}^{\alpha}P_{i,j}^{T}$ is $\alpha-\text{cut}$ values for crisp truth membership, ${}^{\beta}P_{i,j}^{I}$ is $\beta-\text{cut}$ values for crisp indeterminacy membership, and ${}^{\gamma}\hat{P}_{i,j}^{F}$ is the $\gamma-\text{cut}$ values for crisp falsity membership, where $\alpha,\beta,\gamma \in (0,1]$ and the deneutrosophication of NCNs as follows:

$$
\begin{aligned}
{}^{\alpha}P_{i,j}^{T} &= \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1}\left\langle {}^{\alpha}\hat{P}_{i,j}^{T(L)}, {}^{\alpha}\hat{P}_{i,j}^{T(M)}, {}^{\alpha}\hat{P}_{i,j}^{T(R)} \right\rangle \\
{}^{\beta}P_{i,j}^{I} &= \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1}\left\langle {}^{\beta}\hat{P}_{i,j}^{I(L)}, {}^{\beta}\hat{P}_{i,j}^{I(M)}, {}^{\beta}\hat{P}_{i,j}^{I(R)} \right\rangle \\
{}^{\gamma}P_{i,j}^{F} &= \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1}\left\langle {}^{\gamma}\hat{P}_{i,j}^{F(L)}, {}^{\gamma}\hat{P}_{i,j}^{F(M)}, {}^{\gamma}\hat{P}_{i,j}^{F(R)} \right\rangle
\end{aligned}
\tag{9}
$$

where $\left\langle {}^{\alpha}\hat{P}_{i,j}^{T(L)}, {}^{\alpha}\hat{P}_{i,j}^{T(M)}, {}^{\alpha}\hat{P}_{i,j}^{T(R)} \right\rangle$ denoted as left, mean, and right NCN for truth membership at any α values, $\left\langle {}^{\beta}\hat{P}_{i,j}^{I(L)}, {}^{\beta}\hat{P}_{i,j}^{I(M)}, {}^{\beta}\hat{P}_{i,j}^{I(R)} \right\rangle$ denoted as left, mean, and right NCN for indeterminacy membership at any β values and $\left\langle {}^{\gamma}\hat{P}_{i,j}^{F(L)}, {}^{\gamma}\hat{P}_{i,j}^{F(M)}, {}^{\gamma}\hat{P}_{i,j}^{F(R)} \right\rangle$ denoted as left, mean, and right NCN for falsity membership at any γ values.

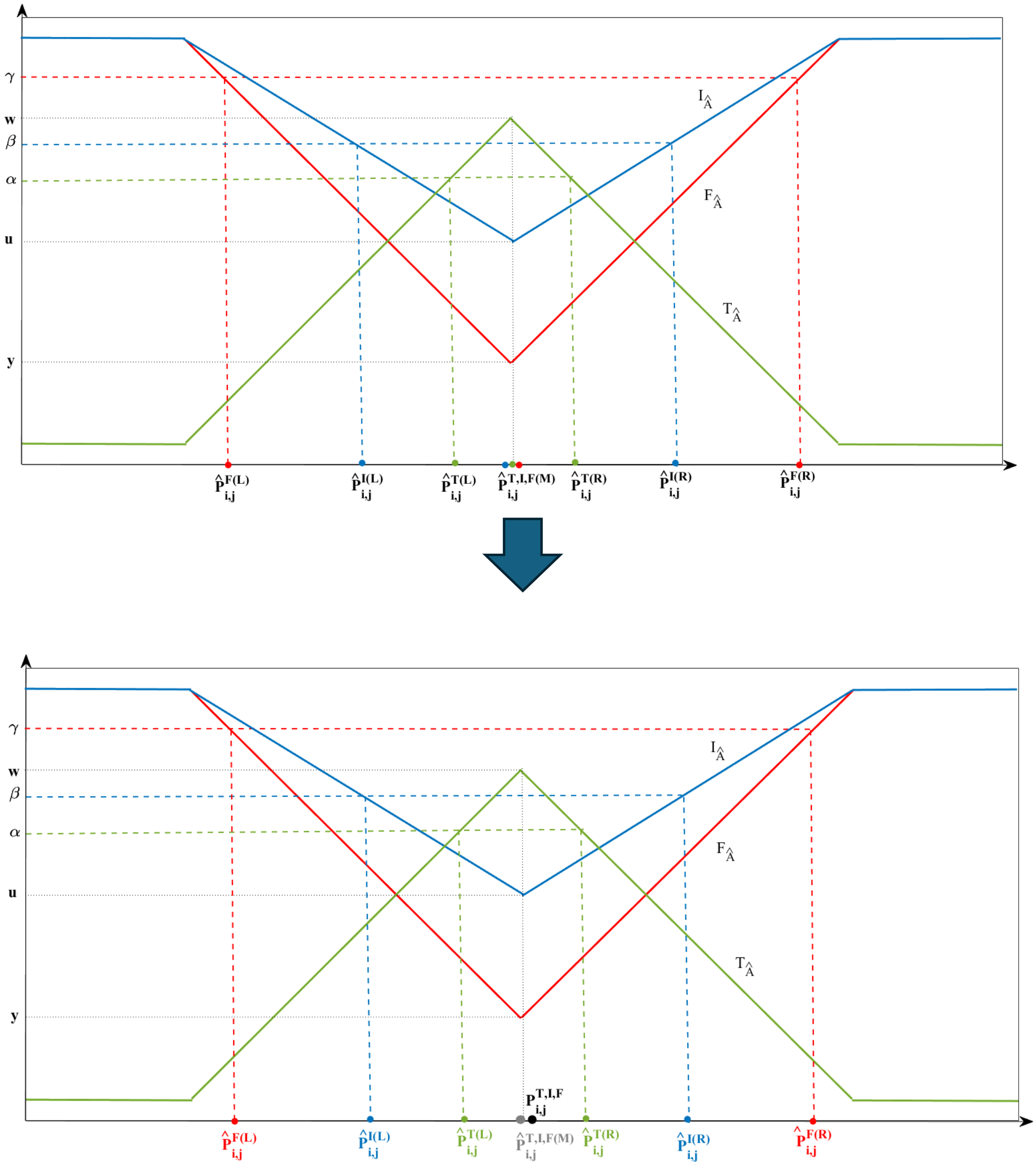

Figure 2. Deneutrosophication of neutrosophic control nets (NCNs)

4. Deneutrosophication of Neutrosophic Bézier Surface (NBS) Approximation Model

Before proceeding with the deneutrosophication of the neutrosophic Bézier surface (NBS) approximation model, the mathematical representation of the NBS approximation model was introduced in [25]. After that, the deneutrosophication process of NBS approximation for each

$\langle \alpha, \beta, \gamma \rangle - \text{cut}$ is discussed. By using the NCNs in Definition 9, the definition of NBS approximation is as follows:

Definition 11 [25] Let $\hat{P}_{i,j}^{T,I,F}$ be the set of NCNs where $i = 1,...,n+1$ and $j = 1,...,m+1$. Then, the NBS denoted as $BS(u,v)^T$ for truth membership, $BS(u,v)^I$ for indeterminacy membership and $BS(u,v)^F$ for falsity membership in the direction u and v. Cartesian Bézier surface is given by

$$\begin{aligned} BS(u,v)^T &= \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} \hat{P}_{i,j}{}^T J_i^n(u) K_j^m(v) \\ BS(u,v)^I &= \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} \hat{P}_{i,j}{}^I J_i^n(u) K_j^m(v) \\ BS(u,v)^F &= \sum_{i=1}^{n+1}\sum_{j=1}^{m+1} \hat{P}_{i,j}{}^F J_i^n(u) K_j^m(v) \end{aligned} \tag{10}$$

where $J_i^n(u)$ and $K_j^m(v)$ respectively are Bernstein basis functions that can be written as follows:

$$\begin{aligned} J_{(n,i)}(u) &= \binom{n}{i} u^i (1-u)^{n-i} \qquad (0)^0 \equiv 1 \\ K_{(m,j)}(v) &= \binom{m}{j} v^j (1-v)^{m-j} \qquad (0)^0 \equiv 1 \end{aligned} \tag{11}$$

$$\begin{aligned} \binom{n}{i} &= \frac{n!}{i!(n-1)!} \qquad (0)^0 \equiv 1 \\ \binom{m}{j} &= \frac{n!}{j!(m-1)!} \qquad (0)^0 \equiv 1 \end{aligned} \tag{12}$$

After that, the $\langle \alpha, \beta, \gamma \rangle - \text{cut}$ of NCNs from Definition, 10 was deneutrosophicated and blended with Equations 11 and 12 to form a deneutrosophication of the NBS approximation model. Therefore, the following is the mathematical representation of the NBS approximation model's deneutrosophication using the average of triangular footprint method:

Definition 12 Let $\hat{P}_{i,j}^{T,I,F}$ be the set of NCNs where $i = 1,...,n+1$ and $j = 1,...,m+1$. The deneutrosophication of NB-sS for truth, indeterminacy, and falsity denoted by $BS(u,v)^{T,I,F}$ and defined as follows:

$$BS(u,v)^{T,I,F} = \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1} \hat{P}_{i,j}{}^{T,I,F} J_i^n(u) K_j^m(v) \tag{13}$$

For each $\langle \alpha, \beta, \gamma \rangle - \text{cut}$ can be written as:

$$\begin{aligned} {}^{\alpha}BS(u,v)^T &= \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1} {}^{\alpha}P_{i,j}{}^T J_i^n(u) K_j^m(v) \\ {}^{\beta}BS(u,v)^I &= \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1} {}^{\beta}P_{i,j}{}^I J_i^n(u) K_j^m(v) \\ {}^{\gamma}BS(u,v)^F &= \frac{1}{3}\sum_{i=1}^{n+1}\sum_{j=1}^{m+1} {}^{\gamma}P_{i,j}{}^F J_i^n(u) K_j^m(v) \end{aligned} \tag{14}$$

where ${}^{\alpha}P_{i,j}^{T}$ is $\alpha-$cut values for crisp truth membership, ${}^{\beta}P_{i,j}^{I}$ is $\beta-$cut values for crisp indeterminacy membership, and ${}^{\gamma}P_{i,j}^{F}$ is the $\gamma-$cut values for crisp falsity membership with $\alpha,\beta,\gamma\in(0,1]$.

The deneutrosophication process of NBCs was constructed by $(\alpha,\beta,\gamma)-$cut operations for left and right NCNs which the (α,β,γ) values are $\alpha=0.4,0.6,0.6,0.7,0.9,0.8,0.8,0.4,0.6,0.7,0.5,0.7$, $\beta=0.2,0.3,0.5,0.3,0.1,0.3,0.1,0.3,0.2,0.2,0.5,0.2$ and $\gamma=0.7,0.4,0.2,0.3,0.3,0.2,0.4,0.6,0.5,0.4,0.3,0.4$ where $\alpha_i,\beta_i,\gamma_i\in(0,1]$. The illustration for the selected (α,β,γ) values are presented in Figure 4-6 with their respective memberships. According to Definition 9, Table 2 displays the values of $\left\langle P_{i,j}^{T},P_{i,j}^{I},P_{i,j}^{F}\right\rangle$ after the deneutrosophication process. In this study, deneutrosophication for x and y data is considered.

Table 2. Left and right neutrosophic control points (NCPs) with their respective membership

Neutrosophic values, $\hat{P}_{i,j}$	**Triangular Neutrosophic Number,** $\left\langle \hat{P}_{i,j}^{(L)},\hat{P}_{i,j}^{(M)}\hat{P}_{i,j}^{(R)}\right\rangle$		**Deneutrosophication Process**				
	x- data	**y- data**	$\alpha-$cut	$\beta-$cut	$\gamma-$cut	$\left\langle P_{i,j}^{T},P_{i,j}^{I},P_{i,j}^{F}\right\rangle$ **x**	**y**
$\hat{P}_{1,1}=(-16,16)$	$\langle -12,-16,-20\rangle$	$\langle 12,16,20\rangle$	$\alpha_1=0.4$	$\beta_1=0.2$	$\gamma_1=0.7$	-16	16
$\hat{P}_{1,2}=(-6,16)$	$\langle -2,-6,-10\rangle$	$\langle 12,16,20\rangle$	$\alpha_2=0.6$	$\beta_3=0.3$	$\gamma_2=0.4$	-6	16
$\hat{P}_{1,3}=(6,16)$	$\langle 2,6,10\rangle$	$\langle 12,16,20\rangle$	$\alpha_3=0.6$	$\beta_3=0.5$	$\gamma_3=0.2$	6	16
$\hat{P}_{1,4}=(16,16)$	$\langle 12,16,20\rangle$	$\langle 12,16,20\rangle$	$\alpha_4=0.7$	$\beta_4=0.3$	$\gamma_4=0.3$	16	16
$\hat{P}_{2,1}=(-16,6)$	$\langle -12,-16,-20\rangle$	$\langle 2,6,10\rangle$	$\alpha_5=0.9$	$\beta_5=0.1$	$\gamma_5=0.3$	-16	6
$\hat{P}_{2,2}=(-6,6)$	$\langle -2,-6,-10\rangle$	$\langle 2,6,10\rangle$	$\alpha_6=0.8$	$\beta_6=0.3$	$\gamma_6=0.2$	-6	6
$\hat{P}_{2,3}=(6,6)$	$\langle 2,6,10\rangle$	$\langle 2,6,10\rangle$	$\alpha_7=0.8$	$\beta_7=0.1$	$\gamma_7=0.4$	6	6
$\hat{P}_{2,4}=(16,6)$	$\langle 12,16,20\rangle$	$\langle 2,6,10\rangle$	$\alpha_8=0.4$	$\beta_8=0.3$	$\gamma_8=0.6$	16	6
$\hat{P}_{3,1}=(-16,-16)$	$\langle -12,-16,-20\rangle$	$\langle -12,-16,-20\rangle$	$\alpha_9=0.6$	$\beta_9=0.2$	$\gamma_9=0.5$	-16	-16
$\hat{P}_{3,2}=(-6,-16)$	$\langle -2,-6,-10\rangle$	$\langle -12,-16,-20\rangle$	$\alpha_{10}=0.7$	$\beta_{10}=0.2$	$\gamma_{10}=0.4$	-6	-16
$\hat{P}_{3,3}=(6,-16)$	$\langle 2,6,10\rangle$	$\langle -12,-16,-20\rangle$	$\alpha_{11}=0.5$	$\beta_{11}=0.5$	$\gamma_{11}=0.3$	6	-16
$\hat{P}_{3,4}=(16,-16)$	$\langle 12,16,20\rangle$	$\langle -12,-16,-20\rangle$	$\alpha_{12}=0.7$	$\beta_{12}=0.2$	$\gamma_{12}=0.4$	16	-16

According to Definition 5, the numerical examples in Table 1 meet the neutrosophic number requirements. Next, using Definition 6, the triangular neutrosophic number, labeled as $\left\langle \hat{P}_i^{(L)},\hat{P}_i^{(M)}\hat{P}_i^{(R)}\right\rangle$, was shown in Table 2. For example, $\hat{P}_{1,1}^{T}=(-16,16)$ as an NCP after going through the triangular neutrosophic number will get $\langle -12,-16,-20\rangle$. Then, by using Definition 10 and Equation 9 which is the deneutrosophication process for $\alpha=0.4$, it will transform into a crisp value, $(-16,16)$.

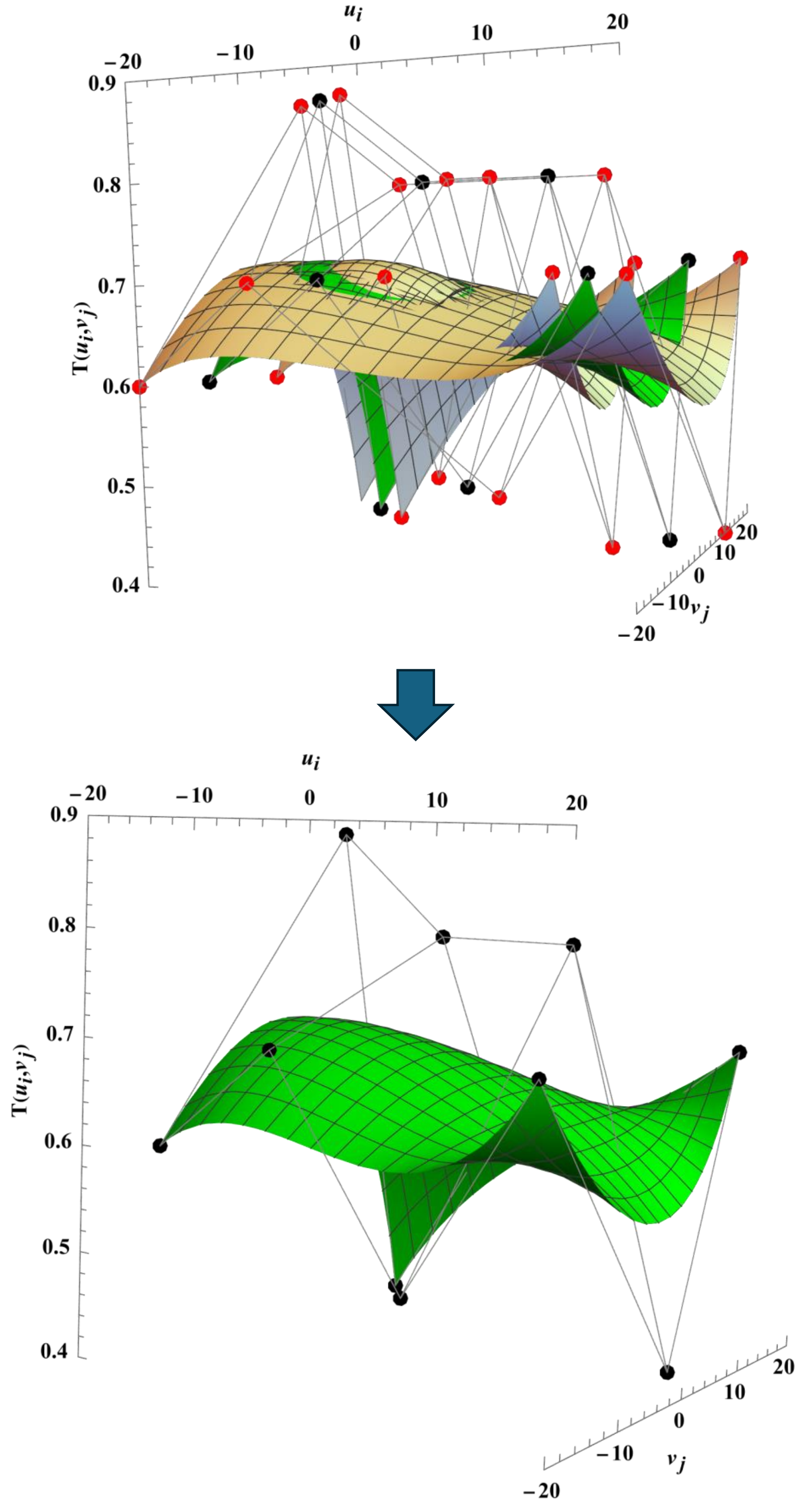

Figure 4. Deneutrosophication of NBSs approximation for truth membership with their respective alpha values

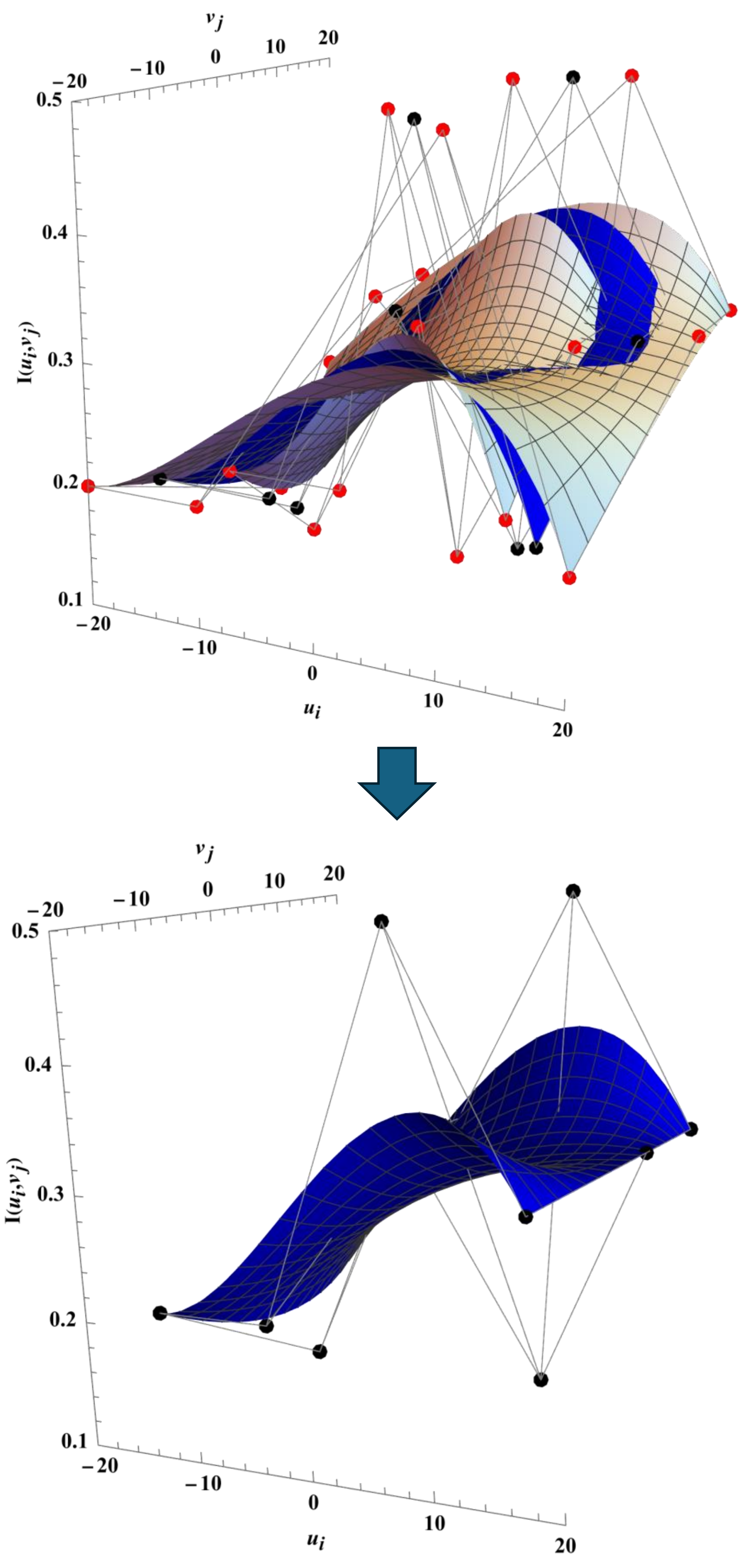

Figure 5. Deneutrosophication of NBSs approximation for indeterminacy membership with their respective betta values

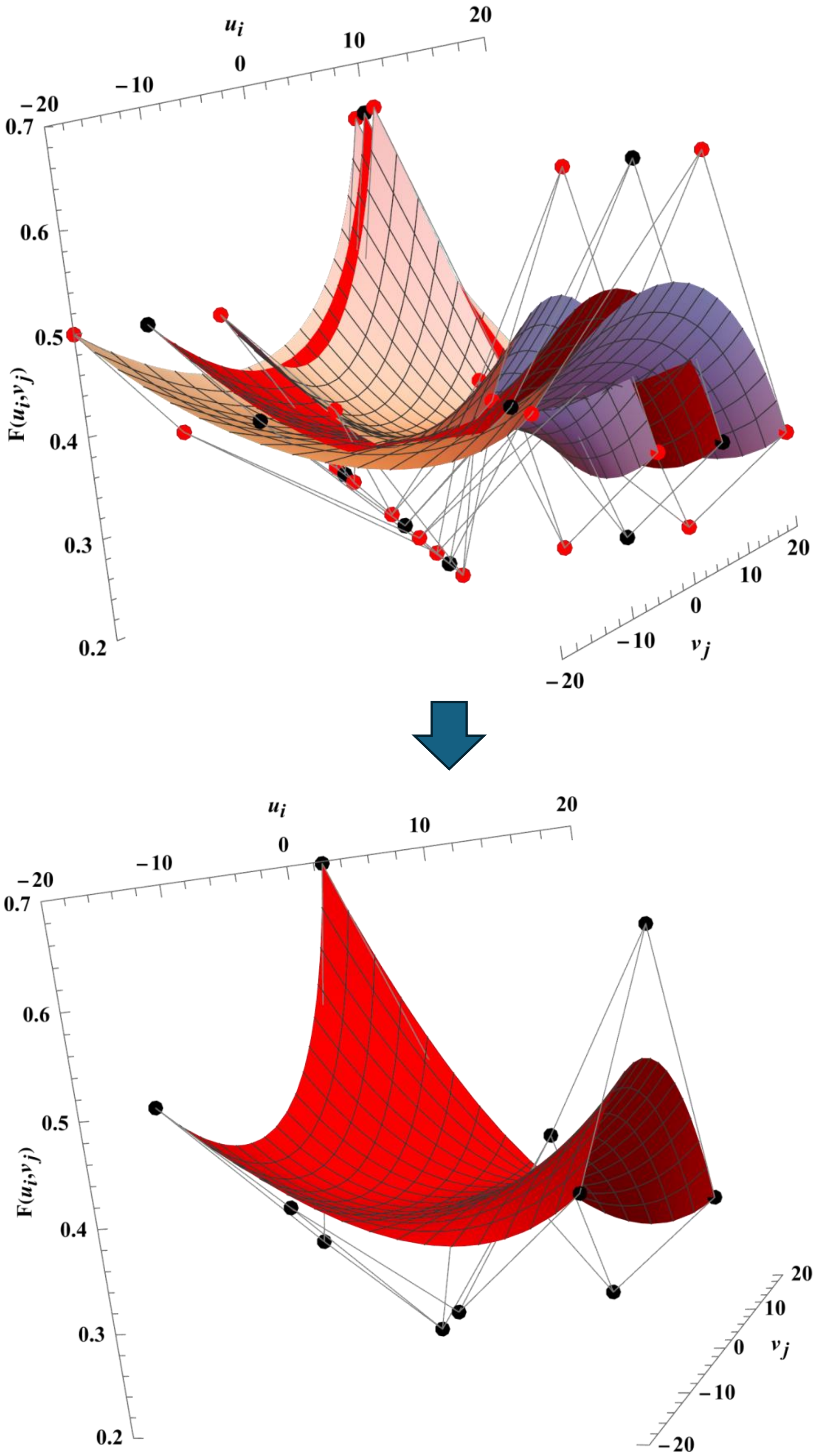

Figure 6. Deneutrosophication of NBSs approximation for falsity membership with their respective gamma values

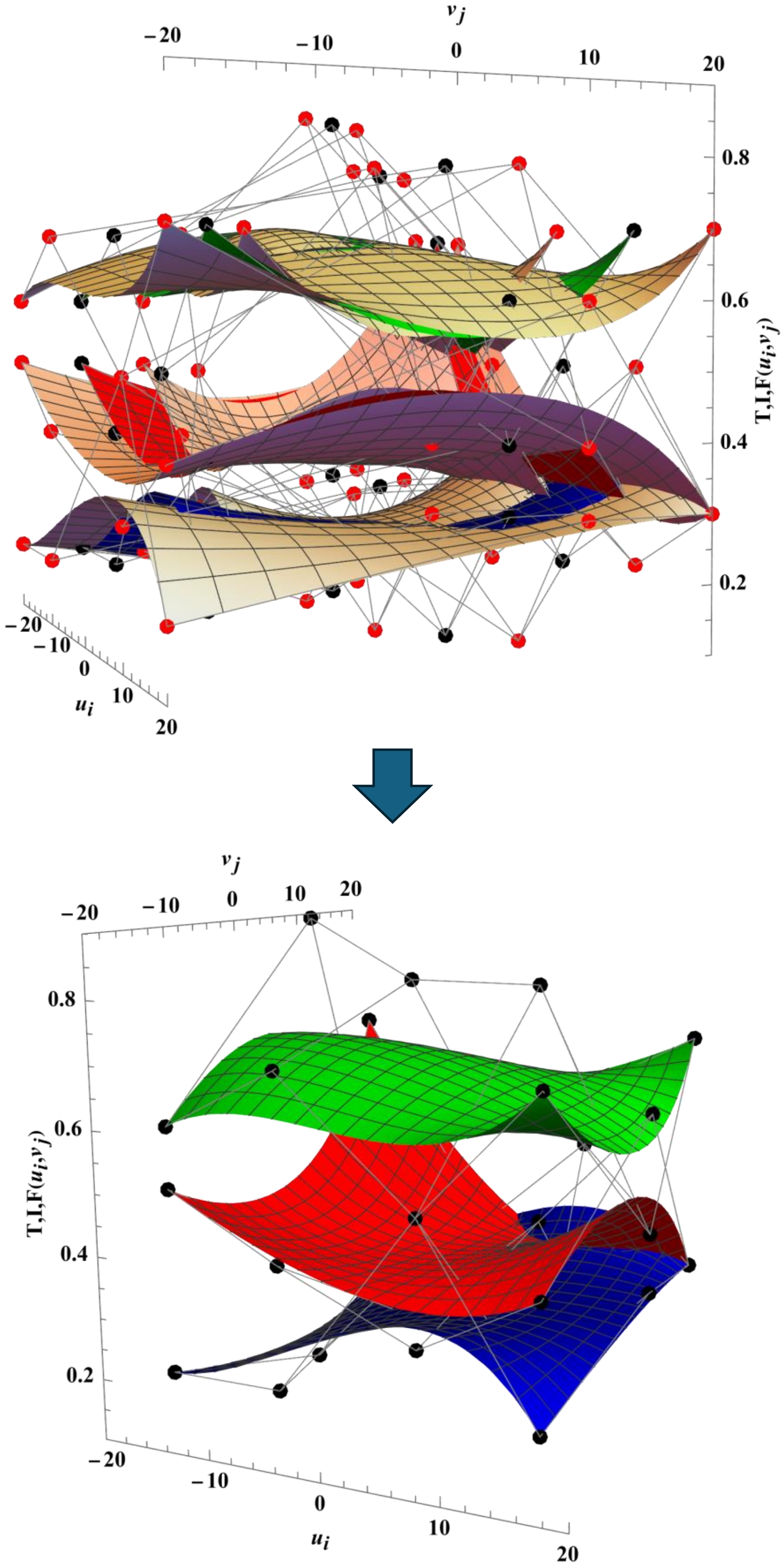

Figure 7. Deneutrosophication of NBSs approximation with their respective memberships and $\langle \alpha, \beta, \gamma \rangle$ values

Finally, Figure 7 shows the distinct surfaces between truth, indeterminacy, and falsity memberships for visualization of deneutrosophication of NBSs approximation for selected $\langle \alpha, \beta, \gamma \rangle$ values. In the figure, the green surface is denoted as NBSs for truth membership, the blue surface is denoted as NBSs for indeterminacy membership, and the red surface is denoted as NBSs for falsity membership. At the end of this section, an algorithm to generate the deneutrosophication of NBSs, the algorithm as shown in Figure 8 is demonstrated.

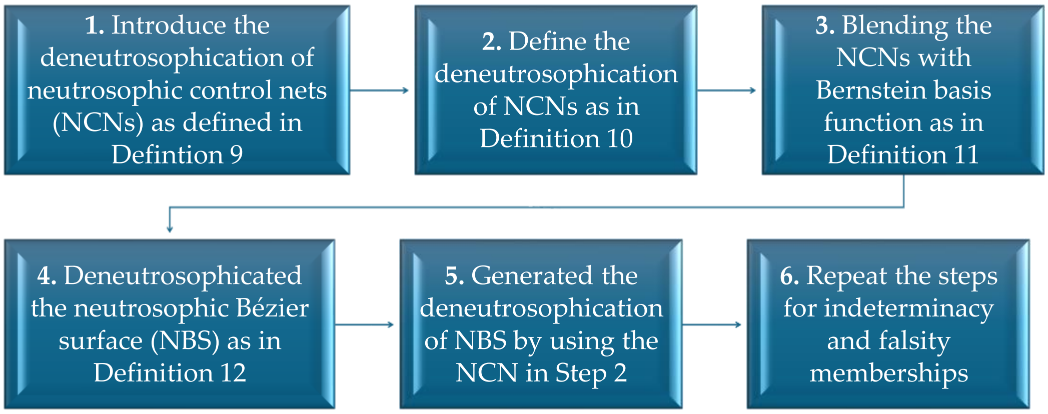

Figure 8. An algorithm to generate the deneutrosophication of neutrosophic Bézier surface approximation models.

6. Discussion and Conclusion

This work has introduced the deneutrosophication of neutrosophic Bézier surface (NBS) approximation model for truth, indeterminacy, and falsity memberships by using an average of triangular footprints. The operation of $\langle \alpha, \beta, \gamma \rangle$-cut can be utilized to describe the degree of truth membership, indeterminacy, and falsity based on the neutrosophic values. The advantages of utilizing this method include the ability to define the crisp values for each $\langle \alpha, \beta, \gamma \rangle$ values on the left and right NCNs of the NBS. As a proposition, this process will come out with a crisp output value from a range based on the neutrosophic feature numbers or the triangular neutrosophic values.

Future researchers can use other surface basis functions, such as the non-uniform rational B-splines (NURBS) geometric model, which could be used to simulate neutrosophic surfaces in the future. Furthermore, this research may be expanded to represent another model such as the B-spline model.

The deneutrosophication of NBS can be utilized in 3D surface modeling to build road surface boundaries, closed boundaries of map perimeter modeling, and calculate the area region based on $\langle \alpha, \beta, \gamma \rangle$ level values. This approach can also be used in engineering applications with uncertain collective data to remodel an entity based on its original uncertainty.

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