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Neutrosophic Log-Gamma Distribution and its Applications to Industrial Growth

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Abstract: This study suggests a novel statistical distribution known as the neutrosophic log-gamma distribution (NLGD) for analyzing the interval value data. The proposed distribution is derived from the transformation method, utilizing the approach of neutrosophic logic. The statistical characteristics of the proposed are studied within a neutrosophic framework. Basic statistical properties such as moments of origin, mode, mean deviation, and other related functions that are commonly employed in statistical applications are derived. In addition to the basic properties of the model, the estimation approach of maximum likelihood is also studied. The derived estimators of the proposed model are further assessed by the simulation method. Comparative research using real-world examples shows that NLGD does a better job of modelling complicated industrial growth and production measures than standard distribution. Application of the proposed model improves our understanding of the theoretical component and our ability to forecast outcomes in statistical applications. This, in turn, leads to improved decision-making and operational efficiency in many industrial sectors.

Keywords: Neutrosophic model, descriptive statistics, uncertainty measures, estimation, simulation.

1. Introduction

The distribution of income size is crucial for all economies when it comes to making decisions on social and economic policies [1] . The distribution of income plays a crucial role in economic and social statistics, serving as the foundation for concentration and Lorenz curves [2], [3] . These curves are essential for measuring inequality and evaluating overall social welfare. Market demand and elasticity are influenced by income distribution, which in turn impacts how firms behave and ultimately affects market equilibrium [4]. Income distribution plays a crucial role in shaping the level of savings within a society, as well as impacting the productive efforts of different societal groups. Understanding the distribution of firm sizes is crucial for economists specializing in industrial organization, government regulators, and courts [5] . In antitrust cases, courts rely on market share data from firms and industries to make informed decisions. Pareto's laws of income distribution laid the foundation for statistical studies on personal income distributions [6]. There

are many statistical distributions used to model income size[6], [7], [8] . One of such distributions is gamma distribution [9], [10]. The Pearson type III distribution, also known as the gamma distribution, is widely utilized in applied statistics for modeling unimodal and positive data. It is commonly employed in analyzing waiting times, income levels, insurance claims, and rainfall patterns.
If the exponential function applied to gamma distribution we obtained the log-gamma distribution which also has very important applications in economics [11-13] . Mathematically, if random variable Y follows gamma distribution with two parameters, then $X = exp(Y)$ follows the log-gamma distribution with probability density function (PDF) given below:

$$f(\mathrm{y}) = \frac{b^p}{\Gamma(p)} \mathrm{y}^{b-1}(\log(\mathrm{y}))^{p-1}, \mathrm{y} \geq 1, \mathrm{p}, \mathrm{b} > 0 \quad (1)$$

where p, and b are both shape parameters of the distribution.
The classical structure of log-gamma distribution is shown in Figure 1.

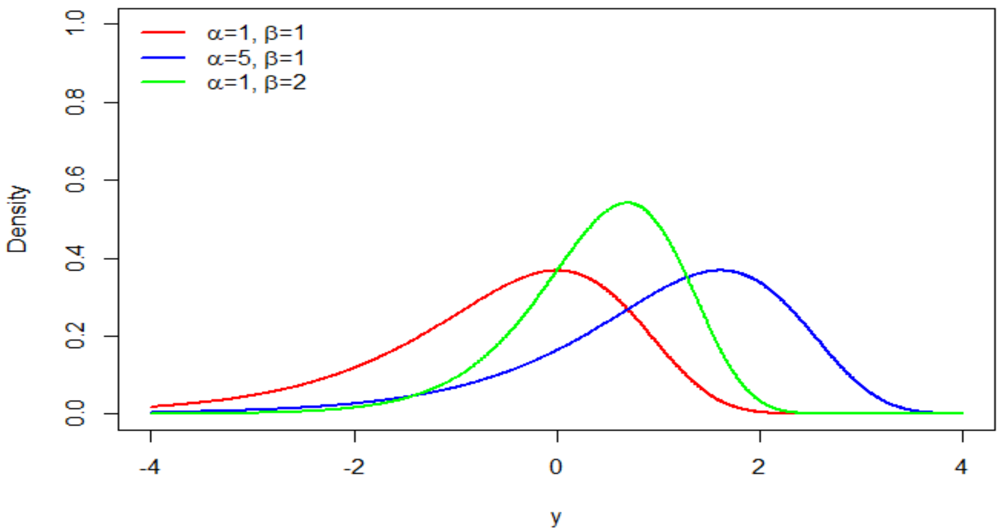

Figure 1: Density plots of the log-gamma distribution

The log gamma distribution is very versatile and can be viewed as the generalized form of the Pareto distribution. Particularly with parameter p = 1, it reduces to the Pareto distribution of type-I. Unlike the Pareto type I distribution, which always has a decreasing density, the log-gamma distribution can have unimodal densities, making it more flexible. The shape of the classical. There are several reasons that log-gamma distribution is very important in statistical applications for business data, finance and engineering studies. The log-gamma distribution is well suited for modelling skewed data. Many real data sets exhibit skewness and log-gamma provides adequate fit compared to other symmetric distributions. In terms of scale and shape parameters log-gamma distribution is symmetric for modeling variety of real data sets and its versatile behaviors [14]. It has a key application in modelling financial data such as profit, loss asset returns, especially when applications exhibit skewness and heavy tails problems. In reliability studies, it is commonly employed to model the lifetime of systems and products. It is highly useful in studying the survival time and in planning maintenance schedules. This also frequently used in medical research for modelling survival data which are often skewed. The log-gamma distribution is also used as prior distribution for many parameters in Bayesian studies. To model extreme events, it is also used in extreme value theory. This helps in understanding risk assessment and mitigation.

Although the log-gamma distribution is useful for skewed and heavy-tailed data, it struggles with imprecise data. Neutrosophic distributions, on the other hand, are tailored to handle uncertainty and imprecision commonly found in real-world situations [15-17]. Neutrosophic collections provide a more thorough approach compared to the traditional collection notion when dealing with ambiguous, vague, and irregular information [18]. The subscription features, namely truth-membership, indeterminacy-membership, and falsity-membership, provide a precise understanding of whether a component belongs to the collection, is uncertain, or does not belong to the collection. This structure also incorporates relevant data and probability distributions to further enhance its efficiency in handling intricate information [19-22]. Neutrosophic statistics, together with possibility distributions, offer valuable methods for analyzing data or uncertainty that is characterized by neutrosophic sets [23-25]. Neutrosophic probability distributions are used to determine the likelihood of events or outcomes in situations where there is ambiguity. They specifically focus on measures such as central tendency, dispersion, and other properties of neutrosophic data [26].

To develop a more adaptable distribution term the NLGD is the primary aim of this study. This neutrosophic model effectively manages the fuzzy applications. The core motivation behind this study is to handle the ambiguous data and enhancing its ability to analyze and interpret such vague information.

This study is organized as follows: Section2 describes the classical model and its extension to neutrosophic model. Section 3 discusses the quantile function and simulation analysis of the NLGD. Section4 examines the estimation strategy for unknown parameters. Section 5 conducts a real data analysis to explain the theoretical part of the proposed model. Finally, Section 6 concludes major findings.

2. Proposed Model

In the neutrosophic framework, the parameters p and b are not fixed numbers rather than belonging to intervals or fuzzy sets. This reflecting the imprecision and uncertainty in the model and can be defined as:

$$f(z;\beta_N,p_N)=\frac{\beta_N^{p_N}}{\Gamma(p_N)}z^{\beta_N-1}(\log z)^{p_N-1},\quad z\geq 1,\ p_N>0,\ \beta_N>0 \qquad (2)$$

where $p_N\epsilon[p_l,p_u]$ and $\beta_N\epsilon[\beta_l,\beta_u]$ are two imprecise shape parameters of the proposed NLGD. The conversion of log-gamma distribution into neutrosophic structure enhanced the capability of the distribution to manage uncertain and imprecise data. The neutrosophic form provide more realistic and flexible data modelling, making it more valuable distribution for handling applications involving ambiguous information. The structure of imprecise distribution can be seen in Figure 2.

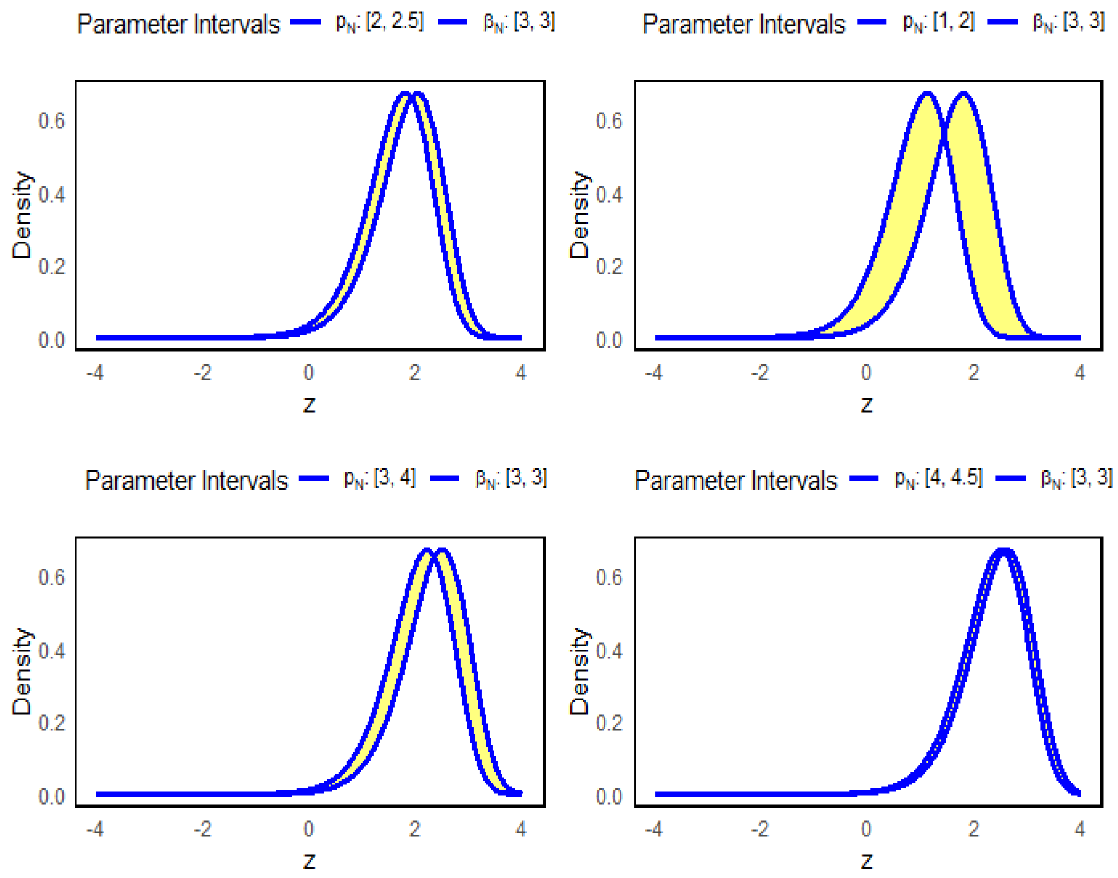

Figure 2: PDF plot of the NLGD for different imprecise parameters setting

Figure 2 visualize the PDF plot of NLGD assuming that shape parameter P_N is imprecise while the β_N is crisp value. Due to not precisely unknown value of shape parameter P_N , uncertainty in the PDF is shown with yellow shaded color each plot. This representation indicates more comprehensive understanding of possible shape of the distribution can take. Neutrosophic PDF helps in visualizing how parametric imprecision affects the distribution, allowing better decision making under uncertain environment. The neutrosophic PDF can have numerous benefits, for example it allows actuaries to view the potential variability in loss distributions that may lead to setting reserves and premiums. Similarly, it also guides the economists to count for uncertainties in economic indicators resulting in more robust forecasting models. Similar to PDF function another associated function is the cumulative distribution function. The neutrosophic CDF function of NLGD can be determined as follows:

$$H(z) = \int_1^z \frac{\beta_N^{p_N}}{\Gamma(p_N)} z^{\beta_N - 1} (\log z)^{p_N - 1} dz \tag{3}$$

Further simplification of (3) yielded:

$$H(z) = \frac{\gamma\left(p_N, \frac{log(z)}{\beta_N}\right)}{Gamma(p_N)} \tag{4}$$

where $\gamma(.)$ representing the incomplete gamma distribution.

The shape of the neutrosophic CD function is depicted in Figure 3.

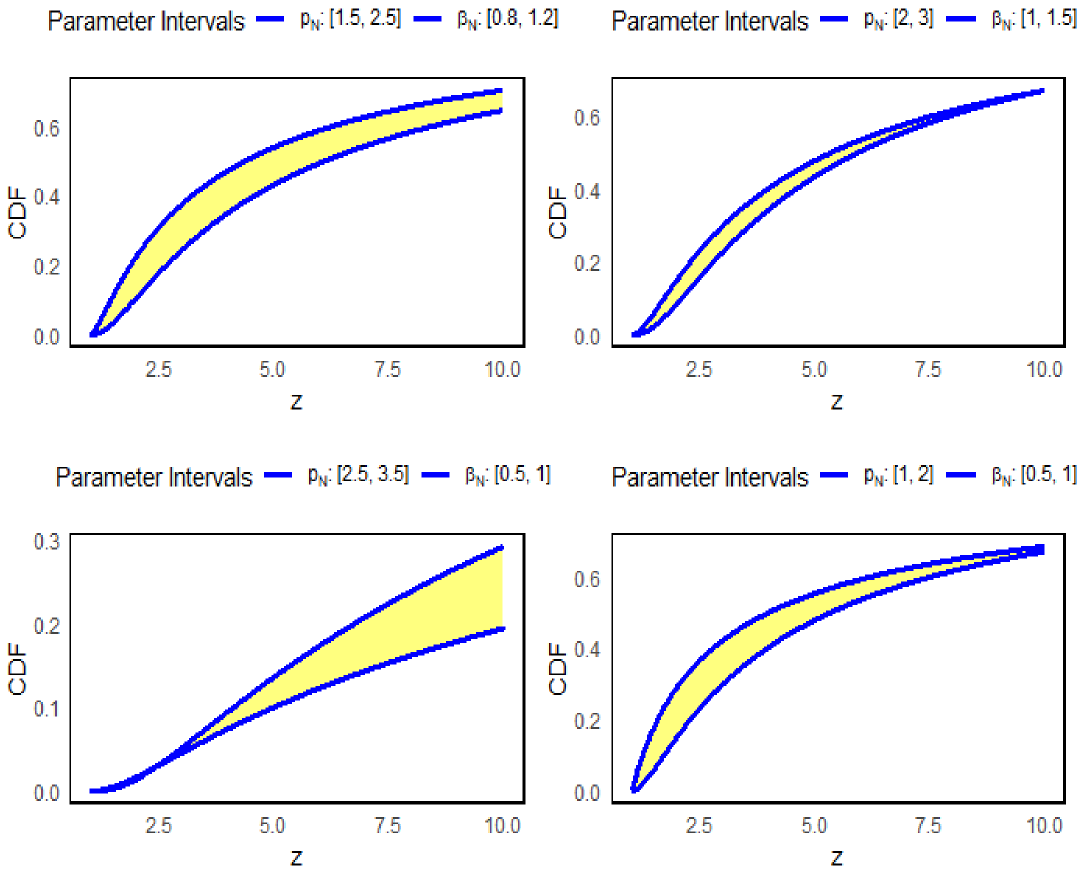

Figure 3: CDF curves of the NLGD with various imprecise parameters.

Figure 3 shows the CDF of the NLGD indicates the cumulative probability of random variable being less than or equal to some particular value. Figure 3 indicates that similar to classic structure plot starts at zero and increases to one with non-decreasing feature. This plot provides a complete description of the NLGD. The yellow area in each plot indicates indeterminacy in the parameters of the distribution. The yellow bounded area encapsulates the possible variations in the CD function due to imprecision in the studied parameters. By plotting the lower and upper bound of the distribution plot, users can assess the potential variability in the cumulative probabilities which quite significant for knowing the behavior of the model over its entire domain.

The other important function related to CDF is the survival function. Survival function is critical concepts used in reliability theory, risk management and survival analysis. This function is important because it provides information about the failure rates or longevity of systems or processes. Mathematically the survival function can easily be established for the NLGD as:

$$S(z) = 1 - H(z) = 1 - \frac{\gamma\left(p_N, \frac{log(z)}{\beta_N}\right)}{Gamma(p_N)} \quad (5)$$

For imprecise values of neutrosophic parameters, the survival function of the NLGD can be depicted in Figure 4.

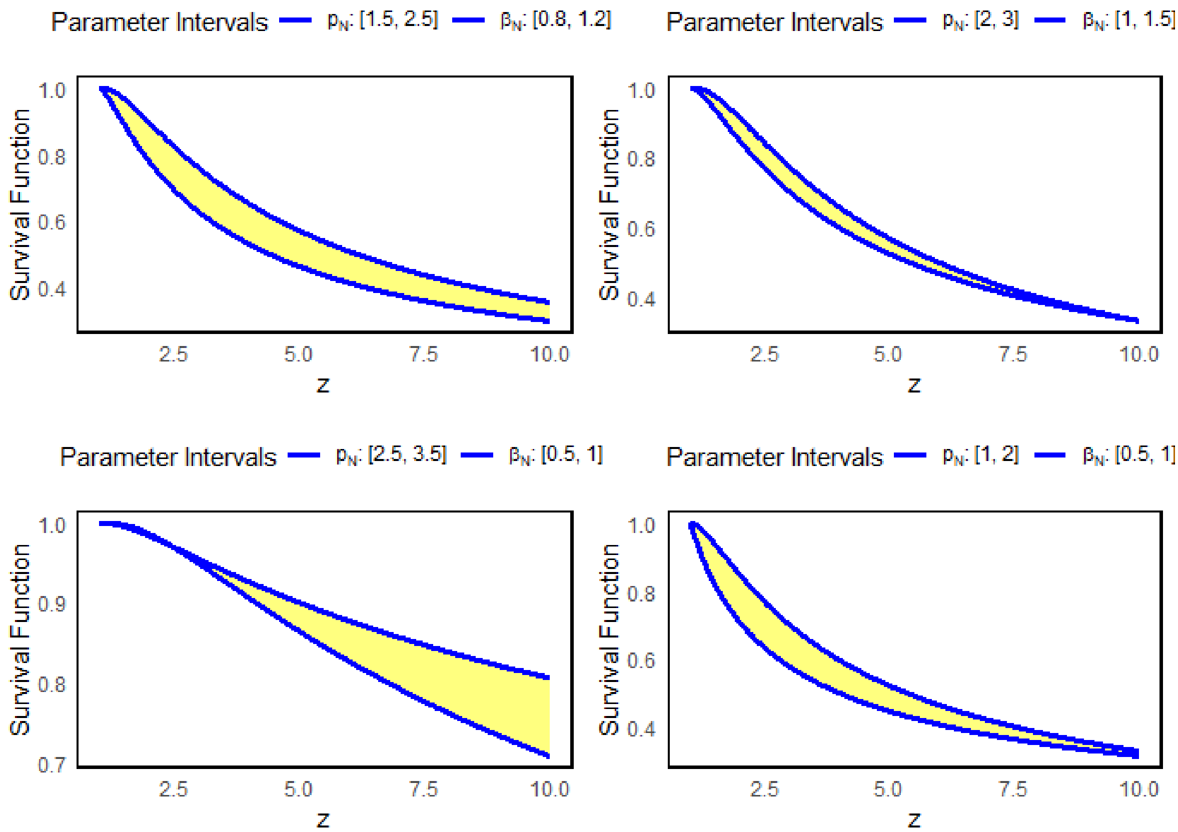

Figure 4: Survival function of the NLGD

As shown in Figure 4, survival function is time dependent function which provides information about the likelihood of systems or component to survival behind the time period t. It is essential information because this information is used in reliability analysis to improve the durability of the products. It helps in making data-driven decisions about warranties of the products and better understanding about the distribution of lifetimes. The ratio between PDF and survival functions provides the hazard function. The hazard function of the NLGD is given by:

$$h(z;\beta_N,p_N)=\frac{\frac{\beta_N^{p_N}}{\Gamma(p_N)}z^{\beta_N-1}(\log z)^{p_N-1}}{1-\frac{\gamma\left(p_N,\frac{\log(z)}{\beta_N}\right)}{\Gamma(p_N)}};\, z\geq 1,\ p_N>0,\ \beta_N>0. \quad (6)$$

Hazard function provides the instantaneous failure rate of component or system at any specific time. It represents the likelihood of death or failure occurring at particular time t given that system or component has been survived up to that time. The hazard function for different values of neutrosophic parameters are shown in Figure 5.

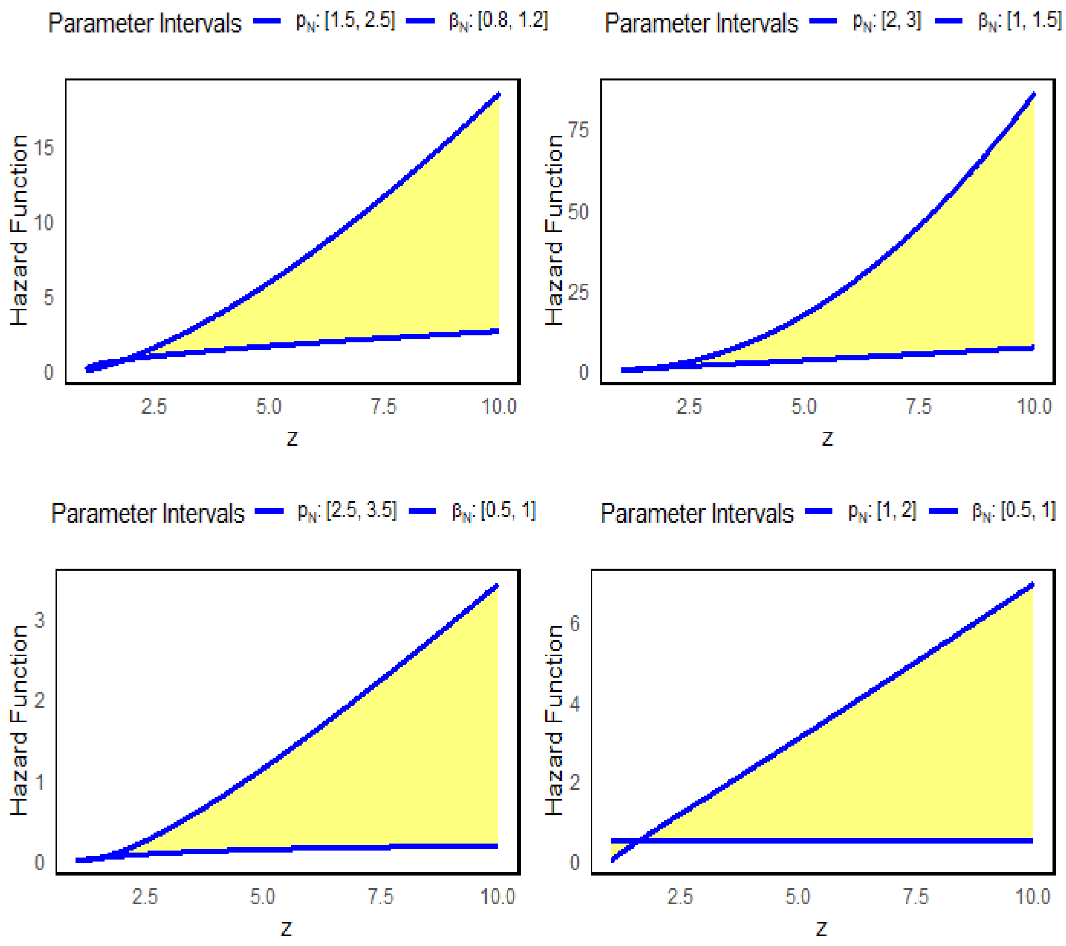

Figure 5: Hazard function of the NLGD for different imprecise parameters setting

As shown in Figure 5, neutrosophic hazard function is different from the classical concept of hazard function. It is represented by a sturdy curve and able to handle vagueness, indeterminacy and uncertainty. This sturdy structure reflects the inherent variability in due to assuming the imprecise values of the distributional parameters.

The kth moment of the NLGD can be written as follows:

$$E(Z^k) = \left(\frac{\beta_N}{\beta_N - k}\right)^{p_N}, \quad k < \beta_N \tag{7}$$

In neutrosophic framework (7) can be derived as follows:

$$E(Z^k) = \int_1^\infty z^k f(z; \beta_N, p_N)\, dz \tag{8}$$

Equation (8) can be further simplified as:

$$E(Z^k) = \int_1^\infty z^k \frac{\beta_N^{p_N}}{\Gamma(p_N)} z^{\beta_N - 1} (\log z)^{p_N - 1}\, dz \tag{9}$$

Solving (9) yielded:

$$E(Z^k) = \frac{\beta_N^{p_N}}{\Gamma(p_N)} \frac{\Gamma(p_N)}{(\beta_N - k)^{p_N}} \tag{10}$$

Hence simplification of (10) provided

$$E(Z^k) = \left(\frac{\beta_N}{\beta_N - k}\right)^{p_N} \tag{11}$$

which is required result.

Now assuming the different of values of k mean and variance can easily be derived.

Thus, the mean of the NLGD is given by:

$$mean = E(Z) = \left(\frac{\beta_N}{\beta_N-1}\right)^{p_N} \quad (12)$$

Similarly, variance from (11) can be written as:

$$\text{var}(Z) = \left(\frac{\beta_N}{\beta_N-2}\right)^{p_N} - \left(\frac{\beta_N}{\beta_N-1}\right)^{2p_N}, \quad \beta_N > 2 \quad (13)$$

Utilizing mean and variance coefficient of variation of the NLGD can be written as:

$$\text{CV} = \frac{\sqrt{\left(\frac{\beta_N}{\beta_N-2}\right)^{p_N} - \left(\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}\right)^2}}{\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}} \quad (14)$$

Similarly, mode of the NLGD can be expressed as:

$$z_{\text{mode}} = \exp\left(\frac{p_N-1}{\beta_N+1}\right) \quad (15)$$

To derive the mode of the NLGD, we need to differentiate the PDF with respect to z and setting it to zero.

i.e.,

$$\frac{d}{dz} f(z) = \frac{\beta_N^{p_N}}{\Gamma(p_N)}\left[(\beta_N - 1) z^{\beta_N-2} (\log z)^{p_N-1} + z^{\beta_N-2} (p_N - 1)(\log z)^{p_N-2}\right] = 0 \quad (16)$$

Simplification of (16) leads to:

$$(\beta_N - 1) \log z + (p_N - 1) = 0 \quad (17)$$

Further simplification yielded:

$$z_{\text{mode}} = \exp\left(\frac{p_N-1}{\beta_N+1}\right) \quad (18)$$

which is required result.

Utilizing (11) , third and fourth moments respectively can be written as:

$$E(Z^3) = \left(\frac{\beta_N}{\beta_N-3}\right)^{p_N} \quad (19)$$

$$E(Z^4) = \left(\frac{\beta_N}{\beta_N-4}\right)^{p_N} \quad (20)$$

Using (19) and (20), coefficients of skewness and kurtosis can be written as:

$$\gamma_1 = \frac{\left(\frac{\beta_N}{\beta_N-3}\right)^{p_N} - 3\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N} \sigma^2 - \left(\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}\right)^3}{\sigma^3} \quad (21)$$

$$\gamma_2 = \frac{\left(\frac{\beta_N}{\beta_N-4}\right)^{p_N} - 4\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}\left(\frac{\beta_N}{\beta_N-3}\right)^{p_N} + 6\left(\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}\right)^2\left(\frac{\beta_N}{\beta_N-2}\right)^{p_N} - 3\left(\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}\right)^4}{\left(\left(\frac{\beta_N}{\beta_N-2}\right)^{p_N} - \left(\left(\frac{\beta_N}{\beta_N-1}\right)^{p_N}\right)^2\right)^2} \quad (22)$$

Likewise other important functions and key statistical properties can be established in the neutrosophic framework. Table 1 shows some basic suggested model based on neutrosophic parameters.

Table 1: Computation of basic properties of proposed model

β_N	p_N	Mean	variance	CV
[2.5,3.5]	[1, 2]	[1,5.44]	[0,48]	[0,6.92]
[3.1,4.0]	[2,4]	[1.06,13.16]	[0,173.73]	[0,12.34]
[4,4.5]	[3,4]	[1.42,5.06]	[0,23.40]	[0,3.24]
[5,5.5]	[4,5]	[1.52, 4.91]	[0,18.38]	[0,2.81]
[6,6.5]	[5, 6]	[1.54, 4.82]	[0,16.02]	[0,2.5]
[7, 7.5]	[7, 8]	[1.67,5.96]	[0, 22.8]	[0,2.84]

Table1 indicates the values in intervals, reflecting the imprecision and vagueness existed in the distribution parameters. Some intervals particularly related to variance and CV include zero, indicating that the distribution can be perfectly predictable in some cases. Generally, CV tends to decrease as values of the parameters increase.

3. Quantile Function

The quantile function of the NLGD can be calculated easily by inversing the CDF function. However, the quantile function of the NLGD does not have no closed form expression that makes it difficult to solve it analytically. Instead, some numerical solution will be applicable to find random numbers generation from the NLGD. Finding the quantile function via inverse CDF method is a fundamental concept in statistical applications and used essentially for generating random samples from the NLGD. Using quantile function on uniform numbers, one can generate random samples from the target model.

In the neutrosophic framework the quantile function can be written as :

$$Q(u) = [Q_L(u), Q_U(u)] \tag{23}$$

The exact expression for (23) is not straightforward and may be expressed as:

$$Q(u) = \left[{Q_L}^{-1}(u), {Q_U}^{-1}(u)\right] \tag{24}$$

To find the numerical solution of (24), the Newton Raphson method may be used. It is an iterative procedure to find successively better approximation to the root of real value function. Given the $F(z)$, the iteratively updates the estimate z_n as follows:

$$z_{n+1} = z_n - \frac{F(z_n)-u}{F(\acute{z_n})} \tag{25}$$

where $F(\acute{z_n})$ is the derivative of $F(z_n)$. This method needs the derivative of CDF which is an essential component of the fundamental equation. The used of Newton Raphson method is most effective in initial guess is chosen closer to true value. To find the random samples from the NLGD, a program written in R has been utilized. Let we assume that the proposed model with $\beta_N = [2.5, 3.5\ ]$ and $p_N = [1, 2]$. The random generated samples with some specific seed setting, are shown in Table 2.

Table 2: Random samples generation from the NLGD.

Random Samples				
[1.31, 1.49]	[1.72, 1.84]	[1.42, 1.58]	[1.81,1.93]	[1.90, 2.00]
[0.88, 1.14]	[1.51, 1.66]	[1.82, 1.94]	[1.53, 1.68]	1.45, 1.62]
[1.93, 2.03]	[1.45, 1.62]	[1.62, 1.76]	[1.54, 1.69]	[1.05, 1.28]

Table 2 show 15 random samples generated through computer program written in R. The R package "VGAM" has been utilized for random samples generation of the NLGD. Table 2 indicates that values are in intervals due to assumed vagueness in the parameters of the proposed model.

4. Estimation Procedure

In this section, analytical results of the NLD for moments, skewness, and kurtosis have been validated using the Monte Carlo simulation. The NLD can be readily simulated in R software to assess the validity of theory-based results.

To estimate the parameters of the NLGD, we can focus on the commonly used method of maximizing the log-likelihood function of the NLGD. This method is well known in statistical analysis and commonly used known as maximum likelihood estimation. In this procedure we try to find the best fit parameters of shape and scale parameters that maximize the likelihood of the observed sample. Let for the data values $Z = z_1, z_2, \dots z_n$ assume to follow the NLGD then the log-likehood function can be written as:

$$\mathcal{L}(\beta_N, p_N; \{z_i\}) = \sum_{i=1}^{n} \left[\log\left(\frac{\beta_N^{p_N}}{\Gamma(p_N)}\right) + (\beta_N - 1)\log z_i + (p_N - 1)\log(\log z_i)\right] \quad (26)$$

Further simplification of (26) yielded:

$$\mathcal{L}(\beta_N, p_N; \{z_i\}) = n\log\left(\frac{\beta_N^{p_N}}{\Gamma(p_N)}\right) + (\beta_N - 1)\sum_{i=1}^{n}\log z_i + (p_N - 1)\sum_{i=1}^{n}\log(\log z_i) \quad (27)$$

Partial derivatives of (27) with respect to β_N and p_N resulted in respectively:

$$\frac{\partial\mathcal{L}}{\partial\beta_N} = n\frac{p_N}{\beta_N} + \sum_{i=1}^{n}\log z_i \quad (28)$$

$$\frac{\partial\mathcal{L}}{\partial p_N} = n\left(\log\beta_N - \frac{\Gamma'(p_N)}{\Gamma(p_N)}\right) + \sum_{i=1}^{n}\log(\log z_i) \quad (29)$$

Now it is evident that analytical solution of (28) and (29) are not possible, instead some numerical approach would be used to find the optimum values of β_N and p_N. We have utilized BFGS optimization method to find the values of β_N and p_N. A program written in R has been utilized to find the optimum values of shape and scale parameters of the NLGD. The objective of this simulation program is to find the optimum estimates and the calculate the corresponding mean squared error (MSE) at various sample sizes. Random sample of sizes $\{n = 25, 50, 75,\ \ 100\ and\ 120\}$

are drawn from the NLGD with $p_N = [1,1]$ and $\beta_N = [2.5,3.5]$. To ensure sufficient statistical power each sample size is simulated 1000 times to find the estimates and corresponding MSE. Results from this simulation are shown in Table 3.

Table 3: Estimated parameters of the NLGD with MSE using simulated data

Sample Size	Estimated (β_N)	Estimated (p_N)	MSE (β_N)	MSE (p_N)
25	[2.54, 4.01]	[2.41, 3.91]	[0.85, 2.58]	[2.16,11.49]
50	[3.75, 3.75]	[1.01, 4.21]	[0.32, 1.30]	[1.12, 6.55]
75	[2.15, 4.09]	[1.76, 4.83]	[0.24, 1.03]	[0.92, 5.19]
100	[1.97, 3.49]	[1.68, 4.21]	[0.22, 0.97]	[0.84, 4.83]
120	[2.11, 2.59]	[1.92, 3.07]	[0.20, 1.00]	[0.83, 4.41]

Table 3 shows the results from the simulation data, where each dataset is simulated for specified sample size. The estimated parameters and their MSE(s) are given in ranges due to assumed indeterminacy in the parameters of underlying model. Results show that as the sample size increases, the estimated values of parameters tend to narrow, indicating precise estimation with larger sample sizes. Overall MSE of the estimated values decrease with increasing sample size. This indicates that more reliable and accurate results are expected with larger sample sizes.

5 Real Data Application

In this section, we have utilized the suggested distribution model on actual carbon dioxide (CO2) data obtained from the World Bank for Saudi Arabia [27]. The release of CO2 has a notable effect on a country's climate. To do high emissions of CO2, air quality in urban areas of Kingdom is highly compromised because of industrial activities. This climate indicator is a focal point of research globally and plays a crucial role in the ongoing changes to the world's climate. The primary cause of carbon emissions stems from the combustion of fossil fuels during industrial activities and deforestation. The Kingdom of Saudi Arabia holds a prominent position in the global oil market, presenting both distinct challenges and opportunities when it comes to managing carbon emissions. It is crucial to study the effects of CO2 on public health and take measures to reduce its emission on a smaller scale. The Kingdom, known for its desert and arid environment, is especially at risk from climate change. The changes in the climate have a profound impact not only on the country's weather patterns but also contribute significantly to heatwaves, global warming, and harsh weather conditions. As a result, these weather phenomena have repercussions on natural resources like water availability, shifts in climate patterns, and affect agricultural productivity. On the other hand, due to major supplier oil, Kingdom highly depends on fossile export. Transitioning to renewable energy sources particularly solar power can help decrease carbon emissions. Utilization of country vast desert zone and solar industry can provide healthy environment. Due to environmental and instrumental factors, carbon emissions values are not always precise and can vary. In order to determine the significance of a proposed method, exact carbon emission values are transformed into interval values using a procedural methodology [28]. The carbon emission values in time span of 2005 to 2020 are given in Table 4.

Table 4: Carbon emission values for Saudi Arabia time period 2005-2020

Carbon emission values				
[12.63,13.21]	[12.42, 14.00]	[13.02,13.84]	[13.32, 15.08]	[13.33, 15.21]
[15.12, 15.21]	[14.85, 15.90]	[15.08, 16.87]	[15.43, 16.53]	[16.36,17.28]
[16.30, 18.21]	[16.34, 17.24]	[15.40, 16.75]	[14.49, 16.63]	[14.60, 14.80]

Interval values in Table 4 show that imprecise values are available for analysis due to technical malfunctions and limitations of measuring devices or sensors. Carbon emissions are usually measured by air sampling procedures or gas emission at specific points. Thus, due limitations of these procedures, collected samples can be considered compromised and not available in exact numbers. If we fit the conventional model of log-gamma distribution on carbon emission values, fitting graph can be seen in Figure 6.

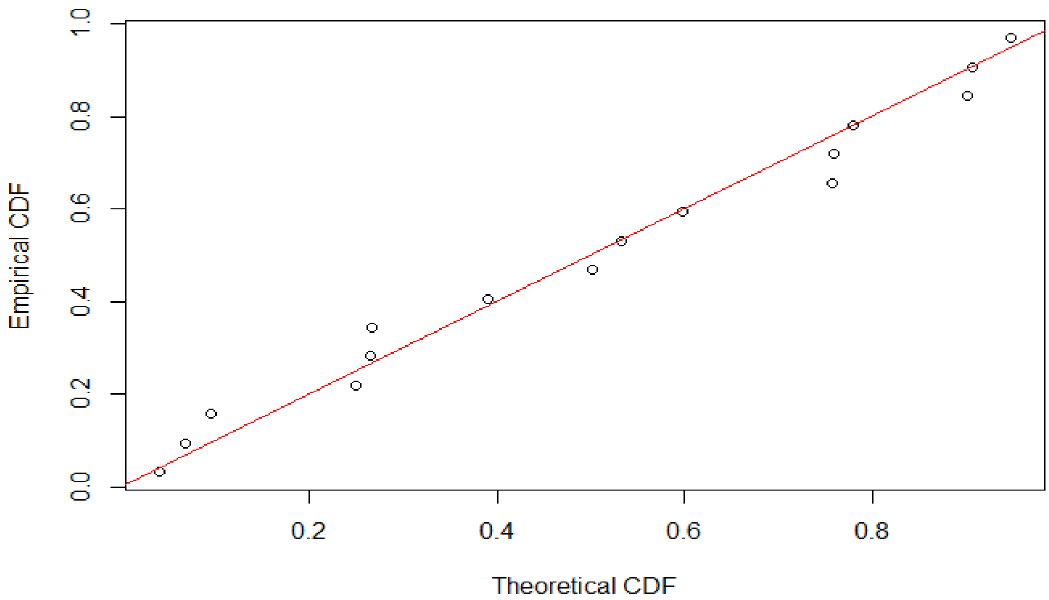

Figure 6: Fitting plot of log-gamma distribution on carbon emissions data

Figure 6 shows that log-gamma distribution is better fit model the carbon emissions data. However, the existing log-gamma distribution could not be used to analysis imprecise data given in Table 4. A statistical description of the data in terms of estimated parameters of the NLGD are given in Table 5.

Table 5: Estimated parameters of the NLGD for carbon emissions data

Statistical measures	Computed values
Estimated (β_N)	[7.49, 8.48]
Estimated (p_N)	[123.75, 123.91]

Results in Table 5 show that estimated values of the parameters are in interval forms due to vagueness in the underlying data. These imprecise estimated values can be used to find the statistical characteristics of the carbon emissions data. The neutrosophic model in this way is more generic as compared to existing model.

6 Conclusions

In this study, a novel distribution called NLGD has been introduced for analyzing neutrosophic data. The NLGD distribution shows great efficacy in analyzing data with interval values. By employing the neutrosophic logic, the suggested distribution has established a statistical model for describing interval characteristics such as mean, mode, variance, and shape coefficients. Key functions like the reliability function and hazard function have been devloped due to the widespread use of the proposed model in reliability analysis. These functions are essential for understanding and analyzing the model utility in survival analysis. The MLE method has been used to estimate the parameters of the proposed model under an indeterminate environment. A simulation has been conducted to evaluate the accuracy of the established model and examine the impact of sample size on the results. Based on the simulation results, it can be concluded that larger sample sizes lead to more precise estimation. At long last, a genuine data set on carbon emissions has been employed to analyze both the theoretical framework of the study and the practical implementation of the proposed model.

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References

1. Y. Atikankul, A. Thongteeraparp, W. Bodhisuwan, and A. Volodin, "The Length-Biased Weighted Lindley Distribution with Applications," Lobachevskii Journal of Mathematics, 41(3), 2020.
2. T. Sitthiyot and K. Holasut, "A simple method for estimating the Lorenz curve," Humanities and Social Sciences Communications, 8(1), 2021.
3. M. Fajar, S. Setiawan, and N. Iriawan, "The New Measures of Lorenz Curve Asymmetry: Formulation and Hypothesis Testing," Decision Making: Applications in Management and Engineering, 7(1), 2024.
4. C. Kroer, A. Peysakhovich, E. Sodomka, and N. E. Stier-Moses, "Computing Large Market Equilibria Using Abstractions," Operational Research, 70(1), 2022.
5. G. Issayeva et al., "The Relationship between Renewable Energy Consumption, CO2 Emissions, Economic Growth, and Industrial Production Index: The Case of Kazakhstan," International Journal of Energy Economics and Policy, 13(6), 2023.
6. Y. Bilan, H. Mishchuk, N. Samoliuk, and H. Yurchyk, "Impact of Income Distribution on Social and Economic Well-Being of the State," Sustainability (Switzerland), 12(1), 2020.
7. C. Panico, A. Pinto, and M. P. Anyul, "Income distribution and the size of the financial sector: A Sraffian analysis," Cambridge Journal of Economics, 36(6), 2012.
8. R. Becerril-Arreola, R. E. Bucklin, and R. Thomadsen, "Effects of income distribution changes on assortment size in the mainstream grocery channel," Management Science, 67(9) 2021.
9. X. Ke, S. Wang, M. Zhou, and H. Ye, "New Approaches on Parameter Estimation of the Gamma Distribution," Mathematics, 11(4), 2023.
10. H. S. Klakattawi, "The Weibull-Gamma distribution: Properties and applications," Entropy, 21(5), 2019.
11. L. J. Halliwell, "The Log-Gamma Distribution and Non-Normal Error," Variance (ISSN 1940-6452), 2017.
12. A. M. M. Ibrahim and Z. Khan, "Neutrosophic Laplace Distribution with Properties and Applications in Decision Making," International Journal of Neutrosophic Science, vol. 23(1), 2024.
13. H. Y. A. Shihabeldeen and Z. Khan, "Neutrosophic Structure of the Log-logistic Model with Applications to Medical Data," International Journal of Neutrosophic Science, vol. 23(1), 2024.
14. M. N. Shahzad and Z. Asghar, "Parameter Estimation of Singh Maddala Distribution by Moments," International Journal of Advanced Statistics and Probability, 1(3), 2013.
15. Smarandache, F. Neutrosophic set-a generalization of the intuitionistic fuzzy set. International Journal of Pure and Applied Mathematics, 24(3), 287, 2005.
16. Broumi, S., Sundareswaran, R., Shanmugapriya, M. ., Nordo, G., Talea, M. ., Bakali, A., and Smarandache, F. Interval- valued fermatean neutrosophic graphs, Decision Making: Applications in Management and Engineering, Collected Papers, 5(2), 176–200, 2022. https://doi.org/10.31181/dmame0311072022b
17. F. Smarandache, X. Zhang, and M. Ali, Algebraic Structures of Neutrosophic Triplets, Neutrosophic Duplets, or Neutrosophic Multisets, Symmetry 2019,11(2), p. 171, 2019
18. Khan, Z., Gulistan, M., Hashim, R., Yaqoob, N., & Chammam, W. Design of S-control chart for neutrosophic data: An application to manufacturing industry. Journal of Intelligent & Fuzzy Systems, 38(4), 4743–4751, 2020.
19. Smarandache, F. A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability. Infinite Study, 2005.
20. Smarandache, F., Introduction to Neutrosophic Statistics, Sitech & Education Publishing, Craiova, 2014, 124 p.
21. Z. Khan, A. Al-Bossly, M. M. A. Almazah, and F. S. Alduais, On Statistical Development of Neutrosophic Gamma Distribution with Applications to Complex Data Analysis, Complexity, vol. 2021, pp. 1-8, 2021.
22. U. Rivieccio, Neutrosophic logics: Prospects and problems, Fuzzy Sets and Systems, vol. 159(4), pp. 1860–1868, Jul. 2008.

23. S. Broumi and F. Smarandache, Several similarity measures of neutrosophic sets, Infinite Study, 410(1), 2013.

24. A. M. M. Ibrahim and Z. Khan, Neutrosophic Laplace Distribution with Properties and Applications in Decision Making, International Journal of Neutrosophic Science, 23(1), 2024

25. H. Y. A. Shihabeldeen and Z. Khan, Neutrosophic Structure of the Log-logistic Model with Applications to Medical Data, International Journal of Neutrosophic Science, vol. 23, no. 1, 2024.

26. F. Smarandache et al., Introduction to neutrosophy and neutrosophic environment, Neutrosophic Set in Medical Image Analysis, pp. 3–29, 2019.

27. World bank data (https://data.worldbank.org/)

28. Khan, Z., Gulistan, M., Chammam, W., Kadry, S., & Nam, Y. A new dispersion control chart for handling neutrosophic data. IEEE Access, 8, 96006-96015, 2020.

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