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Refined Neutrosophic Information Based on Truth, Falsity, Ignorance, Contradiction and Hesitation

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Abstract. In this article, starting from primary representation of neutrosophic information, namely the triplet (μ, ω, ν) made up of the degree of truth μ, degree of indeterminacy ω and degree of falsity ν, we define a refined representation in a penta-valued fuzzy space, described by the index of truth t, index of falsity f, index of ignorance u, index of contradiction c and index of hesitation h. In the proposed penta-valued refined representation the indeterminacy was split into three sub-indeterminacies such as ignorance, contradiction and hesitation. The set of the proposed five indexes represents the similarities of the neutrosophic information (μ, ω, ν) with these particular values: T=(1,0,0), F=(0,0,1), U=(0,0,0), C=(1,0,1) and H=(0.5,1,0.5). This representation can be useful when the neutrosophic information is obtained from bipolar information which is defined by the degree of truth and the degree of falsity to which is added the third parameter, its cumulative degree of imprecision.

Keywords: Neutrosophic information, refined representation, hesitation, contradiction, ignorance, falsity, truth, ambiguity.

1 Introduction

The neutrosophic representation of information was proposed by Florentin Smarandache [6], [13-22] and it is a generalisation of intuitionistic fuzzy representation proposed by Krassimir Atanassov [1-4] and also for fuzzy representation proposed by Lotfi Zadeh [23]. The neutrosophic representation is described by three parameters: degree of truth μ, degree of indeterminacy ω and degree of falsity ν. In this paper we assume that the parameters $\mu, \omega, \nu \in [0,1]$.

The representation space (μ, ω, ν) is a primary space for neutrosophic information. Starting from primary space, it can be derived other more nuanced representations belonging to multi-valued fuzzy spaces where the set of parameters defines fuzzy partitions of unity. In these multi-valued fuzzy spaces, at most four parameters of representation are different from zero while all the others are zero [7], [8], [9], [10].

In the following, the paper has the structure: Section 2 presents previous works: two penta-valued representations. In the first representation, the indeterminacy was split in neutrality, ignorance and saturation while in the second the indeterminacy was split into neutrality, under-definedness and over-definedness; Section 3 presents the construction of two multi-valued representation for bipolar information.

The first is based on Belnap logical values, namely true, false, unknown and contradictory while the second is based on a new logic that was obtained by adding to the Belnap logic the fifth value: ambiguity; Section 4 presents two variants for penta-valued representation of neutrosophic information based on truth, falsity, ignorance, contradiction and hesitation; Section 5 presents a penta-valued logic that uses the following values: true, false, unknown, contradictory and hesitant; Section 6 presents five operators for the penta-valued structures constructed in section 4. Firstly, it was defined two binary operators namely union and intersection, and secondly, three unary operators, namely complement, negation and dual. All these five operators where defined in concordance with the logic presented in the section 5; The last section outlines some conclusions.

2 Previous works

It was constructed two representations using penta-valued fuzzy spaces [7], [8], [9]. One that is based on truth, falsity, neutrality, ignorance and saturation and the other that is based on truth, falsity, neutrality, under-definedness and over-definedness.

Below is a brief overview of these variants.

2.1 Penta-valued representation of neutrosophic information based on truth, falsity, neutrality, ignorance and saturation

We can define a penta-valued partition with five indexes: index of truth, index of falsity, index of neutrality, index of ignorance and index of saturation by:

$$t = \mu - \frac{\min(\mu,\omega) + \min(\mu,\nu)}{2} \quad (2.1.1)$$

$$f = \nu - \frac{\min(\nu,\omega) + \min(\mu,\nu)}{2} \quad (2.1.2)$$

$$n = \omega - \frac{\min(\mu,\omega) + \min(\nu,\omega)}{2} \quad (2.1.3)$$

$$u = 1 - max(\mu,\omega,\nu) \quad (2.1.4)$$

$$s = min(\mu,\omega,\nu) \quad (2.1.5)$$

These five indexes verify the condition of partition of unity, namely:

$$t + f + n + u + s = 1 \quad (2.1.6)$$

Also, there exists the equality:

$$t \cdot f \cdot n = 0 \quad (2.1.7)$$

Having this representation, the neutrosophic information could be *true*, *false*, *neutral*, *unknown*, and *saturated*.
These five information features have the following prototypes: $T = (1,0,0)$; $F = (0,0,1)$; $N = (0,1,0)$; $S = (1,1,1)$; $U = (0,0,0)$. The geometrical representation of this construction can be seen in the figure 1.

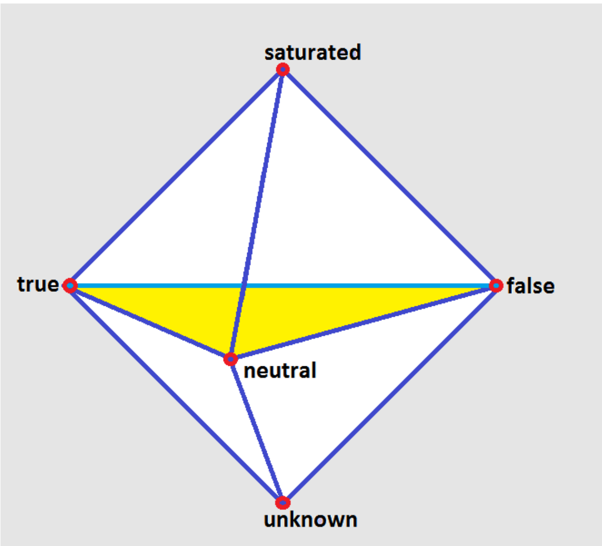

Fig.1. The geometrical representation for the penta-valued space based on true, false, neutral, unknown and saturated.

Also, we can define the inverse transform from the penta-valued space (t, f, n, u, s) to the primary three-valued space (μ, ω, ν) using the next formulae:

$$\mu = t + \min(t, f) + \min(t, n) + s$$

$$\omega = n + \min(t, n) + \min(f, n) + s$$

$$\nu = f + \min(t, f) + \min(f, n) + s$$

2.2 Penta-valued representation of neutrosophic information based on truth, falsity, neutrality, under-definedness and over-definedness

Firstly, we will define the neutrosophic definedness.
Before the definedness construction, we will denote the mean of neutrosophic components:

$$\lambda = \frac{\mu + \nu + \omega}{3} \quad (2.2.1)$$

The neutrosophic definedness is described by a function: $\delta : [0,1] \to [-1,1]$ having the following properties:

i) $\delta(0) = -1$

ii) $\delta\left(\frac{1}{3}\right) = 0$

iii) $\delta(1) = 1$

iv) δ increases with its argument

Here are some examples:

$$\delta(\lambda) = \frac{3\lambda - 1}{1 + \lambda} \quad (2.2.2)$$

$$\delta(\lambda) = 2\sin\left(\lambda \frac{\pi}{2}\right) - 1 \quad (2.2.3)$$

$$\delta(\lambda) = \frac{7\lambda - 3\lambda^2}{2} - 1 \quad (2.2.4)$$

$$\delta(\lambda) = \frac{9\lambda - 3 - |3\lambda - 1|}{4} \quad (2.2.5)$$

$$\delta(\lambda) = \frac{\sqrt{2\lambda} - \sqrt{1-\lambda}}{\sqrt{2\lambda} + \sqrt{1-\lambda}} \quad (2.2.6)$$

If the neutrosophic definedness is positive, the information is inconsistent or over-defined, if it is zero, the neutrosophic information is consistent or complete and if it is negative, the information is incomplete or under-defined. We denote by:

$$\delta^{+} = \max(\delta, 0) \tag{2.2.7}$$

$$\delta^{-} = \max(-\delta, 0) \tag{2.2.8}$$

Using the neutrosophic definedness we define index of *truth*, index of *falsity*, index of *neutrality*, index of *over-definedness* and index of *under-definedness* by:

$$t = \frac{1-\delta^{+}}{3\lambda + \delta^{-}} \cdot \mu \tag{2.2.9}$$

$$n = \frac{1-\delta^{+}}{3\lambda + \delta^{-}} \cdot \omega \tag{2.2.10}$$

$$f = \frac{1-\delta^{+}}{3\lambda + \delta^{-}} \cdot \nu \tag{2.2.11}$$

$$o = \delta^{+} \tag{2.2.12}$$

$$u = \frac{\delta^{-}}{3\lambda + \delta^{-}} \tag{2.2.13}$$

These five parameters verify the condition of fuzzy partition of unity, namely:

$$t + n + f + o + u = 1 \tag{2.2.14}$$

with $u \cdot o = 0$.
Having this representation, the neutrosophic information could be *true*, *false*, *neutral*, *over-defined* and *under-defined*.
For this penta-valued representation the *indeterminacy* has three components: *neutrality*, *over-definedness* and *under-definedness*, namely:

$$i = n + o + u \tag{2.2.15}$$

We must draw attention to the difference between *saturation* that represents the similarity to the vector $(1,1,1)$ and the *over-definedness* that is related to the inequality $\mu + \omega + \nu > 1$. In the same time, for both parameters, the maximum is obtained for $\mu = \omega = \nu = 1$.
Also, the *ignorance* supplies a similarity to the vector $(0,0,0)$ while the *under-definedness* represents a measure of the inequality $\mu + \omega + \nu < 1$ and the maximum is obtained for $\mu = \omega = \nu = 0$.
In figure 2 we can see the geometrical representation of this construction.

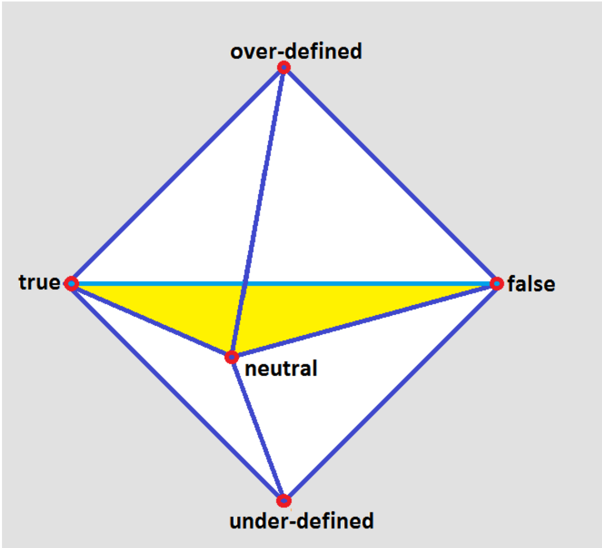

Fig.2. The geometrical representation for the penta-valued space based on true, false, neutral, under-defined and over-defined.

3 Tetra and penta-valued representation of bipolar information

The bipolar information is defined by the degree of truth μ and the degree of falsity ν. Also, it is associated with a degree of certainty and a degree of uncertainty. The bipolar uncertainty can have three features well outlined: ambiguity, ignorance and contradiction. All these three features have implicit values that can be calculated using the bipolar pair (μ, ν).
In the same time, ambiguity, ignorance and contradiction can be considered features belonging to indeterminacy but to an implicit indeterminacy. We can compute the values of these implicit features of indeterminacy. First we calculate the index of ignorance π and index of contradiction κ:

$$\pi = 1\text{-}\min(1, \mu + \nu) \tag{3.1}$$

$$\kappa = \max(1, \mu + \nu) - 1 \tag{3.2}$$

There is the following equality:

$$\mu + \nu + \pi - \kappa = 1 \tag{3.3}$$

which turns into the next tetra valued partition of unity:

$$(\mu - \kappa) + (\nu - \kappa) + \pi + \kappa = 1 \tag{3.4}$$

The four terms form (3.4) are related to the four logical values of Belnap logic: true, false, unknown and contradictory [5]. Further, we extract from the first two terms the bipolar ambiguity α:

$$\alpha = 2 \cdot min(\mu - \kappa, \nu - \kappa) \tag{3.5}$$

The formula (3.5) has the following equivalent forms:

$$\alpha = 2\, min(\mu, \nu) - 2\kappa \tag{3.6}$$

$$\alpha = 1 - |\mu - \nu| - |\mu + \nu - 1| \tag{3.7}$$

$$\alpha = 1 - max(|2\mu - 1|, |2\nu - 1|) \tag{3.8}$$

Moreover, on this way, we get the two components of bipolar certainty: index of truth τ^+ and index of falsity τ^-:

$$\tau^+ = \mu - \kappa - \frac{\alpha}{2} \tag{3.9}$$

$$\tau^- = \nu - \kappa - \frac{\alpha}{2} \tag{3.10}$$

having the following equivalent forms:

$$\tau^+ = \mu - \min(\mu, \nu) \tag{3.11}$$

$$\tau^- = \nu - \min(\mu, \nu) \tag{3.12}$$

So, we obtained a penta-valued representation of bipolar information by $(\tau^+, \tau^-, \alpha, \pi, \kappa)$. The vector components verify the partition of unity condition, namely:

$$\tau^+ + \tau^- + \alpha + \pi + \kappa = 1 \tag{3.13}$$

The bipolar entropy is achieved by adding the components of the uncertainty, namely:

$$e = \alpha + \pi + \kappa \tag{3.14}$$

Any triplet of the form (μ, ν, i) where i is a combination of the entropy components (α, π, κ) does not define a neutrosophic information, it is only a ternary description of bipolar information.
In the following sections, the two representations defined by (3.4) and (3.13) will be used to represent the neutrosophic information in two penta-valued structures.

4 Penta-valued representation of neutrosophic information based on truth, falsity, ignorance, contradiction and hesitation

In this section we present two options for this type of penta-valued representation of neutrosophic information.

4.1 Option (I)

Using the penta-valued partition (3.13), described in Section 3, first, we construct a partition with ten terms for neutrosophic information and then a penta-valued one, thus:

$$(\tau^+ + \tau^- + \alpha + \pi + \kappa)(\omega + 1 - \omega) = 1 \tag{4.1.1}$$

By multipling, we obtain ten terms that describe the following ten logical values: weak true, weak false, neutral, saturated, hesitant, true, false, unknown, contradictory and ambiguous.

$$t_w = \omega\tau^+$$

$$f_w = \omega\tau^-$$

$$n = \omega\pi$$

$$s = \omega\kappa$$

$$h = \omega\alpha$$

$$t = (1 - \omega)\tau^+$$

$$f = (1 - \omega)\tau^-$$

$$u = (1 - \omega)\pi$$

$$c = (1 - \omega)\kappa$$

$$a = (1 - \omega)\alpha$$

The first five terms refer to the upper square of the neutrosophic cube (fig. 3) while the next five refer to the bottom square of the neutrosophic cube (fig. 4).
We distribute equally the first four terms between the fifth and the next four and then the tenth, namely the ambiguity, equally, between true and false and we obtain:

$$t = (1 - \omega)\tau^+ + \frac{\omega\tau^+}{2} + \frac{(1 - \omega)\alpha}{2}$$

$$f = (1 - \omega)\tau^- + \frac{\omega\tau^-}{2} + \frac{(1 - \omega)\alpha}{2}$$

$$u = (1 - \omega)\pi + \frac{\omega\pi}{2}$$

$$c = (1 - \omega)\kappa + \frac{\omega\kappa}{2}$$

$$h = \omega\alpha + \frac{\omega\tau^+}{2} + \frac{\omega\tau^-}{2} + \frac{\omega\pi}{2} + \frac{\omega\kappa}{2}$$

then, we get the following equivalent form for the five final parameters:

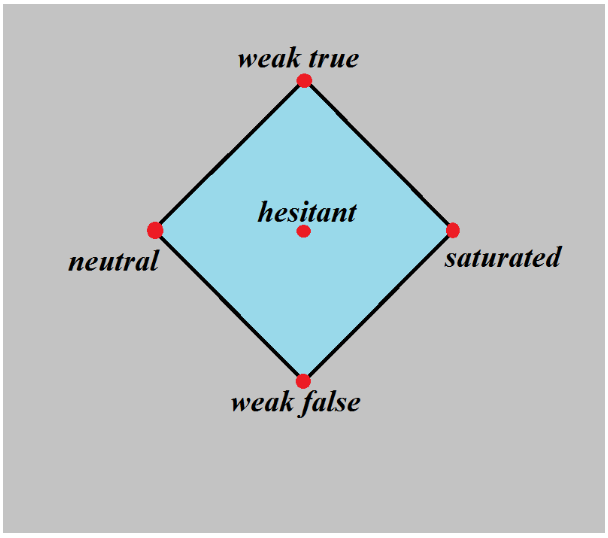

Fig. 3. The upper square of neutrosophic cube and its five logical values.

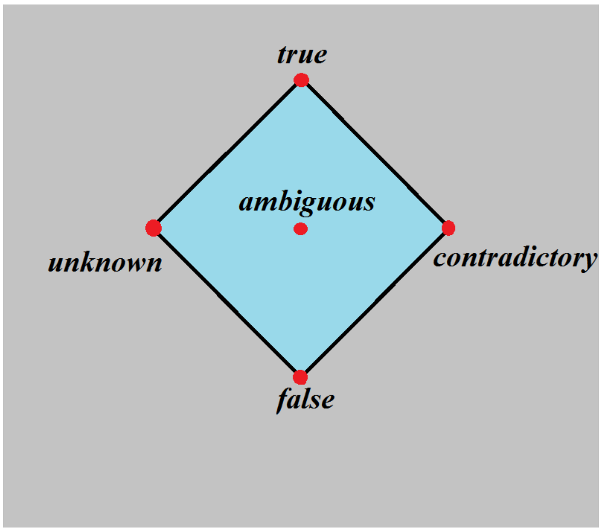

Fig. 4 The bottom square of neutrosophic cube and its five logical values.

$$t = \left(1 - \frac{\omega}{2}\right)(\mu - \kappa) - \frac{\omega\alpha}{4} \quad (4.1.2)$$

$$f = \left(1 - \frac{\omega}{2}\right)(\nu - \kappa) - \frac{\omega\alpha}{4} \quad (4.1.3)$$

$$u = \left(1 - \frac{\omega}{2}\right)\pi \quad (4.1.4)$$

$$c = \left(1 - \frac{\omega}{2}\right)\kappa \quad (4.1.5)$$

$$h = \frac{(1+\alpha)}{2}\omega \quad (4.1.6)$$

The five parameters defined by relations (4.1.2-4.1.6) define a partition of unity:

$$t + f + h + c + u = 1 \quad (4.1.7)$$

Thus, we obtained a penta-valued representation of neutrosophic information based on logical values: true, false, unknown, contradictory and hesitant.
Since $\pi \cdot \kappa = 0$, it results that $u \cdot c = 0$ and hence the conclusion that only four of the five terms from the partition can be distinguished from zero.
Geometric representation of this construction can be seen in figures 5 and 6.

The inverse transform.

Below, we present the inverse transform calculation, namely the transition from penta-valued representation (t, f, h, c, u) to the primary representation (μ, ω, ν).
From formulas (4.1.2) and (4.1.3), it results by subtraction:

$$t - f = \left(1 - \frac{\omega}{2}\right)(\mu - \nu) \quad (4.1.8)$$

From formulas (4.1.4) and (4.1,5), it results by subtraction:

$$c - u = \left(1 - \frac{\omega}{2}\right)(\mu + \nu - 1) \quad (4.1.9)$$

Then from (4.1.2), (4.1.3) and (3.5) it results:

$$2\min(t, f) = (1 - \omega)\alpha \quad (4.1.10)$$

Formula (4.1.6) is equivalent to the following:

$$\frac{2h - \omega}{\omega} = \alpha \quad (4.1.11)$$

Eliminating parameter α between equations (4.1.10) and (4.1.11), we obtained the equation for determining parameter ω:

$$\omega^2 - \omega(1 + 2h + 2\min(t, f)) + 2h = 0 \quad (4.1.12)$$

Note that the second-degree polynomial:

$$p(\omega) = \omega^2 - \omega(1 + 2h + 2\min(t, f)) + 2h \quad (4.1.13)$$

has negative values for $\omega = 1$ and $\omega = 2h$, namely

$$p(1) = p(2h) = -2min(t, f)$$

So, it has a root grater than $max(1,2h)$ and one less than $min(1,2h)$. Also, for $\omega = 0$, it has a positive value, namely $p(0) = 2h$. Therefore, the root belongs to the interval $[0, min(1,2h)]$ and it is defined by formula:

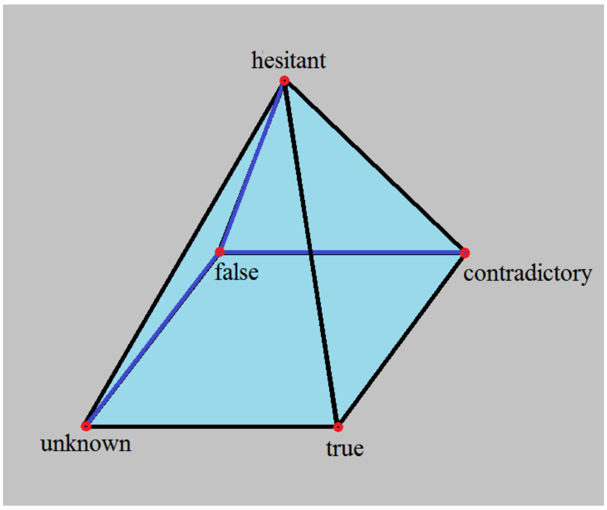

Fig. 5. The geometrical representation of the penta-valued space, based on true, false, unknown, contradictory and hesitant.

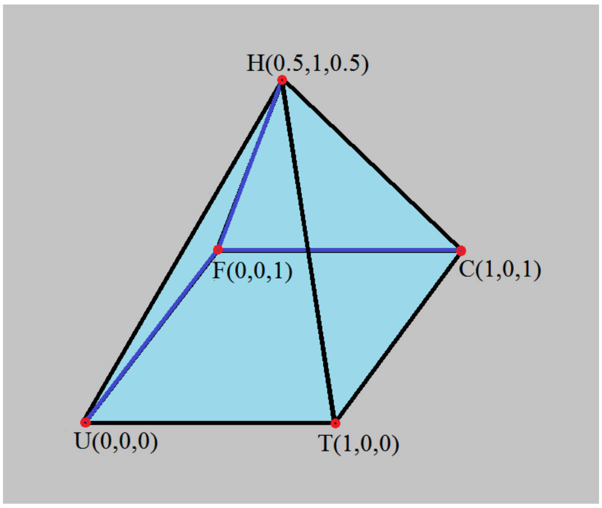

Fig. 6. Geometric representation of prototypes for features: truth, falsity, ignorance, contradiction and hesitation.

$$\omega = \beta - \sqrt{\beta^2 - 2h} \tag{4.1.14}$$

where:

$$\beta = \frac{1}{2} + h + \min(t, f) \tag{4.1.15}$$

We must observe that

$$\beta^2 - 2h \geq \left(\frac{1}{2} + h\right)^2 - 2h = \left(\frac{1}{2} - h\right)^2 \geq 0$$

and hence formula (4.1.14) provides a real value for ω.

Then, from (4.1.8) and (4.1.9), it results the system:

$$\mu - \nu = \frac{t - f}{1 - \frac{\omega}{2}}$$

$$\mu + \nu - 1 = \frac{c - u}{1 - \frac{\omega}{2}}$$

Finally, we obtain formulas for μ and ν.

$$\mu = \frac{1}{2} + \frac{t - f + c - u}{2 - \beta + \sqrt{\beta^2 - 2h}} \tag{4.1.16}$$

$$\nu = \frac{1}{2} + \frac{f - t + c - u}{2 - \beta + \sqrt{\beta^2 - 2h}} \tag{4.1.17}$$

Formulas (4.1.14), (4.1.16) and (4.1.17) represent the re-calculating formulas for the primary space components (μ, ω, ν) namely inverse transformation formulas.

4.2 Option (II)

Using the tetra-valued partition defined by formula (3.4) we obtain:

$$\mu - \kappa - \frac{\alpha\omega}{2} + \nu - \kappa - \frac{\alpha\omega}{2} + \pi + \kappa + \omega = 1 + \omega - \alpha\omega$$

$$\frac{\left(\mu - \kappa - \frac{\alpha\omega}{2}\right) + \left(\nu - \kappa - \frac{\alpha\omega}{2}\right) + \pi + \kappa + \omega}{1 + \omega - \alpha\omega} = 1 \tag{4.2.1}$$

We obtained a penta-valued partition of unity for neutrosophic information. These five terms are related to the following logical values: true, false, unknown, contradictory, hesitation:

$$t = \frac{\mu - \kappa - \frac{\alpha\omega}{2}}{1 + (1 - \alpha)\omega} \tag{4.2.2}$$

$$f = \frac{\nu - \kappa - \frac{\alpha\omega}{2}}{1 + (1 - \alpha)\omega} \tag{4.2.3}$$

$$u = \frac{\pi}{1 + (1 - \alpha)\omega} \tag{4.2.4}$$

$$c = \frac{\kappa}{1 + (1 - \alpha)\omega} \tag{4.2.5}$$

$$h = \frac{\omega}{1 + (1 - \alpha)\omega} \tag{4.2.6}$$

Formula (4.2.1) becomes:

$$t + f + h + c + u = 1 \quad (4.2.7)$$

The inverse transform

From (4.2.2) and (4.2.3) it results:

$$t - f = \frac{\mu - \nu}{1 + (1 - \alpha)\omega} \quad (4.2.8)$$

From (4.2.4) and (4.2.5) it results:

$$c - u = \frac{\mu + \nu - 1}{1 + (1 - \alpha)\omega} \quad (4.2.9)$$

From (4.2.8) and (4.2.9) it results:

$$\alpha = 1 - \frac{(|t - f| + |c - u|)}{1 - \omega(|t - f| + |c - u|)} \quad (4.2.10)$$

from (4.2.6) it results:

$$\frac{1}{\omega} + 1 - \frac{1}{h} = \alpha \quad (4.2.11)$$

Finally, from (4.2.10) and (4.2.11) it results the following equation:

$$(|t - f| + |c - u|)\omega^2 - \omega + h = 0 \quad (4.2.12)$$

Note that the second-degree polynomial defined by:

$$p(\omega) = (|t - f| + |c - u|)\omega^2 - \omega + h$$

has a negative value for $\omega = 1$, namely:

$$p(1) = -2\min(t, f)$$

Hence, it has a root grater than 1 and and another smaller than 1. Also for $\omega = h$ it has a positive value, namely:

$$p(h) = (|t - f| + |c - u|)h^2$$

So the solution belongs to the interval : $[h, 1]$

The value of the parameter ω is given by:

$$\omega = \frac{2h}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}} \quad (4.2.13)$$

From (4.2.11) it results:

$$\alpha = 1 - \frac{2(|t - f| + |c - u|)}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}} \quad (4.2.14)$$

from (4.2.13) and (4.2.14) it results:

$$1 + (1 - \alpha)\omega = \frac{2}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}}$$

Then, from (4.2.8) and (4.2.9) it results:

$$\mu - \nu = \frac{2(t - f)}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}}$$

$$\mu + \nu - 1 = \frac{2(c - u)}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}}$$

Finally, it results for the degree of truth and degree of falsity, the following formulas:

$$\mu = \frac{1}{2} + \frac{t - f + c - u}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}} \quad (4.2.15)$$

$$\nu = \frac{1}{2} + \frac{f - t + c - u}{1 + \sqrt{1 - 4h(|t - f| + |c - u|)}} \quad (4.2.16)$$

The formulae (4.2.15), (4.2.16) and (4.2.13) represent the formulae for recalculating of the primary space components (μ, ω, ν), namely the inverse transformation formulas.

5 Penta-valued logic based on truth, falsity, ignorance, contradiction and hesitation

This five-valued logic is a new one, but is related to our previous works presented in [11], [12].

In the framework of this logic we will consider the following five logical values: *true* t , *false* f , *unknown* u , *contradictory* c, and *hesitant* h. We have obtained these five logical values, adding to the four Belnap logical values the fifth: *hesitant*.

Tables 1, 2, 3, 4, 5, 6 and 7 show the basic operators in this logic.

Table 1. The Union

$\cup$	t	c	h	u	f
t	t	t	t	t	t
c	t	c	h	h	c
h	t	h	h	h	h
u	t	h	h	u	u
f	t	c	h	u	f

Table 2. The intersection.

$\cap$	t	c	h	u	f
t	t	c	h	u	f
c	c	c	h	h	f
h	h	h	h	h	f
u	u	h	h	u	f
f	f	f	f	f	f

The main differences between the proposed logic and the Belnap logic are related to the logical values u and c. We have defined $c \cap u = h$ and $c \cup u = h$ while in the Belnap logic there were defined $c \cap u = f$ and $c \cup u = t$.

Table 3. The complement.

	$\neg$
t	f
c	c
h	h
u	u
f	t

Table 4. The negation.

	$-$
t	f
c	u
h	h
u	c
f	t

Table 5. The dual.

	$\approx$
t	t
c	u
h	h
u	c
f	f

The complement, the negation and the dual are interrelated and there exists the following equalities:

$$\approx x = -\neg x \tag{5.1}$$

$$\neg x = - \approx x \tag{5.2}$$

$$-x = \neg \approx x \tag{5.3}$$

Table 6. The equivalence

$\leftrightarrow$	t	c	h	u	f
t	t	c	h	u	f
c	c	c	h	h	c
h	h	h	h	h	h
u	u	h	h	u	u
f	f	c	h	u	t

The *equivalence* is calculated by:

$$x \leftrightarrow y = (\neg x \cup y) \cap (x \cup \neg y) \tag{5.4}$$

Table 7. The S-implication

$\rightarrow$	t	c	h	u	f
t	t	c	h	u	f
c	t	c	h	h	c
h	t	h	h	h	h
u	t	h	h	u	u
f	t	t	t	t	t

The *S-implication* is calculated by:

$$x \rightarrow y = \neg x \cup y \tag{5.5}$$

6 New operators defined on the penta-valued structure

There be $x = (t, c, h, u, f) \in [0,1]^5$. For this kind of vectors, one defines the union, the intersection, the complement, the negation and the dual operators. The operators are related to those define in [12].

The Union: For two vectors $a, b \in [0,1]^5$, where $a = (t_a, c_a, h_a, u_a, f_a)$, $b = (t_b, c_b, h_b, u_b, f_b)$, one defines the union (disjunction) $d = a \cup b$ by the formula:

$$\begin{aligned} t_d &= t_a \vee t_b \\ c_d &= (c_a + f_a) \wedge (c_b + f_b) - f_a \wedge f_b \\ u_d &= (u_a + f_a) \wedge (u_b + f_b) - f_a \wedge f_b \\ f_d &= f_a \wedge f_b \end{aligned} \tag{6.1}$$

with $h_d = 1 - (t_d + c_d + u_d + f_d)$

The Intersection: For two vectors $a, b \in [0,1]^5$ one defines the intersection (conjunction) $c = a \cap b$ by the formula:

$$\begin{aligned} t_c &= t_a \wedge t_b \\ c_c &= (c_a + t_a) \wedge (c_b + t_b) - t_a \wedge t_b \\ u_c &= (u_a + t_a) \wedge (u_b + t_b) - t_a \wedge t_b \\ f_c &= f_a \vee f_b \end{aligned} \tag{6.2}$$

with $h_c = 1 - (t_c + c_c + u_c + f_c)$

In formulae (6.1) and (6.2), the symbols “$\vee$” and “$\wedge$” represent the maximum and the minimum operators, namely:
$\forall x, y \in [0,1]$,

$$x \vee y = \max(x, y)$$

$$x \wedge y = \min(x, y)$$

The union “$\cup$” and intersection “$\cap$” operators preserve de properties $t + c + u + f \leq 1$ and $u \cdot c = 0$, namely:

$$t_{a \cup b} + c_{a \cup b} + u_{a \cup b} + f_{a \cup b} \leq 1$$

$$c_{a \cup b} \cdot u_{a \cup b} = 0$$

$$t_{a \cap b} + c_{a \cap b} + u_{a \cap b} + f_{a \cap b} \leq 1$$

$$c_{a \cap b} \cdot u_{a \cap b} = 0$$

The Complement: For $x = (t, c, h, u, f) \in [0,1]^5$ one defines the complement x^c by formula:

$$x^c = (f, c, h, u, t) \tag{6.3}$$

The Negation: For $x = (t, c, h, u, f) \in [0,1]^5$ one defines the negation x^n by formula:

$$x^n = (f, u, h, c, t) \tag{6.4}$$

The Dual: For $x = (t, c, h, u, f) \in [0,1]^5$ one defines the dual x^d by formula:

$$x^d = (t, u, h, c, f) \tag{6.5}$$

In the set $\{0,1\}^5$ there are five vectors having the form $x = (t, c, h, u, f)$, which verify the condition $t + f + c + h + u = 1$:
$T = (1,0,0,0,0)$ (*True*), $F = (0,0,0,0,1)$ (*False*), $C = (0,1,0,0,0)$ (*Contradictory*), $U = (0,0,0,1,0)$ (*Unknown*) and $H = (0,0,1,0,0)$ (*Hesitant*).
Using the operators defined by (6.1), (6.2), (6.3), (6.4) and (6.5), the same truth table results as seen in Tables 1, 2, 3, 4, 5, 6 and 7.
Using the complement, the negation and the dual operators defined in the penta-valued space and returning in the primary three-valued space, we find the following equivalent unary operators:

$$(\mu, \omega, \nu)^{\mathrm{c}} = (\nu, \omega, \mu) \tag{6.6}$$

$$(\mu, \omega, \nu)^{\mathrm{n}} = (1 - \mu, \omega, 1 - \nu) \tag{6.7}$$

$$(\mu, \omega, \nu)^{\mathrm{d}} = (1 - \nu, \omega, 1 - \mu) \tag{6.8}$$

Conclusion

In this paper it was presented two new penta-valued structures for neutrosophic information. These structures are based on Belnap logical values, namely true, false, unknown, contradictory plus a fifth, hesitant.
It defines the direct conversion from ternary space to the penta-valued one and also the inverse transform from penta-valued space to the primary one.
There were defined the logical operators for the penta-valued structures: union, intersection, complement, dual and negation.

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