A Review on Recent Development of Neutro-Topology

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Abstract: Smarandache proposed NeutroAlgebra and AntiAlgebra. NeutroAlgebras and AntiAlgebras are a new research topic based on real-world scenarios. He investigated the concepts of neutro- and anti-structure. He demonstrated using NeutroAlgebra concepts that just because a statement is completely true in a classical Algebra does not imply that it is also completely true in a NeutroAlgebra or AntiAlgebra. It is determined by the operations and axioms on which it is based (whether they are completely true, partially true, totally false, or partially or completely indeterminate). This study examines the concepts of Generalised regular Neutro-Topological space and its properties.

Keywords: Neutro-Topology; NeutroClosed sets; NeutroOpen sets; GR-NeutroInterior.

1. Introduction

Topology is a significant subject of Mathematics, hence it is surprising that topology's appreciation was delayed in the history of Mathematics. Topology is the study of space characteristics that are unaffected by continuous deformation.

A key idea in mathematics, set theory, dates back to the work of Russian mathematician George Cantor (1877). We were able to investigate a variety of mathematical ideas thanks to set theory. However, there are a lot of unknowns in our life. The traditional logic of mathematics is frequently insufficient to resolve these difficulties. Then the idea of fuzzy sets was introduced by Zadeh [1]. It is a development of the traditional idea of a set. In his paper, he presented a hypothesis according to which fuzzy sets are sets with imprecise boundaries. In both directions, gradual changes from membership to non membership can be expressed using fuzzy sets. It offers meaningful representations of vague notions in everyday language in addition to a powerful and meaningful way to quantify uncertainties. a value in the discourse universe that indicates the fuzzy set's degree of membership. Real values in the closed range of 0 to 1 are used to represent these membership classifications. Chang [2] discovered and popularized the theory of fuzzy topological spaces. The concepts for creating fuzzy topological spaces were provided by Lowen [3]. He provided the idea of fuzzy compression and two new functions, which allowed for the evident observation of further relationships between fuzzy topological spaces and topological spaces. A unique fuzzy topological space called the product spaces was discussed by Cheng-Ming [4]. He established a type of fuzzy points neighbourhood formation, such as the Q-neighbourhood, which is a crucial idea in fuzzy topological spaces. He also demonstrated how each fuzzy topological space is isomorphic topologically by a specific space of topology.

Atanassov [5] introduced the concept of intuitionistic fuzzy sets as an extension of sets with better applicability. Coker [6] developed the idea of intuitionistic smooth fuzzy topological spaces using the concept of intuitionistic fuzzy sets. The definitions of the intuitionistic smooth fuzzy topological spaces were first presented by Samanta and Mondal [7].

Smarandache [8] introduced the concept of a neutrosophic set for the first time. These concepts have three different degrees: T for membership, I for uncertainty, and F for non-membership. In other words, a situation is treated in neutrosophy in accordance with its trueness, falsity, and uncertainty. As a result, neutrosophic sets and logic enable us to make sense of a variety of uncertainties in our daily lives. On this topic, numerous studies have been conducted. Sahin et al. recently discovered some operations for neutrosophic sets with interval values; Neutrosophic multigroups and applications were researched by Ulucay et al [9]; Q-neutrosophic soft expert set and its application were introduced by Hassan et al [10]. The acquisition of neutrosophic soft expert sets was introduced by Sahin et al [11]; Interval-valued refined neutrosophic sets and their applications were researched by Ulucay et al [12]. Neutosophic set importance on deep transfer learning techniques was obtained by Khalifa et al. [13]; Generalised Hamming similarity measure based on neutrosophic quadraple numbers and its applications were researched by Kargin et al. [14]; In order to assess the quality of online education, Sahin et al. [15] obtain Hausdorff Measures on generalised set valued neutrosophic quadraple numbers and decision-making applications. The foundation for a wide family of novel mathematical ideas, including both their crisp and fuzzy counterparts, was laid by neutrosophy. The concepts of neutrosophic crisp set and neutrosophic crisp topological space were first developed by Salama et al. and Alblowi [16]. Neutron structures and antistructures are defined by Smarandache [17]. An algebraic structure can be divided into three regions, similar to neutrosophic logic: A, the set of elements that satisfy the conditions of the algebraic structure, the truth region; Neutro A, the set of elements that do not meet the conditions of the algebraic structure, the uncertainty region; and anti-A, the set of elements that do not satisfy the conditions of the algebraic structure, the inaccuracy region. By eliminating neutrosophic sets and neutrosophic numbers, the structure of neutrosophic logic has been translated to the structure of classical algebras. The academic world has seen a rise in interest in neutrosophic set theory research in recent years. As a result, it is possible to generate neutro-algebraic structures, which are more broadly structured than classical algebras. Additionally, the region of elements that do not conform to any of the classical algebras is also considered to have anti-algebraic structures. Recent research includes studies on neutro-algebra by Smarandache et al. [18], the neutrosophic triplet of BI-algebras by Razaei et al. [19], neutro-bck-algebra by Smarandache et al. [20], and neutro-hypergroups by Ibrahim et al. [21].

In this paper, we introduce new Generalization of Regular Neutro-open (briefly, GRN-open) sets and Generalised regulat Anti-open set. This new class shows stronger properties in general topological spaces that mean GRN-open sets exists in between the class of regular open sets and the class of open sets. Also, we investigate GRN-neighbourhood, GRN-interior and GRN-closure properties.

2. Preliminaries

Definition 2.1. The NeutroSophication of the Law [22]

1. Let X be a non-empty set and $*$ be a binary operation. For some elements (a, b) ∈ (X, X), (a∗b) ∈ X (degree of well defined (T)) and for other elements (x, y), (p, q) ∈ (X, X); [x∗y is indeterminate (degree of indeterminacy (I)), or p∗q ∉ X (degree of outer-defined (F)], where (T, I, F) is different from (1,0,0) that represents the Classical Law, and from (0,0,1) that represents the Anti Law.

2. In Neutro Algebra, the classical well-defined for binary operation $*$ is divided into three regions: degree of well-defined (T), degree of indeterminacy (I) and degree of outer-defined (F) similar to neutrosophic set and neutrosophic logic.

Definition 2.2. [23]

Let X be the non-empty set and τ be a collection of subsets of X. Then τ is said to be a Neutro Topology on X and the pair (X, τ) is said to be a Neutro Topological space, if at least one of the following conditions hold good:

1. $[(\emptyset_N \in \tau, X_N \notin \tau)\ or\ (X_N \in \tau, \emptyset_N \notin \tau)\ ]$ or $[\emptyset_N, X_N \in_{\sim} \tau]$
2. For some n elements $a_1, a_2, \ldots a_n \in \tau, \bigcap_{i=1}^{n} a_i \in \tau$ [degree of truth T] and for other n elements $b_1, b_2, \ldots b_n \in \tau, p_1, p_2, \ldots p_n \in \tau$; [($\bigcap_{i=1}^{n} b_i \notin \tau$) [degree of falsehood F] or ($\bigcap_{i=1}^{n} p_i$ is indeterminate (degree of indeterminacy I)], where n if finite; ; [where (T, I, F) is different from (1,0,0) that represents the Classical Axiom, and from (0,0,1) that represents the Anti Axiom].
3. For some n elements $a_1, a_2, \ldots a_n \in \tau, \bigcup_{i=1} a_i \in \tau$ [degree of truth T] and for other n elements $b_1, b_2, \ldots b_n \in \tau$, $p_1, p_2, \ldots p_n \in \tau$; [($\bigcup_{i=I} b_i \notin \tau$) [degree of falsehood F] or ($\bigcup_{i=I} p_i$ is indeterminate (degree of indeterminacy I)], where n is finite; [where (T, I, F) is different from (1,0,0) that represents the Classical Axiom, and from (0,0,1) that represents the Anti Axiom].

Definition 2.3. [23]

Let X be the non-empty set and τ be a collection of subsets of X. Then τ is said to be an Anti Topology on X and the pair (X, τ) is said to be an Anti Topological space, if at least one of the following conditions hold good:

1. $\emptyset_N, X_N \notin \tau$
2. For n elements $a_1, a_2, \ldots, a_n \in \tau, \bigcap_{i=1}^{n} a_i \notin \tau$ [degree of falsehood F] where n is finite.
3. For some n elements $a_1, a_2, \ldots, a_n \in \tau, \bigcup_{i=1}^{n} a_i \notin \tau$ [degree of falsehood F] where n is finite.

Remark 2.1. [23]

The symbol "∈_~" will be used for situations where it is an unclear appurtenance (not sure if an element belongs or not to a set). For example, if it is not certain whether "a" is a member of the set P, then it is denoted by a ∈_~ P.

Main Works

3. GR-NeutroOpen sets and their properties

We introduce GR-NeutroOpen sets and investigate some of relationships between existed classes.

Definition 3.1. A NeutroSubset M of space P is called Generalized Regular Neutrosphic Open (briefly, GR-NeutroOpen) set if M = NeuInt(g-NeuCl(M)). We denote the class of sets as GRNO(P).

Firstly we have to prove the existence of new class GR-NeutroOpen sets in topological spaces.

Theorem 3.1. Every regular NeutroOpen set is GR-NeutroOpen set.

Proof. Let M be a regular NeutroOpen set in P. To prove that M is GR- NeutroOpen in P.

We know that

M ⊆ g-NeuCl(M) ⊆ NeuCl(M) that is NeuInt(M) ⊆ NeuInt(g-NeuCl(M)) ⊆ NeuInt(cl(M)).

As M is regular NeutroOpen, M = NeuInt(cl(M)) and NeuInt(M) = M.

Hence M ⊆ NeuInt(g-NeuCl(M)) ⊆ NeuInt(NeuCl(M)) = M,

Thus NeuInt(g-NeuCl(M)) = M. Therefore M is GR- NeutroOpen in P.

The converse of above theorem need not be true.

Example 3.1. Let P = {1,2,3,4} with the topology on it τ = {P,∅,{1},{2},{1,2},{2,3},{1,2,3}}, then sets {2}, {1,2} are NeutroOpen sets but not regular NeutroOpen sets in P.

Theorem 3.2. Every GR-NeutroOpen set is NeutroOpen set.

Proof. Let M be a GR- NeutroOpen set in P. That is M = NeuInt(g- NeuCl(M)). As interior of any subset of P is an NeutroOpen set, therefore M is a NeutroOpen in P.

The converse of above theorem need not be true.

Example 3.2. Let P = {1,2,3,4} with the topology on it

τ ={P,∅,{1},{2},{1,2},{2,3},{1,2,3}}.

Then the set {1,2,3} is NeutroOpen set but not GR- NeutroOpen in P.

Remark 3.1. From Theorem 3.2, we know that every GR- NeutroOpen set is a NeutroOpen set but not conversely. We know that every NeutroOpen set is semi- NeutroOpen but not conversely. Hence every GR- NeutroOpen set is a semi- NeutroOpen set but not conversely.

Remark 3.2. From Theorem 3.2, we know that every NeutroOpen set is a NeutroOpen set but not conversely. We know that every NeutroOpen set is g- NeutroOpen but not conversely. Hence every GR- NeutroOpen set is a g- NeutroOpen set but not conversely.

Theorem 3.3. Intersection of two GR-NeutroOpen sets is a GR- NeutroOpen set in topological spaces.

Proof. Let M and N be two GR- NeutroOpen sets in space P. To prove that M ∩ N is GR- NeutroOpen set in space P, that is to prove that M∩N = NeuInt(g-NeuCl(M∩N)). As M and N are GR-

NeutroOpen sets in P,M = NeuInt(g-NeuCl(M)), N = NeuInt(g-NeuCl (N)). We know that M∩N ⊆ M, g-NeuCl(M∩N) ⊆ g-NeuCl(M) also M∩N ⊆ N, g-NeuCl(M∩N) ⊆ g-NeuCl(N). Which implies NeuInt(g-NeuCl (M ∩ N)) ⊆ NeuInt(g-NeuCl (M)) and NeuInt(g-NeuCl (M ∩ N)) ⊆ NeuInt(g-NeuCl N)). This implies NeuInt(g-NeuCl(M ∩ N))∩ NeuInt(g-NeuCl (M∩N)) ⊆ NeuInt(g-NeuCl (M))∩ NeuInt(g-NeuCl (N)) That is NeuInt(g-NeuCl (M ∩N)) ⊆ NeuInt(g-NeuCl (M))∩ NeuInt(g-NeuCl (N)) = M∩N...(i) M∩N = NeuInt(M)∩NeuInt(N) = NeuInt(M∩N) [M = NeuInt(M) and N = NeuInt(N) because of if M and N are NeutroOpen sets, then every NeutroOpen is NeutroOpen in P] NeuInt(M∩N) ⊆ NeuInt(g-cl(A∩B)). M∩N ⊆ NeuInt(g-NeuCl(M ∩N))...(ii) From (i) and (ii), M ∩N = NeuInt(g-NeuCl (M ∩N)). Hence M ∩N is GR- NeutroOpen set in P.

Remark 3.3. The union of two GR- NeutroOpen sets is generally not a GR- NeutroOpen set in topological spaces.

Example 3.3. Let P = {1,2,3,4} with topology on it

τ = {P,∅,{1},{2},{1,2},{2,3},{1,2,3}}.If M = {1,2} and

N = {2,3} are GR-open sets in P but M ∩ N = {1,2,3} is not GR- NeutroOpen set in P.

Theorem 3.4. If M is a GR- NeutroOpen then NeuInt(M) = M.

Proof. Let M is GR-NeutroOpen. To prove NeuInt(M) = M. We know that every GR- NeutroOpen set is NeutroOpen, that is M is NeutroOpen set then NeuInt(M) = M. The converse of above theorem need not be true.

Example 3.4. Let P = {1,2,3,4} with topology on it τ = {P,∅,{1},{2},{1,2},{1,2,3}}, then GRNO(P) = {P,∅,{1},{2},{1,2}}. Then the Neutro-set M = {1,2,3}, Note that NeuInt(M) = {1,2,3} is not a GR-NeutroOpen set, but it is NeutroOpen set of P.

Theorem 3.5. If M is g-closed and NeutroOpen in P, then M is GR- NeutroOpen in P.

Proof. Let M is g-closed and NeutroOpen in P. To prove that M is GR- NeutroOpen i.e. to prove M = NeuInt(g-NeuCl (M)). Now g-NeuCl(M) = M, because M is g- NeutroOpen set. As NeuInt(g-NeuCl(M)) = NeuInt(M) this implies NeuInt(g-NeuCl (M)) = M, because M is NeutroOpen set. Then M is GR- NeutroOpen in P.

Remark 3.4. Complement of a GR-NeutroOpen set need not be GR- NeutroOpen set.

Example 3.5. Let P={1,2,3,4} with topology on it τ={P,∅,{1},{2},{1,2},{2,3},{1,2,3}}. Note that {1,2} is a

GR- NeutroOpen set. But P − {1,2} = {3} is not a GR- NeutroOpen set in P.

4. GR-NeutroCNeutroClosed sets and their properties

We introduce GR-NeutroClosed sets and investigate some of their properties.

Definition 4.1. A subset M of space P is called Generalized Regular Neutrosophic Closed (briefly, GR-NeutroClosed) set if P − M is GR- NeutroClosed in P. Then its family is denoted as GRNC(P).

This new class of sets properly lies between the class of regular NeutroClosed sets and the class of NeutroClosed sets

Theorem 4.1. A subset M of P is GR- NeutroClosed if and only if M = NeuCl(g-NeuInt(M)).

Proof. (i) Suppose M is GR- NeutroClosed. To prove M = NeuCl(g-Neu¬Int(M)). As M is GR-NeutroClosed, P − Mis GR-NeutroOpen in P, which implies P–M=NeuInt(g-NeuCl(P–M)). P–M = NeuInt(P–g-NeuInt(M)). [because g-NeuCl(P – M) = P – g-NeuCl(M))] = P – NeuCl(g-NeuInt(M)). So (P–M)c = [P–NeuCl(g-NeuInt(M))]c. That is M = NeuCl(g-NeuInt(M)). (ii) Suppose M = NeuCl(g-Int(M)). To prove M is GR- NeutroClosed, [That is to prove P–M is GR-NeutroOpen set]. That is P–M = NeuInt(g¬NeuCl(M). Now given M = NeuCl(g-NeuInt(M)). P − M = P − NeuCl(g--NeuInt(M)). P − M = NeuInt(g-NeuCl(P – M)).implies that P – M is GR-NeutroOpen set that is M is GR- NeutroClosed in P.

Theorem 4.2. Every regular NeutroClosed set is GR- NeutroClosed set.

Proof. Let M be a regular NeutroClosed set in space P. Then Mc is a regular NeutroOpen set. By Theorem 3.1, Mc is GR- NeutroOpen set. Therefore M is a GR- NeutroClosed set in P.

The converse of above theorem need not be true.

Example 4.1. From Example 3.1, the set {3,4} and {1,3,4} are GR- NeutroClosed sets but not regular NeutroClosed in P.

Theorem 4.3. Every GR- NeutroClosed set is NeutroClosed set.

Proof. Let M be a GR- NeutroClosed set in P. Then Mc is a GR- NeutroOpen in P. By Theorem 3.2, Mc is an NeutroOpen set in P. Therefore M is a NeutroClosed set in P.

The converse of above theorem need not be true.

Example 4.2. From Example 3.1, the set {4} is NeutroClosed set but not GR- NeutroClosed set in P.

Remark 4.1. From Theorem 4.3, we have, every GR- NeutroClosed set is a NeutroClosed set but not conversely. Also, every NeutroClosed set is semi- NeutroClosed set but not conversely. Hence every GR- NeutroClosed set is a semi- NeutroClosed set but not conversely.

Remark 4.2. From Theorem 4.3, we have, every GR- NeutroClosed set is a NeutroClosed set but not conversely. Every NeutroClosed set is NeutroClosed but not conversely. Hence every GR-NeutroClosed set is NeutroClosed set but not conversely.

Remark 4.3. From Theorem 4.3, we know that every GR- NeutroClosed set is a NeutroClosed set but not conversely. It is clear that every NeutroClosed set is g- NeutroClosed but not conversely. Hence every GR- NeutroClosed set is a g- NeutroClosed set but not conversely.

Remark 4.4. The following example shows that GR- NeutroClosed sets are independent of ir-NeutroClosed sets, s-NeutroClosed sets and regular semi- NeutroOpen (=regular semi-NeutroClosed) sets.

Example 4.3. Let P = {1,2,3,4,5} with topology on it

τ = {P,∅,{1},{1,4},{2,3},{1,2,3},{1,2,3,4}}. Then

NeutroClosed sets in P are P,Ø,{5},{4,5},{1,4,5},{2,3,5}, {2,3,4,5}.

GR- NeutroClosed sets in P are P,Ø,{4,5},{1,4,5},{2,3,5}, {2,3,4,5}.

π- NeutroClosed sets in P are P,Ø,{5},{1,4,5},{2,3,5}.

s- NeutroClosed sets in P are P,Ø,{5},{1,4,5},{2,3,5}.

regular semi-NeutroOpen sets in P are P,Ø,{1,4},{2,3},{1,4 ,5},{2,3,5}.

Theorem 4.4. Union of two GR- NeutroClosed sets is a GR- NeutroClosed set in topological spaces.

Proof. Let M and N be two GR- NeutroClosed sets in P. To prove that M ∪N = NeuCl(g-NeuInt(M ∪ N)). As M and N are GR- NeutroClosed sets in P, M = NeuCl(g-NeuInt(M)), N = NeuCl(g-NeuInt(N)). We know thatM ⊆ M∪N, g-NeuInt(M) ⊆ g-NeuInt(M∪N) also N ⊆ M∪N, g-¬NeuInt(N) ⊆ g-NeuInt(M ∪N). Which implies NeuCl(g-NeuInt(M)) ⊆ NeuCl(g-NeuInt (M ∪ N)) and NeuCl(g-NeuInt(N)) ⊆ NeuCl(g-NeuInt (M ∪ N)). This implies NeuCl(g-NeuInt (M)) ∪ NeuCl(g-NeuInt (N)) ⊆ NeuCl(g-NeuInt (M ∪N)) ∪ NeuCl(g-NeuInt (M∪N)). That is

NeuCl(g-NeuInt (M))∪ NeuCl(g-NeuInt (N)) ⊆ NeuCl(g-NeuInt (M ∪N))...(i)

M ∪N = NeuCl(M)∪ NeuCl(N) = NeuCl(M ∪N) [M = NeuCl(M) and N = NeuCl(N) and M, N are NeutroClosed sets, because every GR- NeutroClosed is NeutroClosed set] NeuCl(M∪N) ⊒ NeuCl(g-NeuInt (M ∪ N)) i.e. M ∪N ⊒ NeuCl(g-NeuInt (M ∪N))...(ii)

From(i) and (ii), M ∪ N = NeuCl(g-NeuInt (M ∪N)). Hence M ∪N is GR- NeutroClosed set in P. Hence A∪ B is GR- NeutroClosed in X.

Remark 4.5 The intersection of two GR- NeutroClosed sets in topological spaces is generally not a GR- NeutroClosed set.

Example 4.4. From Example 3.1, then sets M = {1,4} and N = {3,4} are GR- NeutroClosed sets in P but M ∩N = {4} is not GR- NeutroClosed set in P.

Theorem 4.5. If M is a GR- NeutroClosed if and only if NeuCl(M) = M.

Proof. If M is GR- NeutroClosed. To prove NeuCl(M) = M. We know that every GR- NeutroClosed set is NeutroClosed set i.e. M is NeutroClosed then NeuCl(M) = M.

The converse of above theorem need not be true.

Example 4.5. Let P = {1,2,3,4} with topology on it τ = {P,Ø,{1},{2},{1,2},{1,2,3}}. Then GRNC(P)={P,Ø,{3,4} ,{1,3,4},{2,3,4}}. Then the set M = {4}. Note that NeuCl(M)= {4} is not a GR-NeutroClosed set, but it is a NeutroClosed set of P.

Theorem 4.6. If M is g-NeutroOpen and NeutroClosed in P, then M is GR- NeutroClosed set in P.

Proof. Let M is g-NeutroOpen and NeutroClosed set in P. To prove that

M is GR- NeutroClosed set i.e. to prove M NeuCl(g-NeuInt (M)). Now g-NeuInt(M) = M, because M is g-NeutroOpen set. As NeuCl(g-NeuInt (M)) = NeuCl(M) this implies NeuCl(g-NeuInt (M)) = M, because M is NeutroClosed set. Then Mis GR- NeutroClosed set in P.

5. GR-NeutroNeighbourhoods and GR-NeutroInterior

Definition 5.1. (i) Let P be a topological space and x ∈ P, A subset N of P is said to be a GR-NeutroNeighbourhood (briefly, GR-NeuNhd) of x if and only if there exists a GR-NeutroOpen set G such that x ∈ G ⊆ N.

(ii) The collection of all GR-NeutroNeighbourhood of x ∈ P is GR-NeutroNeighbourhood system at x and is denoted by GR-N(x).

Definition 5.2. Let M be a subset of P. A point x ∈ M is said to be GR-NeutroInterior point of M if and only if P is a GR-NeutroNeighborhood of x. The set of all GR-NeutroInterior points of M is called the GR-NeutroInterior of M and is denoted as GR-int(M).

Theorem 5.1. If M is a subset of P, then GR-NeutroInt(M) = ∪{G : G is GR-NeutroOpen set, G ⊆ M}.

Proof. Let M be a subset of P. x ∈ GR-NeuInt(A) implies that x is a GR-NeutroInterior point of P i.e. M is a GR-NeuNhd of point x. Then there exists a GR-NeutroOpen set G such that x ∈ G ⊆ A

implies that x ∈ ∪{G : G is GR- NeutroOpen set, G ⊆ M}. Hence

GR-NeuInt(M) = ∪{G : G is GR- NeutroOpen set, G ⊆ M}.

Theorem 5.2. Let P be a topological space and M ⊆ P, then show that M is GR- NeutroOpen set if and only if GR-NeuInt(M) = M.

Proof. Let M be a GR- NeutroOpen set in P. Then clearly the largest GR- NeutroOpen set contained in M, is itself M. Hence GR-NeuInt(M) = M.

Conversely, suppose that M ⊆ P and GR-NeuInt(M) = M. Since GR-NeuInt(M) is a GR-NeutroOpen set in P, it follows that M is a GR- NeutroOpen set in P.

Theorem 5.3. Let M and N are subset of P. Then

1. GR-NeuInt(P) = P and GR-NeuInt(Ø) = Ø.
2. GR-NeuInt(M) ⊆ M.
3. If N is any GR- NeutroOpen set contained in M, then N ⊆ GR-NeuInt(M).
4. If M ⊆ N, then GR-NeuInt(M) ⊆ GR-NeuInt(N).
5. GR-NeuInt(GR-NeuInt(M)=GR-NeuInt(M).

Proof. (1) Since P and Ø are GR- NeutroOpen sets, by Theorem 5.3, GR-NeuInt(P) = ∪{G : G is GR-NeutroOpen set, G ⊆ P} = P ∪ { all GR- NeutroOpen sets } = P. That is GR-NeuInt(P) = P. Since Ø is the only GR-NeutroOpen set contained in Ø, GR-NeuInt(Ø) = Ø.

(2) Let x ∈ GR-NeuInt(A) implies that x is a GR-NeutroInterior point of M. That is Mis a GR-NeuNhd of x i.e. x ∈ M. Thus x ∈ GR-int(A) implies x ∈ A. Hence GR-NeuInt(M) ⊆ M.

(3) Let N be any GR- NeutroOpen set such that N ⊆ M. Let x ∈ N. Since N is a GR-NeutroOpen set contained in M, x is a GR-NeuInterior point of M. That is x ∈ GR-NeuInt(M). Hence N ⊆ GR-NeuInt(M).

(4) Let M and N be subsets of P such that M ⊆ N. Let x ∈ GR-NeuInt(M). Since GR-NeuInt(M) ⊆ M and M ⊆ N, we have GR-NeuInt(M) ⊆ N. Now GR-NeuInt(M) is a GR- NeutroOpen set and GR-NeuInt(N) is the largest GR- NeutroOpen set contained in N, we have to find GR-NeuInt(M) ⊆ GR-NeuInt(N).

(5) Since GR-NeuInt(M) is a GR-NeutroOpen set in P, it follows that GR-NeuInt(GR-NeuInt(M))=GR-NeuInt(M).

Theorem 5.4. If M and N are subsets of P, then GR-NeuInt(M) ∪ GR-NeuInt(N) ⊆ GR-NeuInt(M ∪N).

Proof. We know that M ⊆ M ∪ N and N ⊆ M ∪ N. We have, by Theorem 5.5(iv), GR-NeuInt(A) ⊆ GR- NeuInt(M ∪N) and GR- NeuInt (N) ⊆ GR- NeuInt (M ∪ N). This implies GR- NeuInt (M)∪ GR-NeuInt (N) ⊆ GR- NeuInt(M ∪N).

Theorem 5.5. Let M and N are subsets of P, then GR- NeuInt(M)∩ GR- NeuInt (N)=GR- NeuInt (M ∩N).

Proof. We know that M ∩N ⊆ M and M ∩N ⊆ N. We have, by Theorem 5.5(iv), GR- NeuInt (M ∩ N) ⊆ GR-NeuInt(M) and GR- NeuInt(M ∩ N) ⊆ GR- NeuInt(N).

This implies GR- NeuInt(M ∩ N) ⊆ GR- NeuInt (M)∩ GR- NeuInt(N)...(i)

Again, let x ∈ GR- NeuInt(M)∩GR- NeuInt(N). Then x ∈ GR- NeuInt(M) and x ∈ GR- NeuInt(N).

Hence x is a NeutroInterior point of each of NeutroSets M and N. It follows that M and N are GR-NeuNhd of x, so that their intersection M∩N is also a GR-NeuNhd of x. Hence x ∈ GR NeuInt (M ∩ N). Thus x ∈ GR- NeuInt(M)∩ GR- NeuInt (N) implies that x ∈ GR- NeuInt (M ∩ N). Therefore GR-NeuInt(M)∩ GR- NeuInt(N) ⊆ GR- NeuInt (M ∩N)...(ii)

From (i) and (ii), we get GR- NeuInt (M)∩ GR- NeuInt (N)=GR- NeuInt(M∩ N).

6. GRN-closure and their properties

Using the GR-NeutroClosed sets we can introduce the concept of GR-NeutroClosure operator in topological spaces.

Definition 6.1. Let M be a subset of a space P. We define the GR-NeutroClosure of M to be the intersection of all GR-NeutroClosure sets containing M. Mathematically, GR-cl(M) = ∩ {F M ⊆ F ∈ GRC(P)}.

Theorem 6.1. Let P be any topological space and M ⊆ P, then show that M is GR-NeutroClosure set if and only if GR-cl(M) = M.

Proof. Let M be a GR-NeutroClosed set in P. Then clearly the smallest GR-NeutroClosed set contained in M, is itself M. Hence GR-NeuCl(M) = M.

Conversely, suppose that M ⊆ P and GR-NeuCl(M) = M. Since GR-NeuCl(M) is a GR-NeutroOpen set in P, it follows that M is a GR-NeutroClosed set in P.

Theorem 6.2. Let M and N are subset of P. Then

GR-NeuCl(P) = P and GR-cl(∅) = ∅.

M ⊆ GR-NeuCl(M).

If N is any GR-NeuClosed set contained in M, then GR-NeuCl(M) ⊆ N.

If M ⊆ N, then GR-NeuCl(M) ⊆ GR-NeuCl(N).

GR-NeuCl(GR-NeuCl(A))=GR-NeuCl(M)

Proof. (1) Obviously.

(2) By the definition of GR-NeuClosure of M, it is obvious that M ⊆ GR-NeuCl(M).

(3) Let N be any GR-NeutroClosed set containing M. Since GR-NeuCl(M) is the intersection of all GR-NeutroClosed sets containing M i.e GR-NeuCl(M) is contained in every GR-NeutroClosed set containing M. Hence GR-NeuCl(M) ⊆ N.

(4) Let M and N are NeutroSubsets of P such that M ⊆ N. By the definition of GR-NeutroClosure, GR-NeuCl(N) = ∩{F N ⊆ F ∈ GRC(P)}. If N ⊆ F ∈ GRNC(P), then GR-NeuCl(N) ⊆ F. Since M ⊆ N, M ⊆ N ⊆ F ∈ GRNC(P), we have GR-NeuCl(M) ⊆ F. Therefore GR-NeuCl(M) ⊆ ∩{F N ⊆ F ∈ GRNC(P)}=GR-NeuCl(P). That is GR-NeuCl(M) ⊆ GR-NeuCl(N).

Since GR-NeuCl(M) is a GR-NeutroClosed set in P. It follows that GR-NeuCl(GR-NeuCl(P)) = P.

Theorem 6.3. Let M and N are subsets of P, then GR-NeuCl(M∪ N)= GR-cl(M)∪ GR-NeuCl(N).

Proof. Let M and N are subsets of P. Clearly M ⊆ M ∪N and N ⊆ M ∪ N. We have by the Theorem 6.3(iv), GR-NeuCl(M) ⊆ GR-NeuCl(M ∪N) and GR-NeuCl(N) ⊆ GR-NeuCl(M ∪N). This implies GR-NeuCl(M)∪ GR-NeuCl(N) ⊆ GR-NeuCl(M ∪ N)...(i).

Now to prove that GR-NeuCl(M ∪ N) ⊆ GR-NeuCl(M)∪ GR-NeuCl(N). Let x ∈ GR-NeuCl(M ∪N) and x ∉ GR-NeuCl(M)∪ GR-NeuCl(N). Then there exists GR-NeutroClosed sets M1 and N1 with M ⊆ M1, N ⊆ N1 and x ∉ M1 ∪N1. We have M ∪ N ⊆ M1 ∪N1 and M1 ∪N1 is a GR-NeutroClosed set by Theorem 6.3, such that x ∉ M1 ∪N1. Thus x ∉ GR-NeuCl(M ∪ N) which is contradiction to x ∈ GR-NeuCl(M ∪ N).

Hence GR-NeuCl(M ∪N) ⊆ GR-NeuCl(M)∪ GR-NeuCl(N)...(ii).

From (i) and (ii), we have GR-NeuCl(M ∪N)= GR-NeuCl(M)∪ GR-NeuCl(N).

Theorem 6.4. Let M and N are subsets of P, then GR- NeuCl (M∩ N) ⊆ GR- NeuCl (M)∩ GR- NeuCl (N).

Proof. Let M and N are subsets of P. Clearly M∩N ⊆ M and M ∩N ⊆ N. We have, by Theorem 6.3(iv), GR- NeuCl (M ∩N) ⊆ GR- NeuCl(M) and GR- NeuCl (M ∩N) ⊆ GR- NeuCl(N). This implies GR- NeuCl(M ∩N) ⊆ GR- NeuCl(M)∩ GR- NeuCl (N).

Remark 6.1. In general GR- NeuCl (M)∩ GR- NeuCl (N) ≠ GR- NeuCl (M∩ N), as seen from the following example.

Example 6.1. Consider P = {1,2,3,4}, topology on it τ= {P,Ø,{1},{2},{1,2},{2,3},{1,2,3}}, M = {2,3}, and N= {3,4}, M∩N = {3}, GR- NeuCl (M) = {2,3,4}, GR- NeuCl (N) = {3,4}, GR- NeuCl (M ∩ N) = {3} and GR- NeuCl (M)∩ GR- NeuCl (N) = {3,4}. Therefore GR- NeuCl (M)∩ GR- NeuCl (N) Z GR- NeuCl(M ∩N).

Theorem 6.5. Let M be a subset of P and x ∈ P. Then x ∈ GR- NeuCl (M) if and only if V ∩M ≠ Ø for every GR-NeutroOpen set V containing x.

Conclusion

In this study, new Generalization of Regular Neutro-open sets and Generalized regular Neutro-open set has been studied. Some properties of Regular Neutro-open sets and are studied. Also, properties of GRN-neighbourhood, GRN-interior and GRN-closure properties are investigated. Hope this work will give more benefits for further studies of Neutro-Topology.

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Received: June 15, 2024. Accepted: August 6, 2024