A Note on Generalized Heptagonal Neutrosophic Sets

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Abstract: The concept of a Generalized Heptagonal Neutrosophic set is introduced and its properties are examined in this paper. We also discussed about the interior and closure operators of the Generalized Heptagonal neutrosophic set.

Keywords: neutrosophic Set, Heptagonal neutrosophic set, Generalized Heptagonal neutrosophic set, GHN-interior and GHN-closure operator.

1. Introduction

Introduction and study of fuzzy set theory were done by Zadeh [12]. Atanassov[4] introduced an intuitionistic fuzzy set theory. Coker [5] created later intuitionistic fuzzy topology. Florentine Smarandache [6] established the concept of Neutrosophic Fuzzy set theory in 1999. Truth, falsehood, and indeterminacy are the three components on which he defined the neutrosophic set. Salama and et al. [1-2] derived the neutrosophic topological spaces by transforming the idea of neutrosophic crisp set in 2012. Many scientists working in the fields of partitioned, quadripartitioned, pentapartitioned, heptapartitioned, etc. and they have developed neutrosophic topological spaces recently. Kungumaraj E and et al[8] invented the heptagonal neutrosophic set and heptagonal neutrosophic topological spaces in 2023. Radha R and Gayathri Devi R K [9] introduced the Generalized Quadripartitioned Neutrosophic Set in 2022. We establish and further investigate the notion of a Generalized Heptagonal Neutrosophic set in this study. We also discussed about the Generalized Heptagonal Neutrosophic set's interior and closure operators.

2. Preliminaries

Definition 2.1. [6] Let a non-empty fixed set be X. An element of the form A = {⟨x, αA(x), βA(x), γA(x)⟩: x ∈ X} is known as a neutrosophic set (NS), where αA(x), βA(x), γA(x) represent the degrees of membership, indeterminacy and non-membership respectively, of each element x ∈ X to the set A accordingly.

A neutrosophic set A = {⟨x, αA(x), βA(x), γA(x)⟩: x ∈ X} can be identified as an ordered triple ⟨αA(x), βA(x), γA(x)⟩ in]−0, 1+[on X.

Definition 2.2. [3] A family Ţ of neutrosophic subsets in X that meets the following axioms is a neutrosophic topology (NT) on a non-empty set X.

(Axiom 1) 0N, 1N ∈ Ţ

(Axiom 2) G1 ∩ G2 ∈ Ţ for any G1 , G2 ∈ Ţ

(Axiom 3)∪Gi ∈ Ţ ∀{Gi : i∈ J} ⊆ Ţ

The pair (X, Ţ) is used to represent a neutrosophic topological space Ţ over X.

Definition 2.3. [8] A heptagonal neutrosophic number S is briefed as

S = < [(p, q, r, s, t, u, v); μ], [(p ′, q′ , r′ , s′ , t′ , u′ , v′); ℰ] ,[(p ″, q″, r″, s″, t″, u″, v″); η] > where μ, ℰ, η ∈ [0, 1], where α : R ⇒ [0, μ] denotes the truth membership function, β : R⇒ [ℰ, 1] denotes the indeterminacy membership function and γ : R ⇒ [η, 1] denotes the falsity membership function.

Using ranking technique, heptagonal neutrosophic number is changed as,

$$\lambda = \frac{(p + q + r + s + t + u + v)}{7}$$

$$\mu = \frac{(p' + q' + r' + s' + t' + u' + v')}{7}$$

$$\delta = \frac{(p'' + q'' + r'' + s'' + t'' + u'' + v'')}{7}$$

Then the Heptagonal Neutrosophic set HNS takes the form

A_{HN} = <x; λA_{HN} (x), μA_{HN} (x), δA_{HN} (x) >

Definition 2.4. [8] Assume that X is a non-void set and A_{HN} and B_{HN} be a HNS of the form A_{HN} = <x; λA_{HN} (x), μA_{HN} (x), δA_{HN} (x) >, B_{HN} =<x; λB_{HN} (x), μB_{HN} (x), δB_{HN} (x)>, then their heptagonal neutrosophic number operations may be defined as

- **Inclusive:**
  (i) $A_{HN} \subseteq B_{HN} \Rightarrow \lambda A_{HN}(x) \leq \lambda B_{HN}(x), \mu A_{HN}(x) \geq \mu B_{HN}(x), \delta A_{HN}(x) \geq \delta B_{HN}(x)$, for all x∈X.
  (ii) $B_{HN} \subseteq A_{HN} \Rightarrow \lambda B_{HN}(x) \leq \lambda A_{HN}(x), \mu B_{HN}(x) \geq \mu A_{HN}(x), \delta B_{HN}(x) \geq \delta A_{HN}(x)$, for all x∈X.
- **Union and Intersection:**
  (i) $A_{HN} \cup B_{HN}$ = {<x; ($\lambda A_{HN}(x) \vee \lambda B_{HN}(x), \mu A_{HN}(x) \wedge \mu B_{HN}(x), \delta A_{HN}(x) \wedge \delta B_{HN}(x)$)>}
  (ii) $A_{HN} \cap B_{HN}$ = {<x; ($\lambda A_{HN}(x) \wedge \lambda B_{HN}(x), \mu A_{HN}(x) \vee \mu B_{HN}(x), \delta A_{HN}(x) \vee \delta B_{HN}(x)$)>}
- **Complement:**

  Assume that X is a non-void set and A_{HN} = <x; λA_{HN}(x), μA_{HN}(x), δA_{HN}(x)> be the HNS, then the complement of A is represented by A'_{HN} and it takes the form

  A'_{HN} = <x; δA_{HN} (x), 1−μA_{HN} (x), λA_{HN} (x) > for all x∈X.
- **Universal and Empty set:**

  Let A_{HN} = <x; λA_{HN} (x), μA_{HN} (x), δA_{HN} (x) > be a HNS and the universal set I_A and the null set O_A of A_{HN} is defined by
  (i) I_{HN} = <x; (1, 0, 0)> for all x∈X.
  (ii) O_{HN} = <x; (0, 1, 1)> for all x∈X.

Definition 2.5. [8] A family Ʈ of heptagonal neutrosophic subsets in X that meets the following axioms is a heptagonal neutrosophic topology (HNT) on a non-empty set X.

(HNT1) $I_{HN}(x)$, $O_{HN}(x)$ ∈ Ʈ

(HNT2) ∪ A_i∈ Ʈ , ∀{A_i : i∈ J} ⊆ Ʈ

(HNT3) $A_1 \cap A_2$∈ Ʈ for any A_1, A_2 ∈ Ʈ

The heptagonal neutrosophic topological space Ʈ over X is represented as a pair (X, Ʈ). All the sets in Ʈ are known as heptagonal neutrosophic open set of X and its respective complements are said to be heptagonal neutrosophic closed set of X.

Definition 2.6. [8] Let A be a HNS in HNTS X. Then,

- HN-int(A_{HN}) = ⋃{G_{HN}: G_{HN} is a HNOS in X and $G_{HN} \subseteq A_{HN}$} is referred to heptagonal neutrosophic interior of A. HN-int(A_{HN}) is the largest HN-open subset contained in A_{HN}.
- HN-cl(A_{HN}) = ⋂{K_{HN}: K_{HN} is a HNCS in X and $A_{HN} \subseteq K_{HN}$} is referred to heptagonal neutrosophic closure of A. HN-cl(A_{HN}) is the smallest HN-closed subset containing A_{HN}.

Definition 2.7. [8] Let (X_{HN},Ʈ) and (Y_{HN} ,σ)are two non-empty Heptagonal neutrosophic topological spaces.A map f : $X_{HN} \rightarrow Y_{HN}$ is called a heptagonal neutrosophic continuous function, if each heptagonal neutrosophic open set A_{HN} in Y_{HN} has an inverted image f^{-1}(A_{HN}) that is also a heptagonal neutrosophic open in X_{HN}.

3. Generalized Heptagonal neutrosophic set

we characterize and define a new category of generalized sets in Heptagonal neutrosophic topological spaces in this section.

Definition 3.1: Assume that X is a non-void set and consider A_{HN} be the HNS after ranking technique in definition 2.3, then a generalized heptagonal neutrosophic set GHN(A) is of the form GHN(A) = < x; λGHNA(x), μGHNA(x), δGHNA(x)> where λGHNA(x) is the truth membership degree, μGHNA(x) is the indeterminacy degree and δGHNA(x) is the false membership degree values respectively of each element x∈X to the set A satisfying the condition λGHNA(x) ∧ μGHNA(x) ∧ δGHNA(x) ≤ 0.5.

Example 3.2: Consider X = {x,y} and A_{HN}, B_{HN} ∈ HN(X).

A_{HN} = { < x; (λ: 0.85,0.65,0.55,0.78,0.92,0.63,0.38), (μ: 0.75,0.95,0.63,0.48,0.56,0.88,0.78), (δ: 0.25,0.36,0.45,0.45,0.42,0.72,0.62)>, < y; (λ:0.83,0.65,0.72,0.98,0.66,0.53,0.92), (μ:0.73,0.53,0.45,0.38,0.92,0.75,0.63), (δ:0.45,0.35,0.25,0.35,0.85,0.65,0.15)>} and

B_{HN} = { < x; (λ:0.86,0.73,0.62,0.52,0.93,0.45,1), (μ:0.43,0.39,0.26,0.59,0.58,0.93,0.32), (δ:0.55,0.73,0.62,0.52,0.95,0.89,0.44)>, < y; (λ:0.73,0.62,0.51,0.42,0.33,0.29,0.19), (μ:0.82,0.92,1,0.61,0.54,0.76,0.46), (δ:0.19,0.23,0.63,0.52,0.95,0.82,1)>}

By Ranking Technique, (Definition 2.3)
A_{HN} = { < x; (λ:0.68), (μ:0.72), (δ:0.47)>, < y; (λ:0.76), (μ:0.63), (δ:0.44)>}
B_{HN} = { < x; (λ:0.73), (μ:0.50), (δ:0.67)>, < y; (λ:0.44), (μ:0.73), (δ:0.62)>}

A_{HN}	x	y	B_{HN}	x	y
λ	0.68	0.76	λ	0.73	0.44
μ	0.72	0.63	μ	0.50	0.73
δ	0.47	0.44	δ	0.67	0.62
$\lambda \wedge \mu \wedge \delta$	**0.47**	**0.44**	$\lambda \wedge \mu \wedge \delta$	**0.50**	**0.44**
A_{HN} is a Generalized Heptagonal NS			**B_{HN} is a Generalized Heptagonal NS**		

Definition 3.3: Generalized Heptagonal neutrosophic Set operations

Assume that X is a non-void set and GHN(U) and GHN(V) are HNS of the form
GHN(U) = < a; λGHNU(a), μGHNU(a), δGHNU(a) >,

GHN(V) = < a; λGHNV(a), μGHNV(a), δGHNV(a)>, then the generalized heptagonal neutrosophic number operations may be defined as

- **Inclusive:**

  GHN(U)⊆GHN(V) ⇒ λGHNU(a)≤λGHNV(a), μGHNU(a)≥μGHNV(a), δGHNU(a)≥δGHNV(a), for all a∈X.

Union and Intersection:

GHN(U)⋃GHN(V) = {<a; (λGHNU(a)∨λGHNV(a), μGHNU(a)∧μGHNV(a), δGHNU(a)∧δGHNV(a))>}

GHN(U)⋂GHN(V) = {<a; (λGHNU(a)∧λGHNV(a), μGHNU(a)∨μGHNV(a), δGHNU(a)∨δGHNV(a))>}

Complement:

Assume that X is a non-void set and GHN(A) be the GHNS of the form <a; λGHNA(a), μGHNA(a), δGHNA(a)>, then its complement is represented by GHN(A′) and is defined by

GHN(A′) = < a; (δGHNA(a), 1–μGHNA(a), λGHNA(a)) > for all a∈X.

Universal and Empty set:

The universal set and the null set of GHNS over X is defined by

(i) GHN(Ia) = <a; (1,0,0)> for all a∈X.

(ii) GHN(Oa) = <a; (0,1,1)> for all a∈X.

For simplicity, we consider the GHNS after the ranking technique in the subsequent examples:

Example 3.4:

(i) Consider Y = {p,q,r} and the Generalized heptagonal neutrosophic sets
GHN(A) = {<y: (p,0.52, 0.42, 0.67),(q,0.31,0.75,0.44),(r,0.26,0.68,0.88)>}
GHN(B) = {<y: (p,0.66, 0.33,0.48),(q,0.55,0.66,0.33),(r,0.48,0.32,0.64)>}
Here GHN(A) ⊆ GHN(B), since

{<y: (p, 0.52 ≤ 0.66, 0.42 ≥ 0.33, 0.67 ≥ 0.48),(q, 0.31 ≤ 0.55,0.75 ≥ 0.66,0.44 ≥ 0.33),
(r, 0.26 ≤ 0.48,0.68 ≥ 0.32,0.88 ≥ 0.64)>}

(ii) Consider Y = {p,q,r} and the Generalized heptagonal neutrosophic sets
GHN(C) = {<y: (p,0.43,0.52,0.68),(q,0.91,0.43,0.26),(r,0.85,0.69,0.37)>}
GHN(D) = {<y: (p,0.74,0.68,0.18),(q,0.39,0.45,0.77),(r,0.14,0.52,0.88)>}

GHN(C)⋃GHN(D) = {<y: (p, 0.43∨0.74, 0.52∧0.68, 0.68∧0.18), (q, 0.91∨0.39, 0.43∧0.45, 0.26∧0.77), (r, 0.85∨0.14, 0.69∧0.52, 0.37∧0.88)>} = {<x: (p,0.74,0.52,0.18),(q,0.91,0.43,0.26),(r,0.85,0.52,0.37)>}
GHN(C)⋂GHN(D) = {<y: (p, 0.43∧0.74, 0.52∨0.68, 0.68∨0.18), (q, 0.91∧0.39, 0.43∨0.45, 0.26∨0.77), (r, 0.85∧0.14, 0.69∨0.52, 0.37∨0.88)>} = {<x: (p,0.43,0.68,0.68),(q,0.39,0.45,0.77),(r,0.14,0.69,0.88)>}
GHN(A′) = {<y: (p,0.68,0.48,0.43),(q,0.26,0.57,0.91),(r,0.37,0.31,0.85)>}

GHN(B′) = {<y: (p,0.18,0.32,0.74),(q,0.77,0.55,0.39),(r,0.88,0.48,0.14)>}

Definition 3.5: A family Ʈ of Generalized heptagonal neutrosophic sets adhering to the following axioms is called a Generalized heptagonal neutrosophic topology on a non-empty set Y.

i) GHN(I_X), GHN(O_X) ∈Ʈ.

ii) For any sub collection of the elements of Ʈ, whose union is contained in Ʈ.

iii) For any finite sub collection of the elements of Ʈ, whose intersection is contained in Ʈ.

The pair (Y, Ʈ) is called a Generalized heptagonal neutrosophic topological space (GHNTS) over Y.

Remark 3.6:

1. Every member of Ʈ is referred to be a GHN-open set in X.
2. The set GHN(A) is referred to be a GHN-closed set in X if GHN(A′) is open in Ʈ.

Example 3.7: Consider the Generalized heptagonal neutrosophic sets with Y = {s, t, u}

GHN(D) = {<y: (s,0.25, 0.45, 0.65), (t,0.50, 0.60, 0.70), (u, 0.35, 0.25, 0.15)>}

GHN(E) = {<y: (s,0.33, 0.44, 0.55), (t,0.55, 0.56, 0.57), (u, 0.48, 0.18, 0.12)>}

Here GHN(D)⋃GHN(E) = GHN(E) and GHN(D)⋂GHN(E) = GHN(E)

Hence Ʈ $= \{GHN(Ix), GHN(D), GHN(E), GHN(Ox)\}$ forms a Generalized Heptagonal Neutrosophic Topological space.

Definition 3.8: Let GHN(A) be a GHNS in GHNTS X. Then,

- GHN-int(A) = ⋃ {GHN(F) ; where GHN(F) is GHNO in X and GHN(F) ⊆ GHN(A)} is said to be a generalized heptagonal neutrosophic interior of A. GHN-int(A) is the largest GHN-open subset contained in GHN(A).
- GHN-cl(A) = ⋂ {GHN(K) ; where GHN(K) is GHNC in X and GHN(A) ⊆ GHN(K)} is said to be a generalized heptagonal neutrosophic closure of A. GHN-int(A) is the smallest GHN-closed subset containing GHN(A).

Example 3.9: Consider X = {s,t} and the Generalized heptagonal neutrosophic sets

GHN(F_1) = {<x: (s, 0.4, 0.3, 0.5), (t, 0.1, 0.2, 0.5)>}

GHN(F_2) = {<x: (s, 0.4, 0.4, 0.5), (t, 0.4, 0.3, 0.4)>}

Here GHN(F_1)⋃GHN(F_2) = GHN(F_2) and GHN(F_1)⋂GHN(F_2) = GHN(F_1)

Ʈ $= \{GHN(I_x), GHN(F_1), GHN(F_2), GHN(O_x)\}$ is a Generalized Heptagonal NeutrosophicTopological space, then

GHN(F_1) and GHN(F_2) are GHN-open sets of X, GHN(F_1') and GHN(F_2') are GHN-closed sets of X.

Consider the GHN sets

GHN(A) {<x: (s, 0.3, 0.3, 0.6), (t, 0.3, 0.2, 0.5)>},

GHN(B) {<x: (s, 0.6, 0.7, 0.3), (t, 0.5, 0.8, 0.3)>},

GHN(C) {<x: (s, 0.4, 0.6, 0.5), (t, 0.3, 0.6, 0.9)>} and

GHN(D) {<x: (s, 0.5, 0.4, 0.4), (t, 0.9, 0.4, 0.3)>}

GHN – interior operator		**GHN – closure operator**	
GHN-int(F_1) = $GHN(F_1)$	GHN-int(A) = $GHN(O_x)$	GHN-cl(F_1) = $GHN(I_x)$	GHN-cl(A) = $GHN(I_x)$
GHN-int(F_2) = $GHN(F_2)$	GHN-int(B) = $GHN(O_x)$	GHN-cl(F_2) = $GHN(I_x)$	GHN-cl(B) = $GHN(I_x)$
GHN-int(F_1') = $GHN(O_x)$	GHN-int(C) = $GHN(O_x)$	GHN-cl(F_1') = $GHN(F_1')$	GHN-cl(C) = $GHN(I_x)$
GHN-int(F_2') = $GHN(O_x)$	GHN-int(D) = $GHN(O_x)$	GHN-cl(F_2') = $GHN(F_2')$	GHN-cl(D) = $GHN(I_x)$

Proposition 3.10:

Consider (X, Ţ) be a GHNTS. Therefore, for every two generalized heptagonal neutrosophic subsets GHN(M) and GHN(N) of a GHNTS X we have

(i) GHN-int(M)⊆M
(ii) GHN(M) is a GHNO set in X if and only if GHN-int(M) = M
(iii) GHN-int(GHN-int(M)) = GHN-int(M)
(iv) If M ⊆ N then GHN-int(M) ⊆ GHN-int(N)
(v) GHN-int(M∩N) = GHN-int(M)∩GHN-int(N)
(vi) GHN-int(M)∪GHN-int(N) ⊆ GHN-int(M∪N)

Proof.

(i) Follows from Definition 3.8.

(ii) GHN(M) is a GHNO set in X. Then M ⊆GHN-int(M) and by using (i) we get GHN-int(M) = M. Conversely assume that GHN-int(M) = M. By using Definition 3.8, GHN(M) is a GHNO set in X. Thus (ii) is proved.

(iii) By using (ii), GHN-int(GHN-int(M)) = GHN-int(M). This proves (iii).

(iv) Since M⊆N, by using (i), GHN-int(M)⊆M⊆N. That is GHN-int(M)⊆N. By (iii), GHN-int(GHN-int(M)) ⊆ GHN-int(N). Thus GHN-int(M) ⊆ GHN-int(N). Thus (iv) is proved.

(v) Since M∩N⊆M and M∩N⊆N, by using (iv), GHN-int(M∩N) ⊆ GHN-int(M) and GHN-int(M∩N) ⊆ GHN-int(N). This implies that
GHN-int(M∩N) ⊆ GHN-int(M)∩ GHN-int(N) ---(1).
By(i), GHN-int(M)⊆M and GHN-int(N)⊆N. This implies that
GHN-int(M)∩GHN-int(N) ⊆ M∩N.
Now by (iv), GHN-int(GHN-int(M)∩GHN-int(N)) ⊆ GHN-int(M∩N)
By (1), GHN-int(GHN-int(M))∩GHN-int(GHN-int(N)) ⊆ GHN-int(M∩N).
By (iii), GHN-int(M)∩GHN-int(N) ⊆ GHN-int(M∩N) -----(2).
From (1) and (2), GHN-int(M∩N)=GHN-int(M)∩GHN-int(N). Thus (v) is proved.

(vi) Since M⊆M∪N and M⊆M∪N, by (iv), GHN-int(M)⊆GHN-int(M∪N) and GHN-int(N)⊆GHN-int(M∪N). This implies that,
GHN-int(M)∪GHN-int(N)⊆GHN-int(M∪N). Thus (vi) is proved.

Proposition 3.11: Consider (X, Ʈ) be a GHNTS. Therefore, for every two generalized heptagonal neutrosophic subsets P and Q of a GHNTS X we have

(i) P ⊆ GHN-cl(P)
(ii) P is GHNC set in X if and only if GHN-cl(P) = P
(iii) GHN-cl(GHN-cl(P)) = GHN-cl(P)
(iv) If P ⊆ Q then GHN-cl(P)⊆ GHN-cl(Q)
(v) GHN-cl(P⋂Q) ⊆ GHN-cl(P)⋂GHN-cl(Q)
(vi) GHN-cl(P)⋃GHN-cl(Q) = GHN-cl(P⋃Q)

Proof.

I. Proceed from the definition 3.8.
II. Consider P as a GHNC set in X. Then P contains GHN-cl(P). Now by using (i), we get P = GHN-cl(P). Conversely assume that P = GHN-cl(P). By using Definition 3.8, P is a GHNC set in X. Thus (ii) is proved.
III. By using (ii), GHN-cl(GHN-cl(P)) = GHN-cl(P). This (iii) is proved.
IV. By using (i), Q ⊆ GHN-cl(Q) and since P ⊆ Q, we have P ⊆ GHN-cl(Q). But GHN-cl(P) is the smallest closed set containing P, hence GHN-cl(P) ⊆ GHN-cl(Q). Thus (iv) is proved.
V. Since P⋂Q ⊆ P and P⋂Q ⊆ Q, by using (iv), GHN-cl(P⋂Q) ⊆ GHN-cl(P) and GHN-cl(P⋂Q) ⊆ GHN-cl(Q). This implies that GHN-cl(P⋂Q) ⊆ GHN-cl(P)⋂GHN-cl(Q). Thus (v) is proved.
VI. Since P ⊆ P⋃Q and Q ⊆ P⋃Q, by (iv), GHN-cl(P) ⊆ GHN-cl(P⋃Q) and GHN-cl(Q) ⊆ GHN-cl(P⋃Q). This implies that,
GHN-cl(P) ⋃ GHN-cl(Q) ⊆ GHN-cl(P⋃Q) ------(1)
By(i), P ⊆ GHN-cl(P) and Q ⊆ GHN-cl(Q). This implies that
P⋃Q ⊆ GHN-cl(P) ⋃ GHN-cl(Q).
Now by (iv), GHN-cl (P⋃Q) ⊆ GHN-cl((GHN-cl(P) ⋃ GHN-cl(Q)).
By (1), GHN-cl(P⋃Q) ⊆ GHN-cl(GHN-cl(P)) ⋃ GHN-cl(GHN-cl(Q)).
By (iii), GHN-cl(P⋃Q) ⊆ GHN-cl(P) ⋃ GHN-cl(Q) -----(2).

From (1) and (2), GHN-cl(P⋃Q) = GHN-cl(P) ⋃ GHN-cl(Q).
Thus (vi) is proved.

Proposition 3.12: Consider (X, Ʈ) be a GHNTS. Therefore, for every generalized heptagonal neutrosophic subset U in a GHNTS X we have.

(i) (GHN-int(U))′ = GHN-cl(U′)
(ii) (GHN-cl(U))′ = GHN-int(U′)

Proof.

(i) By definition 3.8, GHN-int (U) = ⋃{ S: S is a GHNO set in X and S ⊆ U}
Taking complement on both sides,
(GHN-int(U))′ = ⋂{ S′: S′ is a GHNC set in X and U′⊆ S′}
Now, replace S′ by L, we get
(GHN-int(U))′ = ⋂{ L : L is a GHNC set in X and U′ ⊆ L}
From the definition 3.8, (GHN-int(A_{HN}))′ = GHN-cl(U′). Thus (i) is proved.
(ii) From (i) Let U′ be the GHNS

We write, (GHN-int(U′))′ = GHN-cl(U)
Taking complement on both sides we get
GHN-int(U′) = (GHN-cl(U))′. Thus (ii) is proved.

Conclusion

The basic operations of generalized heptagonal neutrosophic sets are demonstrated in this article with suitable examples. Further explanations of the concepts of Generalized Heptagonal neutrosophic interior and closure are provided in order to support the GHN topology. The properties of GHN-closed and GHN-open sets of GHN topologies are explained with similar examples. Furthermore, based on GHN topological spaces, continual functions, connectivity, and compact can be developed in the future.

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