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Heptagonal Neutrosophic Quotient Mappings

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Abstract: In 2023, [6] Kungumaraj et al. presented the Heptagonal Neurosophic Number and Heptagonal Neurosophic Topology. Heptagonal neutrosophic numbers are essential because they provide a powerful tool for representing and managing uncertainty in decision-making processes across various domains, offering a more nuanced and versatile approach compared to traditional fuzzy or intuitionistic fuzzy sets. Heptagonal neutrosophic methods differ from other neutrosophic methods primarily in the number of parameters they consider and their applications. Heptagonal neutrosophic numbers consider seven parameters, namely truth, falsity, indeterminacy, neutral, anti-neutral, extra-neutral, and pseudo-neutrality. By de-neutrosophication technique, heptagonal neutrosophic numbers transformed into a crisp neutrosophic values for better outcomes.
The major goal of this research is to investigate the concepts of Heptagonal Neutrosophic (HN) quotient mappings as well as Heptagonal neutrosophic strongly quotient maps and HN*-quotient maps in Heptagonal neutrosophic topological spaces. We provide examples of the fundamental concepts and subsequently we also proved their characterizations.

Keywords: Heptagonal neutrosophic number**;** HN-irresolute map; HN-open map; HN-quotient map; HN strongly quotient map; HN*-quotient map.

1. Introduction

The fuzzy set theory was introduced and studied by Zadeh [11]. An intuitionistic fuzzy set theory was introduced by Atanassov [3]. Later intuitionistic fuzzy topology was developed by Coker [4]. Neutrosophic Fuzzy set theory was introduced by Smarandache [5] in 1999. He defined the neutrosophic set on three components (truth, falsehood, indeterminacy). The Neutrosophic crisp set concept was converted into neutrosophic topological spaces by Salama et al. in [1-2]. In recent years, neutrosophic topological spaces was developed by many scientists in the field of triangular, quadripartitioned, pentapartitioned, heptapartitioned etc. Recently, heptagonal neutrosophic set and heptagonal neutrosophic topological spaces was developed by Kungumaraj E and et al in 2023[6].

Quotient mappings have applications across various areas of mathematics, including algebraic topology, differential geometry, and geometric group theory. Neutrosophic Quotient mappings was first introduced by T. Nandhini and M. Vigneshwaran[8] in 2019. Later Mohana Sundari M and etal[7] and Radha R and etal [9] introduced respectively the Quadripartitioned Neutrosophic Mappings and Pentapartitioned Neutrosophic Quotient Mappings. Recently, Subasree R and etal [10] investigated about the Heptagonal Neutrosophic Semi-open Sets in Heptagonal Neutrosophic Topological Spaces.

In this paper, we presented the following in section-wise, Section 2 provides background information that will help readers understand the study better. Section 3 introduces the concept of heptagonal neutrosophic quotient mappings, HN-strongly quotient maps and HN*-quotient maps along with their key features and instances are given. Section 4 looks at these maps' characterizations as well as the compositions of two of them. The study's final conclusions with illustrations are presented in Section 5 of the conclusion, along with some suggestions for more research.

2. Preliminaries

Definition 2.1. [5] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form A = {⟨x, $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$⟩: x ∈ X} where $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X to the set A.

A Neutrosophic set A = {⟨x, $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$⟩: x ∈ X} can be identified as an ordered triple ⟨ $\alpha_A(x)$, $\beta_A(x)$, $\gamma_A(x)$⟩ in] −0, 1 +[on X.

Definition 2.2. [1] A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

(NT1) 0_N, 1_N∈ τ

(NT2) G_1∩ G_2∈ τ for any G_1 , G_2∈ τ

(NT3)∪G_i∈ τ ∀{G_i : i∈ J} ⊆ τ

The pair (X, τ) is used to represent a neutrosophic topological space τ over X.

Definition 2.3. [6] A heptagonal neutrosophic number S is defined and described as

S = < [(p, q, r, s, t, u, v); μ],[(p ′ , q′ , r′ , s′ , t′ , u′ , v′); $\mathscr{E}$] ,[(p ″, q″, r″, s″, t″, u″, v″); η] > where μ, $\mathscr{E}$, η ∈ [0, 1].The truth membership function α : R ⇒ [0, μ], the indeterminacy membership function β : R⇒ [$\mathscr{E}$, 1], the falsity membership function γ : R ⇒ [η, 1].

Using deneutrosophication technique, heptagonal neutrosophic number is changed as,

$$\lambda = \frac{(p+q+r+s+t+u+v)}{7}$$

$$\mu = \frac{(p'+q'+r'+s'+t'+u'+v')}{7}$$

$$\delta = \frac{(p''+q''+r''+s''+t''+u''+v'')}{7}$$

Then the Heptagonal Neutrosophic set HNS takes the crisp form

A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$ >

Definition 2.4. [6] Let X be a non-empty set and A_{HN} and B_{HN} are HNS of the form A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$ >, B_{HN} =<x; $\lambda B_{HN}(x)$, $\mu B_{HN}(x)$, $\delta B_{HN}(x)$ >, then their heptagonal neutrosophic number operations may be defined as

- **Inclusive:**
  (i) $A_{HN} \subseteq B_{HN} \Rightarrow \lambda A_{HN}(x) \le \lambda B_{HN}(x), \mu A_{HN}(x) \ge \mu B_{HN}(x), \delta A_{HN}(x) \ge \delta B_{HN}(x)$, for all x∈X.
  (ii) $B_{HN} \subseteq A_{HN} \Rightarrow \lambda B_{HN}(x) \le \lambda A_{HN}(x), \mu B_{HN}(x) \ge \mu A_{HN}(x), \delta B_{HN}(x) \ge \delta A_{HN}(x)$, for all x∈X.
- **Union and Intersection:**
  (i) $A_{HN} \cup B_{HN}$ = {<x; $(\lambda A_{HN}(x) \vee \lambda B_{HN}(x), \mu A_{HN}(x) \wedge \mu B_{HN}(x), \delta A_{HN}(x) \wedge \delta B_{HN}(x))$ >}
  (ii) $A_{HN} \cap B_{HN}$ = {<x; $(\lambda A_{HN}(x) \wedge \lambda B_{HN}(x), \mu A_{HN}(x) \vee \mu B_{HN}(x), \delta A_{HN}(x) \vee \delta B_{HN}(x))$ >}
- **Complement:**
  Let X be a non-empty set and A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$> be the HNS, then its complement is denoted by A'_{HN} and is defined by
  A'_{HN} = <x; $\delta A_{HN}(x)$, $1-\mu A_{HN}(x)$, $\lambda A_{HN}(x)$ > for all x∈X.
- **Universal and Empty set:**

Let A_{HN} = <x; $\lambda A_{HN}(x)$, $\mu A_{HN}(x)$, $\delta A_{HN}(x)$ > be a HNS and the universal set I_A and the null set O_A of A_{HN} is defined by

(i) I_{HN} = <x: (1,0,0)> for all $x \in X$.

(ii) O_{HN} = <x: (0,1,1)> for all $x \in X$.

Definition 2.5. [6] A Heptagonal neutrosophic topology (HNT) on a non-empty set X is a family τ of heptagonal neutrosophic subsets in X satisfies the following axioms:

(HNT1) $I_{HN}(x)$, $O_{HN}(x) \in \tau$

(HNT2) $\bigcup A_i \in \tau$, $\forall \{A_i : i \in J\} \subseteq \tau$

(HNT3) $A_1 \cap A_2 \in \tau$ for any A_1 , $A_2 \in \tau$

The pair (X, τ) is used to represent a heptagonal neutrosophic topological space τ over X. The sets in τ are called heptagonal neutrosophic open set of X. The complement of heptagonal neutrosophic open sets are called heptagonal neutrosophic closed set of X.

Definition 2.6. [6] Let A be a HNS in HNTS X. Then,

- HNint(A_{HN}) = $\bigcup\{G_{HN}$: G_{HN} is a HNOS in X and $G_{HN} \subseteq A_{HN}\}$ is called a heptagonal neutrosophic interior of A. It is the largest HN-open subset contained in A_{HN}.
- HNcl(A_{HN}) = $\bigcap\{K_{HN}$: K_{HN} is a HNCS in X and $A_{HN} \subseteq K_{HN}\}$ is called a heptagonal neutrosophic closure of A. It is the smallest HN-closed subset containing A_{HN}.

Definition 2.7.[6]Let (X_{HN},τ) and (Y_{HN} ,σ)are two non-empty Heptagonal neutrosophic topological spaces.A map f: $X_{HN} \rightarrow Y_{HN}$ is called a heptagonal neutrosophic continuous function if the inverse image $\boldsymbol{f^{-1}}$(A_{HN}) of each heptagonal neutrosophic open set A_{HN} in Y_{HN} is heptagonal neutrosophic open in X_{HN}.

3. Heptagonal Neutrosophic Quotient Mappings

In this section, we define a new class of sets in Heptagonal Neutrosophic topological spaces and the quotient mappings.

Definition 3.1: Let A_{HN} be a HNS of a HNTS (X_{HN},τ). Then A_{HN} is said to be

(i) Heptagonal Neutrosophic pre-open [written HN-preO] set of X, if
$A_{HN} \subseteq$ HNint (HNcl(A_{HN})).

(ii) Heptagonal Neutrosophic semi-open [written HN-SO] set of X, if
$A_{HN} \subseteq$ HNcl (HNint(A_{HN})).

(iii) Heptagonal Neutrosophic α-open [written HN-αO] set of X, if
$A_{HN} \subseteq$ HNint (HNcl(HNint (A_{HN}))).

Example 3.2: Let X = {x,y}.Consider

A_{HN} = { < x; (λ:0.8,0.8,0.8,0.8,0.8,0.8,0.8), (μ: 0.3,0.3,0.3,0.3,0.3,0.3,0.3), (δ: 0.5,0.5,0.5,0.5,0.5,0.5,0.5)>, < y; (λ:0.6,0.6,0.6,0.6,0.6,0.6,0.6), (μ:0.5,0.5,0.5,0.5,0.5,0.5,0.5), (δ:0.9,0.9,0.9,0.9,0.9,0.9,0.9)>}

B_{HN} = { < x; (λ: 0.9,0.9,0.9,0.9,0.9,0.9,0.9), (μ:0.2,0.2,0.2,0.2,0.2,0.2,0.2), (δ:0.4,0.4,0.4,0.4,0.4,0.4,0.4)>, < y; (λ: 0.8,0.8,0.8,0.8,0.8,0.8,0.8), (μ: 0.3,0.3,0.3,0.3,0.3,0.3,0.3), (δ:0.7,0.7,0.7,0.7,0.7,0.7,0.7)>} and

C_{HN} = { < x; (λ: 0.5,0.5,0.5,0.5,0.5,0.5,0.5), (μ: 0.3,0.3,0.3,0.3,0.3,0.3,0.3), (δ: 0.5,0.5,0.5,0.5,0.5,0.5,0.5)>, < y; (λ: 0.4,0.4,0.4,0.4,0.4,0.4,0.4), (μ: 0.5,0.5,0.5,0.5,0.5,0.5,0.5), (δ: 0.9,0.9,0.9,0.9,0.9,0.9,0.9)>}

After de-neutrosophication technique in definition 2.3,

A_{HN} = { < x; (λ:0.8), (μ: 0.3), (δ: 0.5)>, < y; (λ:0.6), (μ:0.5), (δ:0.9)>}

B_{HN} = { < x; (λ: 0.9), (μ:0.2), (δ:0.4)>, < y; (λ: 0.8), (μ: 0.3), (δ:0.7)>} and

C_{HN} = { < x; (λ: 0.5), (μ: 0.3), (δ: 0.5)>, < y; (λ: 0.4), (μ: 0.5), (δ: 0.9)>}

τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, A_{HN}⋃B_{HN}, A_{HN}⋂C_{HN}, B_{HN}⋃C_{HN}, A_{HN}⋂B_{HN}, A_{HN}⋂C_{HN}, B_{HN}⋂C_{HN}} be the Heptagonal Neutrosophic topological space.

Consider the other HNS after ranking technique,

D_{HN} = { < x; (λ:0.9), (μ: 0.2), (δ: 0.4)>, < y; (λ:0.7), (μ:0.3), (δ:0.8)>}

E_{HN} = { < x; (λ: 0.6), (μ:0.4), (δ:0.6)>, < y; (λ: 0.3), (μ: 0.6), (δ:0.7)>} and

F_{HN} = { < x; (λ: 0.8), (μ: 0.3), (δ: 0.5)>, < y; (λ: 0.6), (μ: 0.5), (δ: 0.7)>}

Then the HN pre-O sets of X are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, E_{HN}, F_{HN}, E'_{HN}}

HN semi-O sets of X are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, F_{HN}}

HN α-O sets of X are {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}, D_{HN}, F_{HN}}

Remark 3.3: HN α-open set is the smallest set contained in both HN Pre open sets and HN semiopen sets of X.

Definition 3.4. Let (X_{HN},τ) and (Y_{HN},σ)are two non-empty Heptagonal neutrosophic topological spaces. A map f : X_{HN}→ Y_{HN} is called a

(i) HN pre-continuous function, if f^{-1}(A_{HN}) is HN-pre open in (X_{HN},τ), for each HN open set A_{HN} in (Y_{HN} ,σ).
(ii) HN semi-continuous function, if f^{-1} (A_{HN}) is HN-semi open in (X_{HN},τ), for each HN open set A_{HN} in (Y_{HN} ,σ).
(iii) HN α-continuous function, if f^{-1}(A_{HN}) is HN-α open in (X_{HN},τ), for each HN open set A_{HN} in (Y_{HN} ,σ).

Example 3.5: Let X = {x,y} and τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}}, then (X_{HN},τ) be a Heptagonal neutrosophic topological space with

A_{HN} = { < x; (λ:0.8), (μ: 0.3), (δ: 0.5)>, < y; (λ:0.6), (μ:0.5), (δ:0.9)>}

B_{HN} = { < x; (λ: 0.9), (μ:0.2), (δ:0.4)>, < y; (λ: 0.8), (μ: 0.3), (δ:0.7)>}

C_{HN} = { < x; (λ: 0.5), (μ: 0.3), (δ: 0.5)>, < y; (λ: 0.4), (μ: 0.5), (δ: 0.9)>} and

Let Y = {p,q} and σ = {I_{HN}, O_{HN}, U_{HN}, V_{HN}, W_{HN}}, then (Y_{HN},σ) be a Heptagonal neutrosophic topological space with

U_{HN} = { < p; (λ:0.9), (μ: 0.2), (δ: 0.4)>, < q; (λ:0.7), (μ:0.3), (δ:0.7)>}

V_{HN} = { < p; (λ: 0.6), (μ:0.4), (δ:0.6)>, < q; (λ: 0.3), (μ: 0.6), (δ:0.8)>}

W_{HN} = { < p; (λ: 0.8), (μ: 0.3), (δ: 0.5)>, < q; (λ: 0.6), (μ: 0.5), (δ: 0.7)>}

Define a map f : X_{HN}→ Y_{HN} by f(x) = p , f(y) = q, then f is HN pre-continuous map.

Definition 3.6. Let (X_{HN},τ) and (Y_{HN} ,σ)are two non-empty Heptagonal neutrosophic topological spaces.A map f : X_{HN}→ Y_{HN} is called a

(i) HN irresolute map, if f^{-1}(A_{HN}) is HN open in (X_{HN},τ), for each HN open set A_{HN} in (Y_{HN} ,σ).
(ii) HN pre-irresolute map, if f^{-1}(A_{HN}) is HN-pre open in (X_{HN},τ), for each HN–pre open set A_{HN} in (Y_{HN} ,σ).
(iii) HN semi-irresolute map, if f^{-1}(A_{HN}) is HN-semi open in (X_{HN},τ), for each HN–semi open set A_{HN} in (Y_{HN} ,σ).
(iv) HN α-irresolute map, if f^{-1}(A_{HN}) is HN-α open in (X_{HN},τ), for each HN–α open set A_{HN} in (Y_{HN} ,σ).

Example 3.7: Let X = {x,y,z} and τ = {I_{HN}, O_{HN}, A_{HN}}, then (X_{HN},τ) be a Heptagonal neutrosophic topological space with

A_{HN} = { < x; (λ:0.5), (μ: 0.5), (δ: 0.5)>, < y; (λ:0.5), (μ:0.5), (δ:0.5)>, < z; (λ:0.5), (μ:0.5), (δ:0.5)>}

Let Y = {p,q,r} and σ = {I_{HN}, O_{HN}, U_{HN}, V_{HN}, W_{HN}}, then (Y_{HN},σ) be a Heptagonal neutrosophic topological space with

U_{HN} = { < p; (λ:0.3), (μ: 0.3), (δ: 0.3)>, < q; (λ:0.3), (μ:0.3), (δ:0.3)>, < r; (λ:0.3), (μ:0.3), (δ:0.3)>}

Define a map *f* : X_{HN}→ Y_{HN} by *f*(x) = p, *f*(y) = q and *f*(z) = r then *f* is HN pre-irresolute map.

Definition 3.8. Let (X_{HN},τ) and (Y_{HN} ,σ) are two non-empty Heptagonal neutrosophic topological spaces.A map *f* : X_{HN}→ Y_{HN} is called a

(i) HN open map, if *f* (A_{HN}) is HN open in (Y_{HN},σ), for each HN open set A_{HN} in (X_{HN},τ).
(ii) HN pre-open map, if *f* (A_{HN}) is HN-pre open in (Y_{HN},σ), for each HN–pre open set A_{HN} in (X_{HN},τ).
(iii) HN semi-open map, if *f* (A_{HN}) is HN-semi open in (Y_{HN},σ), for each HN–semi open set A_{HN} in (X_{HN},τ).
(iv) HN α-open map, if *f* (A_{HN}) is HN-α open in (Y_{HN},σ), for each HN–α open set A_{HN} in (X_{HN},τ).

Example 3.9: Let X = {x,y} and τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}}, then (X_{HN},τ) be a Heptagonal neutrosophic topological space with

A_{HN} = { < x; (λ:0.8), (μ: 0.3), (δ: 0.5)>, < y; (λ:0.6), (μ:0.5), (δ:0.9)>}
B_{HN} = { < x; (λ: 0.9), (μ:0.2), (δ:0.4)>, < y; (λ: 0.8), (μ: 0.3), (δ:0.7)>}
C_{HN} = { < x; (λ: 0.5), (μ: 0.3), (δ: 0.5)>, < y; (λ: 0.4), (μ: 0.5), (δ: 0.9)>} and
Let Y = {p,q} and σ = {I_{HN}, O_{HN}, U_{HN}, V_{HN}, W_{HN}}, then (Y_{HN},σ) be a Heptagonal neutrosophic topological space with
U_{HN} = { < p; (λ:0.9), (μ: 0.2), (δ: 0.4)>, < q; (λ:0.7), (μ:0.3), (δ:0.7)>}
V_{HN} = { < p; (λ: 0.6), (μ:0.4), (δ:0.6)>, < q; (λ: 0.3), (μ: 0.6), (δ:0.8)>}
W_{HN} = { < p; (λ: 0.8), (μ: 0.3), (δ: 0.5)>, < q; (λ: 0.6), (μ: 0.5), (δ: 0.7)>}
Define a map *f* : X_{HN}→ Y_{HN} by *f*(x) = p, *f*(y) = q and *f*(z) = r, then *f* is a HN pre-open map.

Definition 3.10. Let (X_{HN},τ) and (Y_{HN} ,σ) are two non-empty Heptagonal neutrosophic topological spaces.A map *f* : X_{HN}→ Y_{HN} is called a

(i) HN quotient map, if *f* is both HN-continuous and $\boldsymbol{f^{-1}}$(A_{HN}) is HN open in (X_{HN},τ), implies A_{HN} is HN open set in (Y_{HN} ,σ).
(ii) HN pre-quotient map, if *f* is both HN-pre continuous and $\boldsymbol{f^{-1}}$(A_{HN}) is HN open in (X_{HN},τ), implies A_{HN} is HN-pre open set in (Y_{HN} ,σ).
(iii) HN semi-quotient map, if *f* is both HN-semi continuous and $\boldsymbol{f^{-1}}$(A_{HN}) is HN open in (X_{HN},τ), implies A_{HN} is HN-semi open set in (Y_{HN} ,σ).
(iv) HN α-quotient map, if *f* is both HN-α continuous and $\boldsymbol{f^{-1}}$(A_{HN}) is HN open in (X_{HN},τ), implies A_{HN} is HN-α open set in (Y_{HN} ,σ).

Example 3.11: Let X = {p,q} and τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}}, then (X_{HN},τ) be a Heptagonal neutrosophic topological space with
A_{HN} = { < p; (λ:0.8), (μ: 0.3), (δ: 0.5)>, < q; (λ:0.6), (μ:0.5), (δ:0.9)>}
B_{HN} = { < p; (λ: 0.9), (μ:0.2), (δ:0.4)>, < q; (λ: 0.8), (μ: 0.3), (δ:0.7)>}
C_{HN} = { < p; (λ: 0.5), (μ: 0.3), (δ: 0.5)>, < q; (λ: 0.4), (μ: 0.5), (δ: 0.9)>} and
Let Y = {r,s} and σ = {I_{HN}, O_{HN}, U_{HN}, V_{HN}, W_{HN}}, then (Y_{HN},σ) be a Heptagonal neutrosophic topological space with
U_{HN} = { < r; (λ:0.9), (μ: 0.2), (δ: 0.4)>, < s; (λ:0.7), (μ:0.3), (δ:0.7)>}

V_{HN} = { < r; (λ: 0.6), (μ:0.4), (δ:0.6)>, < s; (λ: 0.3), (μ: 0.6), (δ:0.8)>}

W_{HN} = { < r; (λ: 0.8), (μ: 0.3), (δ: 0.5)>, < s; (λ: 0.6), (μ: 0.5), (δ: 0.7)>}

Define a map *f* : $X_{HN} \rightarrow Y_{HN}$ by *f*(p) = r, *f*(q) = s.

Here *f* is a HN pre-quotient map.

Definition 3.12. Let (X_{HN},τ) and (Y_{HN},σ) are two non-empty heptagonal neutrosophic topological spaces.A map f: $X_{HN} \rightarrow Y_{HN}$ is called a

(i) HN-strongly quotient map, provided A_{HN} is HN-open set in (Y_{HN} ,σ) if and only if f^{-1}(A_{HN}) is HN open in (X_{HN},τ)
(ii) HN-strongly pre-quotient map, provided A_{HN} is HN-open set in (Y_{HN} ,σ) if and only if f^{-1}(A_{HN}) is HN-pre open in (X_{HN},τ).
(iii) HN-strongly semi-quotient map, provided A_{HN} is HN-open set in (Y_{HN} ,σ) if and only if f^{-1}(A_{HN}) is HN-semi open in (X_{HN},τ).
(iv) HN-strongly α-quotient map, provided A_{HN} is HN-open set in (Y_{HN} ,σ) if and only if f^{-1}(A_{HN}) is HN-α open in (X_{HN},τ).

Example 3.13: Let X = {p,q,r} and τ = {I_{HN}, O_{HN}, A_{HN}}, then (X_{HN},τ) be a Heptagonal neutrosophic topological space with

A_{HN} = { < p; (λ:0.5), (μ: 0.6), (δ: 0.4)>, < q; (λ:0.4), (μ:0.5), (δ:0.2)>, < r; (λ:0.7), (μ:0.6), (δ:0.9)>}

Let Y = {a,b,c} and σ = {I_{HN}, O_{HN}, U_{HN}}, then (Y_{HN},σ) be a Heptagonal neutrosophic topological space with

U_{HN} = { < a; (λ:0.5), (μ: 0.5), (δ: 0.5)>, < b; (λ:0.5), (μ:0.5), (δ:0.5)>, < c; (λ:0.5), (μ:0.5), (δ:0.5)>}

Define a map *f* : $X_{HN} \rightarrow Y_{HN}$ by *f*(p) = a, *f*(q) = b, *f*(r) = c.

Here *f* is a HN strongly pre-quotient map.

Definition 3.14. Let (X_{HN},τ) and (Y_{HN} ,σ) are two non-empty Heptagonal neutrosophic topological spaces.A map *f* : $X_{HN} \rightarrow Y_{HN}$ is called a

(i) HN*-quotient map, if *f* is both HN-irresolute and f^{-1}(A_{HN}) is HN open in (X_{HN},τ), implies A_{HN} is HN open set in (Y_{HN} ,σ).
(ii) HN-semi*-quotient map, if *f* is both HN-semi irresolute and f^{-1}(A_{HN}) is HN-semi open in (X_{HN},τ), implies A_{HN} is HN open set in (Y_{HN} ,σ).
(iii) HN-pre*-quotient map, if *f* is both HN-pre irresolute and f^{-1}(A_{HN}) is HN-pre open in (X_{HN},τ), implies A_{HN} is HN open set in (Y_{HN} ,σ).
(iv) HN-α*-quotient map, if *f* is both HN-$\boldsymbol{\alpha}$ irresolute and f^{-1}(A_{HN}) is HN-α open in (X_{HN},τ), implies A_{HN} is HN open set in (Y_{HN} ,σ).

Example 3.15: Let X = {p,q} and τ = {I_{HN}, O_{HN}, A_{HN}, B_{HN}, C_{HN}}, then (X_{HN},τ) be a Heptagonal neutrosophic topological space with

A_{HN} = { < p; (λ:0.8), (μ: 0.3), (δ: 0.5)>, < q; (λ:0.6), (μ:0.5), (δ:0.9)>}

B_{HN} = { < p; (λ: 0.9), (μ:0.2), (δ:0.4)>, < q; (λ: 0.8), (μ: 0.3), (δ:0.7)>}

C_{HN} = { < p; (λ: 0.5), (μ: 0.3), (δ: 0.5)>, < q; (λ: 0.4), (μ: 0.5), (δ: 0.9)>} and

Let Y = {r,s} and σ = {I_{HN}, O_{HN}, U_{HN}, V_{HN}, W_{HN}}, then (Y_{HN},σ) be a Heptagonal neutrosophic topological space with

U_{HN} = { < r; (λ:0.9), (μ: 0.2), (δ: 0.4)>, < s; (λ:0.7), (μ:0.3), (δ:0.7)>}

V_{HN} = { < r; (λ: 0.6), (μ:0.4), (δ:0.6)>, < s; (λ: 0.3), (μ: 0.6), (δ:0.8)>}

W_{HN} = { < r; (λ: 0.8), (μ: 0.3), (δ: 0.5)>, < s; (λ: 0.6), (μ: 0.5), (δ: 0.7)>}

Define a map *f* : $X_{HN} \rightarrow Y_{HN}$ by *f*(p) = r, *f*(q) = s. Here *f* is a HN pre*-quotient map.

4. Characterizations of Heptagonal Neutrosophic Quotient Mappings

In this section, we characterize the Heptagonal Neutrosophic Quotient Mappings and derive some of the results and the composition of two maps.

Theorem 4.1: Let (XHN,τ) and (YHN ,σ) are two non-empty Heptagonal neutrosophic topological spaces. If f : XHN→ YHN is surjective, HN-continuous and HN-open map, then f is a HN-quotient map.

Proof:

We need only to prove that f^{-1}(AHN) is HN-open in XHN implies AHN is a HN–open set in YHN. Let f^{-1}(AHN) is open in XHN. Since f is HN–open map, then $f(f^{-1}$(AHN)) is a HN–open set in YHN. Hence AHN is a HN–open set in YHN, as f is surjective $f(f^{-1}$(AHN)) = AHN. Thus f is a HN–quotient map.

Theorem 4.2: Let (X_{HN},τ) and (Y_{HN} ,σ) are two non-empty Heptagonal neutrosophic topological spaces and f : X_{HN}→ Y_{HN} is a surjective map, then

(i) Every HN-quotient map is HN-pre quotient map.
(ii) Every HN-quotient map is HN-semi quotient map.
(iii) Every HN-quotient map is HN-α quotient map.

Proof.

Let *f* : X_{HN}→ Y_{HN} be a HN-quotient map and since *f* is a HN-continuous function, we have for every HN open set A_{HN} in (Y_{HN} ,σ), f^{-1}(A_{HN}) is HN-open in (X_{HN},τ) and thus, f^{-1}(A_{HN}) is HN-pre open in (X_{HN},τ), because "Every HN open set is HN-pre open set". This implies *f* is a HN-pre continuous function. Now, Let f^{-1}(A_{HN}) is HN-open in (X_{HN},τ), Since *f* is a HN-quotient map, A_{HN} is HN-open set in (Y_{HN} ,σ) and therefore A_{HN} is a HN-pre open set in (Y_{HN} ,σ). Hence *f* is a HN-pre quotient map.

Let *f* : X_{HN}→ Y_{HN} be a HN-quotient map and since *f* is a HN-continuous function, we have for every HN open set A_{HN} in (Y_{HN} ,σ), f^{-1}(A_{HN}) is HN-open in (X_{HN},τ) and thus, f^{-1}(A_{HN}) is HN-semi open in (X_{HN},τ), because "Every HN open set is HN-semi open set". This implies *f* is a HN-semi continuous function. Now, Let f^{-1}(A_{HN}) is HN-open in (X_{HN},τ), Since *f* is a HN-quotient map, A_{HN} is HN-open set in (Y_{HN} ,σ) and therefore A_{HN} is a HN-semi open set in (Y_{HN} ,σ). Hence *f* is a HN-semi quotient map.

Let *f* : X_{HN}→ Y_{HN} be a HN-quotient map and since *f* is a HN-continuous function, we have for every HN open set A_{HN} in (Y_{HN} ,σ), f^{-1}(A_{HN}) is HN-open in (X_{HN},τ) and thus, f^{-1}(A_{HN}) is HN-α open in (X_{HN},τ), because "Every HN open set is HN-α open set". This implies *f* is a HN- α continuous function. Now, Let f^{-1}(A_{HN}) is HN-open in (X_{HN},τ), Since *f* is a HN-quotient map, A_{HN} is HN-open set in (Y_{HN} ,σ) and therefore A_{HN} is a HN- α open set in (Y_{HN} ,σ). Hence *f* is a HN- α quotient map.

Theorem 4.3: Let (X_{HN},τ) and (Y_{HN} ,σ) are two non-empty Heptagonal neutrosophic topological spaces. A surjective map f: X_{HN}→ Y_{HN} is a HN-α quotient map if and only if f is both HN-semi quotient map and HN-pre quotient map.

Proof.

Let *f*: X_{HN}→ Y_{HN} be a HN-α quotient map. Since f is a HN-α continuous, we have f^{-1}(A_{HN}) is HN-α open in (X_{HN},τ), for every HN-open set A_{HN} in (Y_{HN} ,σ). We know that, "Every HN-α open set is both HN-semi and HN-pre open set". Then f^{-1}(A_{HN}) is both HN-semi open and HN-pre open in (X_{HN},τ) and therefore *f* is both HN-semi continuous as well as HN-pre continuous function.

Now, for every HN-α open set A_{HN} in (Y_{HN} ,σ), we have $f^{-1}(A_{HN})$ is HN open in (X_{HN},τ) and since "Every HN-α open set is both HN-semi and HN-pre open set". This implies, for every HN-semi open set A_{HN} in (Y_{HN} ,σ), we have $f^{-1}(A_{HN})$ is HN open in (X_{HN},τ) and for every HN-pre open set A_{HN} in (Y_{HN} ,σ), we have $f^{-1}(A_{HN})$ is HN open in (X_{HN},τ). Hence it proves *f* is both HN-semi quotient map and HN-pre quotient map.

Conversely, let *f* be a HN-semi quotient map and HN-pre quotient map. Since *f* is both HN-semi continuous and HN-pre continuous function, $f^{-1}(A_{HN})$ is both HN-semi open and HN-pre open in (X_{HN},τ), for every HN-open set A_{HN} in (Y_{HN} ,σ). Therefore, $f^{-1}(A_{HN})$ is also a HN-α open set. Thus, *f* is a HN-α continuous function. Now, Since *f* is a neutrosophic semi-quotient map and a pre-quotient map, for every $f^{-1}(A_{HN})$ is HN-open in (X_{HN},τ) implies A_{HN} is a HN-semi open and HN-pre open respectively in (Y_{HN} ,σ), so that A_{HN} is a HN-α open in (Y_{HN} ,σ). Hence, *f* is a HN-α quotient map.

Theorem 4.4: Let (X_{HN},τ), (Y_{HN},σ) and (Z_{HN},ω) are three non-empty heptagonal neutrosophic topological spaces. A surjective map ϕ: X_{HN}→ Y_{HN} is an onto HN-open and HN-pre irresolute map and ψ : Y_{HN}→Z_{HN} be a HN-pre quotient map, then ψ∘ϕ: X_{HN}→Z_{HN} is a HN-pre quotient map.

Proof.

To Prove: ψ∘ϕ is HN-precontinuous

Let A_{HN} be any HN-open set in (Z_{HN},ω) and since ψ be a HN-pre quotient map, then $\psi^{-1}(A_{HN})$ is a HN-pre open in (Y_{HN},σ).Also since ϕ is HN-pre irresolute, we have $\phi^{-1}(\psi^{-1}(A_{HN}))$ is a HN-pre open set in (X_{HN},τ) which implies $(\psi\circ\phi)^{-1}(A_{HN})$ is a HN-pre open set in (X_{HN},τ). Hence ψ∘ϕ is a HN-precontinuous function.

To Prove: $(\psi\circ\phi)^{-1}(B_{HN})$ is a HN-open set in (X_{HN},τ) implies B_{HN} is HN-pre open set in (Z_{HN},ω).

Let $\phi^{-1}(\psi^{-1}(B_{HN}))$ is a HN-pre open set in (X_{HN},τ) and since ϕ is an onto and HN-open map, we have $\phi^{-1}(\psi^{-1}(B_{HN}))$ is HN-open in (Y_{HN},σ). Since ψ be a HN-pre quotient map, we have $\psi^{-1}(B_{HN})$ is HN-open in (Y_{HN},σ). Thus B_{HN} is a HN-pre open set in (Z_{HN},ω). Hence ψ∘ϕ is a HN-pre quotient map.

Corollary 4.5: Let (X_{HN},τ), (Y_{HN},σ) and (Z_{HN},ω) are three non-empty heptagonal neutrosophic topological spaces. A surjective map ϕ: X_{HN}→ Y_{HN} is an onto HN-open and HN-semi irresolute map and ψ : Y_{HN}→Z_{HN} be a HN-semi quotient map, then ψ∘ϕ: X_{HN}→Z_{HN} is a HN-semi quotient map.

Corollary 4.6: Let (X_{HN},τ), (Y_{HN},σ) and (Z_{HN},ω) are three non-empty heptagonal neutrosophic topological spaces. A surjective map ϕ: X_{HN}→ Y_{HN} is an onto HN-open and HN-α irresolute map and ψ : Y_{HN}→Z_{HN}be a HN-α quotient map, then ψ∘ϕ: X_{HN}→Z_{HN} is a HN-α quotient map.

Theorem 4.7: Let (X_{HN},τ) and (Y_{HN} ,σ) are two non-empty Heptagonal neutrosophic topological spaces. A surjective map f: X_{HN}→ Y_{HN} is a HN-strongly pre quotient map and HN-strongly semi quotient map, then f is a HN-strongly α quotient map.

Proof.

Let A_{HN} be a HN-open set in (Y_{HN},σ) and Since f is HN-strongly semi-quotient and HN-strongly pre-quotient, then $f^{-1}(A_{HN})$is HN semi-open as well as HN pre-open. Hence, $f^{-1}(A_{HN})$ is HN-α open in (X_{HN},τ).

Conversely, Let $f^{-1}(A_{HN})$be a HN-αopen set in (X_{HN},τ). Since f is HN strongly semiquotient, for any $f^{-1}(A_{HN})$ is HN-semiopen in (X_{HN},τ). then A_{HN} is HN-open in (Y_{HN},σ). Therefore, it follows that A_{HN} is HN open in (Y_{HN} ,σ) if and only if $f^{-1}(A_{HN})$ is HN-αopen in (X_{HN},τ). So f is a HN-strongly α-quotient map.

Theroem 4.8: Every HN* – quotient map is HN-strongly quotient map.

Proof:

Let f: XHN→ YHN is a HN* – quotient map. To prove f is HN-strongly quotient map.

Let AHN be any HN-open set in (YHN,σ), since f is a HN* – quotient map, f is HN-irresolute and then f^{-1}(AHN) is HN–open in (XHN,τ). This means that AHN is open in (YHN,σ) implies f^{-1}(AHN) is HN–open in (XHN,τ).

Conversely, if f^{-1}(AHN) is HN-open in (XHN,τ) and since f is HN*–quotient map, AHN is an open set in (YHN,σ). This means that f^{-1}(AHN) is HN–open in (XHN,τ) implies AHN is open in (YHN,σ). Hence f is a HN-strongly quotient map.

Corollary 4.9:

(i) Every HN-semi* quotient map is HN-strongly semi quotient map.
(ii) Every HN-pre* quotient map is HN-strongly pre quotient map.
(iii) Every HN-α* quotient map is HN-strongly α quotient map.

Theorem 4.10: The composition of two HN-semi* quotient maps are again HN-semi* quotient.

Proof:

Let (X_{HN},τ), (Y_{HN},σ) and (Z_{HN},ω) are three non-empty heptagonal neutrosophic topological spaces. A surjective map p: $X_{HN}\rightarrow Y_{HN}$ and q : $Y_{HN}\rightarrow Z_{HN}$ be two HN-semi* quotient maps, then $q \circ p$: $X_{HN}\rightarrow Z_{HN}$ is also a HN- semi* quotient map.

First to prove: $q \circ p$: XHN → ZHN is a HN- semi irresolute map.

Let BHN be a HN–semi open set in (ZHN,ω). Since q is HN-semi*quotient, q^{-1}(BHN) is a HN–semi open set in (YHN,σ). Since p is HN-semi*quotient, $p^{-1}(q^{-1}$(BHN)) is HN–semi open in XHN. That is $(q \circ p)^{-1}$ (BHN) is HN–semi open in XHN. Hence $q \circ p$ is HN-semi irresolute.

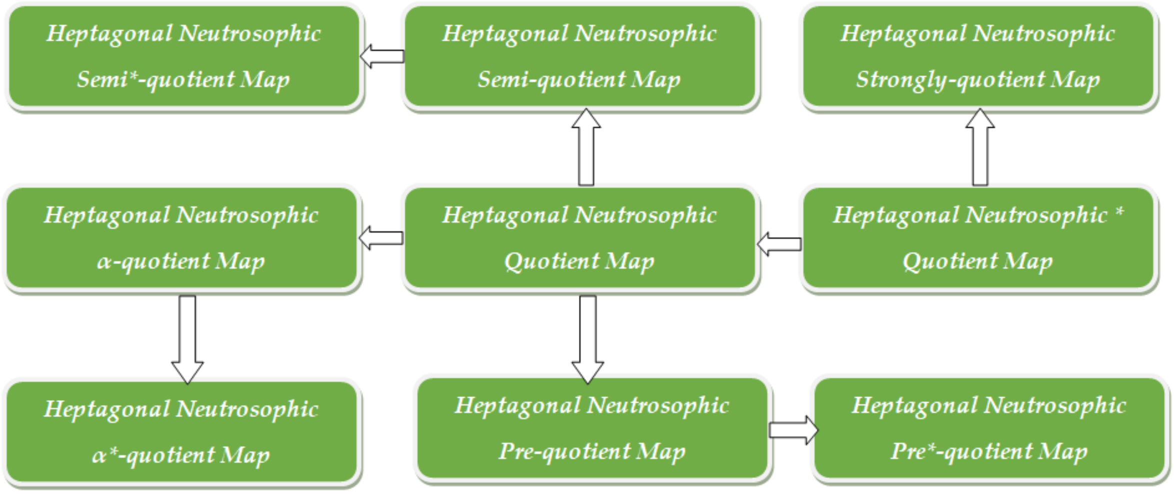
To Prove: $(q \circ p)^{-1}$(BHN) is HN–semi open in XHN implies BHN is open in ZHN.

Let $(q \circ p)^{-1}$ (BHN) is HN-semi open in XHN. That is, $p^{-1}(q^{-1}$(BHN)) is HN-semi open in XHN. Since p is HN-semi*quotient, q^{-1}(BHN) is open in YHN, and hence q^{-1}(BHN) is HN-semi open in YHN. Since q is HN-semi* quotient, BHN is open in ZHN. This implies that $(q \circ p)^{-1}$ (BHN) is HN-semi open in XHN implies BHN is open in ZHN. Hence $q \circ p$ is HN-semi* quotient map.

Corollary 4.11:

(i) The composition of two HN-pre* quotient maps are again HN-pre* quotient.
(ii) The composition of two HN-α* quotient maps are again HN-α* quotient.

Remark 4.12: A brief illustration of this article is as follows:



Conclusion

In this article, we have introduced and studied the concept of Heptagonal Neutrosophic quotient mappings and its characterization. Further, it can be extended in the field of homeomorphism, compactness and connectness and the same can be studied further.

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