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Neutrosophic Industry 5.0 Inventory Model with Technology Driven Demand and Costs Parameters

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Abstract: In this age of technology, the manufacturing sectors are embracing the elements of industry 5.0 to setup a robust kind of production process. This research work proposes a novel neutrosophic production inventory model encompassing the cost parameters of technology inaddition to the conventional inventory costs. In this model the demand is expressed as function of technology of the form $\alpha e^{-\beta t}+\gamma t$ with the coefficients α, β and γdealing with the initial demand, decrease in demand over time and increase in demand with the adoption of new technology. The neutrosophic model developed in this work addresses the decision circumstances of indeterminacy in addition to uncertainty. The primary objective of this paper is to introduce the notion of technology driven demand and new types of costs associated with technology in a neutrosophic modelling environment. The proposed neutrosophic model is simulated and sensitively analyzed to draw inferences of the parameters over the production quantity q(t). The efficiency of this neutrosophic model is determined on making comparative analysis with crisp data sets. The neutrosophic model possesses high degree of flexibility and applicability in technology dominant manufacturing firms facilitating the decision makers to design optimal solutions.

Keywords: Neutrosophic sets, Inventory, Technology driven demand, Optimization

1. Introduction

Industry 5.0[1] is an advancement of industry 4.0 and it primarily focusses on the integration of digital technologies such as IoT (Internet of Things), Artificial Intelligence and big data analytics to evolve intelligent factory frameworks. Industry 4.0 is much concerned on automation whereas Industry 5.0[2] is the next evolution emphasizing on collaborative endeavours between humans and machines. The future manufacturing systems will be certainly embedded into the framework of industry 5.0[3] with key factors such as human-machine collaboration, customization, personalization, sustainability, resilience,flexibility and ethical responsibility.The incorporation of digital oriented technology into production firm costs the manufacturing sectors and this will forman integral part of product production in near future. Inventory modelling is primarily concerned on stock management together with costs optimization. In general, the inventory total cost comprisescosts of ordering, production, holding, shortages, backlog, deterioration, rework, remanufacture and many other related to product production and distribution. However, the modifications in the production system are reflected in the total inventory costs. The developments in the production technology and the augmentations in a production framework incur costs of various kinds and that have to be considered in optimizing the inventory costs.

The production system is expected to be customer centered and function with sustainable consciousness. However, as the demands of the customers are varying with respect to time and with reference to advancements in technology, the production system embeds the facets of technology into their production framework and so as the relevant technology costs with the usual inventory management costs. Inventory modeling with technology driven demand is the key factor of industry 5.0 embraced production firms. The cost parameters considered in modelling may not be of deterministic in nature at all instances as there may exist few indeterminacies in such considerations. To resolve such cases, the theory of neutrosophy is incorporated into inventory modeling to evolve a comprehensive inventory model with neutrosophic parametric representations. Neutrosophic theory developed by Smarandache deals with three- valued function with truth, indeterminacy and falsity membership representations. The objective of this research work is to formulate a neutrosophic inventory model with technology driven demand and technology associated cost parameters. This work also performs a comparative analysis between neutrosophic model and deterministic model to showcase the efficacy of neutrosophic representations of the parameters. The attempt of considering technology dependent demand is a novel aspect of this research work and this has certainly more scope in discussing the nature of the demand under different conditions.

The remaining contents are presented in the following sections. Section 2 consists of the detailed literature review of the works associated with neutrosophic inventory modelling. Section 3 consists of basic definitions of neutrosophy. Section 4 discusses the key aspects of Industry 5.0 and Inventory 5.0. Section 5 develops the inventory model. Section 6 validates the proposed model with numerical examples considering both crisp and neutrosophic representations of the parameters. Section 7 compares the illustrations and draws inferences. Section 8 describes the industrial implications and the last section concludes the work with the ideas and scope of further extension.

2.Literature Review

This section outlines the state of art of neutrosophic inventory models focusing the robustness of neutrosophic representations of inventory parameters. Kar et al[4] modelled inventory model with space constraints using neutrosophic geometric programming. The multi-objective inventory model putforth effectively handled the uncertainty and indeterminacies in inventory management. Mullai and Broumi[5] devised neutrosophic based inventory model without shortages to determine the demand and supply flow in an inventory system. Das and Islam[6] developed fuzzy integrated multi-item inventory model with neutrosophic hesitant fuzzy programming approach to address the challenges of deterioration, space constraints with demand dependent production costs. Mullai and Surya[7] developed neutrosophic inventory backorder problem using triangular neutrosophic numbers considering uncertainty in demand and lead times. Mullai et al[8] formulated a single valued neutrosophic based inventory model with neutrosophic random variable to make accurate estimations of inventory decisive parameters. Islam et al[9] applied neutrosophic hesitant fuzzy programming approach in inventory modeling considering deterioration and space constraints with and without shortages. Mondal et al[10] presented a neutrosophic optimization approach considering inventory policies for seasonal items with logistic-growth demand rates. Rajeswari et al[11] developed reusable based inventory model with octagonal fuzzy neutrosophic numbers. Jdid et al[12] framed a static inventory model with neutrosophic logic and also devised neutrosophic based inventory model with safety reserve.

Sugapriya et al[13] modelled an effective container inventory model with bipolar neutrosophic representations. Garg et al proposed a model for container inventory using trapezoidal bipolar neutrosophic number. Bhavani et al[14] developed a neutrosophic based inventory system with particle swarm optimization algorithm for handling discounts, deterioration items. Sen and Chakrabarthi[15] proposed an industrial based production inventory model with deterioration using neutrosophic fuzzy optimizing approaches. Sarkar and Srivastava[16] presented a multi-item multi-objective neutrosophic model considering sustainability cost parameters. Das and Islam [17]sketched out a neutrosophic based programming approach for optimizing multi-objective shortage follow inventory (SFI) model with ramp demand. Surya and Mullai[18] presented Neutrosophic multi-item inventory control models. Jayanthi [19]developed a neutrosophic fuzzy geometric approach in overage management. Rajeswari [20]employed neutrosophic approaches for optimizing efficient inventory systems. Barman et al[21] modelled rework-based inventory model with neutrosophic representations. Jdid[22] devised neutrosophic static inventory model with economic indicators. Pattnaik et al[23] designed neutrosophic inventory model to handle overage items. Bhavani and Mahapatra[24] proposed an inventory model with generalized triangular neutrosophic cost parameters. Mohanta et al[25] developed neutrosophic inventory model with two-level trade credit policy to handle perishable products. Mohanta[26] framed neutrosophic integrated smart manufacturing-oriented inventory model. Kalaiarasi and Swathi[27] proposed neutrosophic inventory model with quick returns. Miriam et al[28] discussed a neutrosophic rework warehouse inventory model for product distribution considering quality aspects. Dubey et al[29] presented a survey on neutrosophic based inventory problems. Supakar et al [30]developed neutrosophic inventory model and integrated with artificial bee colony algorithm to discuss green production inventory system. Moorthy et al[31] applied neutrosophic logic in inventory management. Surya et al[32] developed neutrosophic inventory model to handle decay items and price dependent demand. Kar et al[33] formulated multi-objective perishable multi-item green neutrosophic inventory models. Das and Islam[34] developed a multi-item inventory model with quadratic demand patterns and with neutrosophic Pythagorean hesitant fuzzy programming. Dubey and Kumar[35] modelled cost effective neutrosophic inventory model.Martin et al[36] designed a neutrosophic based industry 4.0 inventory model integrating neutrosophic logic with inventory cost parameters. These inventory models based on neutrosophy are modelled using different neutrosophic representations. However, neutrosophic trapezoidal interval valued fuzzy numbers are used in inventory optimization. Also, industry 4.0 based neutrosophic inventory model is more industrial centered and this contribution has motivated the authors to explore industry 5.0. Also, the demand nature discussed in the aforementioned inventory models are conventional. This research work has attempted to represent the demand as technology dependent. This work proposes a neutrosophic inventory model integrating the aspects of both industry and inventory 5

3.Preliminaries

This section presents the basic definitions of neutrosophic sets.

3.1 Neutrosophic set[37]

A neutrosophic set is characterized independently by a truth-membership function (x) , an indeterminacy-membership function $\beta(x)$, and a falsity-membership function $\gamma(x)$ and each of the function is defined from $X \rightarrow [0,1]$

3.2 Single valued Neutrosophic set(SVNS)[38]

A SVNS is denoted and defined as $\widetilde{A_N} = \{x, T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x), F_{\widetilde{A_N}}(x)/x\epsilon X\}$, where for each generic point x in X, $T_{\widetilde{A_N}}(x)$ called truth membership function, $I_{\widetilde{A_N}}(x)$ called indeterminacy membership function and $F_{\widetilde{A_N}}(x)$ called falsity membership function in [0,1] and $0 \leq T_{\widetilde{A_N}}(x) + I_{\widetilde{A_N}}(x) + F_{\widetilde{A_N}}(x) \leq 3$. For continuous SNVS$\widetilde{A_N} = \int_{A_N} \langle T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x), F_{\widetilde{A_N}}(x) \rangle / x_i, x_i \epsilon X$.

3.3 Neutrosophic number[38]

Let x be a generic element of a non empty set x. A neutrosophic number $\widetilde{A_N}$ in X is defined as $\widetilde{A_N} = \{$ x, $\langle T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x), F_{\widetilde{A_N}}(x) \rangle / x\epsilon X\}, \forall T_{\widetilde{A_N}}(x), I_{\widetilde{A_N}}(x) and F_{\widetilde{A_N}}(x)$ belongs to $]0^-, 1^+[$ where $T_{\widetilde{A_N}}: X \rightarrow]0^-, 1^+[I,_{\widetilde{A_N}} : X \rightarrow]0^-, 1^+[and F_{\widetilde{A_N}}: X \rightarrow]0^-, 1^+[$ are functions of truth – membership, indeterminacy membership and falsity – membership in $\widetilde{A_N}$respectively with

$0^- \le T_{\widetilde{A_N}}(x) + I_{\widetilde{A_N}}(x) + F_{\widetilde{A_N}}(x) \le 3^+$.

3.4 Interval – valued neutrosophic set[38]

Let X be a nonempty set. Then an interval – valued neutrosophic set [IVNS] $\widetilde{A_N}^{IV}$ of X is defined as:

$\widetilde{A_N}^{IV} = \{\langle x; [T^L_{\widetilde{A_N}}, T^U_{\widetilde{A_N}}], [I^L_{\widetilde{A_N}}, I^U_{\widetilde{A_N}}], [F^L_{\widetilde{A_N}}, F^U_{\widetilde{A_N}}]\rangle : x \epsilon X\}$ where

$[T^L_{\widetilde{A_N}}, T^U_{\widetilde{A_N}}], [I^L_{\widetilde{A_N}}, I^U_{\widetilde{A_N}}] and [F^L_{\widetilde{A_N}}, F^U_{\widetilde{A_N}}] \subset [0,1]\ \forall x \epsilon X\ T^L_{\widetilde{A_N}} = \inf(T_{\widetilde{A_N}}),\ T^U_{\widetilde{A_N}} = \sup(T_{\widetilde{A_N}});\ I^L_{\widetilde{A_N}} = \inf(I_{\widetilde{A_N}}),\ I^U_{\widetilde{A_N}} = \sup(I_{\widetilde{A_N}})$; and $F^L_{\widetilde{A_N}} = \inf(F_{\widetilde{A_N}}), F^U_{\widetilde{A_N}} = \sup(F_{\widetilde{A_N}})$

3.5 Neutrosophic Trapezoidal Interval Valued Number[37]

A single valued trapezoidal Neutrosophic number $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is a special neutrosophic set on the real number set R, whose truth – membership, indeterminacy – membership, and a falsity – membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{(x-a_1)w_{\tilde{a}}}{b_1-a_1}, & (a_1 \le x < b_1) \\ w_{\tilde{a}}, & (b_1 \le x \le c_1) \\ \frac{(d_1-x)w_{\tilde{a}}}{d_1-c_1}, & (c_1 < x \le d_1) \\ 0, & otherwise \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} (b_1 - x + u_{\tilde{a}}(x-a_1))/(b_1-a_1), & (a_1 \le x < b_1) \\ u_{\tilde{a}} & (b_1 \le x \le c_1) \\ \frac{x - c_1 + u_{\tilde{a}}(d_1 - x)}{(d_1 - c_1)}, & (c_1 < x \le d_1) \\ 1, & otherwise \end{cases}$$

and

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b_1 - x + y_{\tilde{a}}(x-a_1))/(b_1-a_1), & (a_1 \le x < b_1) \\ y_{\tilde{a}} & (b_1 \le x \le c_1) \\ \frac{x - c_1 + y_{\tilde{a}}(d_1 - x)}{(d_1 - c_1)}, & (c_1 < x \le d_1) \\ 1, & otherwise \end{cases}$$

respectively.

3.6 Operational Laws on IVTrNeNs[38]

Let $\widetilde{a_{N_1}}^{IV} = \langle [\{(a_1, b_1, c_1, d_1); u_{\widetilde{a_{N_1}}^{IV}}\}, \{(e_1, f_1, g_1, h_1); v_{\widetilde{a_{N_1}}^{IV}}\}, \{(l_1, m_1, n_1, p_1); w_{\widetilde{a_{N_1}}^{IV}}\}] \rangle$ and $\widetilde{a_{N_2}}^{IV} = \langle [\{(a_2, b_2, c_2, d_2); u_{\widetilde{a_{N_2}}^{IV}}\}, \{(e_2, f_2, g_2, h_2); v_{\widetilde{a_{N_2}}^{IV}}\}, \{(l_2, m_2, n_2, p_2); w_{\widetilde{a_{N_2}}^{IV}}\}] \rangle$ be two IVTrNeNs with twelve components, where $u_{\widetilde{a_{N_1}}^{IV}} = [u_{\widetilde{a_{N_1}}^{IV}}^L, u_{\widetilde{a_{N_1}}^{IV}}^U]$; $u_{\widetilde{a_{N_2}}^{IV}} = [u_{\widetilde{a_{N_2}}^{IV}}^L, u_{\widetilde{a_{N_2}}^{IV}}^U]$;

$v_{\widetilde{a_{N_1}}^{IV}} = [v_{\widetilde{a_{N_1}}^{IV}}^L, v_{\widetilde{a_{N_1}}^{IV}}^U]$; $v_{\widetilde{a_{N_2}}^{IV}} = [v_{\widetilde{a_{N_2}}^{IV}}^L, v_{\widetilde{a_{N_2}}^{IV}}^U]$; $w_{\widetilde{a_{N_1}}^{IV}} = [w_{\widetilde{a_{N_1}}^{IV}}^L, w_{\widetilde{a_{N_1}}^{IV}}^U]$;

$w_{\widetilde{a_{N_2}}^{IV}} = [w_{\widetilde{a_{N_2}}^{IV}}^L, w_{\widetilde{a_{N_2}}^{IV}}^U]$, then the following operations hold:

Addition of IVTrNeNs:

$$\widetilde{a_{N_1}}^{IV} + \widetilde{a_{N_2}}^{IV} = \langle \begin{bmatrix} \{(a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2); u_{\widetilde{a_{N_1}}^{IV}} \wedge u_{\widetilde{a_2}^{IV}}\} \\ \{(e_1+e_2, f_1+f_2, g_1+g_2, h_1+h_2); v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}} \\ \{(l_1+l_2, m_1+m_2, n_1+n_2, p_1+p_2); w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}} \end{bmatrix} \rangle$$

Negative of IVTrNeNs:

$$-\widetilde{a_{N_2}}^{IV} = \langle\left[\{-d_2,-c_2,-b_2,-a_2),u_{\widetilde{a_{N_2}}^{IV}}\right\},\{-h_2,-g_2,-f_2,-e_2),v_{\widetilde{a_{N_2}}^{IV}}\},\{-p_2,-n_2,-m_2,-l_2),w_{\widetilde{a_{N_2}}^{IV}}\}]\rangle$$

Subtraction of IVTrNeNs:

$$\widetilde{a_{N_1}}^{IV} - \widetilde{a_{N_2}}^{IV} = \langle\begin{bmatrix}\left\{(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2);u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right\}\\ \{(e_1-h_2,f_1-g_2,g_1-f_2,h_1-e_2);\ v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_{N_2}}^{IV}}\\ \{(l_1-p_2,m_1-n_2,n_1-m_2,p_1-l_2);\ w_{\widetilde{a_{N_1}}^{IV}}\vee w_{\widetilde{a_{N_2}}^{IV}}\end{bmatrix}\rangle$$

Scalar multiplication of SVTrNeN:

$$\lambda\widetilde{a_{N_1}}^{IV} = \begin{cases}\langle\left\{(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1);u_{\widetilde{a_{N_1}}^{IV}}\right\},\left\{(\lambda e_1,\lambda f_1,\lambda g_1,\lambda h_1);v_{\widetilde{a_{N_1}}^{IV}}\right\},\left\{(\lambda l_1,\lambda m_1,\lambda n_1,\lambda p_1);w_{\widetilde{a_{N_1}}^{IV}}\right\}\rangle\ if\lambda>0\\ \langle\left\{(\lambda a_1,\lambda b_1,\lambda c_1,\lambda d_1);u_{\widetilde{a_{N_1}}^{IV}}\right\},\left\{(\lambda e_1,\lambda f_1,\lambda g_1,\lambda h_1);v_{\widetilde{a_{N_1}}^{IV}}\right\},\left\{(\lambda e_1,\lambda f_1,\lambda g_1,\lambda h_1);w_{\widetilde{a_{N_1}}^{IV}}\right\}\rangle\ if\lambda<0\end{cases}$$

Multiplication of SVTrNeN:

$$\widetilde{a_{N_1}}^{IV}.\ \widetilde{a_{N_2}}^{IV} = \begin{cases}\langle(a_1.a_2,b_1.b_2,c_1.c_2,d_1.d_2);(e_1.e_2,f_1.f_2,g_1.g_2,h_1.h_2);(l_1.l_2,m_1.m_2,n_1.n_2,p_1.p_2)\\ \left[u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right],\ [v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_2}^{IV}}],\left[u_{\widetilde{a_{N_1}}^{IV}}\vee u_{\widetilde{a_2}^{IV}}\right]if d_1>0,d_2>0,h_1>0,h_2>0,p_1>0,p_2>0\rangle\\ \langle(a_1.d_2,b_1.c_2,c_1.b_2,d_1.a_2);(e_1.h_2,f_1.g_2,g_1.f_2,h_1.e_2);(l_1.p_2,m_1.n_2,n_1.m_2,p_1.l_2)\\ \left[u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right],\ [v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_2}^{IV}}],\left[u_{\widetilde{a_{N_1}}^{IV}}\vee u_{\widetilde{a_2}^{IV}}\right]if d_1<0,d_2>0,h_1<0,h_2>0,p_1<0,p_2>0\rangle\\ (d_1.d_2,c_1.c_2,b_1.b_2,a_1.a_2);(h_1.h,g_1.g_2,f_1.f_2,e_1.e_2);(p_1.p_2,n_1.n_2,m_1.m_2,l_1.l_2)\\ \langle\left[u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right],\ [v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_2}^{IV}}],\left[u_{\widetilde{a_{N_1}}^{IV}}\vee u_{\widetilde{a_2}^{IV}}\right]if d_1<0,d_2<0,h_1<0,h_2<0,p_1<0,p_2<0\rangle\end{cases}$$

Inverse of SVTrNeN:

$$\boldsymbol{s(\widetilde{a_{N_1}}^{IV})^{-1}} = \frac{\mathbf{1}}{\widetilde{a_{N_1}}^{IV}}\begin{cases}\langle\left(\frac{1}{d_1},\frac{1}{c_1},\frac{1}{b_1},\frac{1}{a_1}\right);\left(\frac{1}{h_1},\frac{1}{g_1},\frac{1}{f_1},\frac{1}{e_1}\right);\left(\frac{1}{p_1},\frac{1}{n},\frac{1}{m_1},\frac{1}{l_1}\right);\boldsymbol{u}_{\widetilde{a_{N_1}}^{IV}},\boldsymbol{v}_{\widetilde{a_{N_1}}^{IV}},\boldsymbol{w}_{\widetilde{a_{N_1}}^{IV}}\rangle,\\ \boldsymbol{if a_1>0,b_1>0,c_1>0,d_1>0,e_1>0,f_1>0,g_1>0,h_1>0,l_1>0,m_1>0,n_1>0,p_1>0}\\ \langle\left(\frac{1}{a_1},\frac{1}{b_1},\frac{1}{c_1},\frac{1}{d_1}\right);\left(\frac{1}{e_1},\frac{1}{f_1},\frac{1}{g_1},\frac{1}{h_1}\right);\left(\frac{1}{l_1},\frac{1}{m_1},\frac{1}{n_1},\frac{1}{p_1}\right);\boldsymbol{u}_{\widetilde{a_{N_1}}^{IV}},\boldsymbol{v}_{\widetilde{a_{N_1}}^{IV}},\boldsymbol{w}_{\widetilde{a_{N_1}}^{IV}}\rangle\\ \boldsymbol{if a_1<0,b_1<0,c_1<0,d_1<0,e_1<0,f_1<0,g_1<0,h_1<0,l_1<0,m_1<0,n_1<0,p_1<0}\end{cases}$$

Division of SVTrNeN:

$$\frac{\widetilde{a_{N_1}}^{IV}}{\widetilde{a_{N_2}}^{IV}} = \begin{cases}\langle\left(\frac{a_1}{d_1},\frac{b_1}{c_1},\frac{c_1}{b_1},\frac{d_1}{a_1}\right),\left(\frac{e_1}{h_1},\frac{f_1}{g_1},\frac{g_1}{f_1},\frac{h_1}{e_1}\right);\left(\frac{l_1}{p_1},\frac{m}{n_1},\frac{n_1}{m_1},\frac{p_1}{l_1}\right);\left[u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right],\left[v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_{N_2}}^{IV}}\right],\left[w_{\widetilde{a_{N_1}}^{IV}}\vee w_{\widetilde{a_{N_2}}^{IV}}\right]\rangle,\\ if d_1>0,d_2>0,h_1>0,h_2>0,p_1>0,p_2>0,\\ \langle\left(\frac{d_2}{d_1},\frac{c_2}{c_1},\frac{b_2}{b_1},\frac{a_2}{a_1}\right),\left(\frac{h_2}{h_1},\frac{g_2}{g_1},\frac{f_2}{f_1},\frac{e_2}{e_1}\right);\left(\frac{p_2}{p_1},\frac{n_2}{n_1},\frac{m_2}{m_1},\frac{l_2}{l_1}\right);\left[u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right],\left[v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_{N_2}}^{IV}}\right],\left[w_{\widetilde{a_{N_1}}^{IV}}\vee w_{\widetilde{a_{N_2}}^{IV}}\right]\rangle, if\\ d_1<0,d_2>0,h_1<0,h_2>0,p_1<0,p_2>0,\\ \langle\left(\frac{d_2}{a_1},\frac{c_2}{b_1},\frac{b_2}{c_1},\frac{a_2}{d_1}\right),\left(\frac{h_2}{e_1},\frac{g_2}{f_1},\frac{f_2}{g_1},\frac{e_2}{h_1}\right);\left(\frac{p_2}{l_1},\frac{n_2}{m_1},\frac{m_2}{n_1},\frac{l_2}{p_1}\right);\left[u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}}\right],\left[v_{\widetilde{a_{N_1}}^{IV}}\vee v_{\widetilde{a_{N_2}}^{IV}}\right],\left[w_{\widetilde{a_{N_1}}^{IV}}\vee w_{\widetilde{a_{N_2}}^{IV}}\right]\rangle, if\\ d_1<0,d_2<0,h_1<0,h_2<0,p_1<0,p_2<0.\end{cases}$$

Where $u_{\widetilde{a_{N_1}}^{IV}}\wedge u_{\widetilde{a_2}^{IV}} = \left[\min\left(u_{\widetilde{a_{N_1}}^{IV}}{}^{L},\ u_{\widetilde{a_{N_2}}^{IV}}{}^{L}\right),\min\left(u_{\widetilde{a_{N_1}}^{IV}}{}^{U},\ u_{\widetilde{a_{N_2}}^{IV}}{}^{U}\right)\right],$

$v_{\widetilde{a_{N_1}}^{IV}} \vee v_{\widetilde{a_{N_2}}^{IV}} = [max\left(v_{\widetilde{a_{N_1}}^{IV}}{}^{L},\ v_{\widetilde{a_{N_2}}^{IV}}{}^{L}\right), max(v_{\widetilde{a_{N_1}}^{IV}}{}^{U},\ v_{\widetilde{a_{N_2}}^{IV}}{}^{U})$ and

$w_{\widetilde{a_{N_1}}^{IV}} \vee w_{\widetilde{a_{N_2}}^{IV}} = [max\left(w_{\widetilde{a_{N_1}}^{IV}}{}^{L},\ w_{\widetilde{a_{N_1}}^{IV}}{}^{L}\right), max(w_{\widetilde{a_{N_1}}^{IV}}{}^{U},\ w_{\widetilde{a_{N_1}}^{IV}}{}^{U})].$

3.7 Score and Accuracy functions of IVTrNeNs[38]

The score function concept is used to find comparison between two IVTrNeNs. Greater of score function value demonstrate the greater of IVTrNeN. According the base of the score and accuracy functions of an IVTrNeN$\widetilde{a_N}^{IV}$canbe defined asfollows:

$$S(\widetilde{a_{N_1}}^{IV}) = \frac{1}{12}\left((8 + (a_1 + b_1 + c_1 + d_1) - (e_1 + f_1 + g_1 + h_1) - (l_1 + m_1 + n_1 + p_1)\right) \times (2 + u_{\widetilde{a_{N_1}}^{IV}}{}^{L} + u_{\widetilde{a_{N_2}}^{IV}}{}^{U} - v_{\widetilde{a_{N_1}}^{IV}}{}^{L} - v_{\widetilde{a_{N_1}}^{IV}}{}^{U} - w_{\widetilde{a_{N_1}}^{IV}}{}^{L} - w_{\widetilde{a_{N_1}}^{IV}}{}^{U}$$

$S(\widetilde{a_{N_1}}^{IV})\epsilon[0,1]$. The accuracy function $A(\widetilde{a_{N_1}}^{IV}) = \frac{1}{4}(a_1 + b_1 + c_1 + d_1 - l_1 - m_1 - n_1 - p_1) \times (2 + u_{\widetilde{a_{N_1}}^{IV}}{}^{L} + u_{\widetilde{a_{N_2}}^{IV}}{}^{U} - v_{\widetilde{a_{N_1}}^{IV}}{}^{L} - v_{\widetilde{a_{N_1}}^{IV}}{}^{U} - w_{\widetilde{a_{N_1}}^{IV}}{}^{L} - w_{\widetilde{a_{N_1}}^{IV}}{}^{U})$

4. Industry 5.0 and Inventory 5.0

Industry 5.0

Industry 5.0 is an enhanced version of industry 4.0 which focuses more on collaboration of both human and machine to evolve a more technology centered scenario. The integration of digital technologies with industrial settingis the prime highlight of this advanced version. The key characteristic features of industry 5.0 are presented pictorially in Fig. 1.

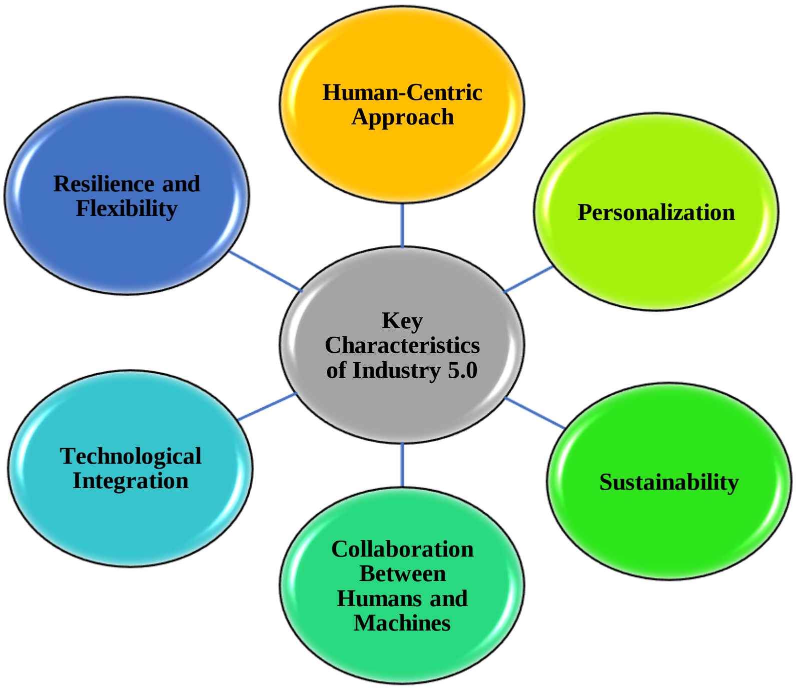

Fig.1. Key Characteristics of Industry 5.0

Human-Centric Approach: Industry 5.0 provides a setting to harmonize the associations between the human and the machines. The competency of the human beings is utilized in resolving the problems by making optimal decisions with the assistance of the machines in handling the tedious tasks.

Personalization: The advanced technical augmentation facilitates customization suiting to the needs of the customers. This is made possible with the collaborative roles of both human and machines enabling a flexible manufacturing process.

Sustainability: Industry 5.0 is more sustainable centric as it facilitates circular economy contributing to waste reduction and environmental sustainability. Resource optimization, remanufacturing and recycling are some of the pivotal roles of this enhanced version.

Collaboration Between Humans and Machines: Industry 5.0 envisages a synergetic relationship between human workers and machines. The working of human with robots in a parallel manner increases the productivity together with the leverage of manpower expertise in the required circumstances.

Technological Integration: Industry 5.0 is circumscribed with advanced technology comprising the approaches of AI, machine learning, data analytics to evolve a more intelligent production system. These well-developed systems are more adaptative in nature and capable of responding to the dynamic needs of the production.

Resilience and Flexibility: Industry 5.0 is potent in adapting to the disruptions occurring in the manufacturing systems thereby ensuring a more flexible and adaptable manufacturing system.

Inventory 5.0

Inventory 5.0 shall be characterized as the enhancement of inventory management in align wit the key components of industry 5.0. The integration of digital technology facets with inventory handling leverages the applications of real-time data analytics, AI and automation making the inventory system more flexible and sustainable.

The key components of Inventory 5.0 are presented in Fig.2 and it explicitly exhibits the attributes of this advanced inventory system.

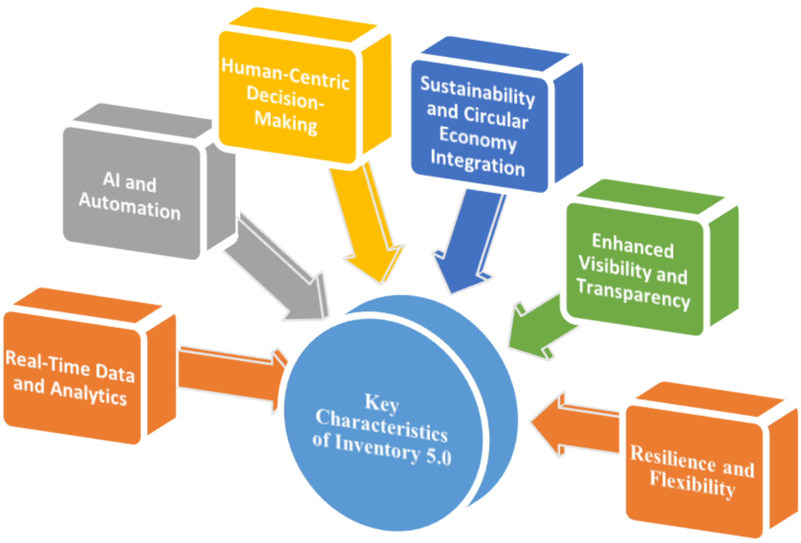

Fig.2. Key Characteristics of Inventory 5.0

On comparing the key components of industry 5.0 and inventory 5.0, it is highly evident that these two phenomena are closely related and there are functioning points of opportunities in align with one another. As both the advanced versions are relatively associated with digital technologies attributing to resilience, flexibility and sustainability, the integration of inventory 5.0 with industry 5.0 may extremely benefit the manufacturing systems.

5. Model Development

This section presents the development of an inventory model characterizing the manufacturing system with technology dependence. Let us consider a technology-oriented inventory system which considers demand to be technology driven and incorporates technology associated cost parameters with other conventional cost parameters. The demand is expressed of the form $\alpha e^{-\beta t}+\gamma t$, where

$\alpha e^{-\beta t}$: Represent the decrease demand over time as initial technology adoption occurs. Here, α is the initial demand due to technologyand β is the rate at which the demand decreases.

γt: Represent the increase demand over time due to continuous market penetration and adoption of new technologies. Here, γ is the coefficient that indicates how demand increases with time.

The model is developed with the assumptions of P > D; no shortages are allowed; D is technology driven.

The notations used in this model are presented as follows,

$\alpha -$ Technology adoption rate coefficient

$\beta -$ Innovation level coefficient

$\gamma -$ Market penetration coefficient

$t -$ Time

P – Production

D – Demand

T_c– Technology Acquisition Costs

$M_c -$ Maintenance and Upgrade Costs

$T_{RC} -$ Training Costs

$I_c -$ Integration Costs

$C_c -$ Cyber security Costs

$D_c -$ Depreciation Costs

T – Total Production time

C_1 – Constant carrying cost per unit time

C_3 – Fixed Ordering cost

$C_2 -$ Purchase Costs

TC – Total Cost

TAC – Total average cost

Let us consider a system of differential equations with technology driven demand

For $0\leq t\leq t_1$

$$\frac{dq(t)}{dt}=P-(\alpha e^{-\beta t}+\gamma t)$$

For $t_1\leq t\leq T$

$$\frac{dq(t)}{dt}=(\alpha e^{-\beta t}+\gamma t)$$

$\alpha e^{-\beta t}$: Represent the decrease demand over time as initial technology adoption occurs. Here, α is the initial demand due to technology,and β is the rate at which the demand decreases.

γt: Represent the increase demand over time due to continuous market penetration and adoption of new technologies. Here, γ is the coefficient that indicates how demand increases with time.

For $0\leq t\leq t_1$

The differential equation is

$$\frac{dq(t)}{dt}=P-(\alpha e^{-\beta t}+\gamma t) \quad (1)$$

This can be solved by integrating both sides with respect to t

$$\int_0^t dq(t)=\int_0^t (P-\alpha e^{-\beta t}-\gamma t)dt$$

$$q(t)-q(0)=Pt+\frac{\alpha e^{-\beta t}}{\beta}-\frac{\gamma t^2}{2}-\frac{\alpha}{\beta}+C_1$$

Given the initial condition $q(0)=0$ (2)

That implies $C_1=0$

Therefore $q(t)=Pt+\frac{\alpha e^{-\beta t}}{\beta}-\frac{\gamma t^2}{2}-\frac{\alpha}{\beta}$ (3)

For $t_1 \le t \le T$

The differential equation is

$\frac{dq(t)}{dt}=(\alpha e^{-\beta t}+\gamma t)$ (4)

This can be solved by integrating both sides with respect to t

$$\int_t^T dq(t)=-\int_t^T(\alpha e^{-\beta t}+\gamma t)dt$$

We use the boundary condition $q(T)=0$ (5)

That implies $C_2=0$

$q(t)=-\frac{\alpha e^{-\beta T}}{\beta}+\frac{\alpha e^{-\beta t}}{\beta}+\frac{\gamma T^2}{2}-\frac{\gamma t^2}{2}$ (6)

Let $q(t_1)=I_{max}$ (7)

Equation (3) and (6) we get

$I_{max}=Pt_1+\frac{\alpha e^{-\beta t_1}}{\beta}-\frac{\gamma {t_1}^2}{2}-\frac{\alpha}{\beta}$

$t_1=\frac{I_{max}}{P-\alpha}$ (8)

$$I_{max}=-\frac{\alpha e^{-\beta T}}{\beta}+\frac{\alpha e^{-\beta t_1}}{\beta}+\frac{\gamma T^2}{2}-\frac{\gamma {t_1}^2}{2}$$

$T-t_1=\frac{I_{max}}{\alpha}$ (9)

$I_{max}=\frac{\alpha(P-\alpha)T}{P}$ (10)

Holding Cost = $C_1\left[\int_0^{t_1}q(t)dt+\int_{t_1}^{T}q(t)dt\right]$

$$=C_1\left(\int_0^{t_1}(P-\alpha)t_1dt+\int_{t_1}^{T}\alpha(T-t_1)dt\right)$$

$$=C_1\left((P-\alpha)\frac{{t_1}^2}{2}+\alpha\frac{(T-t_1)^2}{2}\right)$$

Holding Cost $=\frac{C_1}{2}\left(1-\frac{\alpha}{P}\right)\alpha T^2$ [Using (8), (9) and (10)]

Total Cost = Purchase Costs +Fixed Ordering cost + Holding Cost + Technology Acquisition Costs + Maintenance and Upgrade Costs + Training Costs + Integration Costs + Cyber security Costs + Depreciation Costs

Total Average Cost (T) TC= $\frac{1}{T}\left(C_2+C_3+\frac{C_1}{2}\left(1-\frac{\alpha}{P}\right)\alpha T^2+T_c+M_c+T_{RC}+I_c+C_c+D_c\right)$ ------- (11)

Differentiate with respect to T

$T=\sqrt{\frac{2(C_2+C_3+T_c+M_c+T_{RC}+I_c+C_c+D_c)}{C_1(1-\frac{\alpha}{P})\alpha}}$ ----------------- (12)

6. Illustration

Let us consider an inventory system with the following parameters.

$$\alpha=0.5,\beta=0.1,\gamma=0.05,P=1000\ units,T_c=2000,M_c=1500,T_{Rc}=1000,I_c=500,$$
$$C_c=300,D_c=800,C_1=2,C_2=10000,C_3=500.$$

By using the above expressions of T, TC and t_1,the following values are obtained.

$$T=182.25,t_1=0.091,TC=33199.23$$

Neutrosophic Trapezoidal Fuzzy Numbers for Parameters:

$$\alpha = (0.45,0.5,0.55,0.6; [0.9,1], [0.1,0.15], [0,0.05])$$
$$\beta = (0.08,0.1,0.12,0.14; [0.85,0.95], [0.05,0.1], [0,0.02])$$
$$\gamma = (0.04,0.05,0.06,0.07; [0.8,0.9], [0.1,0.2], [0,0.05])$$
$$P = (950,1000,1050,1100; [0.95,1], [0.05,0.1], [0,0.05])$$
$$T_c = (1900,2000,2100,2200; [0.9,0.95], [0.05,0.1], [0,0.05])$$
$$M_c = (1425,1500,1575,1650; [0.9,0.95], [0.05,0.1], [0,0.05])$$
$$T_{RC} = (950,1000,1050,1100; [0.85,0.95], [0.05,0.1], [0,0.05])$$
$$I_c = (475,500,525,550; [0.85,0.95], [0.05,01], [0,0.05])$$
$$C_c = (285,300,315,330; [0.8,0.9], [0.1,0.15, [0,0.05])$$
$$D_c = (760,800,840,880; [0.85,0.9], [0.1,0.15], [0,0.05])$$
$$C_1 = (1.8,2.,2.2,2.4; [0.95,1], [0.05,0.1], [0,0.05])$$
$$C_2 = (9500,10000,10500,11000; [0.9,0.95], [0.05,0.1], [0,0.05])$$
$$C_3 = (475,500,525,550; [0.9,0.95], [0.05,0.1], [0,0.05])$$
$$I_{max} = (180,200,220,240; [0.9,0.95], [0.05,0.1], 0,0.05])$$
$$T = (148.04,165.49,186.76,212.38; [0.8,0.9], [0.05,0.95], [0,0.05])$$
$$t_1 = (0.164,0.191,0.22,0.25; [0.9,0.95], [0.05,0.1], 0,0.05])$$
$$TC = (24642.38,30319,38518.65,50715.82; [0.8,0.9], [0.05,0.95], [0,0.05])$$

Results and Discussion

The above formulated inventory model is validated with both crisp and interval valued trapezoidal neutrosophic numbers and the results obtained are presented in the Table 1.

Table 1. Result of both crisp and interval valued trapezoidal neutrosophic numbers

Results	Crisp data input	Interval valued trapezoidal neutrosophic data input
T	182.25	(148.04,165.49,186.76,212.38; [0.8,0.9], [0.05,0.95], [0,0.05])
t_1	0.091	(0.164,0.191,0.22,0.25; [0.9,0.95], [0.05,0.1], 0,0.05])
TC	33199.23	(24642.38,30319,38518.65,50715.82; [0.8,0.9], [0.05,0.95], [0,0.05])

8. Sensitivity Analysis

The parameter α are varied and the respective changes on T and TC are determined with both crisp and neutrosophic input.

Case (1) Variations of the parameter α with Crisp data input

The values obtained on varying α are presented in Table 2.

Table 2. Variation of α with Crisp data

α	T	Total Average Cost
0.1	407.451349	81.482121
0.2	288.126020	115.227358
0.3	235.265676	141.117058
0.4	203.756243	162.939793
0.5	182.254241	182.163114
0.6	166.382922	199.539710
0.7	154.048360	215.516737
0.8	144.106257	230.385555
0.9	135.871481	244.348554
1.0	128.905456	257.553101

The respective graphical representations are presented in Fig.3.

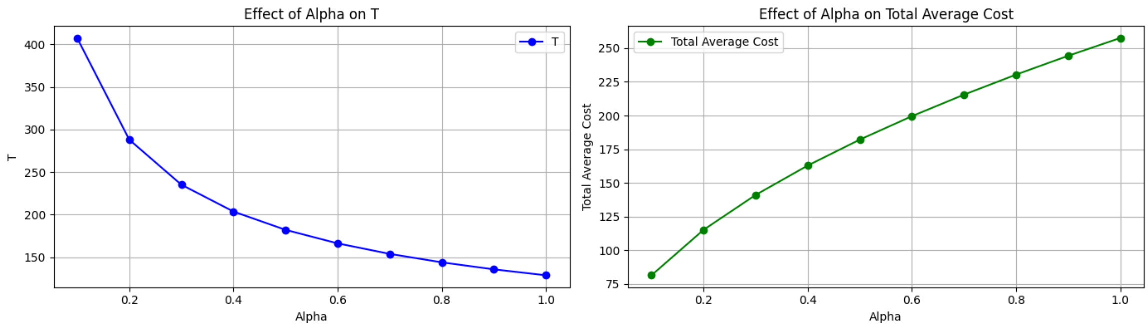

Fig.3. Graphical representations on Effect of α on T and Total Averaege cost using Crisp data

Case (2) Variations of the parameter α with Neutrosophic data input are presented in Table 3.

Table 3. Variation of α with Neutrosophic data

Parameter Value	α	T	Total Average Cost
0	0.400	194.754022	151.260034
1	0.425	188.941646	155.913218
2	0.450	183.620697	160.431261
3	0.475	178.725620	164.825278
4	0.500	174.202484	169.104937

The respective graphical representations are presented in Fig.4.

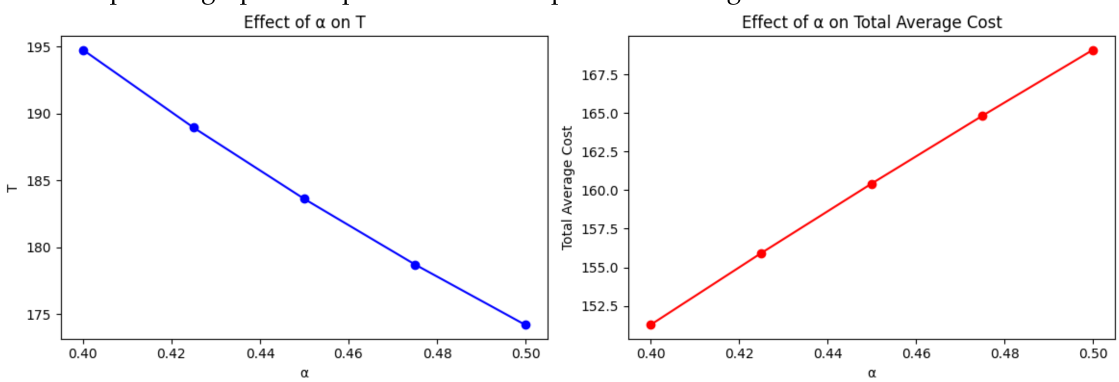

Fig.4. Graphical representations on Effect of α on T and Total Averaege cost using Neutrosophic data

In both the figures[3,4] the effects of α on T and TC are determined. However, the parameter α has positive correlation with TAC and negative correlation with T. The graphical representation of the effect of α is more vivid in terms of neutrosophic data input in comparison with the crisp data input. This shows the efficacy of neutrosophic data representations. In a similar fashion the other cost parametric effects shall be determined.

Case (3) Variations of the parameter α ,β, γ on q(t):

The Sensitivity Analysis for α are presented in the below Table 4.

Table 4. Sensitivity Analysis for α

qα=0.45	qα=0.5	qα=0.55	qα=0.6
100.964620	100.959595	100.954570	100.949545
201.929185	201.919185	201.909185	201.899186
302.893689	302.878765	302.863841	302.848916
403.858129	403.838329	403.818530	403.798731
…	…	…	…
9590.881255	9590.572780	9590.264304	9589.955828
9691.825314	9691.514913	9691.204513	9690.894112
9792.769036	9792.456730	9792.144424	9791.832118
9893.712421	9893.398229	9893.084036	9892.769844
9994.655466	9994.339406	9994.023347	9993.707287

The graphical representation of Sensitivity Analysis for α is represented in fig.5.

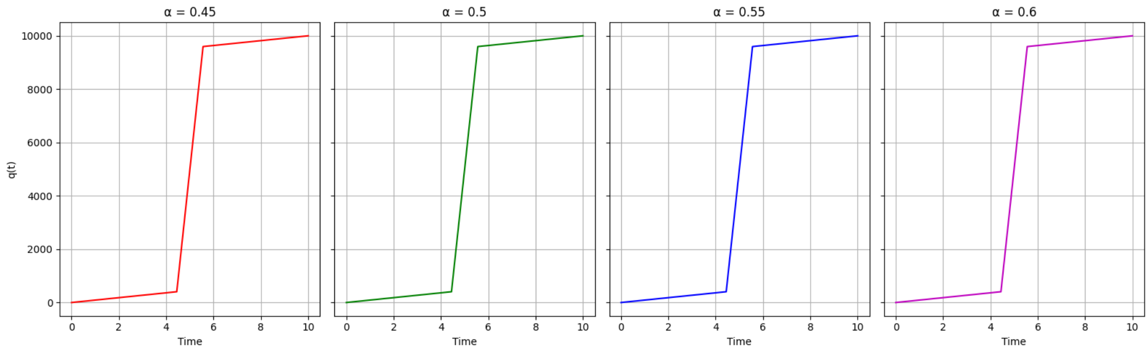

Fig.5. Graphical representation of Sensitivity Analysis for α

The Sensitivity Analysis for β are presented in the below Table 5.

Table 5. Sensitivity Analysis for β

qβ = 0.08	qβ =0.1	qβ = 0.12	qβ = 0.14
100.959544	100.959595	100.959646	100.959696
201.918983	201.919185	201.919386	201.919587
302.878314	302.878765	302.879214	302.879661
403.837533	403.838329	403.839122	403.839910
…	…	…	…
9590.308101	9590.572780	9590.808201	9591.018079
9691.246139	9691.514913	9691.753712	9691.966374
9792.183854	9792.456730	9792.698904	9792.914343
9893.121245	9893.398229	9893.643775	9893.861983

qβ = 0.08	**qβ =0.1**	**qβ = 0.12**	**qβ = 0.14**
9994.058312	9994.339406	9994.588323	9994.809291

The graphical representation of Sensitivity Analysis for β is represented in fig.6.

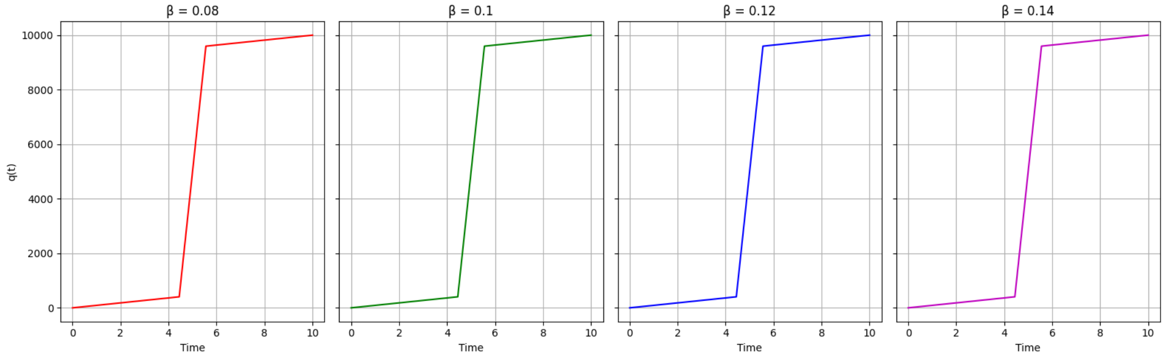

Fig.6. Graphical representation of Sensitivity Analysis for β

The Sensitivity Analysis for β are presented in the below Table 6.

Table 6. Sensitivity Analysis for γ

qγ = 0.04	**qγ = 0.05**	**qγ = 0.06**	**qγ = 0.07**
100.959646	100.959595	100.959544	100.959493
201.919389	201.919185	201.918981	201.918777
302.879224	302.878765	302.878306	302.877847
403.839146	403.838329	403.837513	403.836697
…	…	…	…
9591.033192	9590.572780	9590.112367	9589.651955
9691.985069	9691.514913	9691.044757	9690.574601
9792.936732	9792.456730	9791.976728	9791.496726
9893.888179	9893.398229	9892.908279	9892.418329
9994.839406	9994.339406	9993.839406	9993.339406

The graphical representation of Sensitivity Analysis for γ is represented in fig.7.

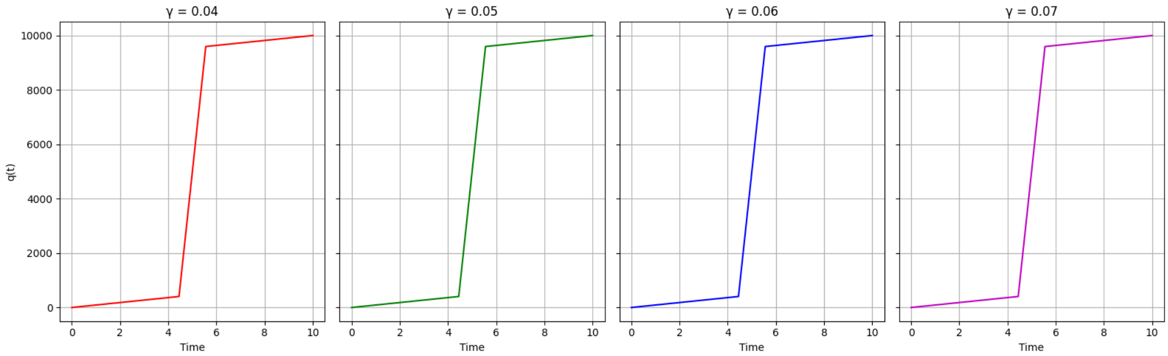

Fig.7. Graphical representation of Sensitivity Analysis for γ

From the above Table [4,5,6] values and respective graphical representations the following inferences on the trend patterns are obtained

The increase in α values causes slight decrease in q(t) over time and then increases rapidly.

The system behaviour is dampened by the impacts of variations in the values of α over a period of time.

The increase in β values causes marginalincrease in q(t) over time with very small variations initially.

The parameter β influences the long-term behaviour and resilience of the system.

The increase in γ values causes decrease in the values of q(t) over a period of time.

The system behaviour is dampened by the impacts of variations in the values of γover a period of time.

All the three parameters α, β and γ causes significant impacts on q(t) enhances the resilience of the system behaviour. The changes in the parameters enables to comprehend the behaviour of the system with respect to output maximization with assurance of the system stability.

9. Industrial Implications

The neutrosophic model with technology dependent demand pattern is more concerned with industry 5.0 and this has high association with industries especially embracing the advanced versions of human -machine collaboration. The discussion of inventory modelling with technology-oriented demand facilitates the decision makers to gain more insights on the costs associated with technology and their influences over the total inventory costs. The variation of the parameters brings the picturization of the demand impacts on inventory decisions. This model also supports manufacturing system with human-machine interferences to devise suitable strategies for cost minimization and profit maximizations. Also, the customer's demand depending on the technology advancements is well reflected in this modelling. The neutrosophic parameter representations model is studied by the influence of the parameters. This neutrosophic model proposed in this work shall be extended by studying the demand patterns influenced by the parameters of sustainability. This work provides room for developing more research ideas of industry and inventory 5.0provides a suitable framework for the managerial to optimize the production based on the demand pattern discussed in this model.

Conclusion

The novel inventory model proposed in this research work with neutrosophic representations is more feasible and comprehensive in nature. The illustration of the model with both crisp and neutrosophic data lays a clear picture of the efficiency of neutrosophic representations over crisp forms. The sensitivity analysis also favours neutrosophic representations. The dynamic behaviour of the model is studied by the influence of the

parameters. This neutrosophic model proposed in this work shall be extended by studying the demand patterns influenced by the parameters of sustainability. This work provides room for developing more research ideas of industry and inventory 5.0

Reference

1. Hussein, G. S., Zaied, A. N. H., & Mohamed, M. (2023). ADM: Appraiser Decision Model for Empowering Industry 5.0-Driven Manufacturers toward Sustainability and Optimization: A Case Study. *Neutrosophic systems with applications*, *11*, 22-30.
2. Alojaiman, B. (2023). Technological modernizations in the industry 5.0 era: A descriptive analysis and future research directions. *Processes*, *11*(5), 1318.
3. Özdemir, V., & Hekim, N. (2018). Birth of industry 5.0: Making sense of big data with artificial intelligence,"the internet of things" and next-generation technology policy. *Omics: a journal of integrative biology*, *22*(1), 65-76.
4. Kar, C., Mondal, B., & Roy, T. K. (2018). An inventory model under space constraint in neutrosophic environment: a neutrosophic geometric programming approach. *Neutrosophic Sets and Systems: An International Book Series in Information Science and Engineering*, *21*(2018), 93-109.
5. Mullai, M., & Broumi, S. (2018). Neutrosophic inventory model without shortages. *Asian Journal of Mathematics and Computer Research*, *23*(4), 214-219.
6. Das, S. K., & Islam, S. (2019). *Fuzzy Multi Item Inventory Model with Deterioration and Demand Dependent Production Cost Under Space Constraint: Neutrosophic Hesitant Fuzzy Programming Approach*. Infinite Study
7. Mullai, M., & Surya, R. (2020). Neutrosophic inventory backorder problem using triangular neutrosophic numbers. *Neutrosophic Sets and Systems*, *31*(1), 148-155.
8. Mullai, M., Sangeetha, K., Surya, R., Kumar, G. M., Jeyabalan, R., & Broumi, S. (2020). *A single valued neutrosophic inventory model with neutrosophic random variable*. Infinite Study
9. Islam, S. (2021). Fuzzy Multi-Objective Inventory Model for Deteriorating Items, with Shortages Under Space Constraint: Neutrosophic Hesitant Fuzzy Programming Approach. *Neutrosophic Operational Research: Methods and Applications*, 197-219.
10. Mondal, B., Garai, A., Mukhopadhyay, A., & Majumder, S. K. (2021). Inventory policies for seasonal items with logistic-growth demand rate under fully permissible delay in payment: a neutrosophic optimization approach. *Soft Computing*, *25*(5), 3725-3750.
11. Rajeswari, S., Sugapriya, C., Nagarajan, D., Broumi, S., & Smarandache, F. (2021). Octagonal fuzzy neutrosophic number and its application to reusable container inventory model with container shrinkage. *Computational and Applied Mathematics*, *40*, 1-20.
12. Jdid, M., Alhabib, R., & Salama, A. A. (2021). The static model of inventory management without a deficit with Neutrosophic logic.
13. Sugapriya, C., Rajeswari, S., Nagarajan, D., & Al Amri, Z. S. M. (2022). An effective container inventory model under bipolar neutrosophic environment. *Neutrosophic Sets and Systems, vol. 50/2022: An International Journal in Information Science and Engineering*, 336.
14. Bhavani, G. D., Georgise, F. B., Mahapatra, G. S., & Maneckshaw, B. (2022). Neutrosophic cost pattern of inventory system with novel demand incorporating deterioration and discount on defective items using particle swarm algorithm. *Computational Intelligence and Neuroscience*, *2022*(1), 7683417.
15. Sen, J. D., & Chakrabarti, T. (2022). An industrial production inventory model with deterioration under neutrosophic fuzzy optimization. *Neuroquantology*, *20*(19), 4399.

16. Sarkar, D., & Srivastava, P. K. (2022). A dynamic optimization approach for multi-item multi-objective fixed-place neutrosophic fuzzy inventory and green management system. *Mathematics in Engineering, Science & Aerospace (MESA)*, *13*(2).
17. Das, K., & Islam, S. (2022). A multi-objective shortage follow inventory (SFI) model involving ramp-type demand, time varying holding cost and a marketing cost under neutrosophic programming approach. *Neutrosophic Sets and Systems*, *49*, 48-69.
18. Surya, R., & Mullai, M. (2022). Neutrosophic multi-item inventory control models with constraints. *Journal of Intelligent & Fuzzy Systems*, *43*(6), 8127-8136.
19. Jayanthi, K. (2022). A Fuzzy Differential Approach to An Overage Management with Customer Acquisition Inventory Problem Via Neutrosophic Fuzzy Geometric Programming Technique. *Journal of Algebraic Statistics*, *13*(3), 215-225
20. Rajeswari, S(2022). Optimization Techniques For Some Efficient Inventory Systems Using Fuzzy And Neutrosophic Models.
21. Barman, H., Roy, S. K., Sakalauskas, L., & Weber, G.W. (2023). Inventory model involving reworking of faulty products with three carbon policies under neutrosophic environment. *Advanced Engineering Informatics*, *57*, 102081.
22. Jdid, M. (2023). Important Neutrosophic Economic Indicators of the Static Model of Inventory Management without Deficit. *Journal of Neutrosophic and Fuzzy Systems (JNFS) Vol*, *5*(01), 08-14.
23. Pattnaik, S., Nayak, M. M., & Kumar, P. (2023). An Inventory Control Model in the Framework of COVID-19 Disruptions Considering Overage Items with Neutrosophic Fuzzy Uncertainty. *Neutrosophic Sets and Systems*, *56*(1), 9.
24. Bhavani, G. D., & Mahapatra, G. S. (2023). Inventory system with generalized triangular neutrosophic cost pattern incorporating maximum life-time-based deterioration and novel demand through PSO. *Soft Computing*, *27*(5), 2385-2402.
25. Mohanta, K., Jha, A. K., Dey, A., & Pal, A. (2023). An application of neutrosophic logic on an inventory model with two-level partial trade credit policy for time-dependent perishable products. *Soft Computing*, *27*(8), 4795-4822.
26. MOHANTA, K., Sarkar, S., Dey, A., De, D., & Pal, A. (2023). Cost Optimization of an Inventory Control Problem in Neutrosophic Environment for Smart Manufacturing
27. Kalaiarasi, K., & Swathi, S. (2023). Neutrosophic Inventory Model With Quick Return For Damaged Materials And Python-Analysis. *2023 Neutrosophic SuperHyperAlgebra And New Types of Topologies*, 219.
28. Renee Miriam, M., Martin, N., Aleeswari, A., & Broumi, S.(2023) Rework Warehouse Inventory Model for Product Distribution with Quality Conservation in Neutrosophic Environment. *Doi: https://doi. org/10.54216/IJNS, 210215*
29. Dubey, A., Dey, A., Broumi, S., & Kumar, R. (2024). A Survey on Neutrosophic Principles for Inventory Management Problem. *Neutrosophic Sets and Systems*, *68*(1), 19.
30. Supakar, P., Manna, A. K., Mahato, S. K., & Bhunia, A. K. (2024). Application of artificial bee colony algorithm on a green production inventory problem with preservation for deteriorating items in neutrosophic fuzzy environment. *International Journal of System Assurance Engineering and Management*, *15*(2), 672-686.
31. Moorthy, C. B., Rajani, D., Pushpalatha, A. P., Ramya, S., Selvaraj, A., & Tiwari, M. (2024). Enhancing Inventory Management through Advanced Technologies and Mathematical Methods: Utilizing Neutrosophic Fuzzy Logic. *Full Length Article*, *24*(4), 50-0.
32. Surya, R., Mullai, M., & Vetrivel, G. (2024). Neutrosophic Inventory System for Decaying Items with Price Dependent Demand. *Neutrosophic Sets and Systems*, *66*, 55-75.
33. Kar, C., De, M., Maiti, M., & Das, P. (2024). Multi-objective perishable multi-item green inventory models with uncertain finite time horizons and constraints by neutrosophic optimisation approach. *International Journal of Mathematics in Operational Research*, *27*(2), 167-198.

34. Das, K., & Islam, S. (2024). A deterministic multi-item inventory model with quadratic demand under neutrosophic and pythagorean hesitant fuzzy programming approach. *Results in Control and Optimization*, *14*, 100367.
35. Dubey, A., & Kumar, R. (2024). Neutrosophic Inventory Management: A Cost-Effective Approach. *Economics*, *18*(1), 20220101
36. Martin, N., Mayan, M. K., & Smarandache, F. (2020). *Neutrosophic optimization of industry 4.0 production inventory model* (Vol. 38). Infinite Study.
37. Martin, N., Mayan, M. K., & Smarandache, F. (2020). *Neutrosophic optimization of industry 4.0 production inventory model* (Vol. 38). Infinite Study.
38. Saini, R. K., Sangal, A., & Ahirwar, A. (2022). A novel approach by using interval-valued trapezoidal neutrosophic numbers in transportation problem. *Neutrosophic Sets Syst*, *51*, 234-253.

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