NSS
Neutrosophic Sets and Systems, Vol. 73, 2024
University of New Mexico

Aczel-Alsina power average aggregation operators of Singlevalued Neutrosophic under confdence levels and theirapplication in multiple attribute decision making

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Abstract. In decision making scenarios, dealing with imprecise information through extensions of fuzzy sets is crucial. Among these extensions, single valued neutrosophic set (SVNS) are especially effective at managing and interpreting such imprecise data. In the current study, decision makers confidence levels, derived from their familiarity with the assessed objects, are combined with the primary data within a neutrosophic framework. This paper focuses on developing innovative confidence single valued neutrosophic (SVN) aggregation operators (AO) that utilize the recently developed Aczel-Alsina (AA) operational laws and power AO (PAO) to capture the interrelationships among aggregated single valued neutrosophic numbers (SVNN). Specifically, it introduces new confidence SVNAA power average AO, namely, confidence SVNAA power weighted and ordered weighted average AO, which integrate the decision maker familiarity with the aggregated arguments. To evaluate the effectiveness of the proposed operators, we perform a comprehensive examination of their desirable properties. Also, we use these suggested operators to establish a innovative approach for SVN multi attribute decision making problems (MADM). A demonstrative example of strategic suppplier selection is provided to validate the proposed approach and highlight its practicality and effectiveness.

Keywords: Single valued neutrosophic sets; Aczel-Alsina; Power aggregation operator; Confidence levels; Average AO; Multiple attribute decision making.

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1. Introduction

In our complex world, managing uncertainty, which arises from navigating imprecision and incomplete data, is essential across various fields, including science, business, and decision making (DM). To address imprecise, contradictory, and incomplete information. Lotfi Zadeh [1] introduced fuzzy set (FS) theory with the membership degree (MD), which is denoted by $\Re$.

Atanassov [2] further developed FS theory with intuitionistic FS (IFS) theory, which incorporates both $\Re$ and non-membership degree (NMD) which is represented by $\hbar$. In IFS, the degree of hesitation is commonly computed as 1-$\Re - \hbar$ which potentially leading to the loss of some uncertainty. As an extension of IFS, Atanassov and Gargov [3] proposed interval valued intuitionistic FS (IVIFS) theory, allowing $\Re$ and $\hbar$ to range over intervals. Further, the generalization of IFS includes pythagorean FS (PyFS) [4], fermatean FS (FFS) [5]and q-rung orthopair FS (q-ROFS) [6]. Cuong et al. [7] introduced picture FS (PFS), by including an dependent neutral MD (∂) along with $\Re$ and $\hbar$. Additionally, the expansion of PFS encompasses spherical FS (SFS) [8], cubical FS (CFS) [9]and T-spherical FS (T-SFS) [10]. Building on IFS, Smarandache introduced the concept of neutrosophic sets (NS) by adding an independent degrees known as the indeterminacy MD (∂) alongside the truth MD ($\Re$) and falsity MD ($\hbar$) over a non-standard interval. Subsequently, Wang et al. [12] developed the single valued neutrosophic set (SVNS) based on the standard real interval [0,1], making it more suitable for computational ease in DM scenarios.

A variety of AO have emerged in the context of different fuzzy environments to enable the combination of evaluated objects, allowing for more effective DM and analysis. Liu and Chen [13] utilized the IF Heronian mean aggregation operator, which is based on Archimedean norms, to aggregate multiple decision matrices in a group DM problem, facilitating the evaluation of multiple perspectives and opinions. Shit and Ghorai [14] proposed FF Dombi AO to solve a MADM. Qiyas et al. [15] investigated Yager operators under PF environment. Ullah et al. [16] explored T-SFS in Hamacher AO.

In recent years, various operators have been designed to integrate confidence and ordering information into the preference components of aggregated data, allowing for more comprehensive and nuanced analysis of complex DM scenarios. In this manner, Dejian Yu [17] developed IF aggregation under confidence levels (CL). Tahir Mahmood et al. [18] established confidence level induced AO based on IF rough sets information. K Rahman et al. [19] proposed confidence based generalized IF AO for group DM. Harish Garg [20] introduced the induced PyF AO and its application to DM process. Manish Kumar [21] made a study on the confidence based q-ROF AO with numerical examples and discussed their applicability in a DM problem. Tanuja Puntaua and Komal [22] introduced the confdence PF AO and applied with a group DM problem. Muhammad Kamran et al. [23] developed CL AO based on SVN rough sets.

Inspired by the literature, the goals of this paper are presented below:

- The confidence an expert has in their assessment significantly impacts the evaluation process and the reliability of the DM outcome.
- There will be an opportunity to incorporate the experts confidence in the evaluated SVN aggregating objects during the DM process.

- Integrating the experts CLs into the exact information within the SVN environment is essential.
- It has been noted that no research has explored the integration of experts CLs with SVN Aczel-Alsina power aggregation operator.
- These objectives have inspired us to develop new confidence-based SVN operators.

The primary contributions of our study are summarized below:

- We propose utilizing SVNAAP weighted and ordered weighted average AO combined with decision makers CLs.
- The essential characteristics of the AO are analyzed.
- We have created SVNMADM approach based on the introduced operators.
- This DM approach has been implemented in the supplier selection process.
- The outcomes are then compared with those obtained from existing SVN average operators documented in the literature.

2. **Preliminaries**

Here, we review some basic definitions related to SVNS within the context of a universal set $\ddot{X}'$.

Definition 2.1. *[12] A SVNS $\breve{\mathbb{C}}$, on the universal set $\ddot{X}'$ is of the form* $\breve{\mathbb{C}} = \{\langle \phi, \Re_{\breve{\mathbb{C}}}(\phi), \partial_{\breve{\mathbb{C}}}(\phi), \hbar_{\breve{\mathbb{C}}}(\phi) | \phi \in \ddot{X}' \rangle\}$ *where* $\Re_{\breve{\mathbb{C}}} : \ddot{X}' \to [0,1]$ *represent the truth membership function,* $\partial_{\breve{\mathbb{C}}} : \ddot{X}' \to [0,1]$ *represent the indeterminacy membership function and* $\hbar_{\breve{\mathbb{C}}} : \ddot{X}' \to [0,1]$ *represent the falsity membership function and* $\Re_{\breve{\mathbb{C}}}(\phi), \partial_{\breve{\mathbb{C}}}(\phi), \hbar_{\mathcal{S}}(\phi) \in [0,1]$ *such that* $0 \leq \Re_{\breve{\mathbb{C}}}(\phi) + \partial_{\breve{\mathbb{C}}}(\phi) + \hbar_{\breve{\mathbb{C}}}(\phi) \leq 3$. *Now we denote the triplets* $\breve{\mathbb{C}} = (\Re_{\breve{\mathbb{C}}}, \partial_{\breve{\mathbb{C}}}, \hbar_{\breve{\mathbb{C}}})$ *as an single valued neutrosophic numbers (SVNN) for simplicity.*

Definition 2.2. *[24] Let* $\breve{\mathbb{C}} = \langle \Re_{\breve{\mathbb{C}}}, \partial_{\breve{\mathbb{C}}}, \hbar_{\breve{\mathbb{C}}} \rangle \in \breve{\mathbb{C}}$ *be a SVNN, then the score function* $\ddot{\mathbb{M}}$ *of* $\breve{\mathbb{C}}$ *is defined as*

$$\ddot{\mathbb{M}}(\breve{\mathbb{C}}) = \frac{2 + \Re_{\breve{\mathbb{C}}} - \partial_{\breve{\mathbb{C}}} - \hbar_{\breve{\mathbb{C}}}}{3} \in [0,1] \tag{1}$$

$$\ddot{\mathbb{M}}(\breve{\mathbb{C}}) = \tfrac{2+0.9-0.7-0.8}{3} = 0.467 \in [0,1]$$

Definition 2.3. *[24] Let* $\breve{\mathbb{C}} = \langle \Re_{\breve{\mathbb{C}}}, \partial_{\breve{\mathbb{C}}}, \hbar_{\breve{\mathbb{C}}} \rangle \in \breve{\mathbb{C}}$ *be a SVNN, then the accuracy function* $\ddot{\mathbb{M}}$ *of* $\breve{\mathbb{C}}$ *is defined as*

$$\ddot{\mathbb{L}}(\breve{\mathbb{C}}) = \Re_{\breve{\mathbb{C}}} - \hbar_{\breve{\mathbb{C}}} \in [-1,1] \tag{2}$$

Definition 2.4. *[24] Let* $\breve{\mathbb{C}}_1 = \langle \Re_{\breve{\mathbb{C}}_1}, \partial_{\breve{\mathbb{C}}_1}, \hbar_{\breve{\mathbb{C}}_1} \rangle$ *and* $\breve{\mathbb{C}}_2 = \langle \Re_{\breve{\mathbb{C}}_2}, \partial_{\breve{\mathbb{C}}_2}, \hbar_{\breve{\mathbb{C}}_2} \rangle$ *be any two SVNNs and* $\ddot{\mathbb{M}}(\breve{\mathbb{C}}_j)$ *and* $\ddot{\mathbb{L}}(\breve{\mathbb{C}}_j)$ *for* $j = 1, 2$ *be their respective score and accuracy values, then we arrive at the following results.*

(1) *If* $\ddot{\mathbb{M}}(\breve{\mathbb{C}}_1) > \ddot{\mathbb{M}}(\breve{\mathbb{C}}_2)$, *then* $\breve{\mathbb{C}}_1 \succ \breve{\mathbb{C}}_2$;

(2) *If* $\ddot{\mathbb{M}}(\breve{\mathbb{C}}_1) < \ddot{\mathbb{M}}(\breve{\mathbb{C}}_2)$, *then* $\breve{\mathbb{C}}_1 \prec \breve{\mathbb{C}}_2$;

(3) *If* $\ddot{\mathbb{M}}(\breve{\mathbb{C}}_1) = \ddot{\mathbb{M}}(\breve{\mathbb{C}}_2)$ *then*

- *If* $\ddot{\mathbb{L}}(\breve{\mathbb{C}}_1) > \ddot{\mathbb{L}}(\breve{\mathbb{C}}_2)$, *then* $\breve{\mathbb{C}}_1 \succ \breve{\mathbb{C}}_2$;
- *If* $\ddot{\mathbb{L}}(\breve{\mathbb{C}}_1) < \ddot{\mathbb{L}}(\breve{\mathbb{C}}_2)$, *then* $\breve{\mathbb{C}}_1 \prec \breve{\mathbb{C}}_2$;
- *If* $\ddot{\mathbb{L}}(\breve{\mathbb{C}}_1) = \ddot{\mathbb{L}}(\breve{\mathbb{C}}_2)$, *then* $\breve{\mathbb{C}}_1 \sim \breve{\mathbb{C}}_2$.

Definition 2.5. *[25] Let* $\breve{\mathbb{C}}_1 = \langle \Re_{\breve{\mathbb{C}}_1}, \partial_{\breve{\mathbb{C}}_1}, \hbar_{\breve{\mathbb{C}}_1} \rangle$ *and* $\breve{\mathbb{C}}_2 = \langle \Re_{\breve{\mathbb{C}}_2}, \partial_{\breve{\mathbb{C}}_2}, \hbar_{\breve{\mathbb{C}}_2} \rangle$ *be any two SVNNs, then the Euclidean distance between them is defined as follows:*

$$\mathfrak{D}(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2) = \sqrt{\frac{1}{3}\{|\Re_{\breve{\mathbb{C}}_1} - \Re_{\breve{\mathbb{C}}_2}|^2 + |\partial_{\breve{\mathbb{C}}_1} - \partial_{\breve{\mathbb{C}}_2}|^2 + |\hbar_{\breve{\mathbb{C}}_1} - \hbar_{\breve{\mathbb{C}}_2}|^2\}} \tag{3}$$

Definition 2.6. *[26] A PAO of dimension n is mapping PAO:* $Q^\rho \to Q$, *according to the following formula.*

$$PAO(\breve{t}_1, \breve{t}_2, \ldots, \breve{t}_\rho) = \frac{\sum_{j=1}^{\rho}(1 + \ddot{E}(\breve{t}_j))\breve{t}_j}{\sum_{j=1}^{\rho} 1 + \ddot{E}(\breve{t}_j)} \tag{4}$$

where $\ddot{E}(\breve{t}_j) = \sum_{h=1,h\neq j}^{\rho} supp\ (\breve{t}_j, \breve{t}_h)$ *and* $(j = 1, 2, \ldots, \rho; h = 1, 2, \ldots, r)$ *which provides the relationship between* $\breve{t}_j$ *and* $\breve{t}_h$ *which must follow the conditions:*

(1) $supp(\breve{t}_j, \breve{t}_h) \in [0, 1]$;

(2) $supp(\breve{t}_j, \breve{t}_h) = supp(\breve{t}_h, \breve{t}_j)$;

(3) $supp(\breve{t}_j, \breve{t}_h) \geq supp(\breve{t}_s, \breve{t}_t)$ *if* $|\breve{t}_j - \breve{t}_h| < |\breve{t}_s - \breve{t}_t|$.

Definition 2.7. *[27] A TN is a function* $\mathfrak{E} : [0, 1]\times$ *[0,1]* $\to$ *[0,1] that fulfills the properties of symmetry, monotonicity, and associativity, and includes an identity element, i.e., for all* $\breve{\ddot{p}}, \breve{\ddot{q}}, \breve{\ddot{n}} \in$ *[0,1]:*

(1) $\mathfrak{E}(\breve{\ddot{p}}, \breve{\ddot{q}}) = \mathfrak{E}(\breve{\ddot{q}}, \breve{\ddot{p}})$;

(2) $\mathfrak{E}(\breve{\ddot{p}}, \breve{\ddot{q}}) \leq \mathfrak{E}(\breve{\ddot{p}}, \breve{\ddot{n}})$ *if* $\breve{\ddot{q}} < \breve{\ddot{n}}$;

(3) $\mathfrak{E}(\breve{\ddot{p}}, \mathfrak{E}(\breve{\ddot{q}}, \breve{\ddot{n}})) = \mathfrak{E}(\mathfrak{E}(\breve{\ddot{p}}, \breve{\ddot{q}}), \breve{\ddot{n}})$;

(4) $\mathfrak{E}(\breve{\ddot{p}}, 1) = \breve{\ddot{p}}$.

The following is a list of some well-known TNs.

(1) *Minimum TN:* $\mathfrak{E}_M(\breve{\ddot{p}}, \breve{\ddot{q}}) = min(\breve{\ddot{p}}, \breve{\ddot{q}})$;

(2) *Product TN:* $\mathfrak{E}_P(\breve{\ddot{p}}, \breve{\ddot{q}}) = \breve{\ddot{p}} \cdot \breve{\ddot{q}}$;

(3) *Lukasiewicz TN:* $\mathfrak{E}_L(\breve{\ddot{p}}, \breve{\ddot{q}}) = max(\breve{\ddot{p}} + \breve{\ddot{q}} - 1, 0)$;

(4) *Drastic TN:* $\mathfrak{E}_D(\breve{\ddot{p}}, \breve{\ddot{q}}) = \begin{pmatrix} \breve{\ddot{p}}, & \text{if } \breve{\ddot{q}}=1 \\ \breve{\ddot{q}}, & \text{if } \breve{\ddot{p}}=1 \\ 0, & \text{otherwise} \end{pmatrix}$;

(5) *Nilpotent minimum:*

$$\mathfrak{E}_{nM}(\breve{\ddot{p}}, \breve{\ddot{q}}) = \begin{pmatrix} min(\breve{\ddot{p}}, \breve{\ddot{q}}) & \text{if } \breve{\ddot{p}} + \breve{\ddot{q}} > 1 \\ 0 & \text{otherwise} \end{pmatrix}.$$

Definition 2.8. *[27] A TCN is a function $\mathfrak{O} : [0,1]\times [0,1] \rightarrow [0,1]$ that fulfills the properties of symmetry, monotonicity, and associativity, and includes an identity element, i.e., for all $\breve{\ddot{p}}, \breve{\ddot{q}}, \breve{\ddot{n}} \in [0,1]$:*

(1) $\mathfrak{O}(\breve{\ddot{p}}, \breve{\ddot{q}}) = \mathfrak{O}(\breve{\ddot{q}}, \breve{\ddot{p}})$;
(2) $\mathfrak{O}(\breve{\ddot{p}}, \breve{\ddot{q}}) \leq \mathfrak{O}(\breve{\ddot{p}}, \breve{\ddot{n}})$ *if* $\breve{\ddot{q}} < \breve{\ddot{n}}$;
(3) $\mathfrak{O}(\breve{\ddot{p}}, \mathfrak{O}(\breve{\ddot{q}}, \breve{\ddot{n}})) = \mathfrak{O}(\mathfrak{O}(\breve{\ddot{p}}, \breve{\ddot{q}}), \breve{\ddot{n}})$;
(4) $\mathfrak{O}(\breve{\ddot{p}}, 0) = \breve{\ddot{p}}$.

The following is a list of some well-known TCNs.

(1) *Maximum TCN:* $\mathfrak{O}_M(\breve{\ddot{p}}, \breve{\ddot{q}}) = max(\breve{\ddot{p}}, \breve{\ddot{q}})$;
(2) *Probabilistic sum TCN:* $\mathfrak{O}_P(\breve{\ddot{p}}, \breve{\ddot{q}}) = \breve{\ddot{p}} + \breve{\ddot{q}} - \breve{\ddot{p}} \cdot \breve{\ddot{q}}$;
(3) *Bounded sum:* $\mathfrak{O}_L(\breve{\ddot{p}}, \breve{\ddot{q}}) = min(\breve{\ddot{p}} + \breve{\ddot{q}}, 1)$;
(4) *Drastic TCN:* $\mathfrak{O}_D(\breve{\ddot{p}}, \breve{\ddot{q}}) = \begin{pmatrix} \breve{\ddot{p}}, & \text{if } \breve{\ddot{q}}=0 \\ \breve{\ddot{q}}, & \text{if } \breve{\ddot{p}}=0 \\ 1, & otherwise \end{pmatrix}$;
(5) *Nilpotent minimum:*
$\mathfrak{O}_{nM}(\breve{\ddot{p}}, \breve{\ddot{q}}) = \begin{pmatrix} max(\breve{\ddot{p}}, \breve{\ddot{q}}) & \text{if } \breve{\ddot{p}} + \breve{\ddot{q}} < 1 \\ 1 & otherwise. \end{pmatrix}$.

Definition 2.9. *[28] AA in early 1982 introduced the concepts of TN and TCN classes for functional equations. The AATN can be defined as follows:*

$$\mathfrak{E}_A^{\phi}(\breve{\ddot{p}}, \breve{\ddot{q}}) = \begin{cases} \mathfrak{E}_D(\breve{\ddot{p}}, \breve{\ddot{q}}), & \text{if } \alpha = 0 \\ min(\breve{\ddot{p}}, \breve{\ddot{q}}), & \text{if } \alpha = \infty \\ e^{-\{(-Ln\breve{\ddot{p}})^{\alpha} + (-Ln\breve{\ddot{q}})^{\alpha}\}^{\frac{1}{\alpha}}}, & otherwise. \end{cases}$$

and the AATCN can be defined as follows:

$$\mathfrak{O}_A^{\phi}(\breve{\ddot{p}}, \breve{\ddot{q}}) = \begin{cases} \mathfrak{O}_D(\breve{\ddot{p}}, \breve{\ddot{q}}); & \text{if } \alpha = 0 \\ max(\breve{\ddot{p}}, \breve{\ddot{q}}); & \text{if } \alpha = \infty \\ e^{-\{(-Ln(1-\breve{\ddot{p}}))^{\alpha} + (-Ln(1-\breve{\ddot{q}}))^{\alpha}\}^{\frac{1}{\alpha}}}, & otherwise. \end{cases}$$

such that $\mathfrak{E}_A^0 = \mathfrak{E}_D, \mathfrak{E}_A^1 = \mathfrak{E}_P, \mathfrak{E}_A^{\infty} = min, \mathfrak{O}_A^0 = \mathfrak{O}_D, \mathfrak{O}_A^1 = \mathfrak{O}_P, \mathfrak{O}_A^{\infty} = max$. *The TN* $\mathfrak{E}_A^{\alpha}$ *and TCN* $\mathfrak{O}_A^{\alpha}$ *are combained to one another for each* $\alpha \in [0, \infty]$. *The class of AATN is strictly increasing, and the class of AATCN is strictly decreasing. The following is the AATN and AATCN operational laws in connection with SVN theory.*

Definition 2.10. *[24] Let* $\breve{\mathbb{C}}_j = \langle \Re_{\breve{\mathbb{C}}_j}, \partial_{\breve{\mathbb{C}}_j}, \hbar_{\breve{\mathbb{C}}_j} \rangle$, $j = 1, 2$ *be two SVNNs,* $\alpha \geq 1$ *and* $K > 0$. *Then, the AATN and AATCN operations of SVNN are defined as:*

(1) $\breve{\mathbb{C}}_1 \oplus \breve{\mathbb{C}}_2 = \langle 1 - e^{-\{(-Ln(1-\Re_{\breve{\mathbb{C}}_1}))^{\alpha} + (-Ln(1-\Re_{\breve{\mathbb{C}}_2}))^{\alpha}\}^{\frac{1}{\alpha}}}, e^{-\{(-Ln(\hbar_{\breve{\mathbb{C}}_1}))^{\alpha} + (-Ln(\hbar_{\breve{\mathbb{C}}_2}))^{\alpha}\}^{\frac{1}{\alpha}}}, e^{-\{(-Ln(\partial_{\breve{\mathbb{C}}_1}))^{\alpha} + (-Ln(\partial_{\breve{\mathbb{C}}_2})))^{\alpha}\}^{\frac{1}{\alpha}}} \rangle$;

(2) $\breve{\mathbb{C}}_1 \otimes \breve{\mathbb{C}}_2 = \langle e^{-\{(-Ln(\Re_{\breve{\mathbb{C}}_1}))^{\alpha}+(-Ln(\Re_{\breve{\mathbb{C}}_2}))^{\alpha}\}^{\frac{1}{\alpha}}}, 1 - e^{-\{(-Ln(1-\hbar_{\breve{\mathbb{C}}_1}))^{\alpha}+(-Ln(1-\hbar_{\breve{\mathbb{C}}_2}))^{\alpha}\}^{\frac{1}{\alpha}}}, 1 - e^{-\{(-Ln(1-\partial_{\breve{\mathbb{C}}_1}))^{\alpha}+(-Ln(1-\partial_{\breve{\mathbb{C}}_2})))^{\alpha}\}^{\frac{1}{\alpha}}}\rangle$;

(3) $K \cdot \breve{\mathbb{C}}_1 = \langle 1 - e^{-\{K(-Ln(1-\Re_{\breve{\mathbb{C}}_1}))^{\alpha}\}^{\frac{1}{\alpha}}}, e^{-\{K(-Ln(\hbar_{\breve{\mathbb{C}}_1}))^{\alpha}\}^{\frac{1}{\alpha}}}, e^{-\{K(-Ln(\partial_{\breve{\mathbb{C}}_1}))^{\alpha}\}^{\frac{1}{\alpha}}}\rangle$;

(4) $\breve{\mathbb{C}}_1^K = \langle e^{-\{K(-Ln(\Re_{\breve{\mathbb{C}}_1}))^{\alpha}\}^{\frac{1}{\alpha}}}, 1 - e^{-\{K(-Ln(1-\hbar_{\breve{\mathbb{C}}_1}))^{\alpha}\}^{\frac{1}{\alpha}}}, 1 - e^{-\{K(-Ln(1-\partial_{\breve{\mathbb{C}}_1}))^{\alpha}\}^{\frac{1}{\alpha}}}\rangle$.

Definition 2.11. *[24] Let* $\breve{\mathbb{C}} = \{\langle \phi, \Re_{\breve{\mathbb{C}}}(\phi), \partial_{\breve{\mathbb{C}}}(\phi), \hbar_{\breve{\mathbb{C}}}(\phi) | \phi \in X\rangle\}$ $\breve{\mathbb{C}}_1 = \{\langle \phi, \Re_{\breve{\mathbb{C}}_1}(\phi), \partial_{\breve{\mathbb{C}}_1}(\phi), \hbar_{\breve{\mathbb{C}}_1}(\phi) | \phi \in X\rangle\}$ *and* $\breve{\mathbb{C}}_2 = \{\langle \phi, \Re_{\breve{\mathbb{C}}_2}(\phi), \partial_{\breve{\mathbb{C}}_2}(\phi), \hbar_{\breve{\mathbb{C}}_2}(\phi) | \phi \in X\rangle\}$ *be any three SVNS, and their set operators are defined as*

(1) $\breve{\mathbb{C}}_1 \subseteq \breve{\mathbb{C}}_2 \Leftrightarrow \Re_{\breve{\mathbb{C}}_1}(\phi) \leq \Re_{\breve{\mathbb{C}}_2}(\phi), \partial_{\breve{\mathbb{C}}_1}(\phi) \leq \partial_{\breve{\mathbb{C}}_2}(\phi)$ *and* $\hbar_{\breve{\mathbb{C}}_1}(\phi) \geq \hbar_{\breve{\mathbb{C}}_2}(\phi) \forall \phi \in X$;

(2) $\breve{\mathbb{C}}_1 \cup \breve{\mathbb{C}}_2 = \{\langle \phi, \{\mathfrak{O}_A\{\Re_{\breve{\mathbb{C}}_1}(\phi), \Re_{\breve{\mathbb{C}}_2}(\phi)\}\}, \{\mathfrak{E}_A\{\partial_{\breve{\mathbb{C}}_1}(\phi), \partial_{\breve{\mathbb{C}}_2}(\phi)\}\}, \{\mathfrak{E}_A\{\hbar_{\breve{\mathbb{C}}_1}(\phi), \hbar_{\breve{\mathbb{C}}_2}\}(\phi)\} | \phi \in X\rangle\}$;

(3) $\breve{\mathbb{C}}_1 \cap \breve{\mathbb{C}}_2 = \{\langle \phi, \{\mathfrak{E}_A\{\Re_{\breve{\mathbb{C}}_1}(\phi), \Re_{\breve{\mathbb{C}}_2}(\phi)\}\}, \{\mathfrak{O}_A\{\partial_{\breve{\mathbb{C}}_1}(\phi), \partial_{\breve{\mathbb{C}}_2}(\phi)\}\}, \{\mathfrak{O}_A\{\hbar_{\breve{\mathbb{C}}_1}(\phi), \hbar_{\breve{\mathbb{C}}_2}(\phi)\}\} | \phi \in X\}\rangle\}$;

(4) $\breve{\mathbb{C}}^c = \{\langle \phi, \hbar_{\breve{\mathbb{C}}}(\phi), \partial_{\breve{\mathbb{C}}}(\phi), \Re_{\breve{\mathbb{C}}}(\phi) | \phi \in X\rangle\}$.

Theorem 2.1. *Let* $\breve{\mathbb{C}}_1 = \langle \Re_{\breve{\mathbb{C}}_1}, \partial_{\breve{\mathbb{C}}_1}, \hbar_{\breve{\mathbb{C}}_1}\rangle$ *and* $\breve{\mathbb{C}}_2 = \langle \Re_{\breve{\mathbb{C}}_2}, \partial_{\breve{\mathbb{C}}_2}, \hbar_{\breve{\mathbb{C}}_2}\rangle$ *be any two SVNNs. Then,*

(1) $\breve{\mathbb{C}}_1 \oplus \breve{\mathbb{C}}_2 = \breve{\mathbb{C}}_2 \oplus \breve{\mathbb{C}}_1$,

(2) $\breve{\mathbb{C}}_1 \otimes \breve{\mathbb{C}}_2 = \breve{\mathbb{C}}_2 \oplus \breve{\mathbb{C}}_1$,

(3) $\Lambda(\breve{\mathbb{C}}_1 \oplus \breve{\mathbb{C}}_2) = \Lambda\breve{\mathbb{C}}_1 \oplus \Lambda\breve{\mathbb{C}}_2, \Lambda \geq 0$,

(4) $\Lambda_1\breve{\mathbb{C}}_1 \oplus \Lambda_2\breve{\mathbb{C}}_1 = (\Lambda_1 + \Lambda_2)\breve{\mathbb{C}}_1, \Lambda_1, \Lambda_2 \geq 0$,

(5) $\breve{\mathbb{C}}_1^{\Lambda} \otimes \breve{\mathbb{C}}_2^{\Lambda} = (\breve{\mathbb{C}}_1 \otimes \breve{\mathbb{C}}_2)^{\Lambda}, \Lambda \geq 0$,

(6) $\breve{\mathbb{C}}_1^{\Lambda_1} \otimes \breve{\mathbb{C}}_2^{\Lambda_2} = (\breve{\mathbb{C}}_1)^(\Lambda_1 + \Lambda_2), \Lambda_1, \Lambda_2 \geq 0$.

Proof. Straightforward. □

3. **Proposed Confidence SVN Aczel-Alsina power aggregation operator**

The current section defines a series of SVN Aczel-Alsina power averaging operators that incorporate CLs with the evaluated SVNNs.

3.1. *Confidence SVN Aczel-Alsina power average aggregation operator*

In this part, we built the confidence SVN weighted and ordered weighted Aczel-Alsina power averaging AO. Additionally, we investigate several fundamental aspects of these proposed operators.

By employing the fundamental operations of AA aggregation tools, we derived appropriate methodologies, including CSVNAAPWAAO, with reliable properties while considering SVNNs. Additionally, we applied a weighted suppport degree throughout our article, using the following equation: $\mathcal{Z}_j = \frac{\tau_j(1+\breve{U}(\breve{\mathbb{C}}_j))}{\sum_{j=1}^{\rho}\tau_j(1+\breve{U}(\breve{\mathbb{C}}_j))}$ where the suppport of $\breve{\mathbb{C}}_j$ is denoted by $\breve{U}(\breve{\mathbb{C}}_j) = \sum_{h=1,h\neq j}^{\rho} \text{supp}(\breve{\mathbb{C}}_j, \breve{\mathbb{C}}_h),\ j = 1,2,\ldots,\rho, h = 1,2,\ldots,r$ and the associated weight vector of $\breve{\mathbb{C}}_j$ is $\tau = (\tau_1,\tau_2,\ldots,\tau_\rho,)^T, j = 1,2,\ldots,\rho, \tau_j > 0, and \sum_{j=1}^{\rho}\tau_j = 1$.

Definition 3.1. *Let $\breve{\mathbb{C}}_j = (\Re_j, \hbar_j, \partial_j)(j = 1,2,\ldots,\rho)$ be a set of SVNNs and $\eta_j$be the CL of $\breve{\mathbb{C}}_j$ with $0 \leq \eta_j \leq 1$. Let $\mathcal{Z} = (\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_\rho)^T$ be the weight vectors for SVNNs with the condition $\sum_{j=1}^{\rho}\mathcal{Z}_j = 1$. Then, the mapping CSVNAAPWAAO: $b^\rho \to b$ operator is given as follows: CSVNAAPWAAO $\{(\breve{\mathbb{C}}_1,\eta_1),(\breve{\mathbb{C}}_2,\eta_2),\ldots,(\breve{\mathbb{C}}_\rho,\eta_\rho)\} = \oplus_{j=1}^{\rho}\mathcal{Z}_j(\eta_j,\breve{\mathbb{C}}_j)$*

$$= \mathcal{Z}_1(\eta_1,\breve{\mathbb{C}}_1) \oplus \mathcal{Z}_2(\eta_2,\breve{\mathbb{C}}_2) \oplus \cdots \oplus \mathcal{Z}_\rho(\eta_\rho,\breve{\mathbb{C}}_\rho). \tag{5}$$

Theorem 3.1. *The aggregated value of the SVNNs $\breve{\mathbb{C}}_j$ for $j = 1,2,\ldots,\rho$ with respect to the weight vector $\mathcal{Z} = (\mathcal{Z}_1,\mathcal{Z}_2,\ldots,\mathcal{Z}_\rho)^T$ and the CL η_j such that $0 \leq \eta_j \leq 1$ obtained using the CSVNAAPWAAO Equation 5 is also a SVNN and is given by CSVNAAPWAAO $\{(\breve{\mathbb{C}}_1,\eta_1),(\breve{\mathbb{C}}_2,\eta_2),\ldots,(\breve{\mathbb{C}}_\rho,\eta_\rho)\} =$*

$$= \langle 1 - e^{-\{\sum_{j=1}^{\rho}(\eta_j\mathcal{Z}_j(-Ln(1-\Re_{\breve{\mathbb{C}}_j}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{\rho}(\eta_j\mathcal{Z}_j(-Ln\hbar_{\breve{\mathbb{C}}_j})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{\rho}(\eta_j\mathcal{Z}_j(-Ln\partial_{\breve{\mathbb{C}}_j})^\alpha)\}^{\frac{1}{\alpha}}} \rangle \tag{6}$$

Proof. By mathematical induction the proof as follows:

(1) For $\rho = 2$, we have CSVNAAPWAAO $((\breve{\mathbb{C}}_1,\eta_1),(\breve{\mathbb{C}}_2,\eta_2)) = \mathcal{Z}(\breve{\mathbb{C}}_1,\eta_1) \oplus \mathcal{Z}(\breve{\mathbb{C}}_2,\eta_2)$. By operational laws, we get $\mathcal{Z}_1(\breve{\mathbb{C}}_1,\eta_1) = \langle 1 - e^{-\{(\eta_1\mathcal{Z}_1(-Ln(1-\Re_{\breve{\mathbb{C}}_1}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_1\mathcal{Z}_1(-Ln\hbar_{\breve{\mathbb{C}}_1})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_1\mathcal{Z}_1(-Ln\partial_{\breve{\mathbb{C}}_1})^\alpha)\}^{\frac{1}{\alpha}}} \rangle$. analogously, for $\mathcal{Z}_2(\breve{\mathbb{C}}_2,\eta_2) = \langle 1 - e^{-\{(\eta_2\mathcal{Z}_2(-Ln(1-\Re_{\breve{\mathbb{C}}_2}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_2\mathcal{Z}_2(-Ln\hbar_{\breve{\mathbb{C}}_2})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_2\mathcal{Z}_2(-Ln\partial_{\breve{\mathbb{C}}_2})^\alpha)\}^{\frac{1}{\alpha}}} \rangle$. CSVNAAPWAAO $((\breve{\mathbb{C}}_1,\eta_1),(\breve{\mathbb{C}}_2,\eta_2)) = \mathcal{Z}(\breve{\mathbb{C}}_1,\eta_1) \oplus \mathcal{Z}(\breve{\mathbb{C}}_2,\eta_2) = \langle 1 - e^{-\{(\eta_1\mathcal{Z}_1(-Ln(1-\Re_{\breve{\mathbb{C}}_1}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_1\mathcal{Z}_1(-Ln\hbar_{\breve{\mathbb{C}}_1})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_1\mathcal{Z}_1(-Ln\partial_{\breve{\mathbb{C}}_1})^\alpha)\}^{\frac{1}{\alpha}}} \rangle \oplus \langle 1 - e^{-\{(\eta_2\mathcal{Z}_2(-Ln(1-\Re_{\breve{\mathbb{C}}_2}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_2\mathcal{Z}_2(-Ln\hbar_{\breve{\mathbb{C}}_2})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_2\mathcal{Z}_2(-Ln\partial_{\breve{\mathbb{C}}_2})^\alpha)\}^{\frac{1}{\alpha}}} \rangle = \langle 1 - e^{-\{(\eta_1\mathcal{Z}_1(-Ln(1-\Re_{\breve{\mathbb{C}}_1}))^\alpha)+(\eta_2\mathcal{Z}_2(-Ln(1-\Re_{\breve{\mathbb{C}}_2}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_1\mathcal{Z}_1(-Ln\hbar_{\breve{\mathbb{C}}_1})^\alpha)+(\eta_2\mathcal{Z}_2(-Ln\hbar_{\breve{\mathbb{C}}_2})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{(\eta_1\mathcal{Z}_1(-Ln\partial_{\breve{\mathbb{C}}_1})^\alpha)+(\eta_2\mathcal{Z}_2(-Ln\partial_{\breve{\mathbb{C}}_2})^\alpha)\}^{\frac{1}{\alpha}}} \rangle = \langle 1 - e^{-\{\sum_{j=1}^{2}(\eta_j\mathcal{Z}_j(-Ln(1-\Re_{\breve{\mathbb{C}}_j}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{2}(\eta_j\mathcal{Z}_j(-Ln\hbar_{\breve{\mathbb{C}}_j})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{2}(\eta_j\mathcal{Z}_j(-Ln\partial_{\breve{\mathbb{C}}_j})^\alpha)\}^{\frac{1}{\alpha}}} \rangle$. Hence, this true for j=2.

(2) Now, supppose that this will be true for j=k. Then we have the following equation: CSVNAAPWAAO $\{(\breve{\mathbb{C}}_1,\eta_1),(\breve{\mathbb{C}}_2,\eta_2),\ldots,(\breve{\mathbb{C}}_k,\eta_k)\} = \langle 1 - e^{-\{\sum_{j=1}^{k}(\eta_k\mathcal{Z}_k(-Ln(1-\Re_{\breve{\mathbb{C}}_k}))^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k}(\eta_k\mathcal{Z}_k(-Ln\hbar_{\breve{\mathbb{C}}_k})^\alpha)\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k}(\eta_k\mathcal{Z}_k(-Ln\partial_{\breve{\mathbb{C}}_k})^\alpha)\}^{\frac{1}{\alpha}}} \rangle$.

Now, we have to show that it also hods for j=k+1 as follows CSVNAAPWAAO $\{(\breve{\mathbb{C}}_1,\eta_1),(\breve{\mathbb{C}}_2,\eta_2),\ldots,(\breve{\mathbb{C}}_k,\eta_k),(\breve{\mathbb{C}}_{k+1},\eta_{k+1})\} = \langle 1 -$

$e^{-\{\sum_{j=1}^{k}(\eta_k \mathcal{Z}_k(-Ln(1-\Re_{\breve{\mathbb{C}}_k}))^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k}(\eta_k \mathcal{Z}_k(-Ln\hbar_{\breve{\mathbb{C}}_k})^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k}(\eta_k \mathcal{Z}_k(-Ln\partial_{\breve{\mathbb{C}}_k})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle \oplus \langle 1 - e^{-\{\sum_{j=1}^{k+1}(\eta_{k+1} \mathcal{Z}_{k+1}(-Ln(1-\Re_{\breve{\mathbb{C}}_{k+1}}))^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k+1}(\eta_{k+1} \mathcal{Z}_{k+1}(-Ln\hbar_{\breve{\mathbb{C}}_{k+1}})^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k+1} k(\eta_{k+1} \mathcal{Z}_{k+1}(-Ln\partial_{\breve{\mathbb{C}}_{k+1}})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle = \langle 1 - e^{-\{\sum_{j=1}^{k+1}(\eta_j \mathcal{Z}_j(-Ln(1-\Re_{\breve{\mathbb{C}}_j}))^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k+1}(\eta_j \mathcal{Z}_j(-Ln\hbar_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{k+1}(\eta_j \mathcal{Z}_j(-Ln\partial_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle$

which is true for j=k+1. □

Example 3.1. Supppose $\breve{\mathbb{C}}_1 = ((0.3, 0.4, 0.2), 0.4), \breve{\mathbb{C}}_2 = ((0.9, 0.8, 0.6), 0.9)$ $\breve{\mathbb{C}}_3 = ((0.7, 0.5, 0.2), 0.6)$ and $\breve{\mathbb{C}}_4 = ((0.9, 0.2, 0.2), 0.7)$ are four SVN numbers along their CL. If we take $\alpha = 3$ and $\tau_j = (0.2, 0.1, 0.3, 0.4)^T$ then, CSVNAAPAAO can be utilized to aggregate the three SVNNs as follows: $D(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2) = 0.476, D(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_3) = 0.238, D(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_4) = 0.365, D(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_3) = 0.311, D(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_4) = 0.416, D(\breve{\mathbb{C}}_3, \breve{\mathbb{C}}_4) = 0.208$. $supp(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2) = 0.524 supp(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_3) = 0.762 supp(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_4) = 0.635, supp(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_3) = 0.689 supp(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_4) = 0.584 supp(\breve{\mathbb{C}}_3, \breve{\mathbb{C}}_4) = 0.792$ then, $\mathcal{Z}_1 = 0.192$, $\mathcal{Z}_2 = 0.092$, $\mathcal{Z}_3 = 0.32$ and $\mathcal{Z}_4 = 0.39$. By using Equation 6, we get CSVNAAPAAO$(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2, \breve{\mathbb{C}}_3) = \langle 0.813, 0.338, 0.268 \rangle$. Employing SVNs allows us to readily demonstrate that the proposed CSVNAAPWAAO fulfills the properties of idempotency, boundedness, and monotonicity, as explained below:

Property 3.1.1. The CSVNAAPWAAO is idempotent. i.e., If $(\breve{\mathbb{C}}_j, \eta_j) = (\breve{\mathbb{C}}, d)$ for all j, then CSVNAAPWAAO $((\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)) = \eta\breve{\mathbb{C}}$.

Proof. Since $\breve{\mathbb{C}}_j = \langle \Re_{\breve{\mathbb{C}}_j}, \hbar_{\breve{\mathbb{C}}_j}, \partial_{\breve{\mathbb{C}}_j} \rangle$, $j = 1, 2, \ldots, \rho$ be the set of SVNNs we can get the following equation: $CSVNAAPWAA((\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)) = \langle 1 - e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln(1-\Re_{\breve{\mathbb{C}}_j}))^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\hbar_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\partial_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle = \langle 1 - e^{-\{(d(-Ln(1-\Re_{\breve{\mathbb{C}}}))^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{(d(-Ln\hbar_{\breve{\mathbb{C}}})^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{(d(-Ln\partial)^{\alpha})\}^{\frac{1}{\alpha}}}\rangle = \langle \Re_{\breve{\mathbb{C}}}, \hbar_{\breve{\mathbb{C}}}, \partial_{\breve{\mathbb{C}}} \rangle = (\eta, \breve{\mathbb{C}})$ □

Property 3.1.2. The CSVNAAPWAAO is boundedness. i.e., For a collection of SVNNs $\breve{\mathbb{C}}_j$ for all $j = 1, 2, \ldots, \rho$ and $\breve{\mathbb{C}}^- = min(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)$ and $\breve{\mathbb{C}}^+ = max(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)$. Then $\breve{\mathbb{C}}^- \leq CSVNAAPWAA((\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)) \leq \breve{\mathbb{C}}^+$.

Proof. Consider $\breve{\mathbb{C}}_j = \langle \Re_{\breve{\mathbb{C}}_j}, \hbar_{\breve{\mathbb{C}}_j}, \partial_{\breve{\mathbb{C}}_j} \rangle$, $j = 1, 2, \ldots, \rho$, be the set of SVNNs. Let $\breve{\mathbb{C}}^- = min(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho) = \langle \Re^-_{\breve{\mathbb{C}}_j}, \hbar^-_{\breve{\mathbb{C}}_j}, \partial^-_{\breve{\mathbb{C}}_j} \rangle$ and $\breve{\mathbb{C}}^+ = max(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)\breve{\mathbb{C}}_j = \langle \Re^+_{\breve{\mathbb{C}}_j}, \hbar^+_{\breve{\mathbb{C}}_j}, \partial^+_{\breve{\mathbb{C}}_j} \rangle$. We have $\Re^-_{\breve{\mathbb{C}}} = \min_j \Re_{\breve{\mathbb{C}}_j}, \hbar^-_{\breve{\mathbb{C}}} = \max_j \hbar_{\breve{\mathbb{C}}_j}, \partial^-_{\breve{\mathbb{C}}} = \max_j \partial_{\breve{\mathbb{C}}_j}, \Re^+_{\breve{\mathbb{C}}} = \max_j \Re_{\breve{\mathbb{C}}_j}, \hbar^+_{\breve{\mathbb{C}}} = \min_j \hbar_{\breve{\mathbb{C}}_j}$ and $\partial^+_{\breve{\mathbb{C}}} = \min_j \partial_{\breve{\mathbb{C}}_j}$. Hence there we have the following subsequent inequalities:

$\langle 1 - e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln(1-\Re^-_{\breve{\mathbb{C}}_j}))^{\alpha})\}^{\frac{1}{\alpha}}}\rangle \leq \langle 1 - e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln(1-\Re_{\breve{\mathbb{C}}_j}))^{\alpha})\}^{\frac{1}{\alpha}}}\rangle \leq \langle 1 -$

$e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln(1-\Re^{+}_{\breve{\mathbb{C}}_j}))^{\alpha})\}^{\frac{1}{\alpha}}}\rangle$; $\langle e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\hbar^{-}_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}} \leq e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\hbar_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}} \leq e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\hbar^{+}_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle$;$\langle e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\partial^{-}_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}} \leq e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\partial_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}} \leq e^{-\{\sum_{j=1}^{\rho}(\eta_j \mathcal{Z}_j(-Ln\partial^{+}_{\breve{\mathbb{C}}_j})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle$. Therefore, $\breve{\mathbb{C}}^{-} \leq CSVNAAPWAA((\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)) \leq \breve{\mathbb{C}}^{+}$. □

Property 3.1.3. The CSVNAAPWAAO is monotonicity. i.e., for any two SVNNs $\breve{\mathbb{C}}_j = \langle \Re_{\breve{\mathbb{C}}_j}, \hbar_{\breve{\mathbb{C}}_j}, \partial_{\breve{\mathbb{C}}_j}\rangle$ and $\breve{\mathbb{C}}'_j = \langle \Re'_{\breve{\mathbb{C}}_j}, \hbar'_{\breve{\mathbb{C}}_j}, \partial'_{\breve{\mathbb{C}}_j}\rangle$ such that $\breve{\mathbb{C}}_j \leq \breve{\mathbb{C}}'_j$ for all $j = 1, 2, \ldots, \rho$. Then CSVNAAPWAAO$(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2, \ldots, \breve{\mathbb{C}}_\rho) \leq$ CSVNAAPWAAO$(\breve{\mathbb{C}}'_1, \breve{\mathbb{C}}'_2, \ldots, \breve{\mathbb{C}}'_\rho)$.

Proof. Due to $\breve{\mathbb{C}}_j \leq \breve{\mathbb{C}}'_j$ for all $j = 1, 2, \ldots, \rho$, there exists $\oplus_{j=1}^{\rho} \mathcal{Z}_j(\breve{\mathbb{C}}_j, \eta_j) \leq \oplus_{j=1}^{\rho} \mathcal{Z}_j(\breve{\mathbb{C}}'_j, \eta_j)$. Thus CSVNAAPWAAO$(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2, \ldots, \breve{\mathbb{C}}_\rho) \leq$ CSVNAAPWAAO$(\breve{\mathbb{C}}'_1, \breve{\mathbb{C}}'_2, \ldots, \breve{\mathbb{C}}'_\rho)$ is true. □

3.1.2. *CSVN Aczel-Alsina power ordered weighted average aggregation operator*

In this part, a novel CSVNAAPOWAAO. This operator considers the ordered weights associated with the aggregated SVNNs.

Definition 3.2. *Let $\breve{\mathbb{C}}_j = (\Re_j, \hbar_j, \partial_j)(j = 1, 2, \ldots, \rho)$ be a set of SVNNs and η_j be the CL of $\breve{\mathbb{C}}_j$ with $0 \leq \eta_j \leq 1$. Let $\mathcal{Z} = (\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_\rho)^T$ be the weight vectors for SVNNs with the condition $\sum_{j=1}^{\rho} \mathcal{Z}_j = 1$. Then, the mapping CSVNAAPOWAAO: $b^\rho \rightarrow b$ operator is given as follows: CSVNAAPOWAAO $\{(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)\} = \oplus_{j=1}^{\rho} \mathcal{Z}_j(\eta_{\vec{\sigma}(j)}, \breve{\mathbb{C}}_{\vec{\sigma}(j)})$*

$$= \mathcal{Z}_1(\eta_{\vec{\sigma}(1)}, \breve{\mathbb{C}}_{\vec{\sigma}(1)}) \oplus \mathcal{Z}_2(\eta_{\vec{\sigma}(2)}, \breve{\mathbb{C}}_{\vec{\sigma}(2)}) \oplus \cdots \oplus \mathcal{Z}_\rho(\eta_{\vec{\sigma}(\rho)}, \breve{\mathbb{C}}_{\vec{\sigma}(\rho)}) \tag{7}$$

where, $(\vec{\vec{\sigma}}(1), \vec{\sigma}(2), \ldots, \vec{\sigma}(\rho))$ is the permutation of $(1, 2, \ldots, \rho)$ with $\breve{\mathbb{C}}_{\vec{\sigma}(j-1)} \leq \breve{\mathbb{C}}_{\vec{\sigma}(j)}$ for all $j = 1, 2, \ldots, \rho$.

Theorem 3.2. *The aggregated value of the SVNNs $\breve{\mathbb{C}}_j$ for $j = 1, 2, \ldots, \rho$ with respect to the weight vector $\mathcal{Z} = (\mathcal{Z}_1, \mathcal{Z}_2, \ldots, \mathcal{Z}_\rho)^T$ and the CL η_j such that $0 \leq \eta_j \leq 1$ obtained using the CSVNAAPOWAAO Equation 7 is also a SVNN and is given by CSVNAAPWAAO $\{(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)\} =$*

$$= \langle 1 - e^{-\{\sum_{j=1}^{\rho}(\eta_{\vec{\sigma}(j)} \mathcal{Z}_j(-Ln(1-\Re_{\breve{\mathbb{C}}_{\vec{\sigma}(j)}}))^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{\rho}(\eta_{\vec{\sigma}(j)} \mathcal{Z}_j(-Ln\hbar_{\breve{\mathbb{C}}_{\vec{\sigma}(j)}})^{\alpha})\}^{\frac{1}{\alpha}}}, e^{-\{\sum_{j=1}^{\rho}(\eta_{\vec{\sigma}(j)} \mathcal{Z}_j(-Ln\partial_{\breve{\mathbb{C}}_{\vec{\sigma}(j)}})^{\alpha})\}^{\frac{1}{\alpha}}}\rangle \tag{8}$$

Proof. The proof of Theorem 3.2 follows the same approach as Theorem 3.1, so it is omitted here. □

Example 3.2. Supppose $\breve{\mathbb{C}}_1 = ((0.3, 0.4, 0.2), 0.4), \breve{\mathbb{C}}_2 = ((0.9, 0.8, 0.6), 0.9)$ $\breve{\mathbb{C}}_3 = ((0.7, 0.5, 0.2), 0.6)$ and $\breve{\mathbb{C}}_4 = ((0.9, 0.2, 0.2), 0.7)$ are four SVN numbers along their CL. If we take $\alpha = 3$ and $\tau_j = (0.2, 0.1, 0.3, 0.4)^T$ then, CSVNAAPAAO can be utilized to aggregate the three SVNNs as follows: $D(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2) = 0.455, D(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_3) = 0.545, D(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_4) = 0.387, D(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_3) = 0.238, D(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_4) = 0.311, D(\breve{\mathbb{C}}_3, \breve{\mathbb{C}}_4) = 0.476$. $supp(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2) = 0.545 supp(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_3) = 0.455 supp(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_4) = 0.613, supp(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_3) = 0.762 supp(\breve{\mathbb{C}}_2, \breve{\mathbb{C}}_4) = 0.689 supp(\breve{\mathbb{C}}_3, \breve{\mathbb{C}}_4) = 0.524$ then, $\mathcal{Z}_1 = 0.188$, $\mathcal{Z}_2 = 0.108$, $\mathcal{Z}_3 = 0.296$ and $\mathcal{Z}_4 = 0.407$. By using Equation 8, we get CSVNAAPAAO$(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2, \breve{\mathbb{C}}_3) = \langle 0.841, 0.418, 0.33 \rangle$. Employing SVNs

allows us to readily demonstrate that the proposed CSVNAAPOWAAO fulfills the properties of idempotency, boundedness, and monotonicity, as explained below.:

Property 3.1.4. The CSVNAAPOWAAO is idempotent. i.e., If $(\breve{\mathbb{C}}_j, \eta_j) = (\breve{\mathbb{C}}, d)$ for all j, then CSVNAAPOWAAO $((\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)) = d\breve{\mathbb{C}}$.

Proof. The proof provided is analogous to that of Property 3.1.1 □

Property 3.1.5. The CSVNAAPOWAAO is boundedness. i.e., For a collection of SVNNs $\breve{\mathbb{C}}_j$ for all $j = 1, 2, \ldots, \rho$ and $\breve{\mathbb{C}}^- = min(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)$ and $\breve{\mathbb{C}}^+ = max(\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)\breve{\mathbb{C}}_j$. Then $\breve{\mathbb{C}}^- \leq CSVNAAPOWAA((\breve{\mathbb{C}}_1, \eta_1), (\breve{\mathbb{C}}_2, \eta_2), \ldots, (\breve{\mathbb{C}}_\rho, \eta_\rho)) \leq \breve{\mathbb{C}}^+$.

Proof. The proof provided is analogous to that of Property 3.1.2 □

Property 3.1.6. The CSVNAAPWAAO is monotonicity. i.e., for any two SVNNs $\breve{\mathbb{C}}_j = \langle \Re_{\breve{\mathbb{C}}_j}, \hbar_{\breve{\mathbb{C}}_j}, \partial_{\breve{\mathbb{C}}_j} \rangle$ and $\breve{\mathbb{C}}'_j = \langle \Re'_{\breve{\mathbb{C}}_j}, \hbar'_{\breve{\mathbb{C}}_j}, \partial'_{\breve{\mathbb{C}}_j} \rangle$ such that $\breve{\mathbb{C}}_j \leq \breve{\mathbb{C}}'_j$ for all $j = 1, 2, \ldots, \rho$. Then CSVNAAPWAAO$(\breve{\mathbb{C}}_1, \breve{\mathbb{C}}_2, \ldots, \breve{\mathbb{C}}_\rho) \leq$ CSVNAAPWAAO$(\breve{\mathbb{C}}'_1, \breve{\mathbb{C}}'_2, \ldots, \breve{\mathbb{C}}'_\rho)$.

Proof. The proof provided is analogous to that of Property 3.1.3 □

4. Evaluation of SVNMADM using proposed operators

This part illustrates how the proposed operators are applied by solving an SVNMADM model.

This section, presents a procedure for solving SVNMADM problems using the proposed operators.

Step 1 Let $\breve{\ddot{S}} = (\breve{\ddot{S}}_1, \breve{\ddot{S}}_2, \ldots, \breve{\ddot{S}}_\kappa)$ be a finite number of alternatives, and $\breve{\ddot{C}} = (\breve{\ddot{C}}_1, \breve{\ddot{C}}_2, \ldots, \breve{\ddot{C}}_\rho)$ be the set of attributes. Let $\tau = (\tau_1, \tau_2, \ldots, \tau_\rho)^T$ be the weight vector of attributes, where $\tau_j \geq 0$, j= 1,2,...,ρ such that $\sum_{j=1}^{\rho} \tau_j = 1$. The SVN decision matrix $D = [\breve{\ddot{S}}_{ij}]_{\kappa\times\rho}$ evaluates the alternatives under each attribute, where $\Re_{ij}, \hbar_{ij}, \partial_{ij}$ indicates the truth, falsity and indeterminacy membership function respectively.

Step 2 To normalize an SVN decision matrix with cost type attribute, use the following Equation. 9

$$D = [\breve{\ddot{S}}_{ij}]_{\kappa\times\rho} = \begin{cases} \langle \Re_{ij}, \hbar_{ij}, \partial_{ij} \rangle & \text{if benefit type} \\ \langle \partial_{ij}, \hbar_{ij}, \Re_{ij} \rangle & \text{if cost type} \end{cases} \tag{9}$$

Step 3 Utilize the suggested operators to aggregate the evaluations for each attribute over all alternatives.

Step 4 Choose the best alternative by ranking the options based on their score values.

5. **Numerical illustration**

Let's consider the practical example of an MADM problem from [29], which involves an organization strategic supppliers under suppply chain risk management in which the five suppliers called the alternatives $\breve{\ddot{S}}_i (i = 1, 2, 3, 4, 5)$ are assessed based on four different attributes $\breve{\ddot{C}}_j (j = 1, 2, 3, 4)$ namely e technology level, service level, risk managing ability and enterprise environment with respect to the weighting vector $\tau = (0.2, 0.1, 0.3, 0.4)^T$.

Step 1 The decision matrix for the MADM problem, featuring confidence-induced SVN preference values evaluated by a decision expert, is presented in Table 1.

Step 2 Since all the attribute are beneficial, there is no need to normalize the confidence SVN decision matrix.

Table 1. Confidence SVN decision matrix evaluated by a decision expert

	$\breve{\ddot{C}}_1$	$\breve{\ddot{C}}_2$	$\breve{\ddot{C}}_3$	$\breve{\ddot{C}}_4$
$\breve{\ddot{S}}_1$	((0.5, 0.8, 0.1),0.3)	((0.6, 0.3, 0.3),0.3)	((0.3, 0.6, 0.1),0.1)	((0.5, 0.7, 0.2),0.1)
$\breve{\ddot{S}}_2$	((0.7, 0.2, 0.1),0.8)	((0.7, 0.2, 0.2),0.8)	((0.7, 0.2, 0.4),0.7)	((0.8, 0.2, 0.1),0.4)
$\breve{\ddot{S}}_3$	((0.6, 0.7, 0.2),0.9)	((0.5, 0.7, 0.3),0.9)	((0.5, 0.3, 0.1),0.2)	((0.6, 0.3, 0.2),0.7)
$\breve{\ddot{S}}_4$	((0.8, 0.1, 0.3),0.7)	((0.6, 0.3, 0.4),0.5)	((0.3, 0.4, 0.2),0.8)	((0.5, 0.6, 0.1),0.8)
$\breve{\ddot{S}}_5$	((0.6, 0.4, 0.4),0.6)	((0.4, 0.8, 0.1),0.8)	((0.7, 0.6, 0.1),0.4)	((0.5, 0.8, 0.2),0.6)

Step 3 Combine all the attribute, each with its own distinct confidence SVN preference value

for each alternative using CSVNAAPWAAO Equation 6 to get the overall SVN $\ddot{\breve{\mathbb{C}}}_i$ of the corresponding $\ddot{\breve{S}}_i$ as $\ddot{\breve{\mathbb{C}}}_1 = \langle 0.241, 0.751, 0.472\rangle$, $\ddot{\breve{\mathbb{C}}}_2 = \langle 0.59, 0.414, 0.332\rangle$, $\ddot{\breve{\mathbb{C}}}_3 = \langle 0.385, 0.616, 0.407\rangle$, $\ddot{\breve{\mathbb{C}}}_4 = \langle 0.2491, 0.386, 0.364\rangle$, $\ddot{\breve{\mathbb{C}}}_5 = \langle 0.368, 0.7, 0.392\rangle$. Combine all the attribute, each with its own distinct confidence SVN preference value for each alternative using CSVNAAPOWAAO Equation 8 to get the overall SVN $\ddot{\breve{\mathbb{C}}}_i$ of the corresponding $\ddot{\breve{S}}_i$ as $\ddot{\breve{\mathbb{C}}}_1 = \langle 0.257, 0.7, 0.52\rangle$, $\ddot{\breve{\mathbb{C}}}_2 = \langle 0.535, 0.414, 0.331\rangle$, $\ddot{\breve{\mathbb{C}}}_3 = \langle 0.347, 0.558, 0.343\rangle$, $\ddot{\breve{\mathbb{C}}}_4 = \langle 0.489, 0.384, 0.386\rangle$, $\ddot{\breve{\mathbb{C}}}_5 = \langle 0.405, 0.732, 0.351\rangle$.

Step 4 Calculate the score values using Equation 1 corresponding to the SVN $\ddot{\breve{\mathbb{C}}}_i$ obtained in Step 3 based on the CSVNAAPWAAO respectively are $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_1) = 0.339$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_2) = 0.587$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_3) = 0.454$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_4) = 0.581$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_5) = 0.427$. Based on CSVNAAPWAAO, the score value .Thus, we have $\ddot{\breve{S}}_2 > \ddot{\breve{S}}_4 > \ddot{\breve{S}}_3 > \ddot{\breve{S}}_5 > \ddot{\breve{S}}_1$. Hence the best alternative is $\ddot{\breve{S}}_2$.

Calculate the score values using Equation 1 corresponding to the SVN $\ddot{\breve{\mathbb{C}}}_i$ obtained in Step 3 based on the CSVNAAPOWAAO respectively are $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_1) = 0.347$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_2) = 0.587$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_3) = 0.482$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_4) = 0.573$, $\ddot{\mathbb{M}}(\ddot{\breve{\mathbb{C}}}_5) = 0.441$. Based on CSVNAAPOWAAO, the score value .Thus, we have $\ddot{\breve{S}}_2 > \ddot{\breve{S}}_4 > \ddot{\breve{S}}_3 > \ddot{\breve{S}}_5 > \ddot{\breve{S}}_1$. Hence the best alternative is $\ddot{\breve{S}}_2$.

6. **Comparative analysis**

In this discussion we compare the overall ranking results achieved with the proposed CSVNAAPWAAO and CSVNAAPOWAAO for the demonstrative example presented in Section 5 against the existing outcomes based on the SVN weighted Bonferroni power average aggregation operator (SVNWBPWAAO). From the Table 2, we observe that the top-ranked

TABLE 2. Comparison of the existing operators with the proposed operators

Method	Operator	Ranking	Best
Guiwu Wei and Zuopeng Zhang [29]	SVNWBPWAAO	$\ddot{\breve{S}}_2 > \ddot{\breve{S}}_4 > \ddot{\breve{S}}_3 > \ddot{\breve{S}}_5 > \ddot{\breve{S}}_1$	$\ddot{\breve{S}}_2$
Proposed	CSVNAAPWAAO	$\ddot{\breve{S}}_2 > \ddot{\breve{S}}_4 > \ddot{\breve{S}}_3 > \ddot{\breve{S}}_5 > \ddot{\breve{S}}_1$	$\ddot{\breve{S}}_2$

alternatives for the proposed operators are the same as those for the existing operators, while the least favorable alternatives remain unchanged for the CSVNAAPWAAO. However, the proposed operators, which incorporate CLs into SVNs, provide a more refined ranking of the alternatives, reflecting the decision maker's subjective familiarity. Additionally, the comparison is visually illustrated in Figure 1.

7. **Conclusion**

This paper presents the development of confidence SVN Aczel-Alsina power average AO, specifically CSVNAAPWAAO and CSVNAAPOWAAO. The fundamental properties of these

Figure 1. Graphical comparison of SVNWBPAAO and CSVNAAPWAAO

proposed AO have also been proven. An important feature of these operators is that they take into account not only the assessed arguments of the decision experts but also the experts' confidence in their assessments. In addition, we developed a SVNMADM method employing the proposed operators and applied it to a real world problem of choosing a supplier system based on four attributes, thereby confirming the validity of our proposal. We also compared our findings with the existing SVNWBPAAO and CSVNAAPWAAOs to emphasize the potential of the proposed operators. Additionally, the results were presented graphically for enhanced clarity.

In the future, this proposed concept can be applied to develop SVN geometric AO and to create a variety of AO for SVNs by integrating probabilistic information, and additional factors.

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Received: June 22, 2024. Accepted: August 15, 2024