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The linear 2-refined neutrosophic differential equations

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Abstract: In this paper, we studied the linear 2-refined neutrosophic differential equations and defined the homogeneous and non-homogeneous 2-refined neutrosophic differential equations. Also to presenting 2-refined neutrosophic differential equation of Bernoulli, which turns into a linear equation and thus facilitates its solution. Apart from talking about how to solve these equations and giving enough examples to support it.

Keywords: integral; differential equation; indeterminacy; Bernoulli; neutrosophic.

1. Introduction and Preliminaries

To describe a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction, Smarandache suggested the neutrosophic Logic as an alternative to the current logics. Smarandache made refined neutrosophic numbers available in the following form: $(a, b_1I_1, b_2I_2, \dots, b_nI_n)$ where $a, b_1, b_2, \dots, b_n \in R \text{ or } C$ [1]

Agboola introduced the concept of refined neutrosophic algebraic structures [2]. Also, the refined neutrosophic rings I was studied in paper [3], where it assumed that I splits into two indeterminacies I_1 [contradiction (true (T) and false (F))] and I_2 [ignorance (true (T) or false (F))]. It then follows logically that: [3]

$$I_1I_1 = {I_1}^2 = I_1 \qquad (1)$$

$$I_2I_2 = {I_2}^2 = I_2 \qquad (2)$$

$$I_1I_2 = I_2I_1 = I_1 \qquad (3)$$

In addition, there are many papers presenting studies on refined neutrosophic numbers [4-5-6-7-8-12-13-14-15] and Mehmet Celik and Ahmed Hatip presented a study on the refined ah-isometry and its applications in refined neutrosophic surfaces. Smarandache discussed neutrosophic indefinite integral (Refined Indeterminacy) [11]

Alhasan also presented several papers on calculus, in which he discussed neutrosophic definite and indefinite integrals. He also presented the most important applications of definite integrals in neutrosophic logic [9-10].

2. Main Discussion

2.1 The homogeneous linear 2-refined neutrosophic differential equations

Definition 1

The general form of the homogeneous linear 2-refined neutrosophic differential equation is given by:

$$\acute{y} + f(x, I_1, I_2)y = 0$$

Whereas $f: D_f \subseteq R \to R_f \cup \{I_1, I_2\}$, while I_1, I_2 are indeterminacy.

$$\acute{y} = -f(x, I_1, I_2)y$$

$$\frac{\acute{y}}{y} = -f(x, I_1, I_2)$$

By integrating the two side, we get:

$$ln\left(\frac{y}{C}\right) = \int -f(x, I_1, I_2)\, dx$$

$$\frac{y}{C} = e^{\int -f(x,I_1,I_2)dx}$$

$$y = C\, e^{\int -f(x,I_1,I_2)dx}$$

Where $C = a_0 + b_0 I_1 + c_0 I_2$ and a_0, b_0, c_0 are real numbers, while I_1, I_2 are indeterminacy.

Example 1

Find the general solution of the following linear 2-refined neutrosophic differential equation:

$$\acute{y} - ((1 - 5I_1 + 2I_2)x + 4 + 7I_2)^3 y = 0$$

Solution:

$$\acute{y} = ((1 - 5I_1 + 2I_2)x + 4 + 7I_2)^3 y$$

$$\frac{\acute{y}}{y} = ((1 - 5I_1 + 2I_2)x + 4 + 7I_2)^3$$

By integrating the two side, we get:

$$ln\left(\frac{y}{C}\right) = \int ((1 - 5I_1 + 2I_2)x + 4 + 7I_2)^3\, dx$$

$$\frac{y}{C} = e^{\int ((1-5I_1+2I_2)x+4+7I_2)^3 dx}$$

$$y = C\, e^{\left(\frac{1}{1-5I_1+2I_2}\right)\frac{((1-5I_1+2I_2)x+4+7I_2)^4}{4}}$$

$$y = C\, e^{\left(1-\frac{5}{6}I_1-\frac{2}{3}I_2\right)\frac{((1-5I_1+2I_2)x+4+7I_2)^4}{4}}$$

Where $C = a_0 + b_0 I_1 + c_0 I_2$ and a_0, b_0, c_0 are real numbers.

Example 2

Find the general solution of the following linear 2-refined neutrosophic differential equation:

$$\acute{y} + sin((4 + 7I_1 - I_2)x - 1 + I_1 - I_2)y = 0$$

Solution:

$$\acute{y} = -sin((4 + 7I_1 - I_2)x - 1 + I_1 - I_2)y$$

$$\frac{\acute{y}}{y} = -sin((4 + 7I_1 - I_2)x - 1 + I_1 - I_2)$$

By integrating the two side, we get:

$$ln\left(\frac{y}{C}\right) = \int -sin((4 + 7I_1 - I_2)x - 1 + I_1 - I_2)\, dx$$

$$\frac{y}{C} = e^{\int -sin((4+7I_1-I_2)x-1+I_1-I_2)dx}$$

$$y = C\, e^{\left(\frac{1}{4+7I_1-I_2}\right)cos((4+7I_1-I_2)x-1+I_1-I_2)}$$

$$y = C\, e^{\left(\frac{1}{4}-\frac{7}{30}I_1+\frac{1}{12}I_2\right)cos((4+7I_1-I_2)x-1+I_1-I_2)}$$

Where $C = a_0 + b_0 I_1 + c_0 I_2$ and a_0, b_0, c_0 are real numbers.

2.2 The non-homogeneous linear 2-refined neutrosophic differential equation

Definition 2

The general form of the non-homogeneous linear 2-refined neutrosophic differential equation is given as:

$$\acute{y} + f(x, I_1, I_2)y = q(x, I_1, I_2) \qquad (1)$$

Where$f: D_f \subseteq R \to R_f \cup \{I_1, I_2\}$, $q: D_q \subseteq R \to R_q \cup \{I_1, I_2\}$, and I_1, I_2 are indeterminacy.

Solution steps:

1) The complement factor of the equation (1) is:

$$\mu(x) = e^{\int f(x,I_1,I_2)dx}$$

2) By multiplying equation (1) by the complement factor, we get:

$$\mu(x)\, \acute{y} + f(x, I_1, I_2)\, \mu(x)y = q(x, I_1, I_2)\, \mu(x)$$

$$\mu(x)\, \acute{y} + f(x, I_1, I_2)e^{\int f(x,I_1,I_2)dx}\, y = q(x, I_1, I_2)\, e^{\int f(x,I_1,I_2)dx}$$

$$\left(y\, \mu(x)\right)' = q(x, I_1, I_2)\, e^{\int f(x,I_1,I_2)dx}$$

By integrating the two side, we get:

$$y = \frac{1}{\mu(x)}\left(\int q(x, I_1, I_2)\, e^{\int f(x,I_1,I_2)dx}\, dx + C\right)$$

Where $C = a_0 + b_0 I_1 + c_0 I_2$ and a_0, b_0, c_0 are real numbers, while I_1, I_2 are indeterminacy.

Example 3

Find the general solution of the following non-homogeneous linear 2-refined neutrosophic differential equation:

$$\acute{y} + \frac{(3 + I_1 - I_2)x - 7I_2}{\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}}\, y = \sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}$$

Solution:

$$f(x, I_1, I_2) = \frac{(3 + I_1 - I_2)x - 7I_2}{\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}} \ , q(x, I_1, I_2) = \sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}$$

The complement factor of the equation is:

$$\mu(x) = e^{\int \frac{(3+I_1-I_2)x-7I_2}{\sqrt{(3+I_1-I_2)x^2-14xI_2}} dx} = \frac{1}{2}\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}$$

By multiplying equation (*) by the complement factor, we get:

$$\mu(x)\, \acute{y} + \frac{(3 + I_1 - I_2)x - 7I_2}{\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}}\, \mu(x) y = \sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}\, \mu(x)$$

$$\mu(x)\, \acute{y} + \frac{(3 + I_1 - I_2)x - 7I_2}{\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}} \left(\frac{1}{2}\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}\right) y = \sqrt{(3 + I_1 - I_2)x^2 - 14xI_2} \left(\frac{1}{2}\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}\right)$$

$$\big(y\, \mu(x)\big)' = \frac{1}{2}\big((3 + I_1 - I_2)x^2 - 14xI_2\big)$$

By integrating the two side, we get:

$$y\, \mu(x) = \frac{1}{2}\big((3 + I_1 - I_2)x^2 - 14xI_2\big)$$

$$y = \frac{1}{\frac{1}{2}\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}} \left(\int \frac{1}{2}\big((3 + I_1 - I_2)x^2 - 14xI_2\big)\, dx\right)$$

$$y = \frac{2}{\sqrt{(3 + I_1 - I_2)x^2 - 14xI_2}} \left(\left(\frac{1}{2} + \frac{1}{6}I_1 - \frac{1}{6}I_2\right)x^3 - 7x^2 I_2 + C\right)$$

Where $C = a_0 + b_0 I_1 + c_0 I_2$ and a_0, b_0, c_0 are real numbers.

3. The 2-refined neutrosophic differential equations that translate into linear

3.1 The Bernoulli's 2-refined neutrosophic differential equation

Definition 2

We call the equation that take the form:

$$\acute{y} + f(x, I_1, I_2)\, y = q(x, I_1, I_2)\, y^n \ ; \ n \neq 0, n \neq 1$$

The 2-refined neutrosophic differential equation of Bernoulli. Let's follow the following steps to solve it:

1) we'll first divide the differential equation by y^n to get:

$$\frac{\acute{y}}{y^n}+\frac{f(x,I_1,I_2)\,y}{y^n}=\frac{q(x,I_1,I_2)\,y^n}{y^n}$$

$$y^{-n}\,\acute{y}+f(x,I_1,I_2)\,y^{1-n}\;=q(x,I_1,I_2)\quad(*)$$

2) Assume that: $w=\;y^{1-n}\quad\Rightarrow\quad\acute{w}=(1-n)\,y^{-n}\acute{y}$

$$\Rightarrow\quad y^{-n}\acute{y}=\frac{1}{1-n}\acute{w}$$

3) by substituting into equation $(*)$, we get:

$$\frac{1}{1-n}\acute{w}+f(x,I_1,I_2)\,w=q(x,I_1,I_2)$$

$$\acute{w}+(1-n)f(x,I_1,I_2)\,w=(1-n)q(x,I_1,I_2)\quad(**)$$

4) The complement factor of the equation $(**)$ is:

$$\mu(x)=e^{(1-n)\int f(x,I_1,I_2)dx}$$

5) By multiplying equation $(**)$by the complement factor, we get:

$$\acute{w}\,\mu(x)+(1-n)f(x,I_1,I_2)\,\mu(x)\,w\;=(1-n)\,q(x,I_1,I_2)\,\mu(x)$$

$$\acute{w}\,\mu(x)+(1-n)f(x,I_1,I_2)\,e^{(1-n)\int f(x,I_1,I_2)dx}\,w\;=(1-n)\,q(x,I_1,I_2)e^{(1-n)\int f(x,I_1,I_2)dx}$$

$$\left(w\,\mu(x)\right)'=(1-n)\,q(x,I_1,I_2)e^{(1-n)\int f(x,I_1,I_2)dx}$$

By integrating the two side, we get:

$$w\;=\frac{1}{\mu(x)}\left(\int(1-n)\,q(x,I_1,I_2)e^{(1-n)\int f(x,I_1,I_2)dx}\,dx+C\right)$$

6) Back to the primary variable y, we get:

$$y^{1-n}=\frac{1}{\mu(x)}\left(\int(1-n)\,q(x,I_1,I_2)e^{(1-n)\int f(x,I_1,I_2)dx}\,dx+C\right)$$

$$\text{Or}\quad y^{n-1}=\frac{e^{(1-n)\int f(x,I_1,I_2)dx}}{\int(1-n)\,q(x,I_1,I_2)e^{(1-n)\int f(x,I_1,I_2)dx}\,dx+C}$$

Where $C=a_0+b_0I_1+c_0I_2$ and a_0,b_0,c_0 are real numbers, while I_1,I_2 are indeterminacy.

Example 4

Find the general solution of the following Bernoulli's 2-refined neutrosophic differential equation:

$$\acute{y} + csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, y = e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)} \sqrt{y}$$

Solution:

$$\frac{\acute{y}}{\sqrt{y}} + csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, \frac{y}{\sqrt{y}} = e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}$$

$$y^{-1/2}\, \acute{y} + csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, y^{1/2} = e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)} \quad (*)$$

$$w = y^{1/2} \quad \Rightarrow \quad \acute{w} = \frac{1}{2} y^{-1/2} \acute{y}$$

$$\Rightarrow \; y^{-1/2}\, \acute{y} = 2\acute{w}$$

By substitution in (*), we get:

$$2\acute{w} + csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, w = e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)} \quad (**)$$

The complement factor is:

$$\mu(x) = e^{(1-n)\int f(x, I_1, I_2) dx}$$

$$\mu(x) = e^{\int \frac{1}{2} csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)\, cot\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right) dx}$$

$$= e^{-\frac{1}{2}\left(\frac{1}{9+11I_1+13I_2}\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}$$

$$= e^{-\frac{1}{2}\left(\frac{1}{9} - \frac{1}{66}I_1 - \frac{13}{198}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}$$

By multiplying equation (**) by the complement factor, we get:

$$2\acute{w}\, \mu(x) + csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, \mu(x)\, w = e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}\, \mu(x)$$

$$\acute{w}\, \mu(x) + \frac{1}{2} csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, \mu(x)\, w = \frac{1}{2} e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}\, \mu(x)$$

$$\acute{w}\, \mu(x) + \frac{1}{2} csc\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, cot\big((9 + 11I_1 + 13I_2)x - 11 + 2I_1 - 4I_2\big)\, e^{-\frac{1}{2}\left(\frac{1}{9} - \frac{1}{66}I_1 - \frac{13}{198}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}\, w = \frac{1}{2} e^{\left(\frac{1}{18} - \frac{1}{132}I_1 - \frac{13}{396}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)} e^{-\frac{1}{2}\left(\frac{1}{9} - \frac{1}{66}I_1 - \frac{13}{198}I_2\right) csc\left((9+11I_1+13I_2)x - 11 + 2I_1 - 4I_2\right)}$$

$$\acute{w}\,\mu(x) + \frac{1}{2}csc\big((9+11I_1+13I_2)x-11+2I_1-4I_2\big)\,cot\big((9+11I_1+13I_2)x-11+2I_1 - 4I_2\big)\,e^{-\frac{1}{2}\left(\frac{1}{9}-\frac{1}{66}I_1-\frac{13}{198}I_2\right)csc\left((9+11I_1+13I_2)x-11+2I_1-4I_2\right)}\,w = \frac{1}{2}e^{\left(\frac{1}{18}-\frac{1}{132}I_1-\frac{13}{396}I_2\right)csc\left((9+11I_1+13I_2)x-11+2I_1-4I_2\right)}e^{-\left(\frac{1}{18}-\frac{1}{132}I_1-\frac{13}{396}I_2\right)csc\left((9+11I_1+13I_2)x-11+2I_1-4I_2\right)}$$

$$\acute{w}\,\mu(x) + \frac{1}{2}csc\big((9+11I_1+13I_2)x-11+2I_1-4I_2\big)\,cot\big((9+11I_1+13I_2)x-11+2I_1 - 4I_2\big)\,e^{-\frac{1}{2}\left(\frac{1}{9}-\frac{1}{66}I_1+\frac{13}{198}I_2\right)csc\left((9+11I_1+13I_2)x-11+2I_1-4I_2\right)}\,w = \frac{1}{2}$$

$$\big(w\,\mu(x)\big)' = \frac{1}{2}$$

By integrating the two side, we get:

$$w\,\mu(x) = \int \frac{1}{2}dx$$

$$w = \frac{1}{\mu(x)}\left(\frac{1}{2}x + C\right)$$

Back to the primary variable y, we get:

$$y^{1/2} = \frac{1}{\mu(x)}\left(\frac{1}{2}x + C\right)$$

$$y^{\frac{1}{2}} = \frac{1}{e^{-\left(\frac{1}{18}-\frac{1}{132}I_1+\frac{13}{396}I_2\right)csc\left((9+11I_1+13I_2)x-11+2I_1-4I_2\right)}}\left(\frac{1}{2}x + C\right)$$

$$\Rightarrow \quad y^{\frac{1}{2}} = \left(\frac{1}{2}x + C\right)e^{\left(\frac{1}{18}-\frac{1}{132}I_1+\frac{13}{396}I_2\right)csc\left((9+11I_1+13I_2)x-11+2I_1-4I_2\right)}$$

Where $C = a_0 + b_0I_1 + c_0I_2$ and a_0, b_0, c_0 are real numbers.

3.2 The 2-refined neutrosophic differential equations takes the following form

$$\acute{f}(y)\frac{dy}{dx} + m(x,I_1,I_2)f(y) = q(x,I_1,I_2) \quad (3)$$

To solve this equation, we follow the following steps:

1) Assume that: $z = f(y) \quad \Rightarrow \quad \acute{z} = (\acute{f}(y)\frac{dy}{dx}$

By substitution into equation (3), we get:

$$\acute{z} + m(x,I_1,I_2)z = q(x,I_1,I_2) \qquad (4)$$

2) The complement factor of the equation (4) is:

$$\mu(x) = e^{\int m(x,I_1,I_2)dx}$$

3) By multiplying equation (4) by the complement factor, we get:

$$\acute{z}\,\mu(x) + m(x,I_1,I_2)\mu(x)\,z = q(x,I_1,I_2)\,\mu(x)$$

$$\acute{z}\,\mu(x) + m(x,I_1,I_2)e^{\int m(x,I_1,I_2)dx}\,\mathrm{z} = q(x,I_1,I_2)\,e^{\int m(x,I_1,I_2)dx}$$

$$\left(z\,\mu(x)\right)' = q(x, I_1, I_2)\; e^{\int m(x,I_1,I_2)dx}$$

$$\left(z\,\mu(x)\right)' = q(x, I_1, I_2)\; e^{\int m(x,I_1,I_2)dx}$$

By integrating the two side, we get:

$$z = \frac{1}{\mu(x)}\left(\int q(x, I_1, I_2)\; e^{\int m(x,I_1,I_2)dx} + C\right)$$

4) Back to the primary variable y, we get:

$$f(y) = \frac{1}{\mu(x)}\left(\int q(x, I_1, I_2)\; e^{\int m(x,I_1,I_2)dx} + C\right)$$

Where $C = a_0 + b_0 I_1 + c_0 I_2$ and a_0, b_0, c_0 are real numbers.

Example 5

Find the general solution of the following neutrosophic differential equation:

$$\cos y\frac{dy}{dx} + cot(x + 1 + 2I_1 - 3I_2)\; \sin y = cos(x + 1 + 2I_1 - 3I_2)\;\; (*)$$

Solution:

$$z = \sin y \quad \Longrightarrow \quad \acute{z} = cosy\,\frac{dy}{dx}$$

By substitution in (*), we get:

$$\acute{z} + cot(x + 1 + 2I_1 - 3I_2)\, z = cos(x + 1 + 2I_1 - 3I_2) \qquad (\acute{*})$$

The complement factor is:

$$\mu(x) = e^{\int cot(x+1+2I_1-3I_2)\,dx}$$

$$= e^{ln\,(sin(x+1+2I_1-3I_2))} = sin(x + 1 + 2I_1 - 3I_2)$$

By multiplying equation $(\acute{*})$ by the complement factor, we get:

$$\acute{z}\,\mu(x) + cot(x + 1 + 2I_1 - 3I_2)\,\mu(x)\,z = \mu(x)\,cos(x + 1 + 2I_1 - 3I_2)$$

$$\acute{z}\,\mu(x) + cot(x + 1 + 2I_1 - 3I_2)\,sin(x + 1 + 2I_1 - 3I_2)\,z = cos(x + 1 + 2I_1 - 3I_2)\,sin(x + 1 + 2I_1 - 3I_2)$$

$$\acute{z}\,\mu(x) + cos(x + 1 + 2I_1 - 3I_2)\,z = cos(x + 1 + 2I_1 - 3I_2)\,sin(x + 1 + 2I_1 - 3I_2)$$

$$\left(z\,\mu(x)\right)' = cos(x + 1 + 2I_1 - 3I_2)\,sin(x + 1 + 2I_1 - 3I_2)$$

By integrating the two side, we get:

$$z\,\mu(x) = \int cos(x + 1 + 2I_1 - 3I_2)\,sin(x + 1 + 2I_1 - 3I_2)dx$$

$$z\,\mu(x) = \int \frac{1}{2} sin(2x + 2 + 4I_1 - 6I_2)dx$$

$$z\,\mu(x) = \frac{-1}{4} cos(2x + 2 + 4I_1 - 6I_2) + C$$

$$z = \frac{1}{sin(x + 1 + 2I_1 - 3I_2)}\left(\frac{-1}{4}cos(2x + 2 + 4I_1 - 6I_2) + C\right)$$

Back to the primary variable y, we get:

$$sin\, y = \frac{1}{sin(x + 1 + 2I_1 - 3I_2)}\left(\frac{-1}{4}cos(2x + 2 + 4I_1 - 6I_2) + C\right)$$

Where $C = a_0 + b_0I_1 + c_0I_2$ and a_0, b_0, c_0 are real numbers.

4. Conclusions

This work is regard as a seminal study in the field of 2-refined neutrosophic differential equations. It included study types of the 2-refined neutrosophic differential equations. In addition, we discussed 2-refined neutrosophic differential equations that transform to linear. The most significant of all was the 2-refined neutrosophic differential equations Bernoulli.

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