Neutrosophic Sets and Systems, Vol. 75, 2025
University of New Mexico

Pairwise Approach to Neutrosophic Soft Bi-Topological Spaces: Exploring Neutrosophic Soft Open Sets and Their Real-Life Applications

Maha Mohammed Saeed[1], Samer Al Ghour[2*], Arif Mehmood[3*], Wahid Rehman[4], Fabio Souto de Azevedo[4], R. Seethalakshi[5], Aamir Abbas[3], M. Kaviyarasu[6*]

[1]Department of Mathematics, Faculty of Sciences, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia; (mmmohammed@kau.edu.sa)

[2*]Department of Mathematics and Statistics, Jordan University of Science and Technology, Irbid, 22110, Jordan; (algore@just.edu.jo)

[3*]Institute of Numerical Sciences, Department of Mathematics Gomal University Dera Ismail Khan, KPK, Pakistan; (mehdaniyal@gmail.com); (abbasaamir113@gmail.com)

[4]Federal University of Rio Grande do Sul Campus Do Vale, Brazil; (wahidkhattak601@gmail.com); (fabio.azevedo@ufrgs.br);

[5]Department of Mathematics (H&R), Rajalakshmi Institute of Technology Kuthambakkam, Chennai 600124, India; (seethamaths79@gmail.com)

[6*]Department of Mathematics, Vel Tech Rangarajan Dr. Sagunthala R &D Institute of Science and Technology, Avadi, Chennai 600062, Tamil Nadu, India; (kavitamilm@gmail.com)

[*]**Corresponding authors email address**; (algore@just.edu.jo);(mehdaniyal@gmail.com)

Abstract: In this article, eight new definitions are introduced in the context of neutrosophic soft bi-topological spaces with respect to the soft points of the space. Among these new definitions, one important concept known as the soft b^{**} open set is selected for a pairwise discussion of different structures. These structures include separation axioms, subspaces, the connections between separation axioms and closures, results concerning interiors and closures, the product of separation axioms, and their relationships with neutrosophic soft coordinate spaces regarding soft points. Finally, the most significant structure known as entropy is discussed, and based on this structure, an example related to engineering is presented. In this example, to achieve effective risk supervision in the field of engineering, certain risks are classified along with specific parameters, and these risks are evaluated by a team of expert engineers. Mehmood et al. [40] proposed a new approach to generalized neutrosophic soft sets in neutrosophic soft topology concerning the soft points of the space and discussed several structures related to this new approach. The references by Ozturk et al. [32-35] led us to this present work, with [32] serving as a significant source of motivation. In this particular work, the author defined neutrosophic soft topological space in an exceptionally novel way.

Keywords: Neutrosophic soft set, neutrosophic soft bi-topology, neutrosophic soft open set, neutrosophic soft b^{**} open sets, neutrosophic soft separation axioms and neutrosophic soft entropy.

1. Introduction

Zadeh [1] proposed afuzzy set (FS) to solve ambiguous information. After that, it has been successfully used in different areas [2-7]. Atanassov [8] extended FS to the intuitionistic fuzzy set (IFS) which is very effective to deal with vagueness of the information. The IFS has the ability to express fuzziness and uncertainty of practical situations which each element concludes membership degree, non-membership degree, and hesitation degree and they are presented by exact numbers. The sum of these three functions is number 1. In general, the degrees may be not exact numbers but interval values. Then the theory of IFS was extended to interval-valued IFS [9]. With the development of the theory of FS and its extension, they have been handled many uncertainties in different real-life problems. But, there are many phenomena cannot be dealt by the FS and its extension. For example, when we ask an expert about a complex statement, he or she may not give the exact answer of the problem and say that the possibility that the statement is true is 0.6, that is false is 0.5 and the degree that he or she is not sure is 0.3. This situation cannot be expressed by FS and IFS, and some new set is needed to express the condition. Neutrosophic set (NS) was first proposed by Smarandache [10, 11] from philosophical point of view. A NS is a set that each element has truth membership degree, indeterminacy membership degree and falsity membership degree. As we known, it was difficult to apply the NS in real science and engineering fields. Smarandache [50] made extension of soft set to hyper soft set, and then to plithogenic hyper soft set. The author generalized the soft set to the hyper soft set by transforming the function F into a multi-attribute function and also introduced the hybrids of crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic hyper-soft Set. Smarandache [51] generalized the soft set to the hyper-soft set by transforming the function F into a multi-attribute function. and in continuation, introduced the hybrids of crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, and Plithogenic Hyper-soft Set Wang et al. [12, 13] proposed single valued neutrosophic set

(SVNS) and interval valued neutrosophic set (IVNS). SVNS and IVNS are the subclasses of simplified neutrosophic set which was proposed by Ye [14] and he presented the operators and relationships of the sets and gave their different properties. Therefore, some new results of SNS appeared [15, 16].

Molodtsov developed soft set theory in a fundamentally different perspective. The application of this theory can be used for meaningfully interpreting real life problems in pure and applied sciences involving imprecise data. Current studies shows that ambiguities in data mining problems can also be solved using soft set theory techniques. The soft set theory could be used to interrogate and extend the idea of probability, fuzzy set, rough set and intuitionistic fuzzy set further. The disadvantage of lack of parameterization tool related to the concepts mentioned above gave a higher realm to soft set theory. In short, unlimited nature of approximate description is the greatest advantage of soft set theory. Molodtsov [17] published the paper titled "Soft set theory: first results", which is considered as the origin of theory of soft sets. Apart from the basic notions of the theory, some of its possible applications and some problems of the future research directions are also discussed in this paper. This theory was further solidified by Maji et al. [18] by defining some fundamentals of the theory such as equality of two soft sets, subset and super set of a soft set etc. As continuation of these ideas, many extensions, hybridizations and extensions were put forward by many authors, some of them are the following: Maji et al. [19] introduced fuzzy soft sets, Wang et al. [20] introduced hesitant fuzzy soft sets and Pei and Miao [21] explore the relationship between fuzzy soft sets and classical information systems. The theory of soft sets is still developing very rapidly both in theoretical as well as application perspectives.

John [22] introduced a soft structure over a set to make a certain discretization of such fundamental mathematical concepts with effectively continuous nature. The benefit is that it provided new tools for the use of the technology of mathematical analysis in real applications catching uncertainty or imperfect data. John [23] studied qualitative properties of certain objects, called topological spaces that are invariant under certain kinds of transformations, called continuous maps. The author also addressed some generalized structures. John [24] have had a quick look into some structures. These are hybridizations including Fuzzy sets, Intuitionistic fuzzy sets, Hesitant fuzzy sets, Rough sets etc.

Al-shami et al. [25] continuously worked on menger spaces and defined a weak type of soft menger spaces, namely, nearly soft menger spaces and gave their complete description using soft s-regular open covers and proved that they coincided with soft menger spaces in the class of soft regular spaces. Also, the authors studied the role of enriched and soft regular spaces in preserving nearly soft meagerness between soft topological spaces and their parametric topological spaces. Finally, the authors established some properties of nearly soft menger spaces with respect to hereditary and topological properties and product spaces. Al-shami et al. [26] sorted out the weakest conditions that preserve some topologically inspired properties and introduced the concept of an infra soft topological spaces which is a collection of subsets that extend the concept of soft topology by dispensing with the postulate that the collection is closed under arbitrary union. The authors studied the basic concepts of infra soft topological spaces such as infra soft open and infra soft closed sets, infra soft interior and infra soft closure operators, and infra soft limit and infra soft boundary points of a soft set. The authors devised some techniques to generate infra soft topologies and explored the concept of continuity between infra soft topological spaces and determined the conditions under which the continuity is preserved between infra soft topological space and its parametric infra topological spaces. Al-shami et al. [27] discussed weak forms of soft separation axioms and fixed soft points. The authors discussed soft separation axioms and other separation axioms with respect generalized open sets. Al-shami et al. [28] introduced the concept of infra soft connected and infra soft locally connected spaces and discussed the behaviors of infra soft connected and infra soft locally connected spaces under infra soft homeomorphism maps and a finite product of soft spaces. The authors discussed component of a soft point and established its main properties. Al-shami [29] redefined the concept of soft mappings to be convenient for studying the topological concepts and notions in different soft structures and introduced the concepts of open, closed, and homeomorphism mappings in the content of infra soft topology. The authors established main properties and investigated the transmission of these concepts between infra soft topology and its parametric infra topologies and defined a quotient infra soft topology and infra soft quotient mappings and studied their main properties with the aid of illustrative examples.

Al-shami [30] discussed infra soft compact and infra soft Lindelof spaces and described them using a family of infra soft closed sets and displayed their main properties. The author focused on studying the “transmission” of these concepts between infra soft topology and classical infra topology which helped us to discover the behaviors of these concepts in infra soft topology using their counterparts in classical infra topology and vice versa. Among the obtained results, these concepts are closed under infra soft homeomorphisms and finite product of soft spaces. Finally, the author introduced the concept of fixed soft points and reveal main characterizations, especially those induced from infra soft compact spaces. Asad et al. [31] originated the notion of soft bi-operators. Mehmood et al. [41] studied and discussed new operations of union, intersection, and complement with the help of vague soft sets in a new way. On the basis of these operations, vague soft topology is re-defined. Pairwise vague soft open sets and pairwise vague soft closed sets are also re-defined in vague soft bitopological structures (VSBTS). Moreover, generalized vague soft open sets are introduced in VSBTS concerning soft points of the space. On the basis of generalized vague soft open sets, separation axioms are also introduced. In continuation, these separations axioms are engaged with other important results in VSBTS. Mehmood et al. [42] installed a new concept of vague soft bi-topological space which is based on generalized vague soft open set and is known as vague soft β open set and on the basis of this new concept few structures are re-generated in vague soft bi-topological space with respect to soft points of the spaces. Abobala [48] discussed maximal and minimal ideals of n-refined neutrosophic rings.

Vadivel et al. [52] introduced a new types of δ-open sets and δ-closed sets in neutrosophic topological spaces which are a stronger form. Also, the notion of neutrosophic δ mappings were introduced and also discuss their relationship between near mappings in neutrosophic topological spaces. Moreover, the authors investigated some of their basic properties and examples in neutrosophic topological spaces.

Ozturk et al. [32] pioneers of new operations on neutrosophic soft sets. These operations are union and intersections and discussed basic results. Ozturk et al. [33] unveiled the concept of neutrosophic soft mapping, neutrosophic soft open mapping and neutrosophic soft homeomorphism on the basis of operation defined in [32]. Some results are secured with best understandable examples. Gunduz et al. [34] introduced separation axioms in neutrosophic soft topological spaces with respect to soft points. Ozturk, Taha Yasin [35] introduced new concepts in neutrosophic soft topological spaces. These concepts are boundary, dense set and neutrosophic soft basis. In addition, the concept of soft subspace on neutrosophic soft topological spaces. Some interesting results are addressed with respect to soft points. Some complicated results are secured with best examples. Acikgoz and Esenbel [53] introduced the concepts of neutrosophic soft δ−interior, neutrosophic soft quasi coincidence, neutrosophic soft q-neighborhood, neutrosophic regular open soft set, neutrosophic soft δ−closure, neutrosophic soft θ−closure, neutrosophic semi open soft set and established that the set of all neutrosophic soft δ−open sets is also a neutrosophic soft topology, which is called the neutrosophic soft δ−topology and the authors obtained equivalent forms of neutrosophic soft δ−continuity. Moreover, the notions of neutrosophic soft δ−compactness and neutrosophic soft locally δ−compactness are defined and their basic properties under neutrosophic soft δ−continuous mappings are investigated.

Mehmood et al. [36] introduced a new approach to neutrosophic soft bi topological spaces relative to soft points of the spaces. Mehmood et al. [43] introduced neutrosophic soft $**_b$ open set with the help of neutrosophic soft α open set and neutrosophic soft β open set and with the help of this new definition some neutrosophic soft separation axioms and neutrosophic soft other separation axioms are addressed and in continuation, soft countability and its engagements with other neutrosophic soft results are also addressed.

Deli and Broumi [37] installed concept of a relation on neutrosophic soft sets which permit to compose two neutrosophic soft sets. It is devised to derive advantageous information through the composition of two neutrosophic soft sets. Then, the authors scanned symmetric, transitive and reflexive neutrosophic soft relations and many related concepts such as equivalent neutrosophic soft set relation, partition of neutrosophic soft sets, equivalence classes, quotient neutrosophic soft sets, neutrosophic soft composition are given and their propositions are discussed. Finally, a decision making technique on neutrosophic soft sets is presented. Bera and Mahapatra [38] installed the new structure to the world of mathematics. This structure is known as neutrosophic soft topology and in addition the authors discussed all the fundamentals. Maji [39] introduced concept neutrosophic soft set. Some definitions and operations are introduced and established some properties. Mehmood et al. [40] launched a new way to attempt generalized neutrosophic soft sets in neutrosophic soft topology concerning neutrosophic soft points of the space and discussed few structures concerning these new approached. Hayat et al. [57] developed a framework based on soft sets, TOPSIS and Shannon entropy to identify customer preferences. TOPSIS provides integrated evaluations to obtain the best concept for two customers, acceptable for both, satisfactory for both, and vise versa. Hayat et al. A simple graph's underlying subgraphs (regular subgraphs, irregular subgraphs, cycles, and trees) were used by Hayat et al. [58] to characterize type 2 soft graphs. The authors then presented regular type 2 soft graphs, irregular type 2 soft graphs, and type 2 soft trees. Moreover, type 2 soft cycles, type 2 soft cut-nodes, and type 2 soft bridges were introduced by the authors. In order to illustrate these new ideas, several operations on type 2 soft trees are shown, along with various instances. Following references Ozturk et al. [32-35] leaded me to this present work.

In section 1, the basic definitions are given which are necessary for the up-coming sections. These basic definitions are neutrosophic soft set, complement of neutrosophic soft set, equality of neutrosophic soft sets, union and intersection of neutrosophic soft sets, absolute neutrosophic soft set, neutrosophic soft topology and neutrosophic soft closer. In section 2, pair-wise neutrosophic soft open sets in neutrosophic soft bi-topological space and for the supportment of this structure an example is given. Intersection of two neutrosophic soft topological space, the non-validity of union of two neutrosophic soft topological spaces is discussed with an example. In this section, new types of open and close sets have been introduced in neutrosophic soft bi topological spaces. Eight new neutrosophic soft open sets are introduced. Among these, neutrosophic soft b^{**} is chosen to generate the results pairwisly in neutrosophic soft bi-topological space with respect to soft points. These results are different neutrosophic soft separations axioms and their interconnection among themselves.

In this section 3, neutrosophic soft limit point, neutrosophic soft interior point, few results on the basis of these definitions are discussed. These results are discussed terms of neutrosophic soft interior and neutrosophic soft closures with respect to soft points. In this section 4, some spaces are discussed in terms of product with respect to soft points of the spaces. Product of neutrosophic soft T_0 spaces, neutrosophic soft T_1 spaces, neutrosophic soft T_2 spaces and their engagement with neutro$s$ophic soft coordinate spaces with respect to soft points are studied under soft b^{**} open sets in neutrosophic soft bi-topological space. Finally the most important structure known as entropy is discussed and on the basis of this structures an example is given which is related to engineering which goes like this, in order to obtain an effective risk supervision in the field of engineering, certain risks are classified along with some parameters and these risks are evaluated by the team of experts (engineers). In the final section some concluding comments are summarized and future planning is given.

2. Preliminaries

In this section the basic definitions are given which are necessary for the up-coming sections. These basic definitions are neutrosophic soft set, complement of neutrosophic soft set, equality of neutrosophic soft sets, union and intersection of neutrosophic soft sets, absolute neutrosophic soft set, neutrosophic soft topology and neutrosophic soft closer.

Definition 1.1[18] Let $\mathcal{X}$ be a universal set and $\mathfrak{K}$ be a set of parameters. Let $P(\mathcal{X})$ denote the set of all neutrosophic sets of $\mathcal{X}$ then a neutrosophic soft set $(\tilde{F},\mathfrak{K})$ o v er $\mathcal{X}$ is a set defined by a set of valued function representing a mapping $\tilde{F}:\mathfrak{K}\to P(\mathcal{X})$ where $\tilde{F}$

is called approximate function of the neutrosophic soft set $(\tilde{F},\mathfrak{R})$. In other words the neutrosophic soft set is a parameterized family of some elements of the set $P(\mathcal{X})$ and therefore it can be written as a set of ordered pairs, $(\tilde{F},\mathfrak{R}) = \left\{\left(\left(e, \langle x, \mathbb{T}_{\dagger(e)}(x), \mathbb{I}_{\dagger(e)}(x), \mathbb{F}_{\dagger(e)}(x)\rangle : x \in \mathcal{X}\right) : e \in \mathfrak{R}\right\}\right\}$,

Where, $\mathbb{T}_{\dagger(e)}(x), \mathbb{I}_{\dagger(e)}(x), \mathbb{F}_{\dagger(e)}(x) \in [0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $\tilde{F}(e)$.Since supremum of each $\mathbb{T}, \mathbb{I}, \mathbb{F}$ is 1 so the inequality $0 \leq \mathbb{T}_{\dagger(e)}(x) + \mathbb{I}_{\dagger(e)}(x) + \mathbb{F}_{\dagger(e)}(x) \leq 3$ is obvious.

Definition 1.2[19] Let $(\tilde{F},\mathfrak{R})$ be a neutrosophic soft set over universal set $\mathcal{X}$ and the complement of $(\tilde{F},\mathfrak{R})$ is denoted by $(\tilde{F},\mathfrak{R})^c$ and is defined by:

$(\tilde{F},\mathfrak{R})^c = \left\{\left(\left(e, \langle x, \mathbb{F}_{\dagger(e)}(x), 1 - \mathbb{I}_{\dagger(e)}(x), \mathbb{T}_{\dagger(e)}(x)\rangle : x \in \mathcal{X}\right) : e \in \mathfrak{R}\right\}\right\}$ and It is clear that $\left((\tilde{F},\mathfrak{R})^c\right)^c = (\tilde{F},\mathfrak{R})$.

Definition 1.3[20] Let $(\tilde{F},\mathfrak{R})$ and $(\tilde{G},\mathfrak{R})$ two neutrosophic soft sets over universal set $\mathcal{X}$. $(\tilde{F},\mathfrak{R})$ is said to be neutrosophic soft sub set of $(\tilde{G},\mathfrak{R})$ if $\mathbb{T}_{\tilde{F}(e)}(x) \leqslant \mathbb{T}_{\tilde{G}(e)}(x)$, $\mathbb{I}_{\tilde{F}(e)}(x) \leqslant \mathbb{I}_{\tilde{G}(e)}(x)$, $\mathbb{F}_{\tilde{F}(e)}(x) \geqslant \mathbb{F}_{\tilde{G}(e)}(x)$, $\forall\, e \in \mathfrak{R}$ and $\forall\, x \in \mathcal{X}$. It is denoted by $(\tilde{F},\mathfrak{R}) \subseteq (\tilde{G},\mathfrak{R})$. $(\tilde{F},\mathfrak{R})$ is is said to be neutrosophic soft equal to $(\tilde{G},\mathfrak{R})$ if $(\tilde{F},\mathfrak{R})$ is neutrosophic soft sub set of $(\tilde{G},\mathfrak{R})$ and $(\tilde{G},\mathfrak{R})$ is neutrosophic soft sub set if $(\tilde{F},\mathfrak{R})$ and is denoted by $(\tilde{F},\mathfrak{R}) = (\tilde{G},\mathfrak{R})$.

Definition 1.4[20] let $(\tilde{F},\mathfrak{R})$ and $(\tilde{G},\mathfrak{R})$ be two neutrosophic soft sub sets over universal set $\mathcal{X}$ such that $((\tilde{F},\mathfrak{R}) \neq (\tilde{G},\mathfrak{R})$ then their union is denoted by $(\tilde{F},\mathfrak{R}) \cup (\tilde{G},\mathfrak{R}) = (\tilde{H},\mathfrak{R})$ and is defined by $(\tilde{H},\mathfrak{R}) = \left\{\left(\left(e, [\![x, \mathbb{T}_{\tilde{H}(e)}(x), \mathbb{I}_{\tilde{H}(e)}(x), \mathbb{F}_{\tilde{H}(e)}(x) : x \in \mathcal{X}]\!]\right) : e \in \mathfrak{R}\right\}\right.$

Where,

$$\mathbb{T}_{\tilde{H}(e)}(x) = max\left[\mathbb{T}_{\tilde{F}(e)}(x), \mathbb{T}_{\tilde{G}(e)}(x)\right]$$

$\mathbb{I}_{\tilde{H}(e)}(x) = max\left[\mathbb{I}_{\tilde{F}(e)}(x), \mathbb{I}_{\tilde{G}(e)}(x)\right]$

$\mathbb{F}_{\tilde{H}(e)}(x) = min\left[\mathbb{F}_{\tilde{F}(e)}(x), \mathbb{F}_{\tilde{G}(e)}(x)\right]$

Definition 1.5 [13] Let $(\tilde{F}, \mathfrak{R})$ and $\tilde{G}, \mathfrak{R})$ be two neutrosophic soft sub sets over the universal set $\mathcal{X}$ such that $(\tilde{F}, \mathfrak{R}) \neq (\tilde{G}, \mathfrak{R})$ then their intersection is denoted by $(\tilde{F}, \mathfrak{R}) \cap (\tilde{G}, \mathfrak{R}) = (\tilde{H}, \mathfrak{R})$ and is defined as follows $(\tilde{H}, \mathfrak{R}) = \{\left(\left(e, \left[\!\left[x, \mathbb{T}_{\tilde{H}(e)}(x), \mathbb{I}_{\tilde{H}(e)}(x), \mathbb{F}_{\tilde{H}(e)}(x) : x \in \mathcal{X}\right]\!\right]\right) : e \in \mathfrak{R}\right\}$ where

$\mathbb{T}_{\tilde{H}(e)}(x) = min\left[\mathbb{T}_{\tilde{F}(e)}(x), \mathbb{T}_{\tilde{G}(e)}(x)\right]$

$\mathbb{I}_{\tilde{H}(e)}(x) = min\left[\mathbb{I}_{\tilde{F}(e)}(x), \mathbb{I}_{\tilde{G}(e)}(x)\right]$

$\mathbb{F}_{\tilde{H}(e)}(x) = max\left[\mathbb{F}_{\tilde{F}(e)}(x), \mathbb{F}_{\tilde{G}(e)}(x)\right]$

Definition 1.6 [16] Neutrosophic soft set $(\tilde{F}, \mathfrak{R})$over the universal set $\mathcal{X}$is said to be a null neutrosophic soft set

If$\mathbb{T}_{\tilde{F}(e)}(x) = 0, \mathbb{I}_{\tilde{F}(e)}(x) = 0, \mathbb{F}_{\tilde{F}(e)}(x) = 1; \forall\, e \in \mathfrak{R}$ and $\forall\, x \in \mathcal{X}$.

It is denoted by $0_{(\mathcal{X}, \mathfrak{R})}$.

Definition 1.7 [21] Neutrosophic soft set $(\tilde{F}, \mathfrak{R})$ over universal set $\mathcal{X}$ is said to be an absolute neutrosophical softness if $\mathbb{T}_{\tilde{F}(e)}(x) = 1, \mathbb{I}_{\tilde{F}(e)}(x) = 1, \mathbb{F}_{\tilde{F}(e)}(x) = 0; \forall\, e \in \mathfrak{R}$ and $\forall\, x \in \mathcal{X}$. It is signified as $1_{(\mathcal{X}, \mathfrak{R})}$ clearly, $0_{(\mathcal{X}, \mathfrak{R})}{}^{c} = 1_{(\mathcal{X}, \mathfrak{R})}$ and $1_{(\mathcal{X}, \mathfrak{R})}{}^{c} = 0_{(\mathcal{X}, \mathfrak{R})}$.

Definition 1.8[13] Let $NSS\ (\mathcal{X})$ be the family of all neutrosophic soft sets over universal set $\mathcal{X}$ and $\tau \subset \mathrm{NSS}\ (\mathcal{X})$ then τ is said to be neutrosophic soft topology on $\mathcal{X}$ if:

(1): $0_{(\mathcal{X}, \mathfrak{R})}, 1_{(\mathcal{X}, \mathfrak{R})}. \in \tau,$

(2): The union of any number of neutrosophic soft sets in τ belongs to τ,

(3): The intersection of a finite number of neutrosophic soft sets in in τ belongs to τ then $(\mathcal{X}, \tau, \mathfrak{K})$ is said to be a neutrosophic soft topological space over $\mathcal{X}$. Each member of τ is said to be a neutrosophic soft open set.

Definition 1.9 [13] *Let* $(\mathcal{X}, \tau, \mathfrak{K})$ be a neutrosophic soft topological space over $\mathcal{X}$ and $(\tilde{F}, \mathfrak{K})$ be a neutrosophic soft set over $\mathcal{X}$ then $(\tilde{F}, \mathfrak{K})$ is supposed to be a neutrosophic soft close set if and only if its complement is a neutrosophic set open set.

2. Neutrosophic Pairwise Approach

In this section, pair-wise neutrosophic soft open sets in neutrosophic soft bi-topological space and for the supportment of this structure an example is given. Intersection of two neutrosophic soft topological space, the non-validity of union of two neutrosophic soft topological spaces is discussed with an example. In this section, new types of open and close sets have been introduced in neutrosophic soft bi topological spaces. Eight new neutrosophic soft open sets are introduced. Among these, neutrosophic soft b^{**} is chosen to generate the results pairwisly in neutrosophic soft bi-topological space with respect to soft points. These results are different neutrosophic soft separations axioms and their interconnection among themselves.

Definition 2.1 If $(\mathcal{X}, \tau_1, \partial)$ and $(\mathcal{X}, \tau_2, \partial)$ are two neutrosophic soft topological space (NSTS) then $(\mathcal{X}, \tau_1, \tau_2, \partial)$ is called neutrosophic soft bi topological space. A neutrosophic soft sub-set $\langle \tilde{\eta}, \partial \rangle$ is said to be neutrosophic soft pairwise open in $(\mathcal{X}, \tau_1, \tau_2, \partial)$ if there exists a neutrosophic soft open set $\langle \widetilde{\eta_1}, \partial \rangle$ in τ_1 neutrosophic soft open set $\langle \widetilde{\eta_2}, \partial \rangle$ in τ_2 such that $\langle \tilde{\eta}, \partial \rangle = \langle \widetilde{\eta_1}, \partial \rangle \tilde{\cup} \langle \widetilde{\eta_2}, \partial \rangle$.

Example 2.2 Let $\mathcal{X} = \{x_1, x_2, x_3\}$, $\partial = \{s_1, s_2\}$ and $\tau_1 = \{0_{(\mathcal{X},\partial)}, 1_{(\mathcal{X},\partial)}, (p_1, \partial), (p_2, \partial)\}$, $\tau_2 = \{0_{(\mathcal{X},\partial)}, 1_{(\mathcal{X},\partial)}, (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial)\}$, where $(p_1, \partial), (p_2, \partial), (\mathbb{J}_1, \partial)$, $(\mathbb{J}_2, \partial)$ being neutrosophic soft sub sets are as following:

$$f_{(p_1,\partial)}(s_1) = \begin{pmatrix} < x_1, 02 \times 10^{-1}, 03 \times 10^{-1}, 08 \times 10^{-1} >, \\ < x_2, 02 \times 10^{-4}, 03 \times 10^{-4}, 08 \times 10^{-4} >, \\ < x_3, 02 \times 10^{-1}, 04 \times 10^{-1}, 03 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\wp_1,\partial)}(\mathfrak{s}_2) = \begin{pmatrix} < x_1, 03 \times 10^{-1}, 02 \times 10^{-1}, 06 \times 10^{-1} >, \\ < x_2, 01 \times 10^{-1}, 05 \times 10^{-1}, 05 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 03 \times 10^{-1}, 04 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\wp_2,\partial)}(\mathfrak{s}_1) = \begin{pmatrix} < x_1, 04 \times 10^{-1}, 03 \times 10^{-1}, 06 \times 10^{-1} >, \\ < x_2, 04 \times 10^{-1}, 05 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 03 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\wp_2,\partial)}(\mathfrak{s}_2) = \begin{pmatrix} < x_1, 03 \times 10^{-1}, 04 \times 10^{-1}, 05 \times 10^{-1} >, \\ < x_2, 02 \times 10^{-1}, 06 \times 10^{-1}, 04 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} > \end{pmatrix}$$

$$f_{(J_1,\partial)}(\mathfrak{s}_1) = \begin{pmatrix} < x_1, 05 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} >, \\ < x_2, 06 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 01 \times 10^{-1} > \end{pmatrix}$$

$$f_{(J_1,\partial)}(\mathfrak{s}_2) = \begin{pmatrix} < x_1, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 07 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 05 \times 10^{-1}, 07 \times 10^{-1}, 01 \times 10^{-1} > \end{pmatrix}$$

$$f_{(J_2,\partial)}(\mathfrak{s}_1) = \begin{pmatrix} < x_1, 01 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 03 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 01 \times 10^{-1}, 02 \times 10^{-1}, 02 \times 10^{-1} > \end{pmatrix}$$

$$f_{(J_2,\partial)}(\mathfrak{s}_2) = \begin{pmatrix} < x_1, 01 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 03 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 01 \times 10^{-1}, 02 \times 10^{-1}, 02 \times 10^{-1} > \end{pmatrix}.$$

Therefore, τ_1 and τ_2 are neutrosophic soft topological spaces on $\mathcal{X}$ and so $(\mathcal{X}, \tau_1, \tau_2, \partial)$ is a neutrosophic soft bi topological spaces.

Theorem 2.3 Let $(\mathcal{X}, \tau_1, \tau_2, \partial)$ be a neutrosophic soft topological space then $\tau_1 \tilde{\cap} \tau_2$ is a neutrosophic soft topological space on $\mathcal{X}$.

Proof. For this we have to verify all the three conditions of neutrosophic soft bi topological space. Conditions (1) and (3) are obvious, for condition (2) , let $\{(\wp_i, \partial); i \in I\} \in \tau_1 \tilde{\cap} \tau_2$ then $(\wp_i, \partial) \in \tau_1$, $(\wp_i, \partial) \in \tau_2$ as τ_1 , τ_2 are neutrosophic soft topological spaces on $\mathcal{X}$, then $\cup_i (\wp_i, \partial) \in \tau_1, \cup_i (\wp_i, \partial) \in \tau_2$. Therefore $\cup_i (\wp_i, \partial) \in \tau_1 \tilde{\cap} \tau_2$.

Remark 2.4 Let $(\mathcal{X}, \tau_1, \tau_2, \partial)$ be a neutrosophic soft topological space then $\tau_1 \tilde{\cup} \tau_2$ need not be a neutrosophic soft topological space on $\mathcal{X}$.

Example 2.5 Let $\mathcal{X} = \{x_1, x_2, x_3\}$, $\partial = \{s_1, s_2\}$ and $\tau_1 = \{0_{(\mathcal{X},\partial)}, 1_{(\mathcal{X},\partial)}, (p_1, \partial), (p_2, \partial), (p_3, \partial)\}$, $\tau_2 = \{0_{(\mathcal{X},\partial)}, 1_{(\mathcal{X},\partial)}, (\mathbb{J}_1, \partial), (\mathbb{J}_2, \partial)\}$, where $(p_1, \partial), (p_2, \partial), (p_3, \partial), (\mathbb{J}_1, \partial)$, $(\mathbb{J}_2, \partial)$and being neutrosophic soft sub sets are as following:

$$f_{(p_1,\partial)}(s_1) = \begin{pmatrix} < x_1, 02 \times 10^{-1}, 03 \times 10^{-1}, 08 \times 10^{-1} >, \\ < x_2, 04 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} >, \\ < x_3, 02 \times 10^{-1}, 04 \times 10^{-1}, 03 \times 10^{-1} > \end{pmatrix}$$

$$f_{(p_1,\partial)}(s_2) = \begin{pmatrix} < x_1, 03 \times 10^{-1}, 02 \times 10^{-1}, 06 \times 10^{-1} >, \\ < x_2, 01 \times 10^{-1}, 05 \times 10^{-1}, 05 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 03 \times 10^{-1}, 05 \times 10^{-1} > \end{pmatrix}$$

$$f_{(p_2,\partial)}(s_1) = \begin{pmatrix} < x_1, 04 \times 10^{-1}, 03 \times 10^{-1}, 06 \times 10^{-1} >, \\ < x_2, 04 \times 10^{-1}, 05 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 03 \times 10^{-1}, 05 \times 10^{-1}, 02 \times 10^{-1} > \end{pmatrix}$$

$$f_{(p_2,\partial)}(s_2) = \begin{pmatrix} < x_1, 03 \times 10^{-1}, 04 \times 10^{-1}, 05 \times 10^{-1} >, \\ < x_2, 02 \times 10^{-1}, 06 \times 10^{-1}, 04 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} > \end{pmatrix}$$

$$f_{(p_3,\partial)}(s_1) = \begin{pmatrix} < x_1, 05 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} >, \\ < x_2, 06 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 00 \times 10^{-1} > \end{pmatrix}$$

$$f_{(p_3,\partial)}(s_2) = \begin{pmatrix} < x_1, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 07 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 05 \times 10^{-1}, 07 \times 10^{-1}, 01 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\mathbb{J}_1,\partial)}(s_1) = \begin{pmatrix} < x_1, 05 \times 10^{-1}, 04 \times 10^{-1}, 04 \times 10^{-1} >, \\ < x_2, 06 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1} >, \\ < x_3, 04 \times 10^{-1}, 06 \times 10^{-1}, 01 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\mathbb{J}_1,\partial)}(s_2) = \begin{pmatrix} < x_1, 04 \times 10^{-1}, 06 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 07 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 05 \times 10^{-1}, 07 \times 10^{-1}, 01 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\mathbb{J}_2,\partial)}(s_1) = \begin{pmatrix} < x_1, 01 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 03 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 01 \times 10^{-1}, 02 \times 10^{-1}, 02 \times 10^{-1} > \end{pmatrix}$$

$$f_{(\mathbb{J}_2,\partial)}(s_2) = \begin{pmatrix} < x_1, 01 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} >, \\ < x_2, 03 \times 10^{-1}, 03 \times 10^{-1}, 03 \times 10^{-1} >, \\ < x_3, 01 \times 10^{-1}, 02 \times 10^{-1}, 02 \times 10^{-1} > \end{pmatrix}.$$

Here $\tau_1 \tilde{\cup} \tau_2 = \{0_{(\mathcal{X},\partial)}, 1_{(\mathcal{X},\partial)}, (\wp_1,\partial), (\wp_2,\partial), (\wp_3,\partial), (\mathbb{J}_1,\partial), (\mathbb{J}_2,\partial)\}$ is not a neutrosophic soft topological space on $\mathcal{X}$.

Definition 2.6 Let $(\mathcal{X},\tau_1,\tau_2,\partial)$ be a neutrosophic soft topological space. Then a neutrosophic soft set

$(\mathbb{J},\partial) = \{(\mathfrak{s}, \{< x, T_{\mathbb{J}(\mathfrak{s})}(x), I_{\mathbb{J}(\mathfrak{s})}(x), F_{\mathbb{J}(\mathfrak{s})}(x) >\}: x \in \mathcal{X}, \mathfrak{s} \in \partial\}$

is called as a pairwise neutrosophic soft open set if there exist a neutrosophic soft open $(\mathbb{J}_1,\partial)$ in τ_1, neutrosophic soft open $(\mathbb{J}_2,\partial)$ in τ_2 such that for all $x \in \mathcal{X}$ such that $(\mathbb{J},\partial) = (\mathbb{J}_1,\partial) \tilde{\cup} (\mathbb{J}_2,\partial) =$

$$\left\{\left(\mathfrak{s}, \left\{\begin{matrix} < x, \max\{T_{\mathbb{J}(\mathfrak{s})}(x), T_{\mathbb{J}(\mathfrak{s})}(x)\}, \\ \max\{I_{\mathbb{J}(\mathfrak{s})}(x), I_{\mathbb{J}(\mathfrak{s})}(x)\}, \min\{F_{\mathbb{J}(\mathfrak{s})}(x), F_{\mathbb{J}(\mathfrak{s})}(x)\} > \end{matrix}\right\}\right): \mathfrak{s} \in \partial\right\}$$

Definition 2.7 Let $(\mathcal{X},\tau_1,\tau_2,\partial)$ be a neutrosophic soft topological space. Then a neutrosophic soft set

$(\mathbb{J},\partial) = \{(\mathfrak{s}, \{< x, T_{\mathbb{J}(\mathfrak{s})}(x), I_{\mathbb{J}(\mathfrak{s})}(x), F_{\mathbb{J}(\mathfrak{s})}(x) >\}): x \in \mathcal{X}, \mathfrak{s} \in \partial\}$

is called as a pairwise neutrosophic soft closed set if $(\mathbb{J},\partial)^c$ is a pairwise neutrosophic soft open . It is clear that $(\mathbb{J},\partial)$is a pairwise neutrosophic soft closed set if there exist a pairwise neutrosophic soft closed $(\mathbb{J}_1,\partial)$ in τ_1, pairwise neutrosophic soft closed $(\mathbb{J}_2,\partial)$ in τ_2 such that for all $x \in \mathcal{X}$ $(\mathbb{J},\partial) = (\mathbb{J}_1,\partial) \tilde{\cap} (\mathbb{J}_2,\partial) =$

$$\left\{\left(\mathfrak{s}, \left\{\begin{matrix} < x, \min\{I_{\mathbb{J}(\mathfrak{s})}(x), T_{\mathbb{J}(\mathfrak{s})}(x)\}, \min\{I_{\mathbb{J}(\mathfrak{s})}(x), I_{\mathbb{J}(\mathfrak{s})}(x)\}, \\ \max\{F_{\mathbb{J}(\mathfrak{s})}(x), F_{\mathbb{J}(\mathfrak{s})}(x)\} > \end{matrix}\right\}\right): \mathfrak{s} \in \partial\right\}$$

The set of all pairwise neutrosophic soft closed in $(\mathcal{X},\tau_1,\tau_2,\partial)$ is denoted by $PNSC\,(\mathcal{X},\tau_1,\tau_2,\partial)$.

Definition 2.8 Let $(\mathcal{X},\tau_1,\theta)$ and $(\mathcal{X},\tau_2,\theta)$ be two neutrosophic soft topological spaces over $\mathcal{X}$ then $(\mathcal{X},\tau_2 \vee \tau_2,\theta)$ is the smallest neutrosophic soft topology on $\mathcal{X}$ that contains $(\mathcal{X},\tau_2 \cup \tau_2,\theta)$

Definition 2.9 Let $(\mathcal{X},\tau_1,\tau_2,\theta)$ be a neutrosophic soft topological space over $\mathcal{X}$ and $(\tilde{\mathfrak{f}},\theta)$ be a neutrosophic soft set over $\mathcal{X}$ then $(\tilde{\mathfrak{f}},\theta)$is said

(1): neutrosophic soft semi-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NScl\left(NSint(\tilde{\mathfrak{f}},\theta)\right)$. (2): neutrosophic soft pre-open if$(\tilde{\mathfrak{f}},\theta) \subseteq NSint\left(NScl(\tilde{\mathfrak{f}},\theta)\right)$.

(3): neutrosophic soft α-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NSint\left(NScl\left(NSint\left(\tilde{\mathfrak{f}},\theta\right)\right)\right)$. (4): neutrosophic soft β-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NScl\left(NSint\left(NScl\left(\tilde{\mathfrak{f}},\theta\right)\right)\right)$.

(5): neutrosophic soft b-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NScl\left(NSint(\tilde{\mathfrak{f}},\theta)\right) \cup NSint\left(NScl(\tilde{\mathfrak{f}},\theta)\right)$. (6): neutrosophic soft $*_b$-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NScl\left(NSint(\tilde{\mathfrak{f}},\theta)\right) \cap NSint\left(NScl(\tilde{\mathfrak{f}},\theta)\right)$. (7): neutrosophic soft b^{**}-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NSint\left(NScl\left(NSint(\tilde{\mathfrak{f}},\theta)\right)\right) \cup NScl\left(NSint\left(NScl(\tilde{\mathfrak{f}},\theta)\right)\right)$. (8): neutrosophic soft $**_b$-open if $(\tilde{\mathfrak{f}},\theta) \subseteq NSint\left(NScl\left(NSint(\tilde{\mathfrak{f}},\theta)\right)\right) \cap NScl\left(NSint\left(NScl(\tilde{\mathfrak{f}},\theta)\right)\right)$.

Definition 2.10 A neutrosophic soft bitopological space $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is called pairwise neutrosophic soft T_0 space if $x_1{}^{e_1}_{(s_1,t_1,u_1)}, x_2{}^{e_2}_{(s_2,t_2,u_2)}$ be two neutrosophic soft points such that $x_1{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_2{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$.

If there exists neutrosophic soft b^{**} open sets $(\tilde{\mathfrak{f}},\theta) \tilde{\in} \tau_1$ and $(\tilde{\mathcal{g}},\theta) \tilde{\in} \tau_2$ such that $x_1{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}},\theta), x_1{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathcal{g}},\theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_2{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}},\theta),\ x_2{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathfrak{f}},\theta) \cong 0_{(\mathcal{X},\theta)}$.

Definition 2.11 A neutrosophic soft bitopological space $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is called pair-wise neutrosophic soft T_1 space if $x_1{}^{e_1}_{(s_1,t_1,u_1)}, x_2{}^{e_2}_{(s_1,t_1,u_1)}$ be two neutrosophic soft points such that $x_1{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_2{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$.

If there exists neutrosophic b^{**} open sets $(\tilde{\mathfrak{f}},\theta) \tilde{\in} \tau_1$, $(\tilde{\mathcal{g}},\theta) \tilde{\in} \tau_2$ such that $x_1{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}},\theta), x_2{}^{e_2}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathcal{g}},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_2{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}},\theta),\ x_2{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathfrak{f}},\theta) \cong 0_{(\mathcal{X},\theta)}$.

Definition 2.12 A neutrosophic soft bitopological space $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is called pair-wise neutrosophic soft T_2 space if ${x_1}^{e_1}_{(s_1,t_1,u_1)}, {x_2}^{e_2}_{(s_1,t_1,u_1)}$ be two neutrosophic soft points such that ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} {x_2}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$.

If there exists neutrosophic soft b^{**} open sets $(\tilde{\mathfrak{f}}, \theta) \tilde{\in} \tau_1$, $(\tilde{\mathcal{g}}, \theta) \tilde{\in} \tau_2$ such that ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$, $(\tilde{\mathfrak{f}}, \theta) \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Definition 2.13 In neutrosophic soft bitopological space $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$

τ_1 said to be neutrosophic soft T_0 space with respect to τ_2 if for each pair of neutrosophic soft points ${x_1}^{e_1}_{(s_1,t_1,u_1)}, {x_2}^{e_2}_{(s_2,t_2,u_2)}$, ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} {x_2}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_1 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**}open set $(\tilde{\mathcal{g}}, \theta)$ such that ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$ and ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$, Similarly, τ_2 said to be neutrosophic soft T_0 space with respect to τ_1 if for each pair of neutrosophic soft points ${x_1}^{e_1}_{(s_1,t_1,u_1)}, {x_2}^{e_2}_{(s_2,t_2,u_2)}$, ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} {x_2}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_2 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_1 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$ and ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Definition 2.14 In neutrosophic soft bitopological space $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$

τ_1 said to be neutrosophic soft T_1 space with respect to τ_2 if for each pair of neutrosophic soft points ${x_1}^{e_1}_{(s_1,t_1,u_1)}, {x_2}^{e_2}_{(s_2,t_2,u_2)}$, ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} {x_2}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_1 neutrosophic soft $b^{**} open\ set$ $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**}open set $(\tilde{\mathcal{g}}, \theta)$ such that ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and

$x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{g},\theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap}(\widetilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$, Similarly, τ_2 said to be neutrosophic soft T_1 space with respect to τ_1 if for each pair of neutrosophic soft points $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_2 neutrosophic soft b^{**} open set $(\tilde{f},\theta)$ and τ_1 neutrosophic soft b^{**} open set $(\widetilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\cap}(\widetilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{g},\theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap}(\widetilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$.

Definition 2.15 In neutrosophic soft bitopological space $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$

τ_1 said to be neutrosophic soft T_2 space with respect to τ_2 if for each pair of neutrosophic soft points $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_1 neutrosophic soft b^{**}open set $(\tilde{f},\theta)$ and τ_2 neutrosophic soft b^{**}open set $(\widetilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{g},\theta)$ $(\tilde{f},\theta) \widetilde{\cap}(\widetilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$, Similarly, τ_2 said to be neutrosophic soft T_2 space with respect to τ_1 if for each pair of neutrosophic soft points $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_2 neutrosophic soft b^{**}open set $(\tilde{f},\theta)$ and τ_1 neutrosophic soft b^{**}open set $(\widetilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{g},\theta)$ $(\tilde{f},\theta) \widetilde{\cap}(\widetilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$.

Theorem 2.16 Let $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ be neutrosophic soft bitopological space. If $\langle \mathcal{X},\tau_1,\theta\rangle$ and $\langle \mathcal{X},\tau_2,\theta\rangle$ are neutrosophic soft T_0 spaces. Then $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ is a pair wise neutrosophic soft T_0 space.

Proof. Let $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ and suppose $\langle \mathcal{X}, \tau_1, \theta \rangle$ is neutrosophic soft T_0 space with respect to $\langle \mathcal{X}, \tau_2, \theta \rangle$ space then there exists τ_1 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Similarly, Let $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ and suppose $\langle \mathcal{X}, \tau_2, \theta \rangle$ is neutrosophic soft T_0 space with respect to $\langle \mathcal{X}, \tau_1, \theta \rangle$ space then there exists τ_2 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_1 neutrosophic soft open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Hence, $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is a pair wise neutrosophic soft T_0 space.

Theorem 2.17 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is a pair wise neutrosophic soft T_0 space then $\langle \mathcal{X}, \tau_1 v \tau_2, \theta \rangle$ is a pair wise neutrosophic soft T_0 space.

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space. If $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$. Since $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is a pair wise neutrosophic soft T_0 space. So $\langle \mathcal{X}, \tau_1, \theta \rangle$ is neutrosophic soft T_0 space with respect to $\langle \mathcal{X}, \tau_2, \theta \rangle$ space then there exists τ_1 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Similarly, Let $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ and suppose $\langle \mathcal{X}, \tau_2, \theta\rangle$ is neutrosophic soft T_0 space with respect to $\langle \mathcal{X}, \tau_1, \theta\rangle$ space then there exists τ_2 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_1 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. In either, case $(\tilde{\mathfrak{f}}, \theta), (\tilde{\mathcal{g}}, \theta) \widetilde{\in} \langle \mathcal{X}, \tau_1 v \tau_2, \theta\rangle$ and hence $\langle \mathcal{X}, \tau_1 v \tau_2, \theta\rangle$ is a pair wise neutrosophic soft T_0 space.

Theorem 2.18 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ be neutrosophic soft bitopological space over $\mathcal{X}$ and Y be a non-empty neutrosophic soft s*ubs*et of $\mathcal{X}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ is pair wise neutrosophic soft T_0 space. Then $\langle Y, \tau_{1Y}, \tau_{2Y}, \theta\rangle$ is pair wise neutrosophic soft T_0 space

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ be neutrosophic soft bitopological space over $\mathcal{X}$ If $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ is pair wise neutrosophic soft T_0 space, then there exist τ_1 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ Such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Now, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} Y$ and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta)$. Hence, where $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} Y \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) = (Y_{\mathfrak{f}}, \theta)$ where $(\tilde{\mathfrak{f}}, \theta) \widetilde{\in} \tau_1$.Consider $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$,this means that $\alpha \in \theta$ for some $\alpha \in \theta$. $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\notin} Y \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) = (Y_{\mathfrak{f}}, \theta)$. There fore τ_{1Y} is neutrosophic soft T_0 space with respect τ_{2Y}. Similarly, can pro*v*ed thatτ_{2Y} is neutrosophic soft T_0 space with respect to τ_{1Y}, that is $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{g},\theta)$ and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$ then $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (Y_{\tilde{g}},\theta)$ and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (Y_{\tilde{g}},\theta) \cong 0_{(\mathcal{X},\theta)}$. Thus $\langle Y, \tau_{1Y}, \tau_{2Y}, \theta\rangle$ is pair wise neutrosophic soft T_0 space.

Theorem 2.19 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ be neutrosophic soft bitopological space. Then $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ is a pair wise neutrosophic soft T_1 space if and only if $\langle \mathcal{X}, \tau_1, \theta\rangle$ and $\langle \mathcal{X}, \tau_2, \theta\rangle$ are neutrosophic soft T_1 space.

Proof. Suppose $\langle \mathcal{X}, \tau_1, \theta\rangle$ and $\langle \mathcal{X}, \tau_2, \theta\rangle$ are neutrosophic soft T_0 spaces. If $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ then

1). τ_1 is a neutrosophic soft T_1 space with respect to τ_2. So there exists τ_1 neutrosophic soft b^{**}open set $(\tilde{f},\theta)$ and τ_2 neutrosophic soft b^{**}open set $(\tilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{f},\theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{f},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{g},\theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{f},\theta) \cong 0_{(\mathcal{X},\theta)}$

2). τ_2 is a neutrosophic soft T_1 space with respect to τ_1. So there exists τ_2 neutrosophic soft b^{**}open set $(\tilde{f},\theta)$ and τ_1 neutrosophic soft b^{**}open set $(\tilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{f},\theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{f},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{g},\theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{f},\theta) \cong 0_{(\mathcal{X},\theta)}$ .All possibilities give the same result. So $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ is a pair wise neutrosophic soft T_1 space.

Conversely, we suppose that $\langle \mathcal{X}, \tau_1, \tau_2, \theta\rangle$ is a pair wise neutrosophic soft T_1 space. Then

1). There exists some neutrosophic soft b^{**}open set $(\tilde{f},\theta) \in \tau_1$ with respect to neutrosophic soft b^{**} open set $(\tilde{g},\theta) \in \tau_2$ such $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{f},\theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{f},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{g},\theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$.

2). there exists some neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta) \in \tau_2$with respect to neutrosophic soft b^{**}open set$(\tilde{\mathcal{g}}, \theta) \in \tau_1$such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Thus$\langle \mathcal{X}, \tau_1, \theta \rangle$ and $\langle \mathcal{X}, \tau_2, \theta \rangle$ are neutrosophic soft T_1 spaces.

Theorem 2.20 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space. if $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_1space then $\langle \mathcal{X}, \tau_1 v \tau_2, \theta \rangle$ is also neutrosophic soft T_1 space.

Proof. If $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ then exists n neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta) \in \tau_1$with respect to neutrosophic soft b^{**}open set $(\tilde{\mathcal{g}}, \theta) \in \tau_2$such that

$x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Similarly, exists neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta) \in \tau_2$with respect to neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta) \in \tau_1$such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

$\Longrightarrow (\tilde{\mathfrak{f}}, \theta), (\tilde{\mathcal{g}}, \theta) \in \tau_1 v \tau_2$. So $\langle \mathcal{X}, \tau_1 v \tau_2, \theta \rangle$ is neutrosophic soft T_1 space.

Theorem 2.21 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space and Y be non-empty sub set of $\mathcal{X}$.If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_1 space then $\langle Y, \tau_{1Y}, \tau_{2Y}, \theta \rangle$ is pair wise neutrosophic soft T_1 space.

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \in Y, x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise soft neutrosophic soft T_1 then there exists τ_1 neutrosophic soft b^{**} open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ su *ch* that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Obviously, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} Y \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong (Y_{\mathfrak{f}}, \theta)$ .Then $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \notin Y \tilde{\sqcap} \tilde{\mathfrak{f}}(\alpha)$ for some $\alpha \in \theta$ .This means that $\alpha \in \theta$ then $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \notin Y \tilde{\sqcap} \tilde{\mathfrak{f}}(\alpha)$ for some $\alpha \in \theta$. Therefore, ${}^{e_2}_{(s_2,t_2,u_2)} \notin Y \tilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong (Y_{\mathfrak{f}}, \theta)$.

Obviously, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \in Y \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) = (Y_{\mathcal{g}}, \theta)$ where $(\tilde{\mathcal{g}}, \theta) \tilde{\in} \tau_2$ .Consider $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and this means that $\alpha \in \theta$ then $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \notin Y \tilde{\sqcap} \mathcal{g}(\alpha)$ for some $\alpha \in \theta$ therefore $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \notin Y \cap (\tilde{\mathcal{g}}, \theta) = (Y_{\mathcal{g}}, \theta)$ hence τ_{1Y} is neutrosophic soft T_1 space with respect $((Y_{\mathcal{g}}, \theta)$ to τ_{2Y}. Similarly, it can be proved that τ_{2Y} is neutrosophic soft T_1 space with respect to τ_{1Y} , that is $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$ and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.Then $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (Y_{\mathcal{g}}, \theta)$ and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} (Y_{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Thus $\langle Y, \tau_{1Y}, \tau_{2Y}, \theta \rangle$ is pair wise neutrosophic soft T_1 space.

Theorem 2.22 Every pair wise neutrosophic soft T_1 space is pair wise neutrosophic soft T_0 space.

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft open bitopological space and if $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that

$x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_1 space and that is, $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_1space with respect to τ_2 and τ_2 is neutrosophic soft T_1 space with respect to τ_1 and if τ_1 is neutrosophic soft T_1 space with respect to τ_2 then there exists a τ_1 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and a τ_2 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Obviously $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Therefore τ_1 is neutrosophic soft T_0 space with respect to τ_2. Similarly, if τ_2 is a neutrosophic soft T_1 space with respect to τ_1 then, there exists τ_2 neutrosophic soft b^{**}open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_1 neutrosophic soft open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta), x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$. Obviously $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{\mathfrak{f}}, \theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\sqcap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ or $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{\mathcal{g}}, \theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ .Therefore τ_2 neutrosophic soft T_0 space with respect to τ_1. Thus $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is a pair wise neutrosophic soft T_0 space.

Theorem 2.23 Every pair wise neutrosophic soft T_2 space is pair wise neutrosophic soft T_1 space

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be a neutrosophic soft bitopological space and if $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that

$x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_2space and that is $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_2 space. If τ_1 is neutrosophic soft T_2 space with respect τ_2 then there exists a τ_1 neutrosophic soft open set $(\tilde{\mathfrak{f}}, \theta)$ and τ_2 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that $(\tilde{\mathfrak{f}}, \theta) \tilde{\in} \tau_1$, $(\tilde{\mathcal{g}}, \theta) \tilde{\in} \tau_2$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$, $(\tilde{\mathfrak{f}}, \theta) \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Obviously $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Therefore τ_1 is neutrosophic soft T_1 space with respect to τ_2. Similarly, if τ_2 is neutrosophic soft T_2 space with respect to τ_1 then there exists a τ_2 n neutrosophic soft b^{**} open set $(\tilde{\mathfrak{f}}, \theta)$ set and τ_1 neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$, $(\tilde{\mathfrak{f}}, \theta) \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Obviously $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\sqcap} (\tilde{\mathcal{g}}, \theta) \cong 0_{(\mathcal{X},\theta)}$.

Therefore τ_2 is neutrosophic soft T_1space with respect to τ_1 thus $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair wise neutrosophic soft T_1space.

Theorem 2.24 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is

Pair wise neutrosophic soft T_2space. Then $\langle \mathcal{X}, \tau_1 \vee \tau_2, \theta \rangle$ also a neutrosophic soft T_2 space.

Proof. Since $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space. If $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\sqcap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$.

τ_1 is neutrosophic soft T_2 space with respect to τ_2 so for each pair of neutrosophic soft points $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_1 neutrosophic soft open set $(\tilde{f},\theta)$ and τ_2 neutrosophic soft b^{**}open set $(\tilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{g},\theta)$ $(\tilde{f},\theta) \widetilde{\cap} (\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$, Similarly, τ_2 is neutrosophic soft T_2 space with respect to τ_1 so for each pair of neutrosophic soft points $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists τ_2 neutrosophic soft b^{**} open set $(\tilde{f},\theta)$ and τ_1 neutrosophic soft b^{**} open set $(\tilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{g},\theta)$ $(\tilde{f},\theta) \widetilde{\cap} (\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$. In each case $(\tilde{f},\theta)$,$(\tilde{g},\theta) \widetilde{\in} \tau_1 \vee \tau_2$ and hence $\langle \mathcal{X}, \tau_1 \vee \tau_2, \theta \rangle$ is neutrosophic soft T_2 space.

Theorem 2.25 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space and Y be a non-empty sub set of $\mathcal{X}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pair-wise neutrosophic soft T_2 space then$\langle Y, \tau_{1Y}, \tau_{2Y}, \theta \rangle$ is pair wise neutrosophic soft T_2 space

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space.Let $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$be neutrosophic soft points then $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$. If $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ is pairwise neutrosophic soft T_2 space, Then there exists τ_1 neutrosophic soft b^{**}open set $(\tilde{f},\theta)$ and τ_2 n neutrosophic soft b^{**}open set $(\tilde{g},\theta)$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\tilde{g},\theta)$ $(\tilde{f},\theta) \widetilde{\cap} (\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$. So for each $\alpha \in \theta$,

$x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} \tilde{f}(\alpha), x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} \tilde{g}(\alpha)$, $\tilde{f}(\alpha) \cap \tilde{g}(\alpha) \cong 0_{(\mathcal{X},\theta)}$ and this implies

$x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} Y\tilde{\cap}\tilde{f}(\alpha), x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \in Y\tilde{\cap}\tilde{g}(\alpha)$, $\tilde{f}(\alpha)\tilde{\cap}\tilde{g}(\alpha) \cong 0_{(\mathcal{X},\theta)}$. Hence

$x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} \langle Y_{\tilde{f}},\theta\rangle, x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} \langle Y_{\tilde{g}},\theta\rangle$

$\langle Y_{\tilde{f}},\theta\rangle \cap \langle Y_{\tilde{g}},\theta\rangle \cong 0_{(\mathcal{X},\theta)}$. Where $\langle Y_{\tilde{f}},\theta\rangle$ is neutrosophic soft b^{**} open set in τ_{1Y}, $\langle Y_{\tilde{g}},\theta\rangle$ is neutrosophic soft b^{**}open set in τ_{2Y} therefore $\tau_{1Y}$is neutrosophic soft T_2 space with respect to τ_{2Y}. Similarly, it can be proved that τ_{2Y} is neutrosophic soft T_2 space with respect to τ_{1Y}. Thus $\langle Y,\tau_{1Y},\tau_{2Y},\theta\rangle$ is pair wise neutrosophic soft T_2 space.

Theorem 2.26 Let $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ be neutrosophic soft bitopological space then the following are equivalent

1). $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ be a pair wise neutrosophic soft T_2 space.

2). Let $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points then $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ then there is a neutrosophic soft b^{**} open set $(\tilde{f},\theta) \tilde{\in} \tau_1$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} \mathcal{X} - (\overline{(\tilde{f},\theta)})^{\tau_1}$

Proof. 1) implies 2) suppose $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ is a pair wise neutrosophic soft T_2 space and for $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ be neutrosophic soft points then $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$,$\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ pair wise neutrosophic soft T_2 space implies there exists neutrosophic soft b^{**}open set $(\tilde{f},\theta) \tilde{\in} \tau_1$ and $(\tilde{g},\theta) \tilde{\in} \tau_2$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{f},\theta) \tilde{\in} \tau_1$ and $(\tilde{g},\theta) \tilde{\in} \tau_2$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{f},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{g},\theta)$ $(\tilde{f},\theta) \tilde{\cap} (\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$. Similarly for the second case so that $((\tilde{f},\theta) \subseteq ((\tilde{g},\theta))^{C}$ also $(\overline{(\tilde{f},\theta)})^{\tau_1}$ is the smallest n neutrosophic soft b^{**}close set in τ_2 that contains $(\tilde{f},\theta)$, $(\tilde{g},\theta)$ and is a neutrosophic soft b^{**} close set $\tilde{\in} \tau_2$.

So $(\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2} \tilde{\subseteq} ((\tilde{\mathcal{g}},\theta))^{c}$, $((\tilde{\mathcal{g}},\theta)) \tilde{\subseteq} ((\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2})^{c}$ .Thus $x_{2\,(\mathfrak{s}_2,\mathfrak{t}_2,\mathfrak{u}_2)}^{e_2} \tilde{\in} (\tilde{\mathcal{g}},\theta) \tilde{\subseteq} ((\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2})^{c}$ or $x_{2\,(\mathfrak{s}_2,\mathfrak{t}_2,\mathfrak{u}_2)}^{e_2} \tilde{\in} \mathcal{X} - (\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2}$. Let $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1}, x_{2\,(\mathfrak{s}_2,\mathfrak{t}_2,\mathfrak{u}_2)}^{e_2}$ be neutrosophic soft points, $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1} \tilde{\cap} x_{2\,(\mathfrak{s}_2,\mathfrak{t}_2,\mathfrak{u}_2)}^{e_2} \cong 0_{(\mathcal{X},\theta)}$ then there exists a neutrosophic soft b^{**} open set $(\tilde{\mathfrak{f}},\theta)$ $\tilde{\in}$ τ_1 such that $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1} \tilde{\in} (\tilde{\mathfrak{f}},\theta)$ and $x_{2\,(\mathfrak{s}_2,\mathfrak{t}_2,\mathfrak{u}_2)}^{e_2} \tilde{\in} \mathcal{X} - (\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2}$ is $(\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2}$ neutrosophic soft b^{**} close set in τ_2 so $(\tilde{\mathcal{g}},\theta) \cong \mathcal{X} - (\overline{\tilde{\mathfrak{f}},\theta})^{\tau_2} \in \tau_2$. Now $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1} \tilde{\in} (\tilde{\mathfrak{f}},\theta)$ and $x_{2\,(\mathfrak{s}_2,\mathfrak{t}_2,\mathfrak{u}_2)}^{e_2} \tilde{\in} (\tilde{\mathcal{g}},\theta)$and $(\tilde{\mathfrak{f}},\theta) \cap (\tilde{\mathcal{g}},\theta) \cong (\tilde{\mathfrak{f}},\theta) \tilde{\cap}(\mathcal{X} - (\overline{\tilde{\mathfrak{f}},\theta})^{\tau_2} \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta)\tilde{\cap}(\mathcal{X} - (\tilde{\mathfrak{f}},\theta)$.

Since $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} (\overline{(\tilde{\mathfrak{f}},\theta)})^{\tau_2}$ and hence $(\tilde{\mathfrak{f}},\theta)\tilde{\cap}(\tilde{\mathcal{g}},\theta) \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta)\tilde{\cap}(\mathcal{X} - (\tilde{\mathfrak{f}},\theta) \cong 0_{(\mathcal{X},\theta)}$ and thus $(\tilde{\mathfrak{f}},\theta) \tilde{\cap}(\tilde{\mathcal{g}},\theta) \cong 0_{(\mathcal{X},\theta)}$.

3. Results in Terms of Interior and Closure

In this section, neutrosophic soft limit point, neutrosophic soft interior point, few results on the basis of these definitions are discussed. These results are discussed terms of neutrosophic soft interior and neutrosophic soft closures with respect to soft points.

Definition 3.1 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space and $(\tilde{\mathfrak{f}},\theta)$ neutrosophic soft open set. A neutrosophic soft point $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1}$ is called a neutrosophic soft limit point of $(\tilde{\mathfrak{f}},\theta)$ if every neutrosophic soft b^{**} open set$(\tilde{\mathcal{g}},\theta)$ where $(\tilde{\mathcal{g}},\theta) \cong (\tilde{\mathcal{g}}_1,\theta)\tilde{\cup}(\tilde{\mathcal{g}}_2,\theta)$ such that $(\tilde{\mathcal{g}}_1,\theta) \tilde{\in} \tau_1$ and $(\tilde{\mathcal{g}}_2,\theta) \tilde{\in} \tau_1$ containing $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1}$contains at least one neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}},\theta)$ other than $x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1}$ that is$\left[(\tilde{\mathcal{g}},\theta) - \left\{ x_{1\,(\mathfrak{s}_1,\mathfrak{t}_1,\mathfrak{u}_1)}^{e_1} \right\}\right] \tilde{\cap} (\tilde{\mathfrak{f}},\theta) \neq 0_{(\mathcal{X},\theta)}$.The set of all neutrosophic soft limit point of $(\tilde{\mathfrak{f}},\theta)$ is called the derived neutrosophic soft set and is denoted by $(\tilde{\mathfrak{f}},\theta)^{d}$. If there exists neutrosophic soft b^{**} open set $(\tilde{g},\theta)$ such that $\left((\tilde{g},\theta) - x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\right)\tilde{\cap}(\tilde{\mathfrak{f}},\theta) \cong 0_{(\mathcal{X},\theta)}$.Then $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}$ is unable to be the neutrosophic soft limit point of $(\tilde{\mathfrak{f}},\theta)$.

Definition 3.2 Let $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ be neutrosophic soft bitopological space and $(\tilde{\mathfrak{f}},\theta)$ neutrosophic soft b^{**} open set. A soft point $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \in (\tilde{\mathfrak{f}},\theta)$ is called neutrosophic soft interior point of $(\tilde{\mathfrak{f}},\theta)$ if we can search out neutrosophic soft b^{**} open set $(\tilde{g},\theta)$, where $(\tilde{g},\theta) \cong (\tilde{g_1},\theta)\tilde{\cup}(\tilde{g_2},\theta)$ such that $(\tilde{g_1},\theta) \in \tau_1$ and $(\tilde{g_2},\theta) \in \tau_1$ containing $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \in (\tilde{g},\theta) \subseteq (\tilde{\mathfrak{f}},\theta)$.

Theorem 3.3 Let $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ be a neutrosophic soft bitopological space. If $(\tilde{\mathfrak{f}},\theta)$, $(\tilde{g},\theta)$ be neutrosophic soft subsets then

(1) $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} (\tilde{g},\theta) \Rightarrow (\tilde{\mathfrak{f}},\theta)^{d} \tilde{\subseteq} (\tilde{g},\theta)^{d}$

(2) $[(\tilde{\mathfrak{f}},\theta)\ \tilde{\cup}\ (\tilde{g},\theta)]^{d} \cong (\tilde{\mathfrak{f}},\theta)^{d}\ \tilde{\cup}\ (\tilde{g},\theta)^{d}$

(3) $[(\tilde{\mathfrak{f}},\theta)\ \tilde{\cap}((\tilde{g},\theta)]^{d} \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta)^{d}\tilde{\cap}(\tilde{g},\theta)^{d}$

Proof. Let $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}},\theta)^{d}$ so for every neutrosophic soft b^{**} open set $(\mathfrak{h},\theta)$ where $(\mathfrak{h},\theta) \cong (\mathfrak{h}_1,\theta)\ \tilde{\cup}\ (\mathfrak{h}_2,\theta)$ such that $(\mathfrak{h}_1,\theta) \tilde{\in} \tau_1$ and $(\mathfrak{h}_2,\theta) \tilde{\in} \tau_2$ contains $x^{e}_{(s,t,u)}$, we have $\left((\mathfrak{h},\theta) - x^{e}_{(s,t,u)}\right)\tilde{\cap}(\tilde{\mathfrak{f}},\theta) \cong 0_{(\mathcal{X},\theta)}$.

But $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} (\tilde{g},\theta)$ and so $\left((\mathfrak{h},\theta) - x^{e}_{(s,t,u)}\right)\tilde{\cap}(\tilde{g},\theta) \cong 0_{(\mathcal{X},\theta)}$ implies $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{g},\theta)^{d}$.

Hence $(\tilde{\mathfrak{f}},\theta)^{d} \tilde{\subseteq} (\tilde{g},\theta)^{d}$ Rest are trivial.

Theorem 3.4 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space and $(\tilde{\mathfrak{f}}, \theta)$ be neutrosophic soft subset, then $(\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$ is neutrosophic soft b^{**} close set.

Proof. We will prove $\left((\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d\right)^c$ is neutrosophic soft b^{**} open set. Let $x^e_{(s,t,u)} \tilde{\in} \left((\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d\right)^c$ implies $x^e_{(s,t,u)} \tilde{\notin} (\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$ implies $x^e_{(s,t,u)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \neq 0_{(\mathcal{X}, \theta)}$, $x^e_{(s,t,u)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta)^d \neq 0_{(\mathcal{X}, \theta)}$ and now $x^e_{(s,t,u)} \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \neq 0_{(\mathcal{X}, \theta)}$ so we can find neutrosophic soft b^{**} open set $(\tilde{\mathcal{g}}, \theta)$ where $(\tilde{\mathcal{g}}, \theta) \cong (\tilde{\mathcal{g}}_1, \theta) \tilde{\cup} (\tilde{\mathcal{g}}_2, \theta)$ such that $(\tilde{\mathcal{g}}_1, \theta) \tilde{\in} \tau_1$ and $(\tilde{\mathcal{g}}_2, \theta) \tilde{\in} \tau_1$ such that $\left((\tilde{\mathcal{g}}, \theta) - x^e_{(s,t,u)}\right) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \neq 0_{(\mathcal{X}, \theta)}$ implies $(\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X}, \theta)}$.....(1). we claim that $(\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta)^d \cong 0_{(\mathcal{X}, \theta)}$ because if $(\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta)^d \ncong 0_{(\mathcal{X}, \theta)}$ then there exists $y^{e'}_{(s',t',u')} \tilde{\in} (\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta)^d$ implies $y^{e'}_{(s',t',u')} \tilde{\in} (\tilde{\mathcal{g}}, \theta)$ and $y^{e'}_{(s',t',u')} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)^d$ implies $y^{e'}_{(s',t',u')}$ is neutrosophic soft limit point of $(\tilde{\mathfrak{f}}, \theta)$ so every neutrosophic sof t b^{**} open set $(\tilde{\mathcal{g}}, \theta)$contains $y^{e'}_{(s',t',u')}$intersects $(\tilde{\mathfrak{f}}, \theta)$in some neutrosophic soft point other than $y^{e'}_{(s',t',u')}$ that is $\left((\tilde{\mathcal{g}}, \theta) - y^{e'}_{(s',t',u')}\right) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \cong 0_{(\mathcal{X}, \theta)}$implies $(\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \neq 0_{(\mathcal{X}, \theta)}$ but it contradicts results (1) $thus (\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathcal{g}}, \theta) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta)^d \cong 0_{(\mathcal{X}, \theta)}$ implies $(\tilde{\mathcal{g}}, \theta) \tilde{\subseteq} \left((\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d\right)^c$ or $x^e_{(s,t,u)} \tilde{\in} (\tilde{\mathcal{g}}, \theta) \tilde{\subseteq} \left((\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d\right)^c$ implies $x^e_{(s,t,u)}$ is neutrosophic soft interior point of $\left((\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d\right)$. Thus $\left((\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d\right)^c$ is neutrosophic soft b^{**} open and consequently $(\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$is neutrosophic soft b^{**} close.

Theorem 3.5 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space and $(\tilde{\mathfrak{f}}, \theta)$ be neutrosophic soft subset, then $(\tilde{\mathfrak{f}}, \theta)$ is neutrosophic soft b^{**} close set if $(\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)$

Proof. We show $(\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)$. Let $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)^d$ implies $x^{e}_{(s,t,u)}$ is neutrosophic soft limit point of $(\tilde{\mathfrak{f}}, \theta)$ so for $\tilde{\in} \tau_1$ *and* $(\widetilde{\mathcal{g}_2}, \theta) \tilde{\in} \tau_2$ contains $x^{e}_{(s,t,u)}$, we have $\left((\tilde{\mathcal{g}}, \theta) - x^{e}_{(s,t,u)}\right) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \tilde{\neq} 0_{(\mathcal{X},\theta)}$ ... (1). We are to show that $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ suppose on contrary that $x^{e}_{(s,t,u)} \tilde{\notin} (\tilde{\mathfrak{f}}, \theta)$ Then $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)^c$. Then $(\tilde{\mathfrak{f}}, \theta)^c$ is neutrosophic *soft* b^{**} open because $(\tilde{\mathfrak{f}}, \theta)$ is neutrosophic soft b^{**} close. Since result (1) is true for all neutrosophic soft b^{**} close sets $(\tilde{\mathcal{g}}, \theta)$ containing $x^{e}_{(s,t,u)}$ so replace $(\tilde{\mathcal{g}}, \theta)$ by $(\tilde{\mathfrak{f}}, \theta)^c$ in (1), we get $((\tilde{\mathfrak{f}}, \theta)^c - x^{e}_{(s,t,u)}) \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \tilde{\neq} 0_{(\mathcal{X},\theta)}$. Obviously $(\tilde{\mathfrak{f}}, \theta)^c - x^{e}_{(s,t,u)} \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)^c$. This implies $(\tilde{\mathfrak{f}}, \theta)^c \tilde{\cap} (\tilde{\mathfrak{f}}, \theta) \tilde{\neq} 0_{(\mathcal{X},\theta)}$. So we are forced to accept that $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ that is, $x^{e}_{(s,t,u)} \in (\tilde{\mathfrak{f}}, \theta)^d$ implies $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ therefore $(\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)$ and conversely let $((\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)$. We are to prove that neutrosophic soft close. $(\tilde{\mathfrak{f}}, \theta)$ such that $(\tilde{\mathfrak{f}}, \theta) \tilde{\neq} (\tilde{\mathfrak{f}}_1, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}_2, \theta)$ such that $(\tilde{\mathfrak{f}}_1, \theta) \tilde{\in} \tau_1$ and $(\tilde{\mathfrak{f}}_2, \theta) \tilde{\in} \tau_2$ such that $(\tilde{\mathfrak{f}}_1, \theta)$ and $(\tilde{\mathfrak{f}}_2, \theta)$ are neutrosophic soft b^{**} close in their respective structures. Let $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)^c$ implies $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}, \theta)$ but $(\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)$ so $x^{e}_{(s,t,u)} \tilde{\notin} (\tilde{\mathfrak{f}}, \theta)^d$ implies $x^{e}_{(s,t,u)}$ is not neutrosophic soft limit point of $(\tilde{\mathfrak{f}}, \theta)$ we can search out neutrosophic soft b^{**}*open set* $(\tilde{\mathfrak{f}}_3, \theta)$ where $(\tilde{\mathfrak{f}}_3, \theta) \tilde{\neq} (\tilde{\mathfrak{f}}_4, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}_5, \theta)$ such that

$(\tilde{\mathfrak{f}}_4, \theta) \tilde{\in} \tau_1$ and $(\tilde{\mathfrak{f}}_5, \theta) \tilde{\in} \tau_2$ such that $\tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)^c$ or $x^{e}_{(s,t,u)} \tilde{\in} (\tilde{\mathfrak{f}}_3, \theta) \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta)^c$ implies $x^{e}_{(s,t,u)}$ is neutrosophic soft interior point of $(\tilde{\mathfrak{f}}, \theta)^c$ implies $(\tilde{\mathfrak{f}}, \theta)^c$ is neutrosophic soft b^{**}open and consequently $(\tilde{\mathfrak{f}}, \theta)$ is neutrosophic soft b^{**}close.

Theorem 3.6 Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space if $(\tilde{\mathfrak{f}}, \theta)$ be neutrosophic soft sub set then $\overline{((\tilde{\mathfrak{f}}, \theta))} \cong (\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$

Proof. Since $\overline{(\tilde{\mathfrak{f}}, \theta)}$ is the smallest neutrosophic soft b^{**} close set containing $(\tilde{\mathfrak{f}}, \theta)$ and so $(\tilde{\mathfrak{f}}, \theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)}$ implies $((\tilde{\mathfrak{f}}, \theta))^d \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)}^d$. Also $\overline{(\tilde{\mathfrak{f}}, \theta)}^d \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)}$.Therefore $(\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)}^d \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)} \Rightarrow (\tilde{\mathfrak{f}}, \theta)^d \subseteq \overline{(\tilde{\mathfrak{f}}, \theta)}$ also $(\tilde{\mathfrak{f}}, \theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)} \Rightarrow$ $(\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)}$... (1)

Again since $(\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$ is a neutrosophic soft b^{**}close set containing $(\tilde{\mathfrak{f}}, \theta)$. *But* $\overline{(\tilde{\mathfrak{f}}, \theta)}$ is the smallest neutrosophic soft close set containing $(\tilde{\mathfrak{f}}, \theta)$, so $(\tilde{\mathfrak{f}}, \theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}}, \theta)} \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$ that is $\overline{(\tilde{\mathfrak{f}}, \theta)} \tilde{\subseteq} (\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$ … (2). *Hence* $\overline{(\tilde{\mathfrak{f}}, \theta)} \cong (\tilde{\mathfrak{f}}, \theta) \tilde{\cup} (\tilde{\mathfrak{f}}, \theta)^d$

Theorem 3.7 *Let* $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bitopological space. If $(\tilde{\mathfrak{f}}, \theta)$ be neutrosophic soft set then $(\tilde{\mathfrak{f}}, \theta)$ is neutrosophic soft b^{**} close set if and only if $\overline{(\tilde{\mathfrak{f}}, \theta)} \cong (\tilde{\mathfrak{f}}, \theta)$.

Proof. Let $(\tilde{\mathfrak{f}},\theta)$ be neutrosophicsoft set then $(\tilde{\mathfrak{f}},\theta)$ is neutrosophic soft b^{**}close set so that $(\tilde{\mathfrak{f}},\theta) \cong (\tilde{\mathfrak{f}}_1,\theta) \tilde{\cup} (\tilde{\mathfrak{f}}_2,\theta)$ such that $(\tilde{\mathfrak{f}}_1,\theta) \in \tau_1$ and $(\tilde{\mathfrak{f}}_2,\theta) \in \tau_1$ and are both neutrosophic soft b^{**}close sets in their respective domain. Then $(\tilde{\mathfrak{f}}_1,\theta)^d \cong (\tilde{\mathfrak{f}}_1,\theta)$this implies $(\tilde{\mathfrak{f}}_1,\theta) \tilde{\cup} (\tilde{\mathfrak{f}}_1,\theta)^d \cong (\tilde{\mathfrak{f}}_1,\theta) \Rightarrow \overline{(\tilde{\mathfrak{f}}_1,\theta)} \cong (\tilde{\mathfrak{f}},\theta)$ and conversely let $\overline{(\tilde{\mathfrak{f}},\theta)} \cong (\tilde{\mathfrak{f}},\theta)$this implies that $(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{\mathfrak{f}},\theta)^d \cong (\tilde{\mathfrak{f}},\theta) \Rightarrow (\tilde{\mathfrak{f}},\theta)^d \cong (\tilde{\mathfrak{f}},\theta) \Rightarrow (\tilde{\mathfrak{f}},\theta)$is neutrosophic soft b^{**}close.

Theorem 3.8 Let $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ be neutrosophic soft bitopological space. If $(\tilde{\mathfrak{f}},\theta)$ and $(\tilde{g},\theta)$ are neutrosophic soft subsets such that $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} (\tilde{g},\theta)$ then i) $\overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\subseteq} \overline{(\tilde{g},\theta)}$ ii) $\overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)} \cong \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)}$ iii) $\overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)} \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta) \tilde{\cap} \overline{(\tilde{g},\theta)}$

Proof. i) If $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} (\tilde{g},\theta) \Rightarrow (\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{\mathfrak{f}},\theta)^d \tilde{\subseteq} (\tilde{g},\theta) \tilde{\cup} (\tilde{g},\theta)^d \Rightarrow \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\subseteq} \overline{(\tilde{g},\theta)}$ ii) since $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta) \Rightarrow \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)}$… (1)

Also since $(\tilde{g},\theta) \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta) \Rightarrow \overline{(\tilde{g},\theta)} \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)}$… (2). Maxing results (1) and (2) we have $\overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)} \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)}$…(3) to get the required result we proceeds as since $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)}$ and $(\tilde{g},\theta) \tilde{\subseteq} \overline{(\tilde{g},\theta)} \Rightarrow (\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)}$ since $\overline{(\tilde{\mathfrak{f}},\theta)}$ *and* $\overline{(\tilde{g},\theta)}$ are neutrosophic soft b^{**}close sets so is $\overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)}$ and it contains $(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)$ but $\overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)}$ is the smallest neutrosophic soft b^{**} close sub set containing $(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)$. So $(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)} \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)}$ $\Rightarrow \overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)} \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)}$ …(4) maxing results (3) and (4) we have $\overline{(\tilde{\mathfrak{f}},\theta) \tilde{\cup} (\tilde{g},\theta)} \cong \overline{(\tilde{\mathfrak{f}},\theta)} \tilde{\cup} \overline{(\tilde{g},\theta)}$

iii) Since $(\tilde{\mathfrak{f}},\theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)}$ ` *and* $(\tilde{g},\theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)} \Longrightarrow (\tilde{\mathfrak{f}},\theta)\, \tilde{\cap}(\tilde{g},\theta) \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)\, \tilde{\cup}\, (\tilde{g},\theta)} \tilde{\subseteq} \overline{(\tilde{\mathfrak{f}},\theta)}\, \tilde{\cup}\, \overline{(\tilde{g},\theta)}$ that is $\overline{(\tilde{\mathfrak{f}},\theta)\, \tilde{\cup}\, (\tilde{g},\theta)} \tilde{\subseteq} (\tilde{\mathfrak{f}},\theta)\tilde{\cap}\overline{(\tilde{g},\theta)}$.

4. Product of Few Mixed Structures

In this section some spaces are discussed in terms of product with respect to soft points of the spaces. Product of neutrosophic soft T_0 spaces, neutrosophic soft T_1 spaces, neutrosophic soft T_2 spaces and their engagement with neutrosophic soft coordinate spaces with respect to soft points are studied under soft b^{**} open sets in neutrosophic soft bi-topological space. Finally, the most important structure known as entropy is discussed and on the basis of this structures an example is given which is related to engineering which goes like this, in order to obtain an effective risk supervision in the field of engineering, certain risks are classified along with some parameters and these risks are evaluated by the team of experts (engineers).

Theorem 4.1. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bi-topological space such that it is neutrosophic soft Hausdorff space if and only if for every two neutrosophic soft points $x1^{e_1}_{(s_1,t_1,u_1)}, x2^{e_2}_{(s_2,t_2,u_2)}$, $x1^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} x2^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists neutrosophic soft b^{**}close sets $(\tilde{l_1},\theta)$ and $(\tilde{l_2},\theta)$ that $x1^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{l_1},\theta), x2^{e_2}_{(s_2,t_2,u_2)} \tilde{\cap}(\tilde{l_1},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $x2^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{l_2},\theta), x1^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap}(\tilde{l_2},\theta) \cong 0_{(\mathcal{X},\theta)}$ and $(\tilde{l_1},\theta)\, \tilde{\cup}\, (\tilde{l_2},\theta) \cong 0_{(\mathcal{X},\theta)}$.

Proof. Let $\langle \mathcal{X}, \tau_1, \tau_2, \theta \rangle$ be neutrosophic soft bi-topological space, then for neutrosophic soft points $x1^{e_1}_{(s_1,t_1,u_1)}, x2^{e_2}_{(s_2,t_2,u_2)}$, $x1^{e_1}_{(s_1,t_1,u_1)} \tilde{\cap} x2^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ we can find two neutrosophic soft b^{**}open sets such that $x1^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{g},\theta)$ and $x2^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{h},\theta)$ such that $(\tilde{\mathcal{g}},\theta)\ \tilde{\sqcap}(\tilde{h},\theta)\cong 0_{(\mathcal{X},\theta)}$. Since $(\tilde{\mathcal{g}},\theta)\ \tilde{\sqcap}(\tilde{h},\theta)\cong 0_{(\mathcal{X},\theta)}$ .So $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\in}(\tilde{\mathcal{g}},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\in}(\tilde{h},\theta)\Longrightarrow$ $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\in}(\tilde{\mathcal{g}},\theta), x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\sqcap}(\tilde{\mathcal{g}},\theta)\cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\in}(\tilde{h},\theta)$, $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\sqcap}(\tilde{h},\theta)\cong 0_{(\mathcal{X},\theta)}$ this implies $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\notin}(\tilde{\mathcal{g}},\theta)^{C}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\in}(\tilde{\mathcal{g}},\theta)^{c}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\notin}(\tilde{h},\theta)^{C}, x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\in}(\tilde{h},\theta)^{c}$ or $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\in}(\tilde{h},\theta)^{c}$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\notin}(\tilde{h},\theta)^{c}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\in}(\tilde{\mathcal{g}},\theta)^{c}, x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\notin}(\tilde{\mathcal{g}},\theta)^{c}$.

Let $(\tilde{l_1},\theta)\cong(\tilde{h},\theta)^{c}$ and $(\tilde{l_2},\theta)=(\tilde{\mathcal{g}},\theta)^{c}$ .Then $(\tilde{l_2},\theta)$ and $(\tilde{l_1},\theta)$ are neutrosophic soft b^{**} close sets and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\in(\tilde{l_1},\theta), x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\tilde{\sqcap}(\tilde{l_1},\theta)\cong 0_{(\mathcal{X},\theta)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\in(\tilde{l_2},\theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\tilde{\sqcap}(\tilde{l_2},\theta)\cong 0_{(\mathcal{X},\theta)}$. Also $(\tilde{l_1},\theta)\ \tilde{\cup}\ (\tilde{l_1},\theta)\cong(\tilde{\mathcal{g}},\theta)^{c}\ \tilde{\cup}\ (\tilde{h},\theta)^{c}\cong\left((\tilde{\mathcal{g}},\theta)\tilde{\sqcap}(\tilde{h},\theta)\right)^{c}\cong\left(0_{(\mathcal{X},\theta)}\right)^{c}\cong\mathcal{X}$. Conversely, let us suppose that for every two neutrosophic soft points $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\tilde{\sqcap}x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\cong 0_{(\mathcal{X},\theta)}$, there exists neutrosophic soft b^{**} close sets $(\tilde{l_1},\theta)$ and $(\tilde{l_2},\theta)$ so $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\in}(\tilde{l_1},\theta)$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\notin}(\tilde{l_1},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\in}(\tilde{l_2},\theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\notin}(\tilde{l_2},\theta)$ and $(\tilde{l_1},\theta)\ \tilde{\cup}\ (\tilde{l_2},\theta)\cong\mathcal{X}$. We prove that $\langle\mathcal{X},\tau_1,\tau_2,\theta\rangle$ be a neutrosophic soft Hausdorff space. We proceed as fallow to catch the required space. $(\tilde{\mathcal{g}},\theta)\cong(\tilde{l_1},\theta)^{c}$ and $(\tilde{h},\theta)=(\tilde{l_2},\theta)^{C}$ then $(\tilde{\mathcal{g}},\theta)$ and $(\tilde{h},\theta)$ are neutrosophic soft b^{**} open sets and $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\in}(\tilde{l_1},\theta), x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\notin}(\tilde{l_1},\theta)$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\ \tilde{\in}(\tilde{l_2},\theta), x_{1}{}^{e_1}_{(s_1,t_1,u_1)}\ \tilde{\notin}(\tilde{l_2},\theta)$ implies that $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}\tilde{\sqcap}(\tilde{l_1},\theta)\cong 0_{(\mathcal{X},\theta)}$ and

${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap}(\tilde{l_2},\theta) \cong 0_{(\mathcal{X},\theta)} \Rightarrow$ ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{l_1},\theta)^c$ and ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{l_2},\theta)^c$ $\Rightarrow {x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{h},\theta)$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{g},\theta)$. Moreover $(\tilde{g},\theta) \tilde{\sqcap}(\tilde{h},\theta) \cong (\tilde{l_2},\theta)^c \tilde{\sqcap}(\tilde{l_1},\theta)^c = \left((\tilde{l_2},\theta)\tilde{\sqcap}(\tilde{l_1},\theta)\right)^c \cong \mathcal{X} \cong 0_{(\mathcal{X},\theta)}$. This proves that for every two soft neutrosophic soft points ${x_1}^{e_1}_{(s_1,t_1,u_1)}$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)}$ such that ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\sqcap} {x_2}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ there exists neutrosophic soft b^{**} open sets $(\tilde{g},\theta)$ and $(\tilde{h},\theta)$ such hat ${x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\tilde{g},\theta)$ and ${x_2}^{e_2}_{(s_2,t_2,u_2)} \tilde{\in} (\tilde{h},\theta)$ such that $(\tilde{g},\theta) \tilde{\sqcap}(\tilde{h},\theta) \cong 0_{(\mathcal{X},\theta)}$

Theorem 4.2 If $\langle \mathcal{X},\tau_1,\tau_2,\theta\rangle$ and $\langle Y,\mathcal{J}_1,\mathcal{J}_2,\theta\rangle$ be two neutrosophic soft Hausdorff space then their product is also neutrosophic soft Hausdorff space.

Proof. Let $(\tilde{\lambda_1},\theta)$ and $(\tilde{\lambda_2},\theta)$ be two neutrosophic soft points such that $(\tilde{\lambda_1},\theta) \cong ({x_1}^{e_1}_{(s_1,t_1,u_1)}, {x_2}^{e_2}_{(s_2,t_2,u_2)})$ and $(\tilde{\lambda_2},\theta) \cong ({y_1}^{e_1}_{(s_1,t_1,u_1)}, {y_2}^{e_2}_{(s_2,t_2,u_2)})$ and $(\tilde{\lambda_1},\theta)\tilde{\sqcap} (\tilde{\lambda_2},\theta) \cong 0_{(\mathcal{X},\theta)}$. Suppose $({x_1}^{e_1}_{(s_1,t_1,u_1)}, {y_1}^{e_1}_{(s_1,t_1,u_1)}) \tilde{\in} \widetilde{\mathcal{X}_\theta}$ and $({x_1}^{e_1}_{(s_1,t_1,u_1)}, {y_1}^{e_1}_{(s_1,t_1,u_1)}) \tilde{\in} \widetilde{Y_\theta}$ if ${x_1}^{e_1}_{(s_1,t_1,u_1)} \neq {y_1}^{e_1}_{(s_1,t_1,u_1)}$ are two distinct points of $\widetilde{\mathcal{X}_\theta}$ but $\widetilde{\mathcal{X}_\theta}$ is given to be neutrosophic soft Hausdorff space so there exists two distinct neutrosophic soft b^{**}open sets $(\widetilde{g_1},\theta)$ and $(\widetilde{h_1},\theta)$ such that $({x_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\widetilde{g_1},\theta)$ and $({y_1}^{e_1}_{(s_1,t_1,u_1)} \tilde{\in} (\widetilde{h_1},\theta)$. Now $(\widetilde{g_1},\theta) \times Y$ and $(\widetilde{h_1},\theta) \times Y$ are two distinct neutrosophic soft b^{**} open sets in $\mathcal{X} \times Y$ containing $(\tilde{\lambda_1},\theta)$ and $(\tilde{\lambda_2},\theta)$ respectively that is $(\tilde{\lambda_1},\theta) \tilde{\in} (\widetilde{g_1},\theta) \times Y$ and $(\tilde{\lambda_1},\theta) \tilde{\in} (\widetilde{h_1},\theta) \times Y$ and $(\widetilde{g_1},\theta) \times Y)\tilde{\sqcap}((\widetilde{h_1},\theta) \times Y) \cong 0_{(\mathcal{X},\theta)}$

Thus $\mathcal{X} \times Y$ is a neutrosophic soft Hausdorff space. Now if $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\neq} y_{2}{}^{e_2}_{(s_2,t_2,u_2)}, x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\cap} y_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ are two neutrosophic soft points of $\mathcal{X}$ but Y is given to neutrosophic soft Hausdorff space so we can find two distinct neutrosophic soft b^{**} open sets $(\widetilde{g_2},\theta)$ and $(\widetilde{h_2},\theta)$ such that $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{g_1},\theta)$ and $y_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{h_2},\theta)$. Now $\mathcal{X} \times (\widetilde{g_2},\theta)$ and $\mathcal{X} \times (\widetilde{h_2},\theta)$ are two distinct neutrosophic soft b^{**} open sets in $\mathcal{X} \times Y$ containing $(\widetilde{\lambda_1},\theta)$ and $(\widetilde{\lambda_2},\theta)$ respectively that is $(\widetilde{\lambda_1},\theta) \in \mathcal{X} \times (\widetilde{g_2},\theta)$ and $(\widetilde{\lambda_2},\theta) \in \mathcal{X} \times (\widetilde{h_2},\theta)$ such that $\mathcal{X} \times (\widetilde{g_2},\theta)) \widetilde{\cap} (\mathcal{X} \times (\widetilde{h_2},\theta) \cong 0_{(\mathcal{X},\theta)}$. Hence the product of any finite number of neutrosophic soft Hausdorff spaces is a neutrosophic soft Hausdorff space.

Theorem 4.3 The product of any number of neutrosophic soft Hausdorff spaces is a neutrosophic soft Hausdorff space.

Proof. Let us suppose that $\mathcal{X} \cong \prod_{i \widetilde{\in} I} \mathcal{X}_i$ provided $\mathcal{X}_i \forall\, i \in I$ is a neutrosophic soft Hausdorff space at its own right. We need to show that $\mathcal{X}$ is neutrosophic soft Hausdorff space. For this let $x_{1}{}^{e_1}_{(s_1,t_1,u_1)}$ and $x_{2}{}^{e_2}_{(s_2,t_2,u_2)}$ such that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\cap} x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \cong 0_{(\mathcal{X},\theta)}$ be two neutrosophic soft points of $\prod_{i \widetilde{\in} I} \mathcal{X}_i$ such that they are disjoint. This implies $x^{e}_{(s,t,u)} \cong (x^{e}_{(s,t,u)_i})_{i \in I}$ and $y^{e'}_{(s',t',u')} \cong (y^{e'}_{(s',t',u')_i})_{i \in I}$ where $x^{e}_{(s,t,u)_i}$ and $y^{e'}_{(s',t',u')_i} \widetilde{\in} \widetilde{\mathcal{X}_{\theta_i}} \forall\, i \in I$. But $x^{e}_{(s,t,u)}$ and $y^{e'}_{(s',t',u')}$ are distinct, so $(x^{e}_{(s,t,u)_i})_{i \in I}$ and $(y^{e'}_{(s',t',u')_i})_{i \in I}$ are distinct which leads to $x^{e}_{(s,t,u)_i}$ and $y^{e'}_{(s',t',u')_i}$ are distinct for some $i \in I$. Since the neutrosophic soft projection $\pi_i : \prod_{i \widetilde{\in} I} \widetilde{\mathcal{X}_{\theta_i}} \rightarrow \widetilde{\mathcal{X}_{\theta_i}}$ defined by

$\pi_1\left(x^{e}_{(s,t,u)}\right) \cong x^{e}_{(s,t,u)_i}$ for all $x^{e}_{(s,t,u)} \cong (x^{e}_{(s,t,u)_i})_{i\in I}$. Since $x^{e}_{(s,t,u)_i}$ and $y^{e'}_{(s',t',u')} \widetilde{\in} \widetilde{\mathcal{X}_{\theta_i}}$ such that $x^{e}_{(s,t,u)_i}$ and $y^{e'}_{(s',t',u')_i}$ are distinct and more over $\widetilde{\mathcal{X}_{\theta_i}}$ is neutrosophic soft Hausdorff space. So there exist neutrosophic soft b^{**} open sets $(\tilde{g}_\iota,\theta)$ and $(\tilde{h}_\iota,\theta)$ in $\mathcal{X}_i$ so that $x_{1}{}^{e_1}_{(s_1,t_1,u_1)} \widetilde{\in} (\widetilde{g_{\iota 1}},\theta)$,$y_{1}{}^{e'_1}_{(s'_1,t'_1,u'_1)} \widetilde{\in} (\widetilde{h_{\iota 1}},\theta)$, $(\widetilde{g_{\iota 1}},\theta) \widetilde{\sqcap}(\widetilde{h_{\iota 1}},\theta) \cong 0_{(\mathcal{X},\theta)}$, $x_{2}{}^{e_2}_{(s_2,t_2,u_2)} \widetilde{\in} (\widetilde{g_{\iota 2}},\theta), y_{2}{}^{e'_2}_{(s'_2,t'_2,u'_2)} \widetilde{\in} (\widetilde{h_{\iota 2}},\theta), (\widetilde{g_{\iota 2}},\theta)\widetilde{\sqcap}(\widetilde{h_{\iota 2}},\theta) \cong 0_{(\mathcal{X},\theta)}$, $x_{3}{}^{e_3}_{(s_3,t_3,u_3)} \widetilde{\in} (\widetilde{g_{\iota 3}},\theta)$, $y_{3}{}^{e'_3}_{(s'_3,t'_3,u'_3)} \widetilde{\in} (\widetilde{h_{\iota 3}},\theta), (\widetilde{g_{\iota 3}},\theta)\widetilde{\sqcap}(\widetilde{h_{\iota 3}},\theta) \cong 0_{(\mathcal{X},\theta)}$ and in general $x^{e}_{(s,t,u)_i} \widetilde{\in} (\tilde{g}_\iota,\theta)$, $y^{e'}_{(s',t',u')_i} \widetilde{\in} (\tilde{h}_\iota,\theta)$ and $(\tilde{g}_\iota,\theta) \widetilde{\sqcap}(\tilde{h}_\iota,\theta) \cong 0_{(\mathcal{X},\theta)} \Longrightarrow \pi_i\left(x^{e}_{(s,t,u)}\right) \widetilde{\in} (\tilde{g}_\iota,\theta)$, $\pi_i\left(y^{e'}_{(s',t',u')}\right) \widetilde{\in} (\tilde{h}_\iota,\theta) \Longrightarrow x^{e}_{(s,t,u)} \widetilde{\in} \pi_i{}^{-1}(\tilde{g}_\iota,\theta)$ and $\left(y^{e'}_{(s',t',u')}\right) \widetilde{\in} \pi_i{}^{-1} (h_i , \theta)$ and let $(\tilde{g},\theta) \cong \pi_i{}^{-1}(\tilde{g}_\iota,\theta)$ and $(\tilde{h},\theta) \cong \pi_i{}^{-1}(\tilde{h}_\iota,\theta)$. Then obviously, $x^{e}_{(s,t,u)} \widetilde{\in} (\tilde{g}_\iota,\theta)$ and $y^{e'}_{(s',t',u')} \widetilde{\in} (\tilde{h},\theta)$ since, $\pi_i{}^{-1}(\tilde{g}_\iota,\theta)$ and $\pi_i{}^{-1}(\tilde{h}_\iota,\theta)$ are neutrosophic soft open in $\prod_{i\widetilde{\in} I} \mathcal{X}_i$, so do $(\tilde{g},\theta)$ and $(\tilde{h},\theta)$ it is enough to show that $(\tilde{g},\theta)\widetilde{\sqcap}(\tilde{h},\theta) \cong 0_{(\mathcal{X},\theta)}$ .Consider $(\tilde{g},\theta)\widetilde{\sqcap}(\tilde{h},\theta) \cong \pi_i{}^{-1}(\tilde{g}_\iota,\theta)$ and $\widetilde{\sqcap}\pi_i{}^{-1}(\tilde{h}_\iota,\theta) \cong \pi_i{}^{-1}((g_\iota,\theta)\widetilde{\sqcap}(\tilde{h}_\iota,\theta)) \cong \pi_i{}^{-1}(0_{(\mathcal{X},\theta)}) \cong 0_{(\mathcal{X},\theta)}$.

Theorem 4.4 Let $\langle \pi,\tilde{\tau}_1,\varpi\rangle$and $\langle \mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle$ be two neutrosophic soft topological spaces on the crisp set $\pi$and $\mathfrak{L}$ respectively. $\langle \pi,\tilde{\tau}_1,\varpi\rangle * \langle \mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle$ be the neutrosophic soft product space, then the neutrosophic soft product space $\times\left[\left(\langle \pi,\tilde{\tau}_1,\varpi\rangle * \langle \mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft b_1^{**} space iff each neutrosophic soft coordinate space $\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^j\right]$ is neutrosophic soft b_1^{**}space.

Proof. Suppose each neutrosophic soft coordinate space $\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^j\right]$ is neutrosophic soft $b^{**}{}_1$ space and let $\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^{j:j\in\lambda}\right]$ be an element of $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$then, $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j \tilde{\in} \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^j\right]$ for each $j \tilde{\in} \Delta$. Since $[\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle]^j$ is neutrosophic soft b_1^{**} space, it follows that $\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j\right]$ is $\tilde{\tau}_1\tilde{\mathfrak{F}}_1$ neutrosophic soft b^{**}closed for each $j \tilde{\in} \Delta$. Now, each neutrosophic soft projection mapping $\mathfrak{H}^j$ being neutrosophic soft continuous, it follows that $(\mathfrak{H}^j)^{-1}\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j\right]$ is soft $\tilde{\tau}_1\tilde{\mathfrak{F}}_1$ neutrosophic soft b^{**} closed is $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ for every $j \tilde{\in} \Delta$. Consequently,$\bigcap_j (\mathfrak{H}^\partial)^{-1}\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j\right] = \left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j$ so, every singleton neutrosophic soft subspace of $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft $\tilde{\tau}_1\tilde{\mathfrak{F}}_1$ neutrosophic soft b^{**}closed . Hence, $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft $^{**}b_1$ space and conversely, let $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft $^{*}b_1$ space and let $\left(\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]\right)^\beta$ be an arbitrary neutrosophic soft coordinate space of $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$. Let $\left[\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^\partial\right]\right]^\beta$and $\left[\left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)^j\right]^\beta$ be any two neutrosophic soft distinct points of $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]^\beta$. Choose $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)$ and $\left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)$ in $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ whose β - th coordinate is $\left[\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j\right]\right]^\beta$ and $\left[\left[\left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)^j\right]\right]$ respect i vely. Since $\left[\left[\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)^j\right]\right]^\beta \neq \left[\left[\left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)^j\right]\right]^\beta$ we have $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right) \succ \left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2')}, \varpi\right)$ or $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right) \prec \left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)$ or $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right) \succ\succ \left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)$ or $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right) \preccurlyeq \left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)$ But $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ being soft neutrosophic soft $b^{**}{}_1$space , corresponding to the neutrosophic soft distinct points $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)}, \varpi\right)$ and $\left(\mathfrak{t}_2{}^{m'}{}_{(p_1',p_2',p_3')}, \varpi\right)$ of

$\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ there exists neutrosophic soft b^{**} open sets $f_{1\varpi}$ and $f_{2\varpi}$ in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ such that $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right) \in f_{1\varpi}$ but $\left(t_2{}^{m/}{}_{(p_1/,p_2/,p_3/)},\varpi\right) \tilde{\notin} f_{1\varpi}$ and $\left(t_2{}^{m/}{}_{(p_1/,p_2/,p_3/)},\varpi\right) \in f_{2\varpi}$ but $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right) \tilde{\notin} f_{2\varpi}$. So, there exists basic neutrosophic soft b^{**} open sets $f_{3\varpi} = \times\{f_{1\varpi}{}^{\alpha} : \alpha \in \Delta\}$ and $f_{4\varpi} = \times\{(f_{2\varpi})^{\alpha} : \alpha \in \Delta\}$ such that $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right) \tilde{\in} f_{3\varpi} \tilde{\subseteq} f_{1\varpi}$ and $\left(t_2{}^{m/}{}_{(p_1/,p_2/,p_3/)},\varpi\right) \tilde{\in} f_{4\varpi} \tilde{\subseteq} (f_{2\varpi})$. Clearly, $\left(t_2{}^{m/}{}_{(p_1/,p_2/,p_3/)},\varpi\right) \notin f_{3\varpi}$ and $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right) \tilde{\notin} f_{4\varpi}$. Thus, $f_{1\varpi}{}^{\beta}$ is neutrosophic soft b^{**} open set containing $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}$ but not $\left(t_2{}^{m/}{}_{(p_1/,p_2/,p_3/)},\varpi\right)^{\beta}$; and, $(f_{2\varpi})^{\beta}$ is neutrosophic soft b^{**}open set containing $\left(t_2{}^{m/}{}_{(p_1/,p_2/,p_3/)},\varpi\right)^{\beta}$ but not $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}$.This shows that $\left(\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$ is neutrosophic soft $b_1{}^{**}$space.

Theorem 4.5 Let $\langle\pi,\tilde{\tau}_1,\varpi\rangle$ and $\langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle$be two neutrosophic soft topological spaces such that they are neutrosophic soft $b_2{}^{**}$space on the crisp set π and $\mathfrak{L}$respectively. $\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle$ be the neutrosophic soft product space, then the product space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft $b_2{}^{**}$ space if and only if each neutrosophic soft coordinate space $\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j}\right]$ is neutrosophic soft $b_2{}^{**}$space.

Proof. Suppose each neutrosophic soft coordinate space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j}\right]$ is neutrosophic soft $b_2{}^{**}$ space and let $\langle t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi^{j:j\in\lambda}\rangle$ and $\langle\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{j:j\in\lambda}\rangle$ be two neutrosophic soft space points of $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ such that $\langle\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{j:j\in\lambda}\rangle \succ \langle\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{j:j\in\lambda}\rangle$ Then, $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta} \succ \left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}$ for some for each $\beta \tilde{\in} \Delta$ where $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}, \left(t_2{}^{m/}{}_{(p_1/,p_2/)},\varpi\right)^{\beta} \tilde{\in} \left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\beta}\right]$. Now, $[\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle]^{\beta}$ is neutrosophic soft $b_2{}^{**}$ space and $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta} \succ \left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}$ are points of $\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\beta}$, So there exists a $\tilde{\tau}_1\tilde{\tilde{\mathfrak{F}}}_1$ neutrosophic soft b^{**} open sets $(f_{1\varpi})^{\beta}$ and $(f_{2\varpi})^{\beta}$ such that $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta} \in (f_{1\varpi})^{\beta}$ and $\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta} \tilde{\in} (f_{2\varpi})^{\beta}$ and $(f_{1\varpi})^{\beta} \tilde{\sqcap} (f_{2\varpi})^{\beta} = 0_{\widetilde{(\pi,\varpi)}}$. Since $\mathfrak{H}^{\beta}\left[\left(\left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}\right)\right] = \left(t_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta} \in (f_{1\varpi})^{\beta}$ and

$\mathfrak{H}^{\beta}\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{\beta}\right) = \left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{\beta} \tilde{\in} (\mathcal{f}_{2\varpi})^{\beta}$ each neutrosophic soft projection mapping π^{β} being neutrosophic soft continuous, it follows that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \in (\mathfrak{H}^{\beta})^{-1}((\mathcal{f}_{1\varpi})^{\beta})$ and $\left(\mathfrak{t}_2{}^{m/}{}_{(\mathfrak{p}_1/,\mathfrak{p}_2/,\mathfrak{p}_3/)},\varpi\right) \tilde{\in} (\mathfrak{H}^{\beta})^{-1}((\mathcal{f}_{2\varpi})^{\beta})$
and $(\mathfrak{H}^{\beta})^{-1}\left[(\mathcal{f}_{1\varpi})^{\beta}\tilde{\sqcap}(\mathcal{f}_{2\varpi})^{\beta}\right] = (\mathfrak{H}^{\beta})^{-1}\left[\widetilde{0_{(\pi,\varpi)}}\right] = \widetilde{0_{(\pi,\varpi)}}$. Moreover by neutrosophic soft continuity of $\mathfrak{H}^{\beta}$, $(\mathfrak{H}^{\beta})^{-1}\left((\mathcal{f}_{1\varpi})^{\beta}\right), (\mathfrak{H}^{\beta})^{-1}\left((\mathcal{f}_{2\varpi})^{\beta}\right)$ are a $\tilde{\tau}_1\tilde{\mathfrak{F}}_1$ neutrosophic soft b^{**} open in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$. Hence, $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft $b_2{}^{**}$space. Conversely, let $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft $b_2{}^{**}$space and let $\left(\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$ be an arbitrary neutrosophic soft coordinate space of $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$. We must show that $j:j\in\lambda^{\beta}$ is neutrosophic soft $b_2{}^{**}$space. Let $\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right) = \langle\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)^{j:j\in\lambda}\rangle$ be neutrosophic soft fixed point of $\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right\}$. Let $(\mathcal{f}_{3\varpi})$ be neutrosophic soft subset of $\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{\partial}\right\}$ consisting of all points of the form $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) = \langle\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{j:j\in\lambda}\rangle$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{j} = \left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)^{\alpha}$ if $\alpha \succ \beta$ and $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{\beta}$ may be any neutrosophic soft point of $\left(\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$. Let $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}$ be the restriction of the neutrosophic soft projection mapping $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}:\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right] \to \left(\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right\}\right)^{\beta}$ to $\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$
such that

$$\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}:\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right) \to \left(\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right\}\right)^{\beta}: \mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}\left(\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)\right)\right) = \mathfrak{H}^{\beta}\left(\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)\right)\right) = \left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)^{\beta}$$

$\forall\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \in \left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$. Then, $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}$ is clearly neutrosophic soft one-one and neutrosophic soft onto. Also, the projection mapping $\mathfrak{H}^{\beta}$ being neutrosophic soft continuous, is a restriction $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}$ is therefore neutrosophic soft continuous. Now, let

$(\not{f}_{1\varpi})$ be any neutrosophic soft b^{**} open set in the neutrosophic soft subspace $\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$. Then, $(\not{f}_{1\varpi}) = \left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right) \cap (\not{f}_{4\varpi})$ for some basic neutrosophic soft b^{**} open set $(\not{f}_{4\varpi})$ in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right]$. Let $(\not{f}_{4\varpi}) = \times\{(\not{f}_{1\varpi})^{\alpha} : \alpha \in \Lambda\}$, where, $(\not{f}_{1\varpi})^{\alpha}$ is neutrosophic soft b^{**} open in $\left(\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right\}\right)^{\alpha}$. Let $(\not{f}_{4\varpi}) = \times\,[\{(\not{f}_{1\varpi})^{\alpha} : \alpha\,\tilde{\in}\,\Lambda\}]$, where $(\not{f}_{1\varpi})^{\alpha}$ is neutrosophic soft b^{**} open in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{\partial}\right]$. And consequently, $(\not{f}_{1\varpi}) = \left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right) \tilde{\sqcap} (\times\,[\{(\not{f}_{1\varpi})^{\alpha} : \alpha\,\tilde{\in}\,\Lambda\}])$, Where $(\not{f}_{1\varpi})^{\alpha}$ is neutrosophic soft b^{**} open in $\left(\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right\}\right)^{\alpha}$ for each $\alpha\,\tilde{\in}\,\Lambda$. Thus, either $(\not{f}_{1\varpi}) = \widetilde{0_{(\pi,\varpi)}}$ or $(\not{f}_{1\varpi}) = \{(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi)\ \tilde{\in}\ (\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi)$: βth coordinate of $(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi)$ in $(\not{f}_{1\varpi})$^β }. Therefore $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}(((\not{f}_{1\varpi})) = \widetilde{0_{(\pi,\varpi)}}$ or $(\not{f}_{1\varpi})^{\beta}$, each one of which is neutrosophic soft b^{**}open. This shows that the neutrosophic soft image under $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}$ of every basic neutrosophic soft b^{**}open set in $\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$ is neutrosophic soft b^{**}open and therefore, $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}$ is neutrosophic soft $*_b$open. Thus, $\mathfrak{H}^{\left((\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)}$, is neutrosophic soft homeomorphism and therefore, $\left(\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$ is the neutrosophic soft homeomorphic image of $\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$. Now, every neutrosophic soft subspace of a neutrosophic soft $b_2{}^{**}$space being neutrosophic soft $b_2{}^{**}$space, $\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$ is neutrosophic soft b^{**}open and therefore, $\mathfrak{H}^{\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)}\left(\mathfrak{t}_2{}^{m//}{}_{(\mathfrak{p}_1//,\mathfrak{p}_2//,\mathfrak{p}_3//)},\varpi\right)$ is neutrosophic soft $b_2{}^{**}$space and so its neutrosophic soft homeomorphic image $\left(\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]\right)$ is soft neutrosophic soft $b_2{}^{**}$space. Hence, each neutrosophic soft coordinate space is neutrosophic soft $b_2{}^{**}$space.

Theorem 4.6 Let $\langle\pi,\tilde{\tau}_1,\varpi\rangle$and $\langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle$ be two neutrosophic soft topological spaces on the crisp set π and $\mathfrak{L}$ respectively. $\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle$be the neutrosophic soft product space, then the product space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft

b^{**}-regular space iff each neutrosophic soft coordinate space $\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right]$ is neutrosophic soft b^{**}regular space.

Proof. Suppose each neutrosophic soft coordinate space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right]$is neutrosophic soft b^{**}-regular space. Let $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) = \langle\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{j:j\in\lambda}\rangle$ be any neutrosophic soft point of the neutrosophic soft product space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ and $\mathscr{f}_{1\varpi}$ be any neutrosophic soft b^{**} open set in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \tilde{\in} \mathscr{f}_{1\varpi}$ then there exists a neutrosophic soft b^{**} open set $(\mathscr{f}_{2\varpi})$ in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \tilde{\in} (\mathscr{f}_{2\varpi}) \tilde{\Subset} \mathscr{f}_{1\varpi}$. Let, $\times\left\{\left((\mathscr{f}_{1\varpi})\right)^{j:j\in\lambda}\right\}$ is the neutrosophic soft product space such that $(\mathscr{f}_{1\varpi})^{j}$ is neutrosophic soft $^{**}b$ open in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right]$. Since each $\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right\}$ is neutrosophic soft b^{**} regular space and $\left((\mathscr{f}_{1\varpi})\right)^{j}$ is neutrosophic soft b^{**} open in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right]$containing $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{j}$ there exists a neutrosophic soft b^{**} open set $\left((\mathscr{f}_{3\varpi})\right)^{j}$ in $\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right\}$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{j} \tilde{\in} \left((\mathscr{f}_{3\varpi})\right)^{j}$ and $\overline{\left((\mathscr{f}_{3\varpi})\right)^{j}} \tilde{\Subset} (\mathscr{f}_{1\varpi})^{j}$. Let $\times\left[\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}\right]$, then$\left[\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}\right]$ is neutrosophic soft b^{**}open set in $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ and contains $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)$. Also, $\overline{\times\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}} = \times\left[\overline{\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}} : j \in \Delta\right]$.Further, since $\overline{\left((\mathscr{f}_{3\varpi})\right)^{j}} \tilde{\Subset} (\mathscr{f}_{1\varpi})^{j}$ for each j, we have $\times\left[\overline{\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}} : j \in \Delta\right] \tilde{\Subset}\times\left[\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}\right]$,this shows that for every neutrosophic soft point $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \tilde{\in} \times \left\{\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right\}$ and every neutrosophic soft b^{**} open set $(\mathscr{f}_{1\varpi})$ containing $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2)},\varpi\right)$ there exists a neutrosophic soft b^{**} open set $\times\left[\overline{\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}} : j \in \Delta\right]$ in $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \tilde{\in} \times\left[\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}\right]$ and $\overline{\times\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}} \tilde{\Subset} \times\left[\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}\right]$. Hence, $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft b^{**}-regular. Conversely, let the non-empty neutrosophic soft product space $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ be neutrosophic soft b^{**}-regular and let $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]^{\beta}$ be an arbitrary neutrosophic soft coordinate space. Then, we must show that it is a neutrosophic soft b^{**}-regular. Let $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{\beta}$ be any neutrosophic soft point of $\left(\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$ and let $(\mathscr{f}_{1\varpi})^{\beta}$ be any neutrosophic soft b^{**}open in $\left(\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{\beta} \tilde{\in} (\mathscr{f}_{1\varpi})^{\beta}$. now, choose, soft element $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)$ in $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ whose βth coordinate in $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{\beta}$. Let $(\mathscr{f}_{1\varpi}) = \mathfrak{H}^{\beta-1}(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right)^{i}$. Then, $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)},\varpi\right) \in (\mathscr{f}_{1\varpi})$ and by neutrosophic soft continuity of $\mathfrak{H}^{\beta}$, $(\mathscr{f}_{1\varpi})$ is neutrosophic soft b^{**} open in $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$. Since, $\times\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$is neutrosophic soft b^{**} -regular space so there exists a neutrosophic s o ft b^{**} open set $\times\left[\overline{\left((\mathscr{f}_{3\varpi})\right)^{j:j\in\lambda}}\right]$ and where each $\left((\mathscr{f}_{3\varpi})\right)^{j}$ is neutrosophic soft b^{**} open in

$\left(\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]\right)^{\beta}$ such that $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)\tilde{\in}\times\left\{\left((f_{3\varpi})\right)^{j:j\in\lambda}\right\}$ and $\overline{\left[\left((f_{3\varpi})\right)^{j:j\in\lambda}\right]}\tilde{\subseteq}(f_{1\varpi})$. Now $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)\tilde{\in}\overline{\left[\left((f_{3\varpi})\right)^{j:j\in\lambda}\right]}\tilde{\subseteq}(f_{1\varpi})$ this implies that $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)\tilde{\in}(f_{1\varpi})=\mathfrak{H}^{\beta-1}((f_{1\varpi})^{\beta})$ implies that $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}\tilde{\in}(f_{1\varpi})$, Moreover, $\overline{\left[\left((f_{3\varpi})\right)^{j:j\in\lambda}\right]}=\left[\left((f_{3\varpi})\right)^{j:j\in\lambda}\right]$ and $\overline{\left[\left((f_{3\varpi})\right)^{j:j\in\lambda}\right]}\tilde{\subseteq}(f_{1\varpi})^{\beta}$ This shows that $\left(\times\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right\}\right)^{\beta}$ is neutrosophic soft b^{**} regular and hence, each coordinate space is neutrosophic soft b^{**}regular.

Theorem 4.7 Let $\langle\pi,\tilde{\tau}_1,\varpi\rangle$ and $\langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle$ be two neutrosophic soft topological spaces on the crisp set π and $\mathfrak{L}$ respectively. $\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle$ be the soft product space, then the product space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic soft b^{**}close regular space if each neutrosophic soft coordinate space $\left\{\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right\}$ is neutrosophic soft close b^{**}- space.

Proof. Let each soft coordinate space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j}\right]$ is neutrosophic soft b^{**}close regular space. Then, we must show that the neutrosophic soft product space $\times\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{j:j\in\lambda}\right]$. Let $f_{1\varpi}$ be any member of the usual neutrosophic soft subbase for the neutrosophic soft product topology and let $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)=\langle\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{j:j\in\lambda}\rangle$ be any neutrosophic soft point in $f_{1\varpi}$ Then $f_{1\varpi}=\mathfrak{H}^{\beta-1}((f_{2\varpi})^{\beta})$ is neutrosophic soft b^{**} open in $\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{\beta}\right]$ and contains $\left(\mathfrak{t}_1{}^{m}{}_{(p_1,p_2,p_3)},\varpi\right)^{\beta}$). Since, $\left[\left(\langle\pi,\tilde{\tau}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\mathfrak{F}}_1,\varpi\rangle\right)^{\beta}\right]$ is neutrosophic soft close regular b^{**} -space there exists a neutrosophic soft mapping

$f: \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{\beta}\right] \longrightarrow [\tilde{0}, \tilde{1}] \times [\tilde{0}, \tilde{1}]$ such that $f\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)^{\beta}\right) = \widetilde{0_{(\pi,\varpi)}}$ and $f\left(\mathfrak{t}_2{}^{m'}{}_{(\mathfrak{p}_1{}',\mathfrak{p}_2{}',\mathfrak{p}_3{}')}, \varpi\right) = \widetilde{1_{(\pi,\varpi)}}$. $\forall\, \mathfrak{t}_2{}^{m'}{}_{(\mathfrak{p}_1{}',\mathfrak{p}_2{}',\mathfrak{p}_3{}')} \tilde{\in} \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{\beta}\right] - (\mathfrak{f}_{2\varpi})^{\beta}$.

Since $\mathfrak{H}^{\beta}$ is neutrosophic soft continuous and f is neutrosophic soft continuous, so the neutrosophic soft composite mapping $(f o \mathfrak{H}^{\beta}): \times \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right] \longrightarrow [\tilde{0}, \tilde{1}] \times [\tilde{0}, \tilde{1}]$ is neutrosophic soft continuous.

Now, if $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \tilde{\in} \mathfrak{f}_{1\varpi}$, then $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \tilde{\in} \mathfrak{H}^{\beta^{-1}}((\mathfrak{f}_{1\varpi})^{\beta})$

$\Longrightarrow \mathfrak{H}^{\beta}\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \in (\mathfrak{f}_{2\varpi})^{\beta} \Longrightarrow f\left[\mathfrak{H}^{\beta}\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)\right)\right] = f\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)^{\beta}\right) = \widetilde{0_{(\pi,\varpi)}}$

$\Longrightarrow (f o \mathfrak{H}^{\beta})\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) = \widetilde{0_{(\pi,\varpi)}}$. Again, if $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \tilde{\in} \times \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right] - \mathfrak{f}_{1\varpi}$, then $\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \tilde{\in} \times \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right] - \mathfrak{f}_{1\varpi} \Longrightarrow \left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \tilde{\in} \mathfrak{H}^{\beta^{-1}}(\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{\beta}\right] - \mathfrak{H}^{\beta^{-1}}((\mathfrak{f}_{2\varpi})^{\beta})$

$\Longrightarrow \left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \in \mathfrak{H}^{\beta^{-1}}(\left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{\beta}\right] - (\mathfrak{f}_{2\varpi})^{\beta}) \Longrightarrow \mathfrak{H}^{\beta}\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)\right) \tilde{\in} \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{\beta}\right] - (\mathfrak{f}_{2\varpi})^{\beta}) \Longrightarrow f\left[\mathfrak{H}^{\beta}\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)\right)\right] = \widetilde{1_{(\pi,\varpi)}} \Longrightarrow (f o \mathfrak{H}^{\beta})\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)\right) = \widetilde{1_{(\pi,\varpi)}}$

Thus,

$$f\left[\mathfrak{H}^{\beta}\left(\left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right)\right)\right] = \begin{bmatrix} \widetilde{0_{(\pi,\varpi)}} = \text{if } \left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \in (\mathfrak{f}_{2\varpi})^{\beta}) \\ \widetilde{0_{(\pi,\varpi)}} = \text{if } \left(\mathfrak{t}_1{}^{m}{}_{(\mathfrak{p}_1,\mathfrak{p}_2,\mathfrak{p}_3)}, \varpi\right) \in \times \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right] - \mathfrak{f}_{1\varpi}. \end{bmatrix}$$

Hence, $\times \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ is neutrosophic close b^{**}regular space and conversely, let the neutrosophic soft product space $\times \left[\left(\langle \pi, \tilde{\tau}_1, \varpi\rangle * \langle \mathfrak{L}, \tilde{\mathfrak{F}}_1, \varpi\rangle\right)^{j:j\in\lambda}\right]$ be neutrosophic soft b^{**}close regular b^{**}- space and let $\times\left[\left(\langle\pi,\tilde{\mathfrak{r}}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\mathfrak{j};\mathfrak{j}\in\lambda}\right]$ be an arbitrary neutrosophic soft coordinate space. Then by continuing, we can show that $\times\left[\left(\langle\pi,\tilde{\mathfrak{r}}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\mathfrak{j};\mathfrak{j}\in\lambda}\right]$ is the neutrosophic soft homeomorphic image of a neutrosophic soft subspace of $\times\left[\left(\langle\pi,\tilde{\mathfrak{r}}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\mathfrak{j};\mathfrak{j}\in\lambda}\right]$. Now, every soft subspace of a neutrosophic soft b^{**} close regular space being a neutrosophic soft b^{**}close regular space and neutrosophic soft homeomorphic image of a neutrosophic soft b^{**}close regular space being neutrosophic soft b^{**}close regular space it follows that $\times\left[\left(\langle\pi,\tilde{\mathfrak{r}}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\mathfrak{j};\mathfrak{j}\in\lambda}\right]$ is neutrosophic soft b^{**}close regular space. Hence, each coordinate space of $\times\left[\left(\langle\pi,\tilde{\mathfrak{r}}_1,\varpi\rangle * \langle\mathfrak{L},\tilde{\tilde{\mathfrak{F}}}_1,\varpi\rangle\right)^{\mathfrak{j};\mathfrak{j}\in\lambda}\right]$ is neutrosophic soft b^{**}close regular space.

Definition 4.8 Let $\hbar: \mathrm{NSS}(\mathbb{U}) \to [0,1]$ be a mapping, where $NSS(\mathbb{U})$ represents the set of all $(NSSs)$ on $\mathbb{U}$. For $\langle\widetilde{Q_1},\varpi\rangle \in NSS(\mathrm{U}), \hbar\langle\widetilde{Q_1},\varpi\rangle$is called the entropy of $\langle\widetilde{Q_1},\varpi\rangle$ if it satisfies the following conditions:

1) $\hbar\hbar\langle\widetilde{Q_1},\varpi\rangle = 0 \Leftrightarrow \forall\mathfrak{p}\in\varpi, \mathfrak{t}\in\mathrm{U}, \mathbb{T}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 0, \mathbb{I}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 0$ and $\mathbb{F}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 1$ or

   $\mathbb{T}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 1, \mathbb{I}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 1$ and $\mathbb{F}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 0$

2) $\hbar\langle\widetilde{Q_1},\varpi\rangle = 1 \;\forall\; \mathfrak{p}\in\varpi, \mathfrak{t}\in\mathbb{U}, \mathbb{T}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = \mathbb{I}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = \mathbb{F}_{Q_1(\mathfrak{p})}(\mathfrak{t}) = 0.5$

3) $\hbar\langle\widetilde{Q_1},\varpi\rangle = \hbar\langle\widetilde{Q_1},\varpi\rangle^c$

4) $\forall\mathfrak{p}\in\varpi, \mathfrak{t}\in\mathbb{U}$ when $\langle\widetilde{Q_1},\varpi\rangle \subseteq \langle\widetilde{Q_2},\varpi\rangle$ and $\mathbb{T}_{\widetilde{Q_2}(\mathfrak{p})}(\mathfrak{t}) \leq \mathbb{F}_{Q_2(\mathfrak{p})}(\mathfrak{t}), \mathbb{I}_{Q_1(\mathfrak{p})}(\mathfrak{t}) \leq \mathbb{I}_{Q_2(\mathfrak{p})}(\mathfrak{t})$if

   $\mathbb{I}_{Q_2(\mathfrak{p})}(\mathfrak{t})) \leq 0.5$ or

   $\langle\widetilde{Q_1},\varpi\rangle \supseteq \langle\widetilde{Q_2},\varpi\rangle$, and $\mathbb{T}_{\widetilde{Q_2}(\mathfrak{p})}(\mathfrak{t}) \geq \mathbb{F}_{\widetilde{Q_2}(\mathfrak{p})}(\mathfrak{t})$, then $\hbar\langle\widetilde{Q_1},\varpi\rangle \leq \hbar\langle\widetilde{Q_2},\varpi\rangle$, $\mathbb{I}_{Q_1(\mathfrak{p})}(\mathfrak{t}) \geq$

   $\mathbb{I}_{Q_2(\mathfrak{p})}(\mathfrak{t})$if $\mathbb{I}_{Q_2(\mathfrak{p})}(\mathfrak{t}) \geq 0.5$ then $\langle\widetilde{Q_1},\varpi\rangle \subseteq \langle\widetilde{Q_2},\varpi\rangle$

Example 4.9 In order to obtain an effective risk supervision in the field of engineering, certain risks are classified along with some parameters and these risks are evaluated by the team of experts (engineers). Assume that there is a set of three experts (engineers) assessing the five different kinds of risks with the set of parameters.

Let $\mathbb{U}$ denote the set of risks $\mathbb{U} = \{\mathfrak{t}_1, \mathfrak{t}_2, \mathfrak{t}_3, \mathfrak{t}_4, \mathfrak{p}_5\}$ and let ϖ denote the set of parameters $\varpi = \{\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4, \mathfrak{p}_5\}$.

The neutrosophic soft set

$\mathscr{f}_{1\varpi}$describes the assessment of engineer 1.

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_1)(\mathfrak{t}_1) = \langle 07 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1}\rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_2)(\mathfrak{t}_1) = \langle 05 \times 10^{-1}, 05 \times 10^{-1}, 05 \times 10^{-1}\rangle,$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_3)(\mathfrak{t}_1) = \langle 07 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1}\rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_4)(\mathfrak{t}_1) = \langle 06 \times 10^{-1}, 07 \times 10^{-1}, 08 \times 10^{-1}\rangle,$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_5)(\mathfrak{t}_1) = \langle 07 \times 10^{-1}, 05 \times 10^{-1}, 04 \times 10^{-1}\rangle.$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_1)(\mathfrak{t}_2) = \langle 08 \times 10^{-1}, 03 \times 10^{-1}, 04 \times 10^{-1}\rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_2)(\mathfrak{t}_2) = \langle 07 \times 10^{-1}, 03 \times 10^{-1}, 02 \times 10^{-1}\rangle,$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_3)(\mathfrak{t}_2) = \langle 06 \times 10^{-1}, 08 \times 10^{-1}, 09 \times 10^{-1}\rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_4)(\mathfrak{t}_2) = \langle 08 \times 10^{-1}, 01 \times 10^{-1}, 09 \times 10^{-1}\rangle,$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_5)(\mathfrak{t}_2) = \langle 08 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1}\rangle.$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_1)(\mathfrak{t}_3) = \langle 04 \times 10^{-1}, 06 \times 10^{-1}, 02 \times 10^{-1}\rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_2)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 07 \times 10^{-1}, 02 \times 10^{-1}\rangle,$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_3)(\mathfrak{t}_3) = \langle 02 \times 10^{-1}, 09 \times 10^{-1}, 02 \times 10^{-1}\rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_4)(\mathfrak{t}_3) = \langle 04 \times 10^{-1}, 06 \times 10^{-1}, 05 \times 10^{-1}\rangle$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_5)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 08 \times 10^{-1}, 07 \times 10^{-1}\rangle.$$

$$\mathscr{f}_{1\varpi}(\mathfrak{p}_1)(\mathfrak{t}_4) = \langle 04 \times 10^{-1}, 05 \times 10^{-1}, 03 \times 10^{-1} \rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_2)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 06 \times 10^{-1}, 07 \times 10^-$$
$$\mathscr{f}_{1\varpi}(\mathfrak{p}_3)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 05 \times 10^{-1}, 04 \times 10^{-1} \rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_4)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 04 \times 10^{-1}, 05 \times 10^-$$
$$\mathscr{f}_{1\varpi}(\mathfrak{p}_5)(\mathfrak{t}_4) = \langle 04 \times 10^{-1}, 08 \times 10^{-1}, 03 \times 10^{-1} \rangle.$$
$$\mathscr{f}_{1\varpi}(\mathfrak{p}_1)(\mathfrak{t}_5) = \langle 03 \times 10^{-1}, 02 \times 10^{-1}, 07 \times 10^{-1} \rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_2)(\mathfrak{t}_5) = \langle 02 \times 10^{-1}, 07 \times 10^{-1}, 01 \times 10^-$$
$$\mathscr{f}_{1\varpi}(\mathfrak{p}_3)(\mathfrak{t}_5) = \langle 03 \times 10^{-1}, 05 \times 10^{-1}, 04 \times 10^{-1} \rangle, \mathscr{f}_{1\varpi}(\mathfrak{p}_4)(\mathfrak{t}_5) = \langle 03 \times 10^{-1}, 06 \times 10^{-1}, 09 \times 10^-$$
$$\mathscr{f}_{1\varpi}(\mathfrak{p}_5)(\mathfrak{t}_5) = \langle 04 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1} \rangle.$$

The neutrosophic soft set $\mathscr{f}_{2\varpi}$ describes the assessment of engineer 2.

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_1)(\mathfrak{t}_1) = \langle 04 \times 10^{-1}, 03 \times 10^{-1}, 02 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_2)(\mathfrak{t}_1) = \langle 03 \times 10^{-1}, 01 \times 10^{-1}, 01 \times 10^-$$
$$\mathscr{f}_{2\varpi}(\mathfrak{p}_3)(\mathfrak{t}_1) = \langle 05 \times 10^{-1}, 05 \times 10^{-1}, 09 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(e_4)(\mathfrak{t}_1) = \langle 04 \times 10^{-1}, 05 \times 10^{-1}, 04 \times 10^-$$
$$\mathscr{f}_{2\varpi}(\mathfrak{p}_5)(\mathfrak{t}_1) = \langle 04 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1} \rangle$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_1)(\mathfrak{t}_2) = \langle 03 \times 10^{-1}, 04 \times 10^{-1}, 05 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_2)(\mathfrak{t}_2) = \langle 03 \times 10^{-1}, 04 \times 10^{-1}, 02 \times 10^{-1} \rangle$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_3)(\mathfrak{t}_2) = \langle 03 \times 10^{-1}, 06 \times 10^{-1}, 07 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_4)(\mathfrak{t}_2) = \langle 04 \times 10^{-1}, 09 \times 10^{-1}, 01 \times 10^{-1} \rangle$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_5)(\mathfrak{t}_2) = \langle 03 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1} \rangle$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_1)(\mathfrak{t}_3) = \langle 04 \times 10^{-1}, 06 \times 10^{-1}, 05 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_2)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 09 \times 10^{-1}, 08 \times 10^{-1} \rangle \mathscr{f}_{2\varpi}(\mathfrak{p}_3)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 08 \times 10^{-1}, 01 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_4)(\mathfrak{t}_3) = \langle 04 \times 10^{-1}, 07 \times 10^{-1}, 08 \times 10^{-1} \rangle,$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_5)(\mathfrak{t}_3) = \langle 02 \times 10^{-1}, 04 \times 10^{-1}, 06 \times 10^{-1} \rangle$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_1)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 02 \times 10^{-1}, 01 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_2)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 01 \times 10^{-1}, 02 \times 10^{-1} \rangle$$

$$\mathscr{f}_{2\varpi}(\mathfrak{p}_3)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 02 \times 10^{-1}, 05 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_4)(\mathfrak{t}_4) = \langle 05 \times 10^{-1}, 03 \times 10^{-1}, 04 \times 10^{-1} \rangle, \mathscr{f}_{2\varpi}(\mathfrak{p}_5)(\mathfrak{t}_4) = \langle 04 \times 10^{-1}, 06 \times 10^{-1}, 08 \times 10^{-1} \rangle$$

$$\digamma_{2\varpi}(\mathfrak{p}_1)(\mathfrak{t}_5) = \langle 04\times10^{-1}, 06\times10^{-1}, 08\times10^{-1}\rangle, \digamma_{2\varpi}(\mathfrak{p}_2)(\mathfrak{t}_5) = \langle 03\times10^{-1}, 07\times10^{-1}, 08\times10^{-1}\rangle$$

$$\digamma_{2\varpi}(\mathfrak{p}_3)(\mathfrak{t}_5) = \langle 04\times10^{-1}, 02\times10^{-1}, 06\times10^{-1}\rangle, \digamma_{2\varpi}(\mathfrak{p}_4)(\mathfrak{t}_5) = \langle 04\times10^{-1}, 02\times10^{-1}, 01\times10^{-1}\rangle$$

$$\digamma_{2\varpi}(\mathfrak{p}_5)(\mathfrak{t}_5) = \langle 03\times10^{-1}, 04\times10^{-1}, 08\times10^{-1}\rangle$$

The neutrosophic soft set $\digamma_{3\varpi}$ describes the assessment of engineer 3.

$$\digamma_{3\varpi}(\mathfrak{p}_1)(\mathfrak{t}_1) = \langle 04\times10^{-1}, 05\times10^{-1}, 01\times10^{-1}\rangle, \digamma_{3\varpi}(\mathfrak{p}_2)(\mathfrak{t}_1) = \langle 03\times10^{-1}, 01\times10^{-1}, 01\times10^{-1}\rangle,$$

$$\digamma_{3\varpi}(\mathfrak{p}_3)(\mathfrak{t}_1) = \langle 03\times10^{-1}, 02\times10^{-1}, 03\times10^{-1}\rangle, \digamma_{3\varpi}(\mathfrak{p}_4)(\mathfrak{t}_1) = \langle 03\times10^{-1}, 04\times10^{-1}, 07\times10^{-1}\rangle,$$

$$\digamma_{3\varpi}(\mathfrak{p}_5)(\mathfrak{t}_1) = \langle 05\times10^{-1}, 05\times10^{-1}, 01\times10^{-1}\rangle$$

$$\digamma_{3\varpi}(\mathfrak{p}_1)(\mathfrak{t}_2) = \langle 03\times10^{-1}, 04\times10^{-1}, 05\times10^{-1}\rangle, \digamma_{3\varpi}(\mathfrak{p}_2)(\mathfrak{t}_2) = \langle 02\times10^{-1}, 01\times10^{-1}, 03\times10^{-1}\rangle,$$

$$\digamma_{3\varpi}(\mathfrak{p}_3)(\mathfrak{t}_2) = \langle 03\times10^{-1}, 02\times10^{-1}, 01\times10^{-1}\rangle, \digamma_{3\varpi}(\mathfrak{p}_4)(\mathfrak{t}_2) = \langle 03\times10^{-1}, 05\times10^{-1}, 06\times10^{-1}\rangle,$$

$$\digamma_{3\varpi}(\mathfrak{p}_5)(\mathfrak{t}_2) = \langle 05\times10^{-1}, 04\times10^{-1}, 01\times10^{-1}\rangle$$

$$\digamma_{3\varpi}(\mathfrak{p}_1)(\mathfrak{t}_3) = \langle 03\times10^{-1}, 07\times10^{-1}, 09\times10^{-1}\rangle, \digamma_{3\varpi}(\mathfrak{p}_2)(\mathfrak{t}_3) = \langle 03\times10^{-1}, 01\times10^{-1}, 05\times10^{-1}\rangle,$$

$$\not{f}_{3\varpi}(\mathfrak{p}_3)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 05 \times 10^{-1}, 04 \times 10^{-1} \rangle, \not{f}_{3\varpi}(\mathfrak{p}_4)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 09 \times 10^{-1}, 08 \times 10^{-1} \rangle,$$

$$\not{f}_{3\varpi}(\mathfrak{p}_5)(\mathfrak{t}_3) = \langle 03 \times 10^{-1}, 01 \times 10^{-1}, 07 \times 10^{-1} \rangle, \not{f}_{3\varpi}(\mathfrak{p}_1)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 02 \times 10^{-1}, 06 \times 10^{-1} \rangle, \not{f}_{3\varpi}(\mathfrak{p}_2)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 03 \times 10^{-1}, 02 \times 10^{-1} \rangle,$$

$$\not{f}_{3\varpi}(\mathfrak{p}_3)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 05 \times 10^{-1}, 06 \times 10^{-1} \rangle, \not{f}_{3\varpi}(\mathfrak{p}_4)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 08 \times 10^{-1}, 09 \times 10^{-1} \rangle,$$

$$\not{f}_{3\varpi}(e_5)(\mathfrak{t}_4) = \langle 03 \times 10^{-1}, 01 \times 10^{-1}, 08 \times 10^{-1} \rangle$$

$$\not{f}_{3\varpi}(e_1)(\mathfrak{t}_5) = \langle 04 \times 10^{-1}, 05 \times 10^{-1}, 09 \times 10^{-1} \rangle, \not{f}_{3\varpi}(e_2)(\mathfrak{t}_5) = \langle 04 \times 10^{-1}, 05 \times 10^{-1}, 09 \times 10^{-1} \rangle,$$

$$\not{f}_{3\varpi}(e_3)(\mathfrak{t}_5) = \langle 04 \times 10^{-1}, 05 \times 10^{-1}, 07 \times 10^{-1} \rangle, \not{f}_{3\varpi}(e_4)(\mathfrak{t}_5) = \langle 03 \times 10^{-1}, 07 \times 10^{-1}, 08 \times 10^{-1} \rangle,$$

$$\not{f}_{3\varpi}(e_5)(\mathfrak{t}_5) = \langle 03 \times 10^{-1}, 02 \times 10^{-1}, 06 \times 10^{-1} \rangle$$

Now

$$\hbar_1\langle \not{f}_{1\varpi} \rangle = 00906 \times 10^{-5}, \hbar_2\langle \not{f}_{1\varpi} \rangle = 01148 \times 10^{-4}, \hbar_3\langle \not{f}_{1\varpi} \rangle = 00984 \times 10^{-4}, \hbar_4\langle \not{f}_{1\varpi} \rangle = 01143 \times 10^{-4}, \hbar_5\langle \not{f}_{1\varpi} \rangle = 00746 \times 10^{-4}$$

$$\hbar_1\langle \not{f}_{2\varpi} \rangle = 01069 \times 10^{-4}, \hbar_2\langle \not{f}_{2\varpi} \rangle = 00646 \times 10^{-4}, \hbar_3\langle \not{f}_{2\varpi} \rangle = 00923 \times 10^{-4}, \hbar_4\langle \not{f}_{2\varpi} \rangle = 00918 \times 10^{-4}, \hbar_5\langle \not{f}_{2\varpi} \rangle = 00857 \times 10^{-4}$$

$$\hbar_1\langle \not{f}_{3\varpi} \rangle = 01031 \times 10^{-4}, \hbar_2\langle \not{f}_{3\varpi} \rangle = 00742 \times 10^{-4}, \hbar_3\langle \not{f}_{3\varpi} \rangle = 01222 \times 10^{-4}, \hbar_4\langle \not{f}_{3\varpi} \rangle = 00822 \times 10^{-4}, \hbar_5\langle \not{f}_{3\varpi} \rangle = 00706 \times 10^{-4}$$

Therefore,

$$\hbar(\langle \not{f}_{1\varpi} \rangle = 00985 \times 10^{-4}, \hbar\langle \not{f}_{2\varpi} \rangle = 0.0883 \times 10^{-4}, \hbar\langle \not{f}_{3\varpi} \rangle = 00905 \times 10^{-4}.$$

Entropy is an significant idea for measuring ambiguous information. The less ambiguity information has the larger possibility to select the best. From the totaling we have $\hbar\langle f_{2\varpi}\rangle \leq \hbar\langle f_{3\varpi}\rangle \leq \hbar\langle f_{21\varpi}\rangle$ and therefore, the engineer 2 has larger opportunity to make the decision on risk supervision than engineer 1 and engineer 3. According to engineer 2 $\hbar_1\langle f_{2\varpi}\rangle = 01069 \times 10^{-4}$ has the largest entropy value between the risks. Hence, engineering risk has to be minimized to have an effective risk supervision system.

Conclusion

Neutrosophic soft set is considered to be the most important set in mathematics. This study concerns the concept of new type of neutrosophic soft open sets in neutrosophic soft bi topological space. Eight new neutrosophic soft open sets are introduced. Among these neutrosophic soft b^{**} open set is chosen to generate the results. In addition, some results are studied in terms of neutrosophic soft interior and neutrosophic soft closures with respect to soft points. Finally, the most important structure known as entropy is discussed and on the basis of this structures an example is given which is related to engineering which goes like this, in order to obtain an effective risk supervision in the field of engineering, certain risks are classified along with some parameters and these risks are evaluated by the team of experts (engineers). In upcoming time, we will try to introduce new structures which will play important role in decision making problem as in similarity measures. In addition to this, we will try to see the pairwise structures and some engineering problems in [44, 45, 46, 47, 49, 54,55, 56] with respect new generliased open sets with respect to soft points and crisp points of the space.

Acknowledgement: We would like to extend our heartfelt and profound gratitude to **Dr. Arif Mehmood**, the principal investigator of our collaborative project. His extraordinary brilliance and insightful guidance have been the cornerstone of our success. Dr. Mehmood has not only illuminated our path with remarkable clarity but has also inspired us to embody professionalism and uphold our core values, even in the face of significant challenges. His unwavering support and steadfast commitment to our team have been truly invaluable. Without his relentless dedication and encouragement, the ambitious goals of this project would have remained unfulfilled. We are deeply appreciative of his exceptional leadership and mentorship throughout this transformative journey.

Author Contributions: All authors read and approved the final manuscript.

Funding Statement: The authors received no specific funding for this study.

Conflicts of Interest: The authors declare that they have no conflicts of interest to report regarding the present study.

Availability of Data and Materials: All the data and materials are provided in the manuscript.

References

[1] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8, no. 3, pp. 338-353, 1965.

[2] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning (I), Information Science, vol. 8, no. 3, pp. 199-249, 1975.

[3] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning (II), Information Science, vol. 8, no. 4, pp. 301-357, 1975.

[4] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning (III), Information Science, vol. 9, no. 1, pp. 43-80, 1975.

[5] L. A. Zadeh, Toward a generalized theory of uncertainty (GTU): an outline, Information Sciences, vol. 172, no. 1-2, pp. 1-40, 2005.

[6] J. Ye, A multicriteria decision-making method using aggregation operators for simplified neutrosophic set, Journal of Intelligent & Fuzzy Systems, vol. 26, no. 5, pp. 2459-2466, 2014.

[7] C. Liu and Y. Luo, Correlated aggregation operators for simplified neutrosophic set and their application in multi-attribute group decision making, Journal of Intelligent & Fuzzy Systems, vol. 30, no. 3, pp. 1755-1761, 2016.

[8] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87-96, 1986.

[9] K. T. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, vol. 31, no. 3, pp. 343-349, 1989.

[10] F. Smarandache, Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth, DE: American Research Press, 1999.

[11] F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic: Neutrosophy, Neutrosophic Set and Neutrosophic Probability. Rehoboth, DE: American Research Press, 2003.

[12] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, Single valued neutrosophic sets, Multi-space Multistruction, no. 4, pp. 410-413, 2010.

[13] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, Interval Neutrosophic Sets and Logic: Theory and Applications in Computing. Phoenix, AZ: Hexis, 2005.

[14] J. Ye, Vector similarity measures of simplified neutrosophic sets and their application in multi criteria decision making, International Journal of Fuzzy Systems, vol. 16, no. 2, pp. 204-211, 2014.

[15] C. Li and, S. Luo, The weighted distance measure based method to neutrosophic multi-attribute group decision making, Mathematical Problems in Engineering, vol. 2016, and article ID 3145341, 2016.

[16] P. Liu and L. Shi, The generalized hybrid weighted average operator based on interval neutrosophic hesitant set and its application to multiple attribute decision making," Neural Computing and Applications, vol. 26, no. 2, pp. 457-471, 2015.

[17] D. Molodtsov. Soft set theory - first results. Comput. Math. Appl. 37(4–5), 19–31 (1999)

[18] P.K. Maji, Biswas, R., Roy, A.R.: Soft set theory. Comput. Math. Appl. 45(4–5), 555–562 (2003)

[19] P.K. Maji., Biswas, R., Roy, A.R.: Fuzzy soft sets. J. Fuzzy Math. 9(3), 589–602 (2001)

[20] F. Wang., Li, X., Chen, X.: Hesitant fuzzy soft set and its applications in multi criteria decision making. J. Appl. Math. 643785, 10 (2014)

[21] D. Pei, D. Miao.: From soft sets to information systems. IEEE International Conference on Granular Computing, 2,617-621((2005)

[22]. S.J. John, Soft Sets. In: Soft Sets. Studies in Fuzziness and Soft Computing, vol 400. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-57654-7_1

[23]. S.J. John, Topological Structures of Soft Sets. In: Soft Sets. Studies in Fuzziness and Soft Computing, vol 400. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-57654-7_3

[24]. S.J. John, Hybrid Structures Involving Soft Sets. In: Soft Sets. Studies in Fuzziness and Soft Computing, vol 400. Springer, Cham (2021). https://doi.org/10.1007/978-3-030-57654-7_5

[25] T. M. Al-sham, D. Ljubisa and R. Kocinac, Nearly soft menger spaces, Journal of Mathematics, pp.1-9, 2020.

[26] T. M. Al-sham, New Soft Structure, Infra soft topological spaces, Mathematical Problems in Engineering, pp. 1-12,2021.

[27] T. M. Al-sham, E. A. Tabl and B. A. Asaad, Weak forms of soft separation axioms and fixed soft points, Fuzzy Information & Engineering, vol.12, no.4, pp.509-528, 2020.

[28] T. M. Al-sham, A. El-Sayed and A. Tabi, Connectedness and local connectedness on infra soft topological spaces, Mathematics, vol. 9, pp.1-15, 2021.

[29] T. M. Al-shami, Homeomorphism and quotient mappings in infra soft topological spaces, Journal of Mathematics, pp.1-10, 2021.

[30] T. M. Al-sham, Infra soft compact spaces and application to fixed point theorem, Journal of Function Spaces, pp.1-9, 2021.

[31] B. A. Asaad, T. M. Al-shami and A. Mhemdi, Bioperators on soft topological spaces, AIMS Mathematics, vol.6, no.11, pp.12471-12490, 2021.

[32] T. Y. Ozturk, C. G. Aras and S. Bayramov, A new approach to operations on neutrosophic soft sets and to neutrosophic soft topological spaces, Communications in Mathematics & Applications, vol.10, no.3, pp.481- 493, 2019.

[33] T. Y. Ozturk, E. Karatas, A. Yolcu, On neutrosophic soft continuous mappings, Turkish Journal of Mathematics, vol. 45,no.1,pp.81-95, 2021.

[34] C. Gunduz, T. Y. Ozturk, S. Bayramov, Separation axioms on neutrosophic soft topological spaces, Turkish Journal of Mathematics, vol. 43, no. 1, pp. 498-510, 2019.

[35] T. Y. Ozturk, Some structures on neutrosophic topological spaces, Applied Mathematics & Nonlinear Sciences, vol.6, no.1, pp.467- 478, 2021.

[36] A. Mehmood, M. Aslam, M. I, Khan, H. Qureshi, C. Park and et al., A new attempt to neutrosophic soft bi-topological spaces, Computer Modeling in Engineering and Sciences, vol.10, no.3, pp.1565-1585, 2022.

[37] I. Deli and S. Broumi, Neutrosophic soft relations and some properties, Annals of Fuzzy Mathematics & Informatics, vol. 9, no.1, pp. 169-182, 2015.

[38] T. Bera and N. K. Mahapatra, Introduction to neutrosophic soft topological space, Opsearch, vol. 54, no. 4, pp.841-867, 2017.

[39] P. K. Maji, Neutrosophic soft set, Annals of Fuzzy Mathematics and Informatics, vol 5, no.1, pp. 157-168, 2013.

[40] A. Mehmood, S. Abdullah, M. M. Al-Shomrani, M.M.I. Khan, O. Thinnukool, Some results in neutrosophic soft open sets, Journal of Function Spaces, vol. 9, no.1, pp. 1-15, 2021.

[41] A. Mehmood, F. Afzal, S. Abdullah, M.I. Khan and S. Gul, An absolutely new attempt to vague soft bi-topological spaces basis at newly defined operations regarding vague soft points, Mathematical Problems in Engineering 2021, Article ID 440854, 11.

[42] A. Mehmood, S. Abdullah and C. Park, A new approach to vague soft bi-topological spaces, Computer Modeling in Engineering & Sciences 131 (2021), 411–428.

[43] A. Mehmood, S.A. Ghour, M. Ishfaq and F. Afzal, A look at soft neutrosophic spaces, J Intell Fuzzy Syst (2021), 6473–6494

[44] A. Mehmood, F. Afzal, M.I. Khan, S. Abdullah and S. Gul, Neutrosophic soft quad structures concerning neutrosophic soft points, Mathematical Problems in Engineering 2022, Article ID 8483632, 12.

[45] A. Mehmood, S.A. Ghour, F. Afzal, G. Nordo, N. Saleem, Comprehensive note on characterization of neutrosophic soft p-open sets in neutrosophic soft quad-topological space, J Intell Fuzzy Syst (2022), 1519–1540.

[46] P. Arulpandy , M. Trinita Pricilla, Bipolar neutrosophic soft contra generalized pre-continuous and contra generalized α-continuous mappings, International Journal of Neutrosophic Science, Vol. 18 , No. 1 , (2022) : 144-157 (Doi : https://doi.org/10.54216/IJNS.180112)

[47] M . Mohan, M .Krishnaswamy, K-algebras on quadripartitioned single valued neutrosophic sets. Journal of Fuzzy Extension and Applications, 1(4), (2020): 325-339. doi: 10.22105/jfea.2020.254945.1026

[48] M. Abobala, A study of maximal and minimal ideals of n-refined neutrosophic rings. Journal of Fuzzy Extension and Applications, 2(1), (2021): 16-22. doi: 10.22105/jfea.2021.270647.1072

[49] V. Kandasamy , I. Kandasamy , F. Smarandache, Neutro Algebra of Substructures of the Semigroups built using Zn and Z+, International Journal of Neutrosophic Science, Vol. 18 , No. 3 , (2022) : 135-156 (Doi : https://doi.org/10.54216/IJNS.1803012)

[50] F. Smarandache, Extension of Soft Set to Hyper soft Set, and then to Plithogenic Hyper soft Set, Neutrosophic Sets and Systems, vol. 22, 2018, pp. 168-170. DOI: 10.5281/zenodo.2159754 http://fs.unm.edu/NSS/ExtensionOfSoftSetToHypersoftSet.

[51] F. Smarandache, Extension of Soft Set to Hyper soft Set, and then to Plithogenic Hyper soft Set (revisited), Octogon Mathematical Magazine, vol. 27, no. 1, April 2019, pp. 413-418.

[52] A Vadivel, M. Seenivasan, C. J. Sundar, An Introduction to δ-open sets in a Neutrosophic Topological Spaces, Journal of Physics: Conference Series , 1724 (2021) 012011, doi:10.1088/1742-6596/1724/1/012011.

[53] A. Acikgoz, F. Esenbel, Neutrosophic Soft δ−Topology and Neutrosophic Soft Compactness, AIP Conference Proceedings 2183, 030002 (2019); https://doi.org/10.1063/1.5136106 Published Online: 06, 2019

[54] Singh, P.K, Turiyam set a fourth dimension data representation, Journal of Applied Mathematics and Physics, 9 (2021) 1821-1828. https://doi.org/10.4236/jamp.2021.97116

[55] Singh, P.K, Data with Turiyam Set for Fourth Dimension Quantum Information Processing, Journal of Neutrosophic and Fuzzy Systems, Vol.1, No. 01, PP. 09-23, 2021

[56] C. Granados, A note on Anti Geometry and Neutro Geometry and their application to real life, Neutrosophic Sets and Systems, Vol. 49, 2022.

[57] Hayat, K., Ali, M.I., Karaaslan, F. B.Y. Cao, M. shah. Design concept evaluation using soft sets based on acceptable and satisfactory levels: an integrated TOPSIS and Shannon entropy. Soft Comput **24**, 2229–2263 (2020). https://doi.org/10.1007/s00500-019-04055-7.

[58] Hayat. K, Y. Bing. Y. Cao, A.I. Muhammad, K. Faruk, Q. Zejian, Characterizations of Certain Types of Type 2 Soft Graphs, Discrete Dynamics in Nature and Society, vol. 2018, Article ID 8535703, 15 pages, 2018. https://doi.org/10.1155/2018/8535703

Received: June 28, 2024. Accepted: Oct 11, 2024