



# Neutrosophic Soft Cubic Refined Sets

Anitha Cruz R<sup>1</sup>

<sup>1</sup>Department of Mathematics PG, PSGR Krishnammal Collee for Women, Coimbatore 641004, India.

[anithacruz@psgrkcw.ac.in](mailto:anithacruz@psgrkcw.ac.in)

\*Correspondence: [anithacruz@psgrkcw.ac.in](mailto:anithacruz@psgrkcw.ac.in);

**Abstract.** This paper presents a novel approach to the Neutrosophic Soft Cubic Refined Set (NSCRS), which serves as the foundation for combining the theory of soft sets, the already-existing Neutrosophic cubic set, and the Neutrosophic refined set and Neutrosophic Soft Cubic Set. Further introduced are the internal neutrosophic soft cubic refined set (INSCR) and external neutrosophic soft cubic refined set (ENSCR), along with their various properties, including P-order, P-union, P-intersection, and R-order, R-union, and R-intersection neutrosophic soft cubic refined set.

**Keywords:** Neutrosophic Soft Cubic Refined Set (NSCRS); Neutrosophic Cubic Refined Set (NCRS); internal neutrosophic soft cubic refined set (INSCR), external neutrosophic soft cubic refined set (ENSCR)

## 1. Introduction

Zadeh [25] introduced Fuzzy sets, which address ambiguity related to imprecise states, perceptions, and preferences. Turksen [19] expanded fuzzy set to an interval valued fuzzy set after Zadeh. There are numerous real-world uses for interval valued fuzzy sets. Several authors have provided its applications in the medical field, including Sambuc [16], Kohout [12], Mukherjee and Sarkar [14]. The concept of cubic sets was first presented by Jun et al. [10] as a combination of fuzzy sets and interval valued fuzzy sets. Later, he [11] expanded on this idea by introducing the concept of neutrosophic cubic sets, which are sets that are neutrosophic in nature. Molodtsov [13] Soft Sets introduce a different approach by combining set theory with parameterized information. A soft set is defined by a collection of parameterized sets, which allows for the handling of uncertainties and variations in the data. Researchers are presenting alternative ideas on a daily basis, attesting to the significance of uncertainties that cannot be characterized by traditional mathematics. Several significant findings on this subject include fuzzy sets, intuitionistic fuzzy sets, ambiguous sets, interval-valued fuzzy sets, rough sets, and

classical probability theory. However, Molodtsov pointed out that each of these theories has inherent problems. In 1999, Smarandache [17] proposed the notion of neutrosophic sets, which are a generalization of fuzzy sets, classical sets, and their fuzzy counterparts, by incorporating an independent indeterminacy-membership function. The neutrosophic set allows for the explicit quantification of indeterminacy and the complete independence of truth-membership (T), indeterminacy-membership (I), and false-membership (F). The idea of interval neutrosophic sets, which are generalizations of fuzzy sets, interval valued fuzzy sets, and neutrosophic sets more flexible, was first presented by Wang et al. [20] after Smarandache [17]. It was discovered that interval neutrosophic sets had advantages over neutrosophic sets. The concept of the neutrosophic soft cubic set was introduced and some of its characteristics were examined in different fields by R.A.Cruz and F.N.Irudhayam [1]- [5]

Yager [24] first presented a novel theory known as the theory of bags, or multisets. Subsequently, multisets were first introduced as helpful structures in a variety of computer science and mathematics domains, including database queries, by Blizard [6] and Calude et al. [9]. Smarandache [18] has presented the definition and applications of n-valued refined neutrosophic logic. N-valued interval neutrosophic sets were later introduced by Said Broumi et al. [15], which underlined their use in medical diagnosis. Anjan Mukherjee [26] introduced refined soft set and discussed the several operations between refined soft sets and soft sets. The issue of determining the membership function is just not there in soft set theory. This facilitates the theory's practical application and makes it convenient. Numerous domains could benefit from the application of soft set theory.

In the realm of advanced mathematical theories, Neutrosophic Soft Cubic Refined Sets represent a sophisticated extension of classical set theory, integrating concepts from neutrosophy, soft set theory, and cubic set. This innovative framework addresses the limitations of traditional sets by incorporating the flexibility of neutrosophy, which deals with the uncertainty and imprecision inherent in real-world data. Neutrosophic Soft Cubic Refined Sets combine the principles of neutrosophy, which allows for the representation of truth, falsity, and indeterminacy in a more nuanced manner, with the soft set theory's ability to handle vague and uncertain information. The cubic aspect introduces a three-dimensional approach to managing these uncertainties, adding another layer of depth to the analysis and interpretation of data. In practical terms, this refined set model enhances the ability to represent and process complex information that may not fit neatly into binary or fuzzy frameworks. It offers a robust tool for addressing real-world problems where data is imprecise, incomplete, or conflicting. This approach finds applications across various fields, including decision-making, information retrieval, and data analysis, providing a more comprehensive and adaptable methodology for

managing uncertainty. Overall, Neutrosophic Soft Cubic Refined Sets offer a powerful and flexible approach for tackling complex problems where traditional methods fall short, paving the way for more accurate and insightful analysis in uncertain environments. A general approach to addressing uncertainty problems is soft set theory. This paper discusses the fundamental aspects of the Neutrosophic soft cubic refined set.

## 2. Preliminaries

### Definition 2.1. [11]

Let  $X$  be a non-empty set. By a cubic set, we mean a structure  $\Xi = \{\langle A(x), \mu(x) \rangle | x \in X\}$  in which  $A$  is an interval valued fuzzy set (IVF) and  $\mu(x)$  is a fuzzy set. It is denoted by  $\langle A, \mu \rangle$ .

### Definition 2.2. [17]

Let  $X$  be an universe. Then a neutrosophic set (NS)  $\lambda$  is an object having the form  $\lambda = \{\langle x : T(x), I(x), F(x) \rangle : x \in X\}$  where the functions  $T, I, F : X \rightarrow ]0, 1+[$  define respectively the degree of truth, the degree of indeterminacy, and the degree of falsehood of the element  $x \in X$  to the set  $\lambda$  with the condition

$$-0 \leq T(x) + I(x) + F(x) \leq 3+ .$$

### Definition 2.3. [20]

Let  $X$  be a non-empty set. An interval neutrosophic set (INS)  $A$  in  $X$  is characterized by the truth-membership function  $A_T$ , the indeterminacy-membership function  $A_I$  and the falsity-membership function  $A_F$ . For each point  $x \in X$ ,  $A_T(x), A_I(x), A_F(x) \subseteq [0, 1]$ .

**Definition 2.4.** [23] Let  $E$  be a universe. A  $n$ -valued neutrosophic sets on  $E$  can be defined as follows:

$$A = \langle x, (T_A^1(x), T_A^2(x), \dots, T_A^p(x)), (I_A^1(x), I_A^2(x), \dots, I_A^p(x)), (F_A^1(x), F_A^2(x), \dots, F_A^p(x)) \rangle : x \in X$$

where

$$T_A^1(x), T_A^2(x), \dots, T_A^p(x), I_A^1(x), I_A^2(x), \dots, I_A^p(x), F_A^1(x), F_A^2(x), \dots, F_A^p(x) : E \longrightarrow [0, 1] \text{ such that}$$

$$0 \leq T_A^i(x) + I_A^i(x) + F_A^i(x) \leq 3 \text{ for } i=1, 2, \dots, p \text{ for any } x \in X,$$

Here,  $(T_A^1(x), T_A^2(x), \dots, T_A^p(x)), (I_A^1(x), I_A^2(x), \dots, I_A^p(x))$  and  $(F_A^1(x), F_A^2(x), \dots, F_A^p(x))$  is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element  $x$ , respectively. Also,  $P$  is called the dimension of  $n$ -valued neutrosophic sets (NVNS)  $A$ .

## 3. NEUTROSOPHIC CUBIC REFINED SET

In this section, Neutrosophic cubic refined set, complement, union, and intersection are defined in this paper related to Neutrosophic soft cubic refined set.

**Definition 3.1.** Let X be a universe. A neutrosophic cubic refined set (NCRS ) on X is defined as  $A = (\mathcal{A}, \Lambda)$  where

$$\begin{aligned} \mathcal{A} = \{ < x, ([T_A^{-1}(x), T_A^{+1}(x)], [T_A^{-2}(x), T_A^{+2}(x)], \dots, [T_A^{-p}(x), T_A^{+p}(x)]) \\ ([I_A^{-1}(x), I_A^{+1}(x)], [I_A^{-2}(x), I_A^{+2}(x)], \dots, [I_A^{-q}(x), I_A^{+q}(x)]), \\ ([F_A^{-1}(x), F_A^{+1}(x)], [F_A^{-2}(x), F_A^{+2}(x)], \dots, [F_A^{-r}(x), F_A^{+r}(x)]) >: x \in X \} \end{aligned}$$

$$\Lambda = \{ < x, (t_A^1(x), t_A^2(x), \dots, t_A^p(x)), (i_A^1(x), i_A^2(x), \dots, i_A^q(x)), (f_A^1(x), f_A^2(x), \dots, f_A^r(x)) >: x \in X \}$$

where  $T_A^{-l}(x), T_A^{+l}(x), I_A^{-m}(x), I_A^{+m}(x), F_A^{-n}(x), F_A^{+n}(x), t_A^l(x), i_A^m(x), f_A^n(x) \in [0, 1]$ , such that  $0 \leq T_A^{+l}(x) + I_A^{+m}(x) + F_A^{+n}(x) \leq 3, 0 \leq t_A^l(x) + i_A^m(x) + f_A^n(x) \leq 3$  for all  $l = 1, 2, \dots, p; m = 1, 2, \dots, q; n = 1, 2, \dots, r$ .

In our study, we focus only on the case  $p=q=r$ , where p is called the dimension of neutrosophic cubic refined set (NCRS). The set of all neutrosophic cubic refined sets on X is denoted by  $NCRS(X)$ .

**Definition 3.2.** The complement of  $A = (A, \wedge)$  is denoted by  $A^c$  and is defined as

$$\begin{aligned} A^c = (A^c, A^c) \\ A^c = \{ < x, ([1 - T_A^{+1}(x), 1 - T_A^{-1}(x)], [1 - T_A^{+2}(x), 1 - T_A^{-2}(x)], \dots, [1 - T_A^{+p}(x), 1 - T_A^{-p}(x)]), \\ ([1 - I_A^{+1}(x), 1 - I_A^{-1}(x)], [1 - I_A^{+2}(x), 1 - I_A^{-2}(x)], \dots, [1 - I_A^{+q}(x), 1 - I_A^{-q}(x)]), \\ ([1 - F_A^{+1}(x), 1 - F_A^{-1}(x)], [1 - F_A^{+2}(x), 1 - F_A^{-2}(x)], \dots, [1 - F_A^{+p}(x), 1 - F_A^{-r}(x)]) >: x \in X \} \\ \wedge^c = \{ < x, (1 - T_A^1(x), 1 - T_A^2(x), \dots, 1 - T_A^p(x)), (1 - I_A^1(x), 1 - I_A^2(x), \dots, 1 - I_A^p(x)), \\ (1 - F_A^1(x), 1 - F_A^2(x), \dots, 1 - F_A^p(x)) >: x \in X \} \end{aligned}$$

**Definition 3.3.** Let  $A = (\mathcal{A}, \Lambda), B = (\mathcal{B}, \Psi)$  be two neutrosophic cubic refined set. Then, union of A and B, denoted by  $A \cup B = (\mathcal{A} \cup \mathcal{B}, \Lambda \cup \Psi)$ , is defined as

$$\begin{aligned} \mathcal{A} \cup \mathcal{B} = \{ < x, ([T_A^{-1}(x) \vee T_B^{-1}(x), T_A^{+1}(x) \vee T_B^{+1}(x)], [T_A^{-2}(x) \vee T_B^{-2}(x)], [T_A^{+2}(x) \vee T_B^{+2}(x)], \dots, \\ [T_A^{-p}(x) \vee T_B^{-p}(x), T_A^{+p}(x) \vee T_B^{+p}(x)]), ([I_A^{-1}(x) \vee I_B^{-1}(x), I_A^{+1}(x) \vee I_B^{+1}(x)], \\ [I_A^{-2}(x) \vee I_B^{-2}(x)], [I_A^{+2}(x) \vee I_B^{+2}(x)], \dots, [I_A^{-p}(x) \vee I_B^{-p}(x), I_A^{+p}(x) \vee I_B^{+p}(x)]), \\ ([F_A^{-1}(x) \vee F_B^{-1}(x), F_A^{+1}(x) \vee F_B^{+1}(x)], [F_A^{-2}(x) \vee F_B^{-2}(x)], [F_A^{+2}(x) \vee F_B^{+2}(x)], \dots, \\ [F_A^{-p}(x) \vee F_B^{-p}(x), F_A^{+p}(x) \vee F_B^{+p}(x)]) > \} \\ \Lambda \cup \Psi = \{ < x, ([t_A^1(x) \vee t_B^1(x), t_A^2(x) \vee t_B^2(x)], \dots, t_A^p(x) \vee t_B^p(x)], \\ ([i_A^1(x) \vee i_B^1(x), i_A^2(x) \vee i_B^2(x)], \dots, i_A^p(x) \vee i_B^p(x)], \\ ([f_A^1(x) \vee f_B^1(x), f_A^2(x) \vee f_B^2(x)], \dots, f_A^p(x) \vee f_B^p(x)]) > \} \end{aligned}$$

**Definition 3.4.** Let  $A = (\mathcal{A}, \Lambda)$ ,  $B = (\mathcal{B}, \Psi)$  be two neutrosophic cubic refined sets. Then intersection of A and B, denoted by  $A \cap B$ , is defined as

$$\begin{aligned} \mathcal{A} \cap \mathcal{B} = & \left\{ \langle x, ([T_A^{-1}(x) \wedge T_B^{-1}(x), T_A^{+1}(x) \wedge T_B^{+1}(x)], [T_A^{-2}(x) \wedge T_B^{-2}(x)], [T_A^{+2}(x) \wedge T_B^{+2}(x)], \dots, \right. \\ & [T_A^{-p}(x) \wedge T_B^{-p}(x), T_A^{+p}(x) \wedge T_B^{+p}(x)]), ([I_A^{-1}(x) \wedge I_B^{-1}(x), I_A^{+1}(x) \wedge I_B^{+1}(x)], \\ & [I_A^{-2}(x) \wedge I_B^{-2}(x)], [I_A^{+2}(x) \wedge I_B^{+2}(x)], \dots, [I_A^{-p}(x) \wedge I_B^{-p}(x), I_A^{+p}(x) \wedge I_B^{+p}(x)]), \\ & ([F_A^{-1}(x) \wedge F_B^{-1}(x), F_A^{+1}(x) \wedge F_B^{+1}(x)], [F_A^{-2}(x) \wedge F_B^{-2}(x)], [F_A^{+2}(x) \wedge F_B^{+2}(x)], \dots, \\ & \left. [F_A^{-p}(x) \wedge F_B^{-p}(x), F_A^{+p}(x) \wedge F_B^{+p}(x)]) \right\} \\ \Lambda \cap \Psi = & \left\{ \langle x, ([t_A^1(x) \wedge t_B^1(x), [t_A^2(x) \wedge t_B^2(x)], \dots, t_A^p(x) \wedge t_B^p(x)]), \right. \\ & ([i_A^1(x) \wedge i_B^1(x), [i_A^2(x) \vee i_B^2(x)], \dots, i_A^p(x) \wedge i_B^p(x)]), \\ & \left. ([f_A^1(x) \wedge f_B^1(x), [f_A^2(x) \wedge f_B^2(x)], \dots, f_A^p(x) \wedge f_B^p(x)]) \right\} \end{aligned}$$

**4. NEUTROSOPHIC SOFT CUBIC REFINED SET**

In this section presents the fundamental definitions of Neutrosophic soft cubic refined set, Complement, Union, Intersection, P-order, R-order, P-union, R union, INSCR and ENSCRS.

**Definition 4.1.** Let  $W$  be an initial universe set. Let  $NSCRS(W)$  denote the set of all neutrosophic cubic refined sets and  $E$  be the set of parameters. Let  $N \subset E$  then the pair  $(P, N)$  is termed to be the neutrosophic soft cubic refined set over  $W$  where  $P$  is a mapping given by  $P : N \rightarrow NCR(W)$ .

**Example 4.2.** Trisha wants to buy a washing machine so she can wash her clothes. She must assess a special washing machine that meets the requirements of a regular machine. Assume that  $W = \{w1, w2, w3, w4\}$  represents a variety of machine brands and  $E = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$  be the set of parameters, where  $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$  represent Expensive, Beautiful, Cheap, Very expensive. Let  $N = \{\alpha_2, \alpha_3\} \subseteq E$  Then, the neutrosophic soft cubic refined set,  $(P, N) = \{P(\alpha_i) = \{\langle w, A_{\alpha_i}, \Lambda_{\alpha_i} \rangle : w \in W\} \alpha_i \in N\}, i = 1, 2, 3, 4 \in W$  is represented in tabular form Table 1

**Example 4.3.** From the above example for  $W = \{w_1\}$  and  $P(\alpha_1)$  be neutrosophic refined cubic sets in  $W$  is also represented as  $(P, N) = \{P(\alpha_1) = \{\langle w_1, A_{\alpha_1}, \Lambda_{\alpha_1} \rangle : w_1 \in W\} \alpha_1 \in N\}$   
 $A_{\alpha_1} = \{\langle w_1, \{([0.1, 0.2], [0.3, 0.6], [0.6, 0.8]),$   
 $([0, 0.5], [0.4, 0.7], [0.4, 0.5]), ([0.2, 0.3][0.5][0.4, 0.6]) \rangle\}$   
 $\Lambda_{\alpha_1} = \{\langle w_1, (0.1, 0.3, 0.7)(0.3, 0.5, 0.4)(0.1, 0.3, 0.5) \rangle\}$

**Definition 4.4.** The complement of a neutrosophic soft cubic refined set  $(P, N) = \{\{\langle w, A_{\alpha_i}, \Lambda_{\alpha_i} \rangle : w \in W\} \alpha_i \in N\}$  is denoted by  $(P, N)^c$  where  $P^c : N \rightarrow NCR(W)$  and

$$(P, N)^c = \{(\{ \langle w, A_{\alpha_i}^c, \Lambda_{\alpha_i}^c \rangle : w \in W \} \alpha_i \in N \}$$

$$A_{\alpha_i}^c = \{ \langle w, ([1 - T_{A_{\alpha_i}}^{+1}(w), 1 - T_{A_{\alpha_i}}^{-1}(w)], [1 - T_{A_{\alpha_i}}^{+2}(w), 1 - T_{A_{\alpha_i}}^{-2}(w)], \dots, [1 - T_{A_{\alpha_i}}^{+p}(w), 1 - T_{A_{\alpha_i}}^{-p}(w)]), \\ ([1 - I_{A_{\alpha_i}}^{+1}(w), 1 - I_{A_{\alpha_i}}^{-1}(w)], [1 - I_{A_{\alpha_i}}^{+2}(w), 1 - I_{A_{\alpha_i}}^{-2}(w)], \dots, [1 - I_{A_{\alpha_i}}^{+q}(w), 1 - I_{A_{\alpha_i}}^{-q}(w)]), \\ ([1 - F_{A_{\alpha_i}}^{+1}(w), 1 - F_{A_{\alpha_i}}^{-1}(w)], [1 - F_{A_{\alpha_i}}^{+2}(w), 1 - F_{A_{\alpha_i}}^{-2}(w)], \dots, [1 - F_{A_{\alpha_i}}^{+p}(w), 1 - F_{A_{\alpha_i}}^{-r}(w)]) \rangle : w \in W \}$$

$$\Lambda_{\alpha_i}^c = \{ \langle w, (1 - t_{A_{\alpha_i}}^1(w), 1 - t_{A_{\alpha_i}}^2(w), \dots, 1 - t_{A_{\alpha_i}}^p(w)), (1 - i_{A_{\alpha_i}}^1(w), 1 - i_{A_{\alpha_i}}^2(w), \\ \dots, 1 - i_{A_{\alpha_i}}^p(w)), (1 - f_{A_{\alpha_i}}^1(w), 1 - f_{A_{\alpha_i}}^2(w), \dots, 1 - f_{A_{\alpha_i}}^p(w)) \rangle : w \in W \}$$

**Definition 4.5.** Let  $(P, N)$  be neutrosophic soft cubic refined set over  $W$ .

- (i)  $(P, N)$  is called absolute or universal neutrosophic soft cubic refined set  $W$  if  $P(e) = \hat{W}$  for all  $\alpha \in E$ . We denote it by  $W$ .
  - (ii)  $(P, N)$  is called null or empty neutrosophic soft cubic set  $U$  if  $P(e) = \hat{\Phi}$  for all  $\alpha \in E$ . We denote it by  $\Phi$ .
- Obviously  $\Phi^c = W$  and  $W^c = \Phi$ .

**Definition 4.6.** Let  $(P, N)$  and  $(Q, M)$  be two neutrosophic soft cubic refined set. Then the union of  $(P, N)$  and  $(Q, M)$  is denoted by  $P(\alpha_i) \cup Q(\alpha_i)$  and is defined by  $(H, C) = (H(\alpha_i)$  where  $C = N \cup Mand(H, C) = (P, N) \cup (Q, M)$   $P(\alpha_i) \cup Q(\alpha_i) = H_{\alpha_i} =$

**Table 1.** The tabular representation of  $(P, N)$

$N$	$\alpha_1$	$\alpha_2$	$\alpha_3$
$w_1$	$([0.1,0.2],[0.3,.6],[0.6,.8])$ $([0.0.5],[0.4,0.7],[0.4,0.5])$ $([0.2,0.3][0.0.5][0.4,0.6])$ $((0.1,0.3,0.7)(0.3,0.5,0.4)(0.1,0.3,0.5)$	$([0,0.5],[0.2,0.6],[0,0.4])$ $([0.0.5],[0.2,0.6],[0.4,0.5])$ $([0,0.7][0.3,0.7][0.3,0.5])$ $((0.7,0.3,0.2)(0.3,0.5,0.4)(0.8,0.3,0.1)$	$([0.1,0.3][0.3,0.5],[0,0.7])$ $([0.0,0.3][1.0.3],[0,0.5])$ $([0,0.6][0.4,0.5][0.3,0.4])$ $([0,0.6][0.4,0.5][0.3,0.4])(0.1,0.1,0.9)$
$w_2$	$([0.2,0.4],[0.2,0.3][0,0.5])$ $([0.6,0.7][0,0.5][0.1,0.5])$ $([0.3,0.6][0.2,0.7][0,0.3])$ $(0.6,0.4,0.3)(0.2,0.3,0.7)(0.4,0.3,0.1)$	$([0.4,0.6][0.1,0.5][0.2,0.3])$ $([0.4,0.7][0.4,0.6][0.3,0.4])$ $([0.2,0.3][0.3,0.5][0.5,0.7])$ $(0.4,0.1,0.7)(0.3,0.3,0.5)(0.6,0.4,0.2)$	$([0.2,0.3][0.4,0.5][0.1,0.2])$ $([0.3,0.5][0.2,0.5][0.2,.3])$ $([0.4,0.5][0.2,0.5][0,0.3])$ $(0.3,0.2,0.6)(0.4,0.5,0.1)(0.1,0.1,0.7)$
$w_3$	$([0.1,0.4],[0.1,0.3][.6,.8])$ $([.5,.6][0,.5][.1,.5])$ $([0.4,.6][.2,.6][0,.3])$ $(0.4,0.4,0.5)(0.1,0.3,0.6)(0.6,0.3,0.1)$	$([.5,.6][0,.6][.2,.3])$ $([0.3,.6][.4,.6][.3,.4])$ $([.1,.3][.2,.3][.5,.6])$ $(0.4,0.1,0.6)(0.3,0.4,0.5)(0.8,0.3,0.1)$	$([.3,.4][0.3,0.5][.1,.2])$ $([0.1,.5][.3,.5][0.1,.3])$ $([.4,.5][.3,.5][0.2,.3])$ $(0.3,0.1,0.4)(0.5,0.5,0.2)(0.1,0.2,0.6)$
$w_4$	$([0.2,0.4],[0.2,0.3][0,.5][.7,.8])$ $([.6,.8][0,.5][.1,.5])$ $([0.3,.6][.2,.8][0,.3])$ $(0.6,0.4,0.3)(0.1,0.3,0.8)(0.6,0.3,0.1)$	$([.5,.6][0,.6][.1,.3])$ $([0.3,.8][0.3,.6][.3,.4])$ $([.1,.3][.2,.3][.5,.8])$ $(0.4,0.1,0.8)(0.3,0.4,0.5)(0.8,0.3,0.1)$	$([.2,.5][0.4,.5][.2,.3])$ $([0.3,.5][.2,.5][0,.3])$ $([0.2,.5][.1,.5][0.2,.3])$ $(0.4,0.2,0.6)(0.8,0.5,0.1)(0.1,0.1,0.9)$

$\{\langle w, (A_{\alpha_i} \cup B_{\alpha_i}), (\Lambda_{\alpha_i} \cup \Psi_{\alpha_i}) \rangle : w \in W\} \alpha_i \in N \cap M$  where

$$\begin{aligned}
 A_{\alpha_i} \cup B_{\alpha_i} = & \left\{ \langle w, ([T_{A_{\alpha_i}}^{-1}(w) \vee T_{B_{\alpha_i}}^{-1}(w), T_{A_{\alpha_i}}^{+1}(w) \vee T_{B_{\alpha_i}}^{+1}(w)], [T_{A_{\alpha_i}}^{-2}(w) \vee T_{B_{\alpha_i}}^{-2}(w)], [T_{A_{\alpha_i}}^{+2}(w) \vee T_{B_{\alpha_i}}^{+2}(w)], \dots, \right. \\
 & [T_{A_{\alpha_i}}^{-p}(w) \vee T_{B_{\alpha_i}}^{-p}(w), T_{A_{\alpha_i}}^{+p}(w) \vee T_{B_{\alpha_i}}^{+p}(w)], ([I_{A_{\alpha_i}}^{-1}(w) \vee I_{B_{\alpha_i}}^{-1}(w), I_{A_{\alpha_i}}^{+1}(w) \vee I_{B_{\alpha_i}}^{+1}(w)], \\
 & [I_{A_{\alpha_i}}^{-2}(w) \vee I_{B_{\alpha_i}}^{-2}(w)], [I_{A_{\alpha_i}}^{+2}(w) \vee I_{B_{\alpha_i}}^{+2}(w)], \dots, [I_{A_{\alpha_i}}^{-p}(w) \vee I_{B_{\alpha_i}}^{-p}(w), I_{A_{\alpha_i}}^{+p}(w) \vee I_{B_{\alpha_i}}^{+p}(w)], \\
 & ([F_{A_{\alpha_i}}^{-1}(w) \vee F_{B_{\alpha_i}}^{-1}(w), F_{A_{\alpha_i}}^{+1}(w) \vee F_{B_{\alpha_i}}^{+1}(w)], [F_{A_{\alpha_i}}^{-2}(w) \vee F_{B_{\alpha_i}}^{-2}(w)], [F_{A_{\alpha_i}}^{+2}(w) \vee F_{B_{\alpha_i}}^{+2}(w)], \dots, \\
 & \left. [F_{A_{\alpha_i}}^{-p}(w) \vee F_{B_{\alpha_i}}^{-p}(w), F_{A_{\alpha_i}}^{+p}(w) \vee F_{B_{\alpha_i}}^{+p}(w)] \right\} \\
 \Lambda_{\alpha_i} \cup \Psi_{\alpha_i} = & \left\{ \langle w, ([t_{A_{\alpha_i}}^1(w) \vee t_{B_{\alpha_i}}^1(w), [t_{A_{\alpha_i}}^2(w) \vee t_{B_{\alpha_i}}^2(w)], \dots, t_{A_{\alpha_i}}^p(w) \vee t_{B_{\alpha_i}}^p(w)], \right. \\
 & ([i_{A_{\alpha_i}}^1(w) \vee i_{B_{\alpha_i}}^1(w), [i_{A_{\alpha_i}}^2(w) \vee i_{B_{\alpha_i}}^2(w)], \dots, i_{A_{\alpha_i}}^p(w) \vee i_{B_{\alpha_i}}^p(w)], \\
 & \left. ([f_{A_{\alpha_i}}^1(w) \vee f_{B_{\alpha_i}}^1(w), [f_{A_{\alpha_i}}^2(w) \vee f_{B_{\alpha_i}}^2(w)], \dots, f_{A_{\alpha_i}}^p(w) \vee f_{B_{\alpha_i}}^p(w)] \right\}
 \end{aligned}$$

**Definition 4.7.** Let  $(P, N)$  and  $(Q, M)$  be two neutrosophic soft cubic refined set. Then the intersection of  $(P, N)$  and  $(Q, M)$  is denoted by  $P(\alpha_i) \cap Q(\alpha_i)$  and is defined by  $(H, C) = (H_{\alpha_i}, C)$  where  $C = N \cup M$  and  $(H, C) = (P, N) \cap (Q, M)$

$$P(\alpha_i) \cap Q(\alpha_i) = H_{\alpha_i} = \{\langle w, (A_{\alpha_i} \cap B_{\alpha_i}), (\Lambda_{\alpha_i} \cap \Psi_{\alpha_i}) \rangle : w \in W\} \alpha_i \in N \cap M \text{ where}$$

$$\begin{aligned}
 A_{\alpha_i} \cap B_{\alpha_i} = & \left\{ \langle w, ([T_{A_{\alpha_i}}^{-1}(w) \wedge T_{B_{\alpha_i}}^{-1}(w), T_{A_{\alpha_i}}^{+1}(w) \wedge T_{B_{\alpha_i}}^{+1}(w)], [T_{A_{\alpha_i}}^{-2}(w) \wedge T_{B_{\alpha_i}}^{-2}(w)], [T_{A_{\alpha_i}}^{+2}(w) \wedge T_{B_{\alpha_i}}^{+2}(w)], \dots, \right. \\
 & [T_{A_{\alpha_i}}^{-p}(w) \wedge T_{B_{\alpha_i}}^{-p}(w), T_{A_{\alpha_i}}^{+p}(w) \wedge T_{B_{\alpha_i}}^{+p}(w)], ([I_{A_{\alpha_i}}^{-1}(w) \wedge I_{B_{\alpha_i}}^{-1}(w), I_{A_{\alpha_i}}^{+1}(w) \wedge I_{B_{\alpha_i}}^{+1}(w)], \\
 & [I_{A_{\alpha_i}}^{-2}(w) \wedge I_{B_{\alpha_i}}^{-2}(w)], [I_{A_{\alpha_i}}^{+2}(w) \wedge I_{B_{\alpha_i}}^{+2}(w)], \dots, [I_{A_{\alpha_i}}^{-p}(w) \wedge I_{B_{\alpha_i}}^{-p}(w), I_{A_{\alpha_i}}^{+p}(w) \wedge I_{B_{\alpha_i}}^{+p}(w)], \\
 & ([F_{A_{\alpha_i}}^{-1}(w) \wedge F_{B_{\alpha_i}}^{-1}(w), F_{A_{\alpha_i}}^{+1}(w) \wedge F_{B_{\alpha_i}}^{+1}(w)], [F_{A_{\alpha_i}}^{-2}(w) \wedge F_{B_{\alpha_i}}^{-2}(w)], [F_{A_{\alpha_i}}^{+2}(w) \wedge F_{B_{\alpha_i}}^{+2}(w)], \dots, \\
 & \left. [F_{A_{\alpha_i}}^{-p}(w) \wedge F_{B_{\alpha_i}}^{-p}(w), F_{A_{\alpha_i}}^{+p}(w) \wedge F_{B_{\alpha_i}}^{+p}(w)] \right\} \\
 \Lambda_{\alpha_i} \cap \Psi_{\alpha_i} = & \left\{ \langle w, ([t_{A_{\alpha_i}}^1(w) \wedge t_{B_{\alpha_i}}^1(w), [t_{A_{\alpha_i}}^2(w) \wedge t_{B_{\alpha_i}}^2(w)], \dots, t_{A_{\alpha_i}}^p(w) \wedge t_{B_{\alpha_i}}^p(w)], \right. \\
 & ([i_{A_{\alpha_i}}^1(w) \wedge i_{B_{\alpha_i}}^1(w), [i_{A_{\alpha_i}}^2(w) \wedge i_{B_{\alpha_i}}^2(w)], \dots, i_{A_{\alpha_i}}^p(w) \wedge i_{B_{\alpha_i}}^p(w)], \\
 & \left. ([f_{A_{\alpha_i}}^1(w) \wedge f_{B_{\alpha_i}}^1(w), [f_{A_{\alpha_i}}^2(w) \wedge f_{B_{\alpha_i}}^2(w)], \dots, f_{A_{\alpha_i}}^p(w) \wedge f_{B_{\alpha_i}}^p(w)] \right\}
 \end{aligned}$$

**Definition 4.8.** A neutrosophic soft cubic refined set  $(P, N)$  is contained in the other neutrosophic soft cubic refined set  $(P, M)$ , denoted by  $(P, N) \subseteq (P, M)$  i.e.  $(P, N) \subseteq (P, M)$ , if and only if  $A_{\alpha_i}(w) \subseteq B_{\alpha_i}(w)$  and  $\Lambda_{\alpha_i}(w) \subseteq \Psi_{\alpha_i}(w)$  where

$$\begin{aligned}
 A_{\alpha_i}(w) \subseteq B_{\alpha_i}(w) \Rightarrow & T_{A_{\alpha_i}}^{-k}(w) \leq T_{B_{\alpha_i}}^{-k}(w), T_{A_{\alpha_i}}^{+k}(w) \leq T_{B_{\alpha_i}}^{+k}(w), \\
 & I_{A_{\alpha_i}}^{-k}(w) \geq I_{B_{\alpha_i}}^{-k}(w), I_{A_{\alpha_i}}^{+k}(w) \geq I_{B_{\alpha_i}}^{+k}(w), \\
 & F_{A_{\alpha_i}}^{-k}(w) \geq F_{B_{\alpha_i}}^{-k}(w), F_{A_{\alpha_i}}^{+k}(w) \geq F_{B_{\alpha_i}}^{+k}(w), \\
 \Lambda_{\alpha_i}(w) \subseteq \Psi_{\alpha_i}(w) \Rightarrow & t_{A_{\alpha_i}}^k(w) \leq t_{B_{\alpha_i}}^k(w), i_{A_{\alpha_i}}^k(w) \geq i_{B_{\alpha_i}}^k(w), \\
 & f_{A_{\alpha_i}}^k(w) \geq f_{B_{\alpha_i}}^k(w), \text{ for all } w \in W, i = 1, 2, \dots, p.
 \end{aligned}$$

**Definition 4.9.** Let  $W$  be an universal set. A neutrosophic soft cubic refined set  $(P, N)$  in  $W$  is said to be

- truth-internal (briefly,  $T$ -internal)NSCRS if the following inequality is valid

$$(\forall w \in W, \alpha_i \in N)(T_{A_{\alpha_i}}^{-k}(w) \leq t_{A_{\alpha_i}}^k(w) \leq T_{A_{\alpha_i}}^{+k}(w)), \tag{1}$$

- indeterminacy-internal (briefly,  $I$ -internal)NSCRS if the following inequality is valid

$$(\forall w \in W, \alpha_i \in N)(I_{A_{\alpha_i}}^{-k}(w) \leq i_{A_{\alpha_i}}^k(w) \leq I_{A_{\alpha_i}}^{+k}(w)), \tag{2}$$

- falsity-internal (briefly,  $F$ -internal)NSCRS if the following inequality is valid

$$(\forall w \in W, \alpha_i \in N)(F_{A_{\alpha_i}}^{-k}(w) \leq f_{A_{\alpha_i}}^k(w) \leq F_{A_{\alpha_i}}^{+k}(w)), \tag{3}$$

If a neutrosophic soft refined cubic set in  $W$  satisfies (1), (2) and (3) we say that  $(P, M)$  is an internal neutrosophic soft cubic refined set (INSCRCS) in  $W$ .

**Definition 4.10.** Let  $W$  be an universal set. A neutrosophic soft cubic refined set  $(P, N)$  in  $W$  is said to be

- truth-external (briefly,  $T$ -external)NSCRS if the following inequality is valid

$$(\forall w \in W, \alpha_i \in N)t_{A_{\alpha_i}}^k \notin [T_{A_{\alpha_i}}^{-k}, T_{A_{\alpha_i}}^{+k}], \tag{4}$$

- indeterminacy-external (briefly,  $I$ -external)NSCRS if the following inequality is valid

$$(\forall w \in W, \alpha_i \in N)i_{A_{\alpha_i}}^k \notin [I_{A_{\alpha_i}}^{-k}, I_{A_{\alpha_i}}^{+k}], \tag{5}$$

- falsity-external (briefly,  $F$ -external)NSCRS if the following inequality is valid

$$(\forall w \in W, \alpha_i \in N)f_{A_{\alpha_i}}^k \notin [F_{A_{\alpha_i}}^{-k}, F_{A_{\alpha_i}}^{+k}], \tag{6}$$

If a neutrosophic soft refined cubic set in  $W$  satisfies (4), (5) and (6) we say that  $(P, N)$  is an external neutrosophic soft cubic refined set (ENSCRCS) in  $W$ .

**Theorem 4.11.** Let  $(P, N) = \{\{\langle w, A_{\alpha_i}, \lambda_{\alpha_i} \rangle : w \in W\} \mid \alpha_i \in N\}$  be a neutrosophic soft cubic refined set in  $W$  which is not an ENSCRCS. Then, there exists at least one  $\alpha_i \in N$  for which there exist some  $w \in W$  such that  $t_{A_{\alpha_i}}^k(w) \in [T_{A_{\alpha_i}}^{-k}(w), T_{A_{\alpha_i}}^{+k}(w)]$ ,  $i_{A_{\alpha_i}}^k(w) \in [I_{A_{\alpha_i}}^{-k}(w), I_{A_{\alpha_i}}^{+k}(w)]$ ,  $f_{A_{\alpha_i}}^k(w) \in [F_{A_{\alpha_i}}^{-k}(w), F_{A_{\alpha_i}}^{+k}(w)]$ ,



*Proof.* By the definition of an external neutrosophic soft cubic refined set (ENSCRS) we know that

$$\begin{aligned} t_{A_{\alpha_i}}^k &\notin [T_{A_{\alpha_i}}^{-k}, T_{A_{\alpha_i}}^{+k}], \\ i_{A_{\alpha_i}}^k &\notin [I_{A_{\alpha_i}}^{-k}, I_{A_{\alpha_i}}^{+k}], \\ f_{A_{\alpha_i}}^k &\notin [F_{A_{\alpha_i}}^{-k}, F_{A_{\alpha_i}}^{+k}], \end{aligned}$$

for all  $w \in W$ , corresponding to each  $\alpha_i \in N$ . But given that  $(P, N)$  is not ENSCRS so far at least one  $\alpha_i \in N$  there exists some  $w \in W$  such that

$$\begin{aligned} T_{A_{\alpha_i}}^{-k}(w) &\leq t_{A_{\alpha_i}}^k(w) \leq T_{A_{\alpha_i}}^{+k}(w). \\ I_{A_{\alpha_i}}^{-k}(w) &\leq i_{A_{\alpha_i}}^k(w) \leq I_{A_{\alpha_i}}^{+k}(w). \\ F_{A_{\alpha_i}}^{-k}(w) &\leq f_{A_{\alpha_i}}^k(w) \leq F_{A_{\alpha_i}}^{+k}(w). \end{aligned}$$

Hence the result.  $\square$

**Theorem 4.12.** *Let  $(P, N)$  be a neutrosophic soft cubic refined set in  $W$ . If  $(P, N)$  is both  $T$ -internal soft cubic refined set and  $T$ -external soft cubic refined set in  $W$ , then  $(\forall w \in W, \alpha_i \in N)$*

$$t_{A_{\alpha_i}}^k(w) \in \left\{ T_{A_{\alpha_i}}^{-k}(w)/w \in W, \alpha_i \in N \right\} \cup \left\{ T_{A_{\alpha_i}}^{+k}(w)/w \in W, \alpha_i \in N \right\} \tag{7}$$

*Proof.* Consider the definitions 4.9 and 4.10 which implies that

$$T_{A_{\alpha_i}}^{-k}(w) \leq t_{A_{\alpha_i}}^k(w) \leq T_{A_{\alpha_i}}^{+k}(w) \text{ and } t_{A_{\alpha_i}}^k(w) \notin [T_{A_{\alpha_i}}^{-k}(w), T_{A_{\alpha_i}}^{+k}(w)] \text{ for all } w \in W, \alpha_i \in N.$$

Then it follows that  $t_{A_{\alpha_i}}^k(w) = T_{A_{\alpha_i}}^{-k}(w)$  or  $t_{A_{\alpha_i}}^k(w) = T_{A_{\alpha_i}}^{+k}(w)$ , and hence  $t_{A_{\alpha_i}}^k(w) \in \left\{ T_{A_{\alpha_i}}^{-k}(w)/w \in W, \alpha_i \in N \right\} \cup \left\{ T_{A_{\alpha_i}}^{+k}(w)/w \in W, \alpha_i \in N \right\}$

Hence proved Similarly, the following Theorems hold for the indeterminate and falsity values.

$\square$

**Theorem 4.13.** *Let  $(P, N)$  be a neutrosophic soft cubic refined set in a non-empty set  $W$ . If  $(P, N)$  is both  $I$ - internal NSCRS and  $I$ - external NSCRS, then  $(\forall w \in W, \alpha_i \in N), i_{A_{\alpha_i}}^k(w) \in \left\{ I_{A_{\alpha_i}}^{-k}(w)/w \in W, \alpha_i \in N \right\} \cup \left\{ I_{A_{\alpha_i}}^{+k}(w)/w \in W, \alpha_i \in N \right\}$*

**Theorem 4.14.** *Let  $(P, N)$  be a neutrosophic soft cubic refined set in a non-empty set  $W$ . If  $(P, N)$  is both  $F$ - Internal NSCRS and  $F$ - External NSCRS, then  $(\forall w \in W, \alpha_i \in N), f_{A_{\alpha_i}}^k(w) \in \left\{ F_{A_{\alpha_i}}^{-k}(w)/w \in W, \alpha_i \in N \right\} \cup \left\{ F_{A_{\alpha_i}}^{+k}(w)/w \in W, \alpha_i \in N \right\}$*

**Definition 4.15.** Let  $(P, N) = \{P(\alpha_i) = \{\langle w, A_{\alpha_i}, \Lambda_{\alpha_i} \rangle : w \in W\} \mid \alpha_i \in N\}$  and

$$(Q, M) = \{Q(\alpha_i) = \{\langle w, B_{\alpha_i}, \Psi_{\alpha_i} \rangle : w \in W\} \mid \alpha_i \in M\}$$

be two neutrosophic soft cubic refined sets in  $W$ . Let  $N$  and  $M$  be any two subsets of  $E$  (set of parameters), then we have the following

- (1)  $(P, N) = (Q, M)$  if and only if the following conditions are satisfied
  - (a)  $N = M$  and
  - (b)  $P(\alpha_i) = Q(\alpha_i)$  for all  $\alpha_i \in N$  if and only if  $A_{\alpha_i} = B_{\alpha_i}$  and  $\Lambda_{\alpha_i} = \Psi_{\alpha_i}$  for all  $w \in W$  corresponding to each  $\alpha_i \in N$ .
- (2) If  $(P, N)$  and  $(Q, M)$  are two neutrosophic soft cubic refined sets then we define and denote  $P$ -order as  $(P, N) \subseteq_P (Q, M)$  if and only if the following conditions are satisfied
  - (a)  $N \subseteq M$  and
  - (b)  $P(\alpha_i) \leq_P Q(\alpha_i)$  for all  $\alpha_i \in N$  if and only if  $A_{\alpha_i} \subseteq B_{\alpha_i}$  and  $\Lambda_{\alpha_i} \leq \Psi_{\alpha_i}$  for all  $w \in W$  corresponding to each  $\alpha_i \in M$ .
- (3) If  $(P, N)$  and  $(Q, M)$  are two neutrosophic soft cubic refined sets then we define and denote  $R$ -order as  $(P, N) \subseteq_R (Q, M)$  if and only if the following conditions are satisfied
  - (a)  $N \subseteq M$  and
  - (b)  $P(\alpha_i) \leq_R Q(\alpha_i)$  for all  $\alpha_i \in N$  if and only if  $A_{\alpha_i} \subseteq B_{\alpha_i}$  and  $\Lambda_{\alpha_i} \geq \Psi_{\alpha_i}$  for all  $w \in W$  corresponding to each  $\alpha_i \in N$ .

Definitions of the  $P$ -union,  $P$ -intersection,  $R$ -union and  $R$ -intersection of neutrosophic soft cubic refined sets as follows:

**Definition 4.16.** Let  $(P, N)$  and  $(Q, M)$  be two neutrosophic soft cubic refined sets (NSCRS) in  $W$  where  $N$  and  $M$  are any two subsets of the parameters set  $E$ . Then we define  $P$ -union as  $(P, N) \cup_P (Q, M) = (H, C)$  where  $C = N \cup M$

$$H(\alpha_i) = \left\{ \begin{array}{ll} P(\alpha_i) & \text{if } \alpha_i \in N - M \\ Q(\alpha_i) & \text{if } \alpha_i \in M - N \\ P(\alpha_i) \cup_P Q(\alpha_i) & \text{if } \alpha_i \in N \cap M \end{array} \right\}$$

where  $P(\alpha_i) \cup_P Q(\alpha_i)$  is defined as

$$P(\alpha_i) \cup_P Q(\alpha_i) = \{\langle w, \{A_{\alpha_i} \cup B_{\alpha_i}\}, (\Lambda_{\alpha_i} \cup \Psi_{\alpha_i}) \rangle : w \in W\} \mid \alpha_i \in N \cap M$$

**Definition 4.17.** Let  $(P, N)$  and  $(Q, M)$  be two neutrosophic soft cubic refined sets (NSCRS) in  $W$  where  $N$  and  $M$  are any subsets of the parameter set  $E$ . Then we define  $P$ -intersection

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as  $(P, N) \cap_P (Q, M) = (H, C)$  where  $C = M \cap N$ ,  $H(\alpha_i) = P(\alpha_i) \cap_P Q(\alpha_i)$  and  $\alpha_i \in M \cap N$ . Here  $P(\alpha_i) \cap_P Q(\alpha_i)$  is defined as

$$P(\alpha_i) \cap_P Q(\alpha_i) = H(\alpha_i) = \{ \langle w, (A_{\alpha_i} \cap B_{\alpha_i}), (\lambda_{\alpha_i} \cap \Psi_{\alpha_i}) \rangle : w \in W \} \alpha_i \in N \cap M$$

**Definition 4.18.** Let  $(P, N)$  and  $(Q, M)$  be two neutrosophic soft set cubic sets (NSCS) in  $W$  where  $N$  and  $M$  are any subsets of the parameter set  $E$ . Then we define  $R$ -union of neutrosophic soft cubic refined set as  $(P, N) \cup_R (Q, M) = (H, C)$  where  $C = N \cup M$ ,

$$H(\alpha_i) = \left\{ \begin{array}{ll} P(\alpha_i) & \text{if } \alpha_i \in N - M \\ Q(\alpha_i) & \text{if } \alpha_i \in M - N \\ P(\alpha_i) \cup_R Q(\alpha_i) & \text{if } \alpha_i \in M \cap N \end{array} \right\}$$

where  $P(\alpha_i) \cup_R Q(\alpha_i)$  is defined as

$$P(\alpha_i) \cup_R Q(\alpha_i) = \{ \langle w, \{A_{\alpha_i} \cup B_{\alpha_i}\}, (\Lambda_{\alpha_i} \cap \Psi_{\alpha_i}) \rangle : w \in W \} \alpha_i \in M \cap N$$

**Definition 4.19.** Let  $(P, N)$  and  $(Q, M)$  be two neutrosophic soft cubic refined sets (NSCRS) in  $W$  where  $N$  and  $W$  are any subsets of the parameter set  $E$ . Then we define  $R$ -intersection of neutrosophic soft cubic refined set as  $(P, N) \cap_R (Q, M) = (H, C)$  where  $C = N \cap M$ ,  $H(\alpha_i) = P(\alpha_i) \cap_R Q(\alpha_i)$ ,  $H(\alpha_i) = P(\alpha_i) \cap_R Q(\alpha_i)$  and  $\alpha_i \in M \cap N$ . Here  $P(\alpha_i) \cap_R Q(\alpha_i)$  is defined as

$$P(\alpha_i) \cap_R Q(\alpha_i) = H(\alpha_i) = \{ \langle w, \{A_{\alpha_i} \cap B_{\alpha_i}\}, (\Lambda_{\alpha_i} \cup \Psi_{\alpha_i}) \rangle : w \in W \} \alpha_i \in N \cap M$$

### 5. Conclusions

As research and applications continue to evolve, the Neutrosophic Soft Cubic Refined Sets framework stands out for its ability to handle complex data with greater accuracy and insight. Its development marks a significant step forward in the ongoing quest for more effective and adaptable tools in the mathematical and data sciences, promising to contribute meaningfully to the advancement of both theoretical and practical applications. In this paper we study the concept of Neutrosophic Cubic Refined Set (NSCRS) and extended to Neutrosophic Soft Cubic Refined Set (NSCRS), which is the combination of the theory of soft sets, Neutrosophic cubic set, Neutrosophic refined set and Neutrosophic Soft Cubic Set. Further studied the INSCR and ENSCRS, along with their various properties, including P-order, P-union, P-intersection, and R-order, R-union, and R-intersection neutrosophic soft cubic refined set. With the motivation of ideas presented in this paper, one can think of similarity measures, Cartesian products, and relations on Neutrosophic refined cubic soft sets. Further studies on the topology generated by the Neutrosophic refined cubic soft sets. It is hoped that the combinations of Neutrosophic refined cubic soft sets with topology and rough sets will generate potentially interesting some

new research direction.

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