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Cubic Spherical Neutrosophic Sets for Advanced Decision-Making

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Abstract. In the context of aggregation operators and multiple criteria decision-making, this article presents the idea of cubic spherical neutrosophic sets. The notion of neutrosophic informations are transform into a geometric sphere by determining its center and radius. This advanced geometrical representation extends traditional neutrosophic informations. This study defines and discusses weighted additive and weighted geometric aggregation operators tailored to cubic spherical neutrosophic sets that are vital for handling complex and uncertain information. Practical example, such as evaluating fertilizer brands for coconut farming, illustrate their application in decision-making contexts. By integrating cubic spherical neutrosophic sets into multiple criteria decision-making frameworks, decision makers can effectively manage uncertainty and make informed decisions. When multiple stakeholders are involved in the decision-making process, averaging their decision values may not accurately reflect a true perspective. This multiple criteria decision-making approach overcomes the limitations of traditional averaging method. Theoretical discussions and practical examples contribute to advancing the understanding and application of multiple criteria decision-making, enhancing the reliability of decision support systems.

Keywords: Neutrosophic set; cubic spherical neutrosophic set; cubic spherical neutrosophic aggregation operators; multi criteria decision making.

1. Introduction

In 1998, F. Smarandache [19, 20] introduced and examined neutrosophic sets (NSs) as an extension of Atanassov's [6] theory of Intuitionistic fuzzy sets. Since then, various generalizations of NSs have emerged, finding applications across diverse fields. Particularly in Multiple Criteria Decision Making (MCDM), NSs and their variants play a crucial role. In 2014, Peng et al. [17] proposed an outranking approach tailored for MCDM problems within a simplified neutrosophic framework. They also devised a ranking approach utilizing outranking relations of simplified neutrosophic numbers to address

MCDM issues, demonstrating their method through illustrative examples. In 2015, Majumdar [15] introduced concepts of distance and similarity between NSs, vital for identifying interacting segments in datasets. Moreover, the notion of entropy was proposed to quantify uncertainty in neutrosophic sets, underlining its importance in decision-making. In the same year, Deli, et al. [10] introduced bipolar NSs and associated operations, proposing functions for comparison and aggregation. They also developed operators and functions for a bipolar neutrosophic multiple criteria decision-making approach. Jun Ye [23] introduced trapezoidal NSs, proposing aggregation operators and a decision-making method for problems represented by trapezoidal neutrosophic numbers. In 2016, Biswas et al. [7] introduced a new method for multi attribute group decision-making, extending the preference technique to single-valued neutrosophic settings. Ratings of alternatives per attribute are expressed as single-valued NSs, reflecting decision makers views. A weighted averaging operator based on single-valued NSs aggregates opinions, forming a unified consensus on criteria and alternatives importance. In 2018, Ali et al. [3] explored interval complex NSs and their application in green supplier selection, demonstrating efficiency through real dataset examples from Thuan Yen JSC. Neutrosophic sets and their extensions find diverse applications across various domains such as medical diagnosis [9, 18], medical robotics engineering [16], healthcare services supply chain [5], disaster risk management [1], handling imperfect and incomplete information [2], information processing [8], pattern recognition [4, 9], image segmentation [13], and addressing financial issues [11]. The notion of CSNS was introduced and studied by Gomathi et al. [12] as a geometrical representation of collection of NSs. Recently Krishnaprakash [14] et al. introduced cubic spherical neutrosophic Archimedean triangular norms(ATN) and conorms (ATCN), also applied the concepts in MCDM with an example of selection of electric truck.

This article delves into the introduction of CSNSs, a novel advancement in aggregation operators and multiple criteria decision-making (MCDM). CSNSs offer a transformative approach by converting neutrosophic data into geometric spheres, thereby establishing a geometric representation with discernible centers and radii. This sophisticated representation extends the capabilities of traditional neutrosophic sets, enabling a more comprehensive understanding of uncertain information. The study proceeds to define and explore weighted additive and weighted geometric aggregation operators specifically tailored to CSNSs. These operators play a vital role in navigating the complexities of uncertain information, providing decision-makers with robust tools for informed decision-making. To illustrate the practical application of CSNSs, the article presents examples such as the evaluation of fertilizer brands for coconut farming. Through these examples, decision-makers gain insight into how these innovative methodologies can be effectively employed in decision-making contexts. By integrating CSNSs into MCDM frameworks, decision-makers can effectively manage uncertainty and make informed decisions across diverse domains. Theoretical discussions, coupled with practical examples, contribute to advancing the understanding and application of CSNSs, thereby enhancing the reliability of decision support systems. In essence, this article serves as a catalyst for the adoption of innovative methodologies, ensuring robust decision-making processes amidst uncertainty in various domains. Through the exploration of CSNSs, decision-makers are empowered to navigate complex decision landscapes with confidence and precision.

The following contributions are made in the field of neutrosophic sets and MCDM:

(1) Weighted additive and weighted geometric aggregation operators tailored to cubic spherical neutrosophic sets are defined and discussed.
(2) The applicability of weighted additive and weighted geometric aggregation operators in multiple criteria decision-making (MCDM) contexts are demonstrated.
(3) Development of comprehensive framework for integrating CSNSs into MCDM has been made.
(4) Practical examples to illustrate their effectiveness in real-world decision-making have been provided.
(5) To overcome the limitations of traditional averaging methods when multiple stakeholders are involved in decision-making are illustrated.

2. **Preliminaries**

Let Γ be the universal set containing elements known as Neutrosophic Sets [19] (NSs). Each $\epsilon_i \in \Gamma$ is defined as $\mathcal{N}_{\epsilon_\imath} = \{\langle \epsilon_\imath, \mathbb{T}(\epsilon_\imath), \mathbb{I}(\epsilon_\imath), \mathbb{F}(\epsilon_\imath)\rangle | \epsilon_\imath \in \Gamma\}$, where $\mathbb{T}(\epsilon_\imath)$, $\mathbb{F}(\epsilon_\imath)$, $\mathbb{I}(\epsilon_\imath) : \Gamma \to [0,1]$ represent the degrees of membership, non-membership, and indeterminacy of ϵ_i. These degrees satisfy $0 \leq \mathbb{T}(\epsilon_i) + \mathbb{I}(\epsilon_\imath) + \mathbb{F}(\epsilon_i) \leq 3$ for all $\epsilon_\imath \in \Gamma$ and $\imath = 1, 2, 3, ..., n$. Let Γ be the universal set containing elements known as Cubic Spherical Neutrosophic sets [12] (CSNSs). Each $\epsilon_\imath \in \Gamma$ is defined as $\delta_{\mathbb{R}} = \{\langle \epsilon_\imath, csn\mathbb{T}(\epsilon_\imath), csn\mathbb{I}(\epsilon_\imath), csn\mathbb{F}(\epsilon_\imath); \mathbb{R}\rangle : \epsilon_\imath \in \Gamma\}$, where $csn\mathbb{T}(\epsilon)$, $csn\mathbb{F}(\epsilon)$, $csn\mathbb{I}(\epsilon), \mathbb{R} : \Gamma \to [0,1]$ represent the degrees of membership, non-membership, indeterminacy and radius of $\delta_{\mathbb{R}}$. These degrees satisfy $0 \leq csn\mathbb{T}(\epsilon_i) + csn\mathbb{I}(\epsilon_i) + csn\mathbb{F}(\epsilon_i) \leq 3$ for all $\epsilon_\imath \in \Gamma$ and $\imath = 1, 2, \ldots, k$. Where the center

$$\langle csn\mathbb{T}(\epsilon_\imath), csn\mathbb{I}(\epsilon_\imath), csn\mathbb{F}(\epsilon_\imath)\rangle = \left\langle \frac{\sum_{\jmath=1}^{k_\imath} \mathbb{T}_{\imath,\jmath}}{k_\imath}, \frac{\sum_{\jmath=1}^{k_\imath} \mathbb{I}_{\imath,\jmath}}{k_\imath}, \frac{\sum_{\jmath=1}^{k_\imath} \mathbb{F}_{\imath,\jmath}}{k_\imath} \right\rangle \tag{1}$$

and the radius

$$\mathbb{R}_\imath = \min\left\{ \max_{1\leq \jmath \leq k_\imath} \sqrt{(csn\mathbb{T}(\epsilon_\imath) - \mathbb{T}_{\imath,\jmath})^2 + (csn\mathbb{I}(\epsilon_\imath) - \mathbb{I}_{\imath,\jmath})^2 + (csn\mathbb{F}(\epsilon_\imath) - \mathbb{F}_{\imath,\jmath})^2}, 1 \right\}. \tag{2}$$

For example, choose $X = \{\alpha, \beta\}$ and $\delta_1, \delta_2 \in NS(X)$ such that
$\delta_1 = \{\langle \alpha, 0.88, 0.33, 0.22\rangle, \langle \alpha, 0.77, 0.44, 0.11\rangle, \langle \alpha, 0.55, 0.44, 0.22\rangle, \langle \alpha, 0.66, 0.55, 0.33\rangle\}$ and $\delta_2 = \{\langle \beta, 0.66, 0.22, 0.11\rangle, \langle \beta, 0.88, 0.11, 0.22\rangle, \langle \beta, 0.88, 0.33, 0.11\rangle, \langle \beta, 0.99, 0.44, 0.22\rangle\}$. Then the CSNSs are $\delta_{(R_1)} = \{\langle \alpha, 0.72, 0.44, 0.22; 0.20\rangle : \alpha \in X\}$ and
$\delta_{(R_2)} = \{\langle \beta, 0.85, 0.28, 0.17; 0.22\rangle : \beta \in X\}$.

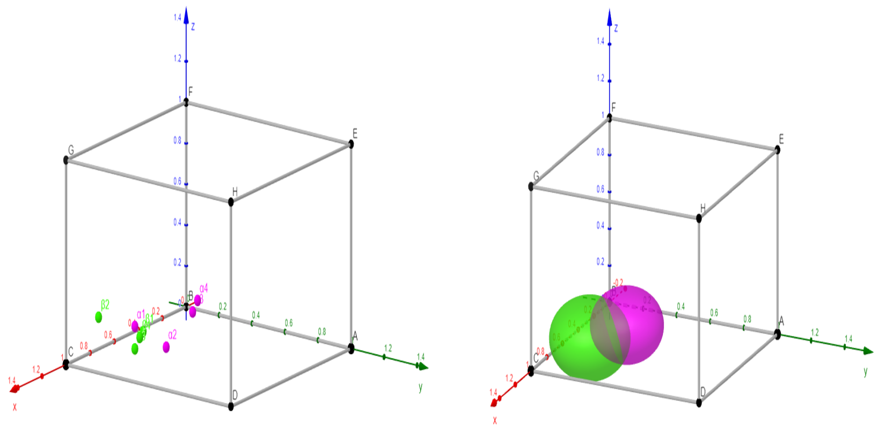

FIGURE 1. The geomentric representation of neutrosophic sets and CSNSs

For any two CSNSs δ_1 and δ_2 defined as $\delta_1 = \langle csn\mathbb{T}_{\delta_1}, csn\mathbb{I}_{\delta_1}, csn\mathbb{F}_{\delta_1}; \mathbb{R}_{\delta_1}\rangle$ and $\delta_2 = \langle csn\mathbb{T}_{\delta_2}, csn\mathbb{I}_{\delta_2}, csn\mathbb{F}_{\delta_2}; \mathbb{R}_{\delta_2}\rangle$, the cosine distance [12] is defined by: $\cos(\delta_1, \delta_2) =$

$$1 - \frac{csn\mathbb{T}_{\delta_1} \cdot csn\mathbb{T}_{\delta_2} + csn\mathbb{I}_{\delta_1} \cdot csn\mathbb{I}_{\delta_2} + csn\mathbb{F}_{\delta_1} \cdot csn\mathbb{F}_{\delta_2}}{\|csn\mathbb{T}_{\delta_1}\| \cdot \|csn\mathbb{T}_{\delta_2}\| + \|csn\mathbb{I}_{\delta_1}\| \cdot \|csn\mathbb{I}_{\delta_2}\| + \|csn\mathbb{F}_{\delta_1}\| \cdot \|csn\mathbb{F}_{\delta_2}\|} \times \frac{|\mathbb{R}_{\delta_1} - \mathbb{R}_{\delta_2}|}{\max(\mathbb{R}_{\delta_1}, \mathbb{R}_{\delta_2})}$$

Lemma 2.1. *[12] Let* $\delta_1 = \langle csn\mathbb{T}_{\delta_1}, csn\mathbb{I}_{\delta_1}, csn\mathbb{F}_{\delta_1}; \mathbb{R}_{\delta_1}\rangle$ *and* $\delta_2 = \langle csn\mathbb{T}_{\delta_2}, csn\mathbb{I}_{\delta_2}, csn\mathbb{F}_{\delta_2}; \mathbb{R}_{\delta_2}\rangle$ *be two CSNS over* $\mathbb{X}$ *and* $\alpha > 0$. *The subsequent operations are then described as follows:*

(1) $\delta_1 \oplus \delta_2 = \langle csn\mathbb{T}_{\delta_1} + csn\mathbb{T}_{\delta_2} - csn\mathbb{T}_{\delta_1}csn\mathbb{T}_{\delta_2}, csn\mathbb{I}_{\delta_1}csn\mathbb{I}_{\delta_2}, csn\mathbb{F}_{\delta_1}csn\mathbb{F}_{\delta_2}; \mathbb{R}_{\delta_1} + \mathbb{R}_{\delta_2} - \mathbb{R}_{\delta_1}\mathbb{R}_{\delta_2}\rangle$.

(2) $\delta_1 \otimes \delta_2 = \langle csn\mathbb{T}_{\delta_1}csn\mathbb{T}_{\delta_2}, csn\mathbb{I}_{\delta_1} + csn\mathbb{I}_{\delta_2} - csn\mathbb{I}_{\delta_1}csn\mathbb{I}_{\delta_2}, csn\mathbb{F}_{\delta_1} + csn\mathbb{F}_{\delta_2} - csn\mathbb{F}_{\delta_1}csn\mathbb{F}_{\delta_2}; \mathbb{R}_{\delta_1}\mathbb{R}_{\delta_2}\rangle$.

(3) $\alpha\delta_1 = \langle 1 - (1 - csn\mathbb{T}_{\delta_1})^{\alpha}, (csn\mathbb{I}_{\delta_1})^{\alpha}, (csn\mathbb{F}_{\delta_1})^{\alpha}; 1 - (1 - \mathbb{R}_{\delta_1})^{\alpha}\rangle$.

(4) ${\delta_1}^{\alpha} = \left\langle csn\mathbb{T}_{\delta_1}^{\alpha}, 1 - (1 - csn\mathbb{I}_{\delta_1})^{\alpha}, 1 - (1 - csn\mathbb{F}_{\delta_1})^{\alpha}; \mathbb{R}_{\delta_1}^{\alpha}\right\rangle$.

3. Cubic Spherical Neutrosophic Aggregation Operators

Definition 3.1. Let δ_ε $(\varepsilon = 1, 2, ..., \lambda)$ be a CSNSs. Then the cubic spherical neutrosophic weighted

(1) arithmetic operator is $CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) = \sum_{\varepsilon=1}^{\lambda} \omega'_\varepsilon \delta_\varepsilon$,

(2) geometric operator is $CSNWGO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) = \prod_{\varepsilon=1}^{\lambda} \omega'_\varepsilon \delta_\varepsilon$,

where $\omega'_\epsilon = (\omega'_1, \omega'_2, ..., \omega'_\lambda)^T$ is the weight vector of $\delta_\varepsilon(\varepsilon = 1, 2, ..., \lambda)$, $\omega'_\varepsilon \in [0, 1]$ and $\sum_{\varepsilon=1}^{\lambda} \omega'_\varepsilon = 1$.

Theorem 3.2. *For a CSNS* $\delta_\varepsilon(\varepsilon = 1, 2, ..., \lambda)$, *we have the following result:*
$CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) =$

$$\left\langle 1 - \prod_{\varepsilon=1}^{\lambda}(1 - csn\mathbb{T}_{\delta_\varepsilon})^{\omega'_\varepsilon}, \prod_{\varepsilon=1}^{\lambda}(csn\mathbb{I}_{\delta_\varepsilon})^{\omega'_\varepsilon}, \prod_{\varepsilon=1}^{\lambda}(csn\mathbb{F}_{\delta_\varepsilon})^{\omega'_\varepsilon}; 1 - \prod_{\varepsilon=1}^{\lambda}(1 - \mathbb{R}_{\delta_\varepsilon})^{\omega'_\varepsilon}\right\rangle \quad (3)$$

where $\omega'_\epsilon = (\omega'_1, \omega'_2, ..., \omega'_\lambda)^T$ *is the weight vector of* $\delta_\varepsilon(\varepsilon = 1, 2, ..., \lambda)$, $\omega'_\varepsilon \in [0, 1]$ *and* $\sum_{\varepsilon=1}^{\lambda} \omega'_\varepsilon = 1$.

Proof: *Mathematical induction can be used to prove the Theorem.*

Case 1: when $\lambda = 2$, then

$$\omega_1'\delta_1 = \left\langle 1-(1-csn\mathbb{T}_{\delta_1})^{\omega_1'}, (csn\mathbb{I}_{\delta_1})^{\omega_1'}, (csn\mathbb{F}_{\delta_1})^{\omega_1'}; 1-(1-\mathbb{R}_{\delta_1})^{\omega_1'} \right\rangle,$$
$$\omega_2'\delta_2 = \left\langle 1-(1-csn\mathbb{T}_{\delta_2})^{\omega_2'}, (csn\mathbb{I}_{\delta_2})^{\omega_2'}, (csn\mathbb{F}_{\delta_2})^{\omega_2'}; 1-(1-\mathbb{R}_{\delta_2})^{\omega_2'} \right\rangle.$$

Thus,

$$CSNWAO_{\omega'}(\delta_1, \delta_2) = \omega_1'\delta_1 + \omega_2'\delta_2$$
$$= \Big\langle 2-(1-csn\mathbb{T}_{\delta_1})^{\omega_1'} - (1-csn\mathbb{T}_{\delta_2})^{\omega_2'} - (1-(1-csn\mathbb{T}_{\delta_1})^{\omega_1'})(1-(1-csn\mathbb{T}_{\delta_2})^{\omega_2'}),$$
$$csn\mathbb{I}_{\delta_1}^{\omega_1'} + csn\mathbb{I}_{\delta_2}^{\omega_2'}, csn\mathbb{F}_{\delta_1}^{\omega_1'} + csn\mathbb{F}_{\delta_2}^{\omega_2'}; 2-(1-cs\mathbb{R}_{\delta_1})^{\omega_1'} - (1-cs\mathbb{R}_{\delta_2})^{\omega_2'}$$
$$-(1-(1-cs\mathbb{R}_{\delta_1})^{\omega_1'})(1-(1-cs\mathbb{R}_{\delta_2})^{\omega_2'}) \Big\rangle$$
$$= \Big\langle 1-(1-csn\mathbb{T}_{\delta_1})^{\omega_1'}(1-csn\mathbb{T}_{\delta_2})^{\omega_2'}, csn\mathbb{I}_{\delta_1}^{\omega_1'} + csn\mathbb{I}_{\delta_2}^{\omega_2'}, csn\mathbb{F}_{\delta_1}^{\omega_1'} + csn\mathbb{F}_{\delta_2}^{\omega_2'};$$
$$1-(1-\mathbb{R}_{\delta_1})^{\omega_1'}(1-\mathbb{R}_{\delta_2})^{\omega_2'} \Big\rangle$$

Case 2: when $\lambda = z$, then

$$CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_z) = \left\langle 1-\prod_{\varepsilon=1}^{z}(1-csn\mathbb{T}_{\delta_\varepsilon})^{\omega_\varepsilon'}, \prod_{\varepsilon=1}^{z}(csn\mathbb{I}_{\delta_\varepsilon})^{\omega_\varepsilon'}, \prod_{\varepsilon=1}^{z}(csn\mathbb{F}_{\delta_\varepsilon})^{\omega_\varepsilon'}; \right.$$
$$\left. 1-\prod_{\varepsilon=1}^{z}(1-\mathbb{R}_{\delta_\varepsilon})^{\omega_\varepsilon'} \right\rangle$$

Case 3: when $\lambda = z+1$, then

$$CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_{z+1})$$
$$= \left\langle 1-\prod_{\varepsilon=1}^{z}(1-csn\mathbb{T}_{\delta_\varepsilon})^{\omega_\varepsilon'} + (1-(1-csn\mathbb{T}_{\delta_{z+1}})^{\omega_{z+1}'}) - (1-\prod_{\varepsilon=1}^{z}(1-csn\mathbb{T}_{\delta_\varepsilon})^{\omega_\varepsilon'}) \right.$$
$$(1-(1-csn\mathbb{T}_{\delta_\varepsilon})^{\omega_{z+1}'}), \prod_{\varepsilon=1}^{z} csn\mathbb{I}_{\delta_\varepsilon}^{\omega_\varepsilon'} csn\mathbb{I}_{\delta_{z+1}}^{\omega_{z+1}'}, \prod_{\varepsilon=1}^{z} csn\mathbb{F}_{\delta_\varepsilon}^{\omega_\varepsilon'} csn\mathbb{F}_{\delta_{z+1}}^{\omega_{z+1}'}; 1-\prod_{\varepsilon=1}^{z}(1-\mathbb{R}_{\delta_\varepsilon})^{\omega_\varepsilon'} +$$
$$\left. (1-(1-\mathbb{R}_{\delta_{z+1}})^{\omega_{z+1}'}) - (1-\prod_{\varepsilon=1}^{z}(1-\mathbb{R}_{\delta_\varepsilon})^{\omega_\varepsilon'})(1-(1-\mathbb{R}_{\delta_\varepsilon})^{\omega_{z+1}'}) \right\rangle$$
$$= \left\langle 1-\prod_{\varepsilon=1}^{z+1}(1-csn\mathbb{T}_{\delta_\varepsilon})^{\omega_\varepsilon'}, \prod_{\varepsilon=1}^{z+1} csn\mathbb{I}_{\delta_\varepsilon}^{\omega_\varepsilon'}, \prod_{\varepsilon=1}^{z+1} csn\mathbb{F}_{\delta_\varepsilon}^{\omega_\varepsilon'}; 1-\prod_{\varepsilon=1}^{z+1}(1-\mathbb{R}_{\delta_\varepsilon})^{\omega_\varepsilon'} \right\rangle$$

In light of the aforementioned findings, equation (4) results for every λ, The proof is now complete. The following characteristics of the $CSNWAO$ operator are evident:

(1) **Idempotency :** Let δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ be a collection of CSNSs. If δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ is equal, that is $\delta_\varepsilon = \lambda$ for $\varepsilon = 1, 2, \ldots, , \lambda$, then $CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) = \lambda$.

(2) **Boundedness :** Let δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ be a collection of CSNSs,
$A^- = \langle \min_\varepsilon csn\mathbb{T}_{\delta_\varepsilon}, \max_\varepsilon csn\mathbb{I}_{\delta_\varepsilon}, \max_\varepsilon csn\mathbb{F}_{\delta_\varepsilon}; \min_\varepsilon \mathbb{R}_{\delta_\varepsilon}\rangle$ and
$A^+ = \langle \max_\varepsilon csn\mathbb{T}_{\delta_\varepsilon}, \min_\varepsilon csn\mathbb{I}_{\delta_\varepsilon}, \min_\varepsilon csn\mathbb{F}_{\delta_\varepsilon}; \max_\varepsilon \mathbb{R}_{\delta_\varepsilon}\rangle$
for $(\varepsilon = 1, 2, \ldots, \lambda)$, then $A^- \subseteq CSNWAO \subseteq A^+$.

(3) **Monotonity :** Let δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ be a collection of CSNSs. If $\delta_\varepsilon \subseteq \delta^*_\varepsilon$ for $\varepsilon = 1, 2, \ldots, , \lambda$, then $CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) \subseteq CSNWAO_{\omega'}(\delta^*_1, \delta^*_2, \ldots, \delta^*_\lambda)$.

Theorem 3.3. *For a CSNS $\delta_\varepsilon(\varepsilon = 1, 2, ..., \lambda)$, we have the following result:*
$CSNWGO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) =$

$$\left\langle \prod_{\varepsilon=1}^{\lambda} csn\mathbb{T}_{\delta_\varepsilon}^{\omega'_\varepsilon}, 1 - \prod_{\varepsilon=1}^{\lambda}(1 - csn\mathbb{I}_{\delta_\varepsilon})^{\omega'_\varepsilon}, 1 - \prod_{\varepsilon=1}^{\lambda}(1 - csn\mathbb{F}_{\delta_\varepsilon})^{\omega'_\varepsilon}; \prod_{\varepsilon=1}^{\lambda} \mathbb{R}_{\delta_\varepsilon}^{\omega'_\varepsilon} \right\rangle \tag{4}$$

where $\omega'_\epsilon = (\omega'_1, \omega'_2, ..., \omega'_\lambda)^T$ is the weight vector of $\delta_\varepsilon(\varepsilon = 1, 2, ..., \lambda)$, $\omega'_\varepsilon \in [0, 1]$ and $\sum_{\varepsilon=1}^{\lambda} \omega'_\varepsilon = 1$.

By the similar manner, we will prove Theorem 2.3.

(1) **Idempotency :** Let δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ be a collection of CSNSs. Ifδ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ is equal, that is $\delta_\varepsilon = A$ for $\varepsilon = 1, 2, \ldots, , \lambda$, then $CSNWGO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) = A$.

(2) **Boundedness :** Let δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ be a collection of CSNSs,
$A^- = \langle \min_\varepsilon csn\mathbb{T}_{\delta_\varepsilon}, \max_\varepsilon csn\mathbb{I}_{\delta_\varepsilon}, \max_\varepsilon csn\mathbb{F}_{\delta_\varepsilon}; \min_\varepsilon \mathbb{R}_{\delta_\varepsilon}\rangle$ and
$A^+ = \langle \max_\varepsilon csn\mathbb{T}_{\delta_\varepsilon}, \min_\varepsilon csn\mathbb{I}_{\delta_\varepsilon}, \min_\varepsilon csn\mathbb{F}_{\delta_\varepsilon}; \max_\varepsilon \mathbb{R}_{\delta_\varepsilon}\rangle$
for $(\varepsilon = 1, 2, \ldots, \lambda)$, then $A^- \subseteq CSNWGO \subseteq A^+$.

(3) **Monotonity :** Let δ_ε $(\varepsilon = 1, 2, \ldots, \lambda)$ be a collection of CSNSs. If $\delta_\varepsilon \subseteq \delta^*_\varepsilon$ for $\varepsilon = 1, 2, \ldots, , \lambda$, then $CSNWGO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) \subseteq CSNWGO_{\omega'}(\delta^*_1, \delta^*_2, \ldots, \delta^*_\lambda)$.

4. Model for Multi Criteria Decision Making in Cubic Spherical Neutrosophic sets

In this section, we propose a Multi-Criteria Decision Making (MCDM) approach using the cubic spherical neutrosophic CSNWA and CSNWGA operators. When multiple decision makers are involved in the decision-making process, simply averaging decision values may not accurately represent their collective perspective. The cubic spherical neutrosophic approach addresses the limitations of traditional averaging methods. We apply this approach to evaluate the usefulness of emerging technology commercialization.

Let $\bigstar = \{\bigstar_1, \bigstar_2 \ldots \bigstar_\lambda\}$ be a set of alternatives and $\emptyset = \{\emptyset_1, \emptyset_2 \ldots \emptyset_\lambda\}$ be a set of criteria. Suppose $(\delta_{\alpha\varepsilon})_{m\times n} = (csn\mathbb{T}_{\delta_\alpha}(\nu_\varepsilon), csn\mathbb{I}_{\delta_\alpha}(\nu_\varepsilon), csn\mathbb{F}_{\delta_\alpha}(\nu_\varepsilon))_{m\times n}$ is a neutrosopic decision matrix, where $csn\mathbb{T}_{\delta_\alpha}(\nu_\varepsilon)$ is the degree of membership of alternatives $\bigstar_\epsilon$, $csn\mathbb{I}_{\delta_\alpha}(\nu_\varepsilon)$ is the degree of neutral membership of alternatives $\bigstar_\epsilon$, and $csn\mathbb{F}_{\delta_\alpha}(\nu_\varepsilon)$ is the degree non-membership of alternatives $\bigstar_\epsilon$, each alternatives $\bigstar_\epsilon$ satisfy $0 \leq csn\mathbb{T}_{\delta_\alpha}(\nu_\varepsilon) + csn\mathbb{I}_{\delta_\alpha}(\nu_\varepsilon) + csn\mathbb{F}_{\delta_\alpha}(\nu_\varepsilon)) \leq 3$.

We propose the following algorithm to solve MCDM problem with cubic spherical neutrosophic information using cubic spherical neutrosophic CSNWA and CSNWGA operators.

Step 1: Start.

Step 2: Input: The available alternatives.

Step 3: Employ the decision information in the form of a matrix
$(\delta_{\alpha\varepsilon})_{m\times n} = (csn\mathbb{T}_{\delta_\alpha}(\nu_\varepsilon), csn\mathbb{I}_{\delta_\alpha}(\nu_\varepsilon), csn\mathbb{F}_{\delta_\alpha}(\nu_\varepsilon))_{m\times n}$.

$$(\delta_{\alpha\varepsilon})_{m\times n} = \begin{bmatrix} \langle csn\mathbb{T}_{11}, csn\mathbb{I}_{11}, csn\mathbb{F}_{11}\rangle & ... & \langle csn\mathbb{T}_{1\kappa}, csn\mathbb{I}_{1\kappa}, csn\mathbb{F}_{1\kappa}\rangle \\ \langle csn\mathbb{T}_{21}, csn\mathbb{I}_{21}, csn\mathbb{F}_{21}\rangle & ... & \langle csn\mathbb{T}_{2\kappa}, csn\mathbb{I}_{2\kappa}, csn\mathbb{F}_{2\kappa}\rangle \\ . & . & . \\ . & . & . \\ . & . & . \\ \langle csn\mathbb{T}_{\lambda 1}, csn\mathbb{I}_{\lambda 1}, csn\mathbb{F}_{\lambda 1}\rangle & ... & \langle csn\mathbb{T}_{\lambda\kappa}, csn\mathbb{I}_{\lambda\kappa}, csn\mathbb{F}_{\lambda\kappa}\rangle \end{bmatrix}$$

Step 4: For each alternative $\bigstar_\epsilon$, $(\epsilon = 1, 2 \ldots, \lambda)$ construct the cubic spherical neutrosophic set

$$\delta_\alpha = \{\langle \nu_\varepsilon, csn\mathbb{T}_{\delta_\alpha}(\nu_\varepsilon), csn\mathbb{I}_{\delta_\alpha}(\nu_\varepsilon), csn\mathbb{F}_{\delta_\alpha}(\nu_\varepsilon); \mathbb{R}_{\delta_\alpha}(\nu_\varepsilon)\rangle : \nu_\varepsilon \in \nu\}$$

$$(\delta_{\alpha\varepsilon})_{m\times n} = \begin{bmatrix} \langle csn\mathbb{T}_{11}, csn\mathbb{I}_{11}, csn\mathbb{F}_{11}; \mathbb{R}_{11}\rangle & ... & \langle csn\mathbb{T}_{1\kappa}, csn\mathbb{I}_{1\kappa}, csn\mathbb{F}_{1\kappa}; \mathbb{R}_{1\kappa}\rangle \\ \langle csn\mathbb{T}_{21}, csn\mathbb{I}_{21}, csn\mathbb{F}_{21}; \mathbb{R}_{21}\rangle & ... & \langle csn\mathbb{T}_{2\kappa}, csn\mathbb{I}_{2\kappa}, csn\mathbb{F}_{2\kappa}; \mathbb{R}_{2\kappa}\rangle \\ . & . & . \\ . & . & . \\ . & . & . \\ \langle csn\mathbb{T}_{\lambda 1}, csn\mathbb{I}_{\lambda 1}, csn\mathbb{F}_{\lambda 1}; \mathbb{R}_{\lambda 1}\rangle & ... & \langle csn\mathbb{T}_{\lambda\kappa}, csn\mathbb{I}_{\lambda\kappa}, csn\mathbb{F}_{\lambda\kappa}; \mathbb{R}_{\lambda\kappa}\rangle \end{bmatrix}$$

where

$$\langle csn\mathbb{T}(\epsilon_\imath), csn\mathbb{I}(\epsilon_\imath), csn\mathbb{F}(\epsilon_\imath)\rangle = \left\langle \frac{\sum_{\jmath=1}^{k_\imath} \mathbb{T}_{\imath,\jmath}}{k_\imath}, \frac{\sum_{\jmath=1}^{k_\imath} \mathbb{I}_{\imath,\jmath}}{k_\imath}, \frac{\sum_{\jmath=1}^{k_\imath} \mathbb{F}_{\imath,\jmath}}{k_\imath} \right\rangle$$

is the center of $\delta_{\alpha\varepsilon}$ and

$$\mathbb{R}_\imath = \min\left\{ \max_{1\le\jmath\le k_\imath} \sqrt{(csn\mathbb{T}(\epsilon_\imath) - \mathbb{T}_{\imath,\jmath})^2 + (csn\mathbb{I}(\epsilon_\imath) - \mathbb{I}_{\imath,\jmath})^2 + (csn\mathbb{F}(\epsilon_\imath) - \mathbb{F}_{\imath,\jmath})^2}, 1 \right\}$$

is the radius of the cubic spherical neutrosophic set $\delta_{\alpha\varepsilon}$ for all $\epsilon = 1, 2, \ldots \lambda$ from the decision matrix $(\delta_{\alpha\varepsilon})_{m\times n}$.

Step 5: Operate $CSNWAO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) =$

$$\left\langle 1 - \prod_{\varepsilon=1}^{\lambda}(1 - csn\mathbb{T}_{\delta_\varepsilon})^{\omega'_\varepsilon}, \prod_{\varepsilon=1}^{\lambda}(csn\mathbb{I}_{\delta_\varepsilon})^{\omega'_\varepsilon}, \prod_{\varepsilon=1}^{\lambda}(csn\mathbb{F}_{\delta_\varepsilon})^{\omega'_\varepsilon}; 1 - \prod_{\varepsilon=1}^{\lambda}(1 - \mathbb{R}_{\delta_\varepsilon})^{\omega'_\varepsilon} \right\rangle$$

and $CSNWGO_{\omega'}(\delta_1, \delta_2, \ldots, \delta_\lambda) =$

$$\left\langle \prod_{\varepsilon=1}^{\lambda} csn\mathbb{T}_{\delta_\varepsilon}^{\omega'_\varepsilon}, 1 - \prod_{\varepsilon=1}^{\lambda}(1 - csn\mathbb{I}_{\delta_\varepsilon})^{\omega'_\varepsilon}, 1 - \prod_{\varepsilon=1}^{\lambda}(1 - csn\mathbb{F}_{\delta_\varepsilon})^{\omega'_\varepsilon}; \prod_{\varepsilon=1}^{\lambda} \mathbb{R}_{\delta_\varepsilon}^{\omega'_\varepsilon} \right\rangle$$

to obtain the overall preference values of the alternative $\bigstar_\epsilon(\epsilon = 1, 2, \ldots \lambda)$.

Step 6: Calculate the cosine distance $\cos(\delta_1, \delta_2) =$

$$1 - \frac{csn\mathbb{T}_{\delta_1} \cdot csn\mathbb{T}_{\delta_2} + csn\mathbb{I}_{\delta_1} \cdot csn\mathbb{I}_{\delta_2} + csn\mathbb{F}_{\delta_1} \cdot csn\mathbb{F}_{\delta_2}}{\|csn\mathbb{T}_{\delta_1}\| \cdot \|csn\mathbb{T}_{\delta_2}\| + \|csn\mathbb{I}_{\delta_1}\| \cdot \|csn\mathbb{I}_{\delta_2}\| + \|csn\mathbb{F}_{\delta_1}\| \cdot \|csn\mathbb{F}_{\delta_2}\|} \times \frac{|\mathbb{R}_{\delta_1} - \mathbb{R}_{\delta_2}|}{\max(\mathbb{R}_{\delta_1}, \mathbb{R}_{\delta_2})}$$

where $\delta_2 = (1, 0, 0; 1)$ is the ideal sphere.

Step 7: The shortest distance value of $\cos(\delta_1, \delta_2)$ is the better alternative $\bigstar_{\epsilon}$, because it is close to the ideal alternative δ_2.

Step 8: Rank the alternatives $\bigstar_{\epsilon}, (\epsilon = 1, 2, \ldots \lambda)$ based on the cubic spherical neutrosophic CSNWA and CSNWGA operators evaluations and cosine distance.

Step 9: Output : Best alternative.

Step 10: End.

5. **Numerical Example**

The coconut industry in India is vital for rural economies, providing employment and contributing to food security and industrial raw materials. Enhanced productivity and diversified product applications continue to strengthen the economic impact of coconuts both nationally and globally. India cultivates several varieties of coconuts, categorized into tall and dwarf types. Some notable varieties include:

Tall Varieties

- **West Coast Tall (WCT)**: Known for its high yield and adaptability to various climatic conditions. It produces around 80-100 nuts per palm annually.
- **East Coast Tall (ECT)**: Another high-yielding variety with an annual production of 70-90 nuts per palm.
- **Tiptur Tall**: Commonly grown in Karnataka, yielding around 60-80 nuts per palm annually.

Dwarf Varieties

- **Chowghat Orange Dwarf (COD)**: Popular for its early bearing and high yield, producing around 50-60 nuts per palm annually.
- **Malayan Yellow Dwarf (MYD)**: Known for its high productivity, yielding 60-70 nuts per palm annually.
- **Gangabondam**: An early bearing variety with an average yield of 40-50 nuts per palm annually.

Hybrid Varieties

- **Chandrasankara (WCT x COD)**: Combines the high yield of tall varieties and the early bearing of dwarf varieties, producing around 100-120 nuts per palm annually.
- **Kerasankara (ECT x MYD)**: Another hybrid with high productivity, yielding 90-110 nuts per palm annually.

The selection of fertilizer for coconut trees is essential for maximizing yield, sustaining tree health, and ensuring the productivity of coconut plantations. Fertilizers provide vital nutrients like nitrogen, phosphorus, potassium, magnesium, and micronutrients, fostering vigorous vegetative growth, enhancing fruit development, and increasing fruit-bearing spikes for higher yields over time. Balanced fertilization also helps coconut trees withstand environmental stressors such as drought, salinity, and temperature fluctuations, improving resilience and reducing susceptibility to diseases and pests. Moreover, proper fertilizer selection contributes to the quality of coconut products, influencing their nutritional composition, flavor profile, and market value, including copra, coconut oil, and coconut water. Sustainable fertilizer practices are crucial for long-term viability, preserving soil fertility, minimizing nutrient runoff, and protecting water quality. Economically, effective fertilizer selection leads to increased farm income, improved livelihoods, and enhanced economic resilience for coconut-growing communities. Overall, the careful selection of fertilizers is integral to the sustainability, productivity, and economic success of coconut plantations, underscoring its critical importance in coconut farming management.

Criteria for Selecting Fertilizer:

- **Beneficiary Criteria:**
  - **Nutrient Content ($\emptyset_1$) :** Coconut trees require a balanced supply of essential nutrients such as nitrogen (N), phosphorus (P), potassium (K), magnesium (Mg), and micronutrients like zinc (Zn) and iron (Fe) for healthy growth and fruit development.
  - **Reputation of Brand ($\emptyset_5$) :** The reputation and reliability of the fertilizer brand reflect its quality and effectiveness. Farmers often consider the track record of a brand in delivering consistent results and addressing specific crop needs.
  - **Availability ($\emptyset_6$) :** Accessibility to the chosen fertilizer is essential for timely application and uninterrupted supply. Factors such as distribution networks, local availability, and logistical considerations influence the suitability of the fertilizer.
- **Non-Beneficiary Criteria:**
  - **Cost ($\emptyset_2$) :** Cost-effectiveness plays a significant role in selecting fertilizer. The price of the fertilizer should align with the budget constraints of the coconut farmer while ensuring optimal yield.
  - **Environmental Impact ($\emptyset_3$) :** Sustainable farming practices emphasize the importance of minimizing environmental impact. Fertilizers should be chosen based on their potential for leaching, runoff, and contribution to pollution.
  - **Ease of Application ($\emptyset_4$) :** The ease of application influences the practicality of fertilizer use. Factors such as application method, frequency, and compatibility with existing farming practices determine the convenience of application.

Methodologies for Selecting Decision Makers in Evaluating Fertilizer Brands for Coconut Farming

The selection of decision makers for evaluating fertilizer brands in coconut farming is a crucial step in agricultural research and decision-making processes. This article presents various methodologies and considerations for identifying suitable decision makers tasked with assessing the best fertilizer options among four brands to optimize coconut yield. The following methods can be considered:

- **Expertise and Experience:** Identifying individuals with expertise and experience in agriculture, particularly in coconut tree cultivation or related crops, such as farmers, agronomists, agricultural researchers, or extension agents knowledgeable about fertilizer selection and its impact on crop yield.
- **Stakeholder Representation:** Ensuring that decision makers represent diverse stakeholders involved in coconut farming, including farmers, agricultural cooperative members, extension officers, representatives from agricultural input suppliers, and agricultural researchers.
- **Diverse Perspectives:** Aim for diversity in decision makers to incorporate a range of perspectives and insights, considering factors such as age, gender, education level, farming practices, and geographic location to ensure a broad representation of views and experiences.
- **Involvement in the Coconut Farming Community:** Selecting decision makers actively engaged in the coconut farming community with a vested interest in improving crop yield and profitability. This may include members of coconut growers' associations, agricultural cooperatives, or local farming communities.
- **Commitment and Availability:** Choosing decision makers committed to actively participating in the selection process, with the time and availability to attend meetings, review information about fertilizer brands, and engage in discussions to make informed decisions.

Once potential decision makers are identified based on these criteria, inviting them to participate in the selection process is essential. Clear communication of the objectives, evaluation criteria, and expected level of involvement is necessary to ensure transparency and collaboration among decision makers. Encouraging open dialogue and collaboration among decision makers will facilitate a thorough and fair evaluation of fertilizer brands.

5.1. *Decision-Maker Evaluation of Fertilizer Brands for Coconut Farming: Assessing Performance Across Six Criteria*

Linguistic terms are crucial in decision-making, allowing qualitative expression of preferences and perceptions. They create a flexible framework for communication, enhancing understanding and consensus-building among stakeholders. By promoting clarity and transparency, linguistic terms enrich decision-making processes, capturing the nuanced nature of human perceptions and preferences.

Linguistic terms	Symbolic Notation	$\mathcal{N} \times 10^{-2}$
Very good	$\star_1$	$(90, 10, 10)$
Good	$\star_2$	$(80, 20, 15)$
Fair	$\star_3$	$(50, 40, 45)$
Bad	$\star_4$	$(35, 60, 70)$
Very bad	$\star_5$	$(10, 80, 90)$

TABLE 1. Linguistic terms for rating of attributes.

Step 1: Four Decision Makers DM1, DM2, DM3 and DM4 evaluates four fertilizer brands $\bigstar_1$, $\bigstar_2$, $\bigstar_3$ and $\bigstar_4$ with six criteria $\emptyset_1$ = Nutrient Content, $\emptyset_2$ = Cost, $\emptyset_3$ = Environmental Impact, $\emptyset_4$ = Ease of Application, $\emptyset_5$ = Reputation of Brand and $\emptyset_6$ = Availability to select the best fertilizer brand for coconut forming. Each evaluators decisions in linguistic phrase are given in Table 2. The neutrosophic numbers that match to the linguistic phrases in Table 1 will be substituted in Table 2. Linguistic terms are replaced with their corresponding neutrosophic number are in Table 3.

Step 2: The normalized decision matrix Table 4 represents the evaluation scores provided by decision makers for each fertilizer brand across the three beneficial $\emptyset_1$, $\emptyset_5$, $\emptyset_6$ and three non beneficial criteria $\emptyset_2$, $\emptyset_3$ and $\emptyset_4$.

Step 3: The process of converting normalized neutrosophic set values into cubic spherical neutrosophic values involves determining their center and radius. Using Equations 1 & 2 we transform the decision makers decisions into cubic spherical neutrosophic numbers, which is represented in Table 5.

Step 4: The weight for each criteria is $\emptyset_1 = 0.0935$, $\emptyset_2 = 0.1594$, $\emptyset_3 = 0.1812$, $\emptyset_4 = 0.2106$, $\emptyset_5 = 0.1812$, $\emptyset_6 = 0.1741$. The Table 6 represents the cubic spherical neutrosophic weighted arithmetic and geometric operators on the calculated CSNSs.

Step 5: The Table 7 illustrates the cosine distances computed between the ideal alternative $\bigstar_{\mathcal{I}}$=(1,0,0;1) and the Cubic Spherical Neutrosophic Weighted Arithmetic Operator (CSNWAO) and the Cubic Spherical Neutrosophic Weighted Geometric Operator (CSNWGO). These distances serve as quantitative measures of alignment between the evaluated operators and the ideal solution for fertilizer selection in coconut farming. Lower cosine distances indicate closer resemblance and alignment with the ideal criteria, suggesting higher suitability for guiding fertilizer selection decisions. Analysis of the results enables stakeholders to refine their decision-making strategies, prioritize options that closely match the ideal criteria, and optimize coconut yield while ensuring sustainable agricultural practices.

Step 6: The ranking of alternatives $\bigstar_\epsilon$ $(\epsilon = 1, 2, 3, 4)$ and the best alternative are given in Table 8

DM's	Brand	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	$\emptyset_4$	$\emptyset_5$	$\emptyset_6$	DM's	Brand	$\emptyset_1$	$\emptyset_2$	$\emptyset_3$	$\emptyset_4$	$\emptyset_5$	$\emptyset_6$
	$\bigstar_1$	$\star_1$	$\star_2$	$\star_2$	$\star_3$	$\star_1$	$\star_2$		$\bigstar_1$	$\star_2$	$\star_1$	$\star_2$	$\star_3$	$\star_2$	$\star_1$
DM1	$\bigstar_2$	$\star_2$	$\star_2$	$\star_2$	$\star_1$	$\star_2$	$\star_2$	DM2	$\bigstar_2$	$\star_3$	$\star_2$	$\star_2$	$\star_1$	$\star_3$	$\star_2$
	$\bigstar_3$	$\star_2$	$\star_1$	$\star_3$	$\star_4$	$\star_2$	$\star_1$		$\bigstar_3$	$\star_1$	$\star_3$	$\star_2$	$\star_2$	$\star_1$	$\star_3$
	$\bigstar_4$	$\star_3$	$\star_3$	$\star_1$	$\star_2$	$\star_3$	$\star_3$		$\bigstar_4$	$\star_1$	$\star_2$	$\star_3$	$\star_2$	$\star_1$	$\star_2$
	$\bigstar_1$	$\star_3$	$\star_2$	$\star_1$	$\star_2$	$\star_3$	$\star_2$		$\bigstar_1$	$\star_2$	$\star_1$	$\star_2$	$\star_3$	$\star_2$	$\star_1$
DM3	$\bigstar_2$	$\star_1$	$\star_2$	$\star_3$	$\star_3$	$\star_1$	$\star_1$	DM4	$\bigstar_2$	$\star_3$	$\star_3$	$\star_2$	$\star_2$	$\star_3$	$\star_3$
	$\bigstar_3$	$\star_2$	$\star_1$	$\star_1$	$\star_4$	$\star_2$	$\star_1$		$\bigstar_3$	$\star_2$	$\star_1$	$\star_1$	$\star_2$	$\star_2$	$\star_1$
	$\bigstar_4$	$\star_2$	$\star_3$	$\star_3$	$\star_1$	$\star_2$	$\star_3$		$\bigstar_4$	$\star_2$	$\star_2$	$\star_3$	$\star_3$	$\star_2$	$\star_1$

TABLE 2. Linguistic term rating of decision makers

DM's	Brand	$\emptyset_1$ ($\mathcal{N} \times 10^{-2}$)	$\emptyset_2$ ($\mathcal{N} \times 10^{-2}$)	$\emptyset_3$ ($\mathcal{N} \times 10^{-2}$)	$\emptyset_4$ ($\mathcal{N} \times 10^{-2}$)	$\emptyset_5$ ($\mathcal{N} \times 10^{-2}$)	$\emptyset_6$ ($\mathcal{N} \times 10^{-2}$)
	$\bigstar_1$	(90, 10, 10)	(80, 20, 15)	(80, 20, 15)	(50, 40, 45)	(90, 10, 10)	(80, 20, 15)
DM1	$\bigstar_2$	(80, 20, 15)	(80, 20, 15)	(80, 20, 15)	(90, 10, 10)	(80, 20, 15)	(80, 20, 15)
	$\bigstar_3$	(80, 20, 15)	(90, 10, 10)	(50, 40, 45)	(35, 60, 70)	(80, 20, 15)	(90, 10, 10)
	$\bigstar_4$	(50, 40, 45)	(50, 40, 45)	(90, 10, 10)	(80, 20, 15)	(50, 40, 45)	(50, 40, 45)
	$\bigstar_1$	(80, 20, 15)	(90, 10, 10)	(80, 20, 15)	(50, 40, 45)	(80, 20, 15)	(90, 10, 10)
DM2	$\bigstar_2$	(50, 40, 45)	(80, 20, 15)	(80, 20, 15)	(90, 10, 10)	(50, 40, 45)	(80, 20, 15)
	$\bigstar_3$	(90, 10, 10)	(50, 40, 45)	(80, 20, 15)	(80, 20, 15)	(90, 10, 10)	(50, 40, 45)
	$\bigstar_4$	(90, 10, 10)	(80, 20, 15)	(50, 40, 45)	(80, 20, 15)	(90, 10, 10)	(80, 20, 15)
	$\bigstar_1$	(50, 40, 45)	(80, 20, 15)	(90, 10, 10)	(80, 20, 15)	(50, 40, 45)	(80, 20, 15)
DM3	$\bigstar_2$	(90, 10, 10)	(80, 20, 15)	(50, 40, 45)	(50, 40, 45)	(90, 10, 10)	(90, 10, 10)
	$\bigstar_3$	(80, 20, 15)	(90, 10, 10)	(90, 10, 10)	(35, 60, 70)	(80, 20, 15)	(90, 10, 10)
	$\bigstar_4$	(80, 20, 15)	(50, 40, 45)	(50, 40, 45)	90, 10, 10)	(80, 20, 15)	(50, 40, 45)
	$\bigstar_1$	(80, 20, 15)	(90, 10, 10)	(80, 20, 15)	(50, 40, 45)	(80, 20, 15)	(90, 10, 10)
DM4	$\bigstar_2$	(50, 40, 45)	(50, 40, 45)	(80, 20, 15)	(80, 20, 15)	(50, 40, 45)	(50, 40, 45)
	$\bigstar_3$	(80, 20, 15)	(90, 10, 10)	(90, 10, 10)	(80, 20, 15)	(80, 20, 15)	(90, 10, 10)
	$\bigstar_4$	(80, 20, 15)	(80, 20, 15)	(50, 40, 45)	(50, 40, 45)	(80, 20, 15)	(90, 10, 10)

TABLE 3. Neutrosophic values of decision makers rankings

Comparative Analysis and Limitations

Our findings are contrasted with those of Biswas et al. [7], Ye's [22] Gomathi et al. [12], Krishnaprakash et al. [14], and their provided visualization. Table 6 displays the order of rating. The ranking results of the suggested method and the current methods are clearly nearly identical. This confirms even further that the suggested techniques are applicable.

In MCDM, the decision maker's involvement plays a crucial role in determining the weights and preferences associated with different criteria. It has been suggested that the decision maker's influence

DM's	Brand	$\emptyset_1$ $(\mathcal{N} \times 10^{-2})$	$\emptyset_2$ $(\mathcal{N} \times 10^{-2})$	$\emptyset_3$ $(\mathcal{N} \times 10^{-2})$	$\emptyset_4$ $(\mathcal{N} \times 10^{-2})$	$\emptyset_5$ $(\mathcal{N} \times 10^{-2})$	$\emptyset_6$ $(\mathcal{N} \times 10^{-2})$
	$\bigstar_1$	$(90, 10, 10)$	$(15, 20, 80)$	$(15, 20, 80)$	$(45, 40, 50)$	$(90, 10, 10)$	$(80, 20, 15)$
DM1	$\bigstar_2$	$(80, 20, 15)$	$(15, 20, 80)$	$(15, 20, 80)$	$(10, 10, 90)$	$(80, 20, 15)$	$(80, 20, 15)$
	$\bigstar_3$	$(80, 20, 15)$	$(10, 10, 90)$	$(45, 40, 50)$	$(70, 60, 35)$	$(80, 20, 15)$	$(90, 10, 10)$
	$\bigstar_4$	$(50, 40, 45)$	$(45, 40, 50)$	$(10, 10, 90)$	$(15, 20, 80)$	$(50, 40, 45)$	$(50, 40, 45)$
	$\bigstar_1$	$(80, 20, 15)$	$(10, 10, 90)$	$(15, 20, 80)$	$(45, 40, 50)$	$(80, 20, 15)$	$(90, 10, 10)$
DM2	$\bigstar_2$	$(50, 40, 45)$	$(15, 20, 80)$	$(15, 20, 80)$	$(10, 10, 90)$	$(50, 40, 45)$	$(80, 20, 15)$
	$\bigstar_3$	$(90, 10, 10)$	$(45, 40, 50)$	$(15, 20, 80)$	$(15, 20, 80)$	$(90, 10, 10)$	$(50, 40, 45)$
	$\bigstar_4$	$(90, 10, 10)$	$(15, 20, 80)$	$(45, 40, 50)$	$(15, 20, 80)$	$(90, 10, 10)$	$(80, 20, 15)$
	$\bigstar_1$	$(50, 40, 45)$	$(15, 20, 80)$	$(10, 10, 90)$	$(15, 20, 80)$	$(50, 40, 45)$	$(80, 20, 15)$
DM3	$\bigstar_2$	$(90, 10, 10)$	$(15, 20, 80)$	$(45, 40, 50)$	$(45, 40, 50)$	$(90, 10, 10)$	$(90, 10, 10)$
	$\bigstar_3$	$(80, 20, 15)$	$(10, 10, 90)$	$(10, 10, 90)$	$(70, 60, 35)$	$(80, 20, 15)$	$(90, 10, 10)$
	$\bigstar_4$	$(80, 20, 15)$	$(45, 40, 50)$	$(45, 40, 50)$	$(10, 10, 90)$	$(80, 20, 15)$	$(50, 40, 45)$
	$\bigstar_1$	$(80, 20, 15)$	$(10, 10, 90)$	$(15, 20, 80)$	$(45, 40, 50)$	$(80, 20, 15)$	$(90, 10, 10)$
DM4	$\bigstar_2$	$(50, 40, 45)$	$(45, 40, 50)$	$(15, 20, 80)$	$(15, 20, 80)$	$(50, 40, 45)$	$(50, 40, 45)$
	$\bigstar_3$	$(80, 20, 15)$	$(10, 10, 90)$	$(10, 10, 90)$	$(15, 20, 80)$	$(80, 20, 15)$	$(90, 10, 10)$
	$\bigstar_4$	$(80, 20, 15)$	$(15, 20, 80)$	$(45, 40, 50)$	$(45, 40, 50)$	$(80, 20, 15)$	$(90, 10, 10)$

TABLE 4. Normalized neutrosophic values each alternatives $\bigstar_\epsilon$.

Brand	$\emptyset_1$ $(\delta_{\mathbb{R}} \times 10^{-2})$	$\emptyset_2$ $(\delta_{\mathbb{R}} \times 10^{-2})$	$\emptyset_3$ $(\delta_{\mathbb{R}} \times 10^{-2})$
$\bigstar_1$	(75, 23, 21; 39)	(13, 15, 85; 8)	(14, 18, 83; 11)
$\bigstar_2$	(68, 28, 29; 34)	(23, 25, 73; 35)	(23, 25, 73; 35)
$\bigstar_3$	(83, 18, 14; 11)	(19, 18, 80; 46)	(20, 20, 78; 42)
$\bigstar_4$	(75, 23, 21; 39)	(30, 30, 65; 23)	(36, 33, 60; 46)
	$\emptyset_4$ $(\delta_{\mathbb{R}} \times 10^{-2})$	$\emptyset_5$ $(\delta_{\mathbb{R}} \times 10^{-2})$	$\emptyset_6$ $(\delta_{\mathbb{R}} \times 10^{-2})$
$\bigstar_1$	(38, 35, 58; 35)	(75, 23, 21; 39)	(85, 15, 13; 8)
$\bigstar_2$	(20, 20, 78; 42)	(68, 28, 29; 34)	(75, 23, 21; 39)
$\bigstar_3$	(43, 40, 58; 41)	(83, 18, 14; 11)	(80, 18, 19; 46)
$\bigstar_4$	(21, 23, 75; 39)	(75, 23, 21; 39)	(68, 28, 29; 34)

TABLE 5. Cubic spherical neutrosophic representation of each alternatives $\bigstar_\epsilon$.

Brand	CSNWAO $(\delta_{\mathbb{R}} \times 10^{-2})$	CSNWGO $(\delta_{\mathbb{R}} \times 10^{-2})$
$\bigstar_1$	(58, 21, 38; 24)	(29, 22, 59; 18)
$\bigstar_2$	(50, 24, 46; 37)	(31, 24, 59; 37)
$\bigstar_3$	(61, 21, 36; 36)	(33, 23, 54; 30)
$\bigstar_4$	(53, 26, 42; 37)	(34, 26, 53; 36)

TABLE 6. CSNWA and CSNWG operators values for fertilizer brands $\bigstar_\epsilon$

	$cos(\bigstar_1, \bigstar_{\mathcal{I}})$	$cos(\bigstar_2, \bigstar_{\mathcal{I}})$	$cos(\bigstar_3, \bigstar_{\mathcal{I}})$	$cos(\bigstar_4, \bigstar_{\mathcal{I}})$
CSNWAO	0.329	0.566	0.47	0.539
CSNWGO	0.585	0.685	0.584	0.625

TABLE 7. Cosine distance between each alternatives $\bigstar_\epsilon$ and ideal sphere $\bigstar_{\mathcal{I}}$

Method	Ranking	Best Brand
$\mathbb{TOPSIS}$ [7]	$\bigstar_2 > \bigstar_3 > \bigstar_4 > \bigstar_1$	$\bigstar_1$
$\mathbb{SNWAA}$ [22]	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{SNWGA}$ [22]	$\bigstar_2 > \bigstar_1 > \bigstar_3 > \bigstar_4$	$\bigstar_4$
$\mathbb{CSNWAAO}$ [12]	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWGAO}$ [12]	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWG}_\rho^A$ [14]	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWG}_\wp^A$ [14]	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWA}_\rho^A$ [14]	$\bigstar_2 > \bigstar_3 > \bigstar_4 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWA}_\wp^A$ [14]	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWAO}$	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$
$\mathbb{CSNWGO}$	$\bigstar_2 > \bigstar_4 > \bigstar_3 > \bigstar_1$	$\bigstar_1$

TABLE 8. Comparative Analysis of Ranking

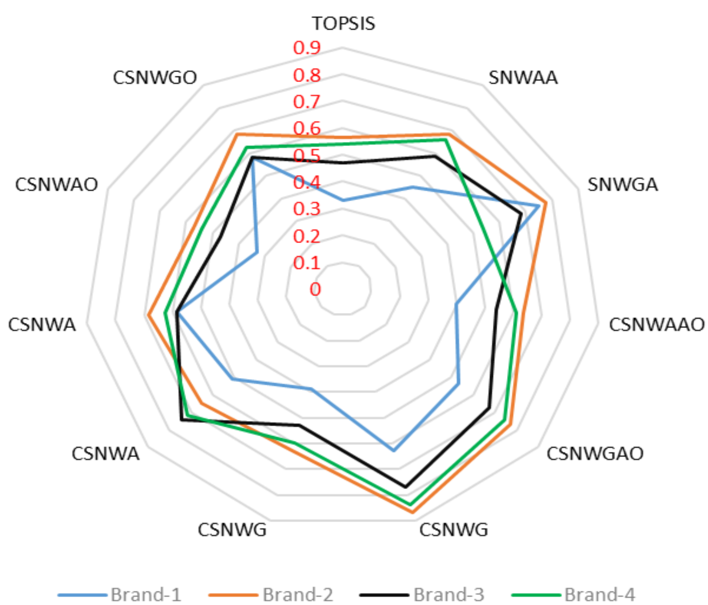

FIGURE 2. Comparative Analysis of Ranking

should be greater than one to enable the creation of a sphere representation in CSNSs. This requirement reflects the need for a significant level of involvement to ensure the meaningful representation of preferences and uncertainties. Understanding the limitations of CSNSs is essential for their effective utilization in MCDM. By addressing constraints such as the eccentricity requirement and the decision maker's involvement, researchers and practitioners can enhance the applicability and reliability of CSNS in real-world decision-making contexts.

6. **Conclusion and Future Work**

In our investigation, we introduced novel cubic spherical neutrosophic aggregation operators within the agricultural domain, particularly focusing on the intricate task of fertilizer selection for coconut trees. By introducing these operators, we aimed to streamline the evaluation process by considering multiple criteria such as nutrient content, cost, environmental impact, brand reputation, and availability. Through the application of CSNSs, we developed both additive and geometric aggregation operators, providing decision-makers with a comprehensive framework to assess and rank fertilizer alternatives. This innovative approach empowers farmers and agricultural practitioners to make informed decisions tailored to their specific needs and sustainability goals, ultimately contributing to optimized crop yields and environmental conservation in coconut farming and beyond.

Looking forward, our research paves the way for further exploration and refinement of cubic spherical neutrosophic aggregation operators in diverse agricultural contexts. Future endeavors will focus on extending the applicability of these operators to address broader agricultural decision-making challenges, including crop selection, pest management, irrigation strategies, and post-harvest practices. Moreover, we aim to develop tailored decision support systems and tools that cater to the unique requirements of farmers, extension agents, and agricultural stakeholders. By advancing the integration of innovative decision-making methodologies into agricultural practices, we strive to promote sustainable and resilient food systems while empowering farmers to make informed choices for improved productivity and livelihoods.

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