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Hausdorf Space in Neutrosophic Ideal Topological Spaces: Applications in decision Making

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Abstract. This paper presents the concept of neutrosophic $\mathfrak{I}$–Hausdorffness in the context of neutrosophic ideal topological spaces, along with various related theorems. Additionally, we provide a practical example showcasing the application of neutrosophic ideals in facilitating decision-making in uncertain environnments.

Keywords: Neutrosophic topological space; Neutrosophic ideal; Neutrosophic Hausdorff space, Neutrosophic closure.

1. Introduction

Smarandache [9] introduced the concept of a neutrosophic set as a generalization of the intuitionistic fuzzy set. Meanwhile, Salama et al. [11–13] introduced the concepts of neutrosophic crisp sets and neutrosophic crisp relations within neutrosophic set theory. Additionally, introduced neutrosophic topology, highlighting several of its characteristics. Also they defined neutrosophic crisp topology and explored some of its properties. Other researchers, such as Wang et al. [14] introduced the concept of a single-valued neutrosophic set. Kim et al. [4] have explored the notions of single-valued neutrosophic partition, single-valued neutrosophic equivalence relation, and single-valued neutrosophic relation.

The introduction of ideals into this framework further refines the structure of neutrosophic topological spaces. An ideal in a topological space is a collection of subsets that is closed under the formation of smaller subsets and finite unions. In neutrosophic ideal topology, ideals are used to define certain "ideal" open sets, providing a way to analyze the space's properties more precisely. Numerous authors [6–8] have conducted research on ideal topological spaces.

In [15] The authors introduced the notion of single-valued neutrosophic ideals sets in Šostak's sense, which is considered as a generalization of fuzzy ideals in Šostak's sense and intuitionistic fuzzy ideals. Also, the concept of single-valued neutrosophic ideal open local function is also introduced for a single-valued neutrosophic topological space. The basic structure, especially a basis for such generated single-valued neutrosophic topologies and several relations between different single-valued neutrosophic ideals and single-valued neutrosophic topologies, are discussed.

In Section 2, we review some definitions and results from the literature and establish certain results needed for subsequent discussions. In Section 3, We introduce the concept of I-Hausdorffness within the context of ideal topological spaces and establish several related results. In Section 4, An example is also presented to demonstrare the applicability of neutrosophic ideals in addressing decision-making challanges in uncertain situations.

2. **Preliminaries**

In this section, we give all basic definitions and results which we need to go through our work. First we give the definition of a neutrosophic set [10, 12].

Definition 2.1. *Let X be a nonempty set and $I = [0,1]$. A neutrosophic set N on X is a mapping defined as $N = \langle T_N, I_N, F_N\rangle : X \to \zeta$, where $\zeta = I^3$ and $T_N, I_N, F_N : X \to I$ such that $0 \leq T_N + I_N + F_N \leq 3$.*

We denote the set of all neutrosophic sets of X by ζ^X and the neutrosophic sets $\langle 0,1,1\rangle$ and $\langle 1,0,0\rangle$ by 0_X and 1_X respectively and $\langle 0,0,0\rangle$ and $\langle 1,1,1\rangle$ by $\bar{0}$ and $\bar{1}$ respectively. Let $(r,s,t),(l,m,n) \in \zeta$, then

- $(r,s,t) \sqcup (l,m,n) = (r \vee l, s \wedge m, t \wedge n)$;
- $(r,s,t) \sqcap (l,m,n) = (r \wedge l, s \vee m, t \vee n)$;
- $(r,s,t) \sqsubseteq (l,m,n) = (r \leq l, s \geq m, t \geq n)$;
- $(r,s,t) \sqsupseteq (l,m,n) = (r \geq l, s \leq m, t \leq n)$.

Definition 2.2. *[10, 12] Let X be a non-empty set and let $N, M \in \zeta^X$ be given by $N = \langle T_N, I_N, F_N\rangle$ and $\mathtt{M} = \langle T_M, I_M, F_M\rangle$. Then*

- *the complement of N denoted by N^c is given by*

$$N^c = \langle 1 - T_N, 1 - I_N, 1 - F_N\rangle$$

- *the union of N and M denoted by $N \sqcup M$ is an neutrosophic set in X given by*

$$N \sqcup M = \langle T_N \vee T_M, I_N \wedge I_M, F_N \wedge F_M\rangle$$

- *the intersection of N and M denoted by $N \sqcap M$ is an neutrosophic set in X given by*
$$N \sqcap M = \langle T_N \wedge T_M, I_N \vee I_M, F_N \vee F_M \rangle$$
- *the product of N and M denoted by $N \times M$ is given by*
$$(N \times M)(x, y) = N(x) \sqcap M(y), \forall\ (x, y) \in X \times Y.$$
- *we say that $N \sqsubseteq M$ if $T_N \leq T_M, I_N \geq I_M, F_N \geq F_M$.*

For an any arbitrary collection $\{N_i\}_{i \in J} \subseteq \zeta^X$ of neutrosophic sets the union and intersection is given by

- $\bigsqcup_{i \in J} N_i = \left\langle \bigvee_{i \in J} T_{N_i}, \bigwedge_{i \in J} I_{N_i}, \bigwedge_{i \in J} F_{N_i} \right\rangle$
- $\bigsqcap_{i \in J} N_i = \left\langle \bigwedge_{i \in J} T_{N_i}, \bigvee_{i \in J} I_{N_i}, \bigvee_{i \in J} F_{N_i} \right\rangle$.

Definition 2.3. *Let X be a nonempty set and $x \in X$. If $r \in (0, 1], s \in [0, 1)$ and $t \in [0, 1)$, then a neutrosophic point $x_{r,s,t}$ in X given by*

$$x_{r,s,t}(y) = \begin{cases} (r, s, t), & \text{if } x = y, \\ (0, 1, 1), & \text{otherwise.} \end{cases}$$

We say $x_{r,s,t} \in N$ if $x_{r,s,t} \sqsubseteq N$. To avoid the ambiguity, we denote the set of all neutrosophic points by $pt(\zeta^X)$.

Definition 2.4. *A neutrosophic set N is said to be quasi coincident with another neutrosophic set M, denoted by NqM if there exists an element $x \in X$ such that $T_N(x) + T_M(x) > 1$ or $I_N(x) + I_M(x) < 1$ or $F_N(x) + F_M(x) < 1$. If M is not quasi coincident with N, then we write $M\bar{q}N$.*

Definition 2.5. *[2] Let X be a nonempty set. Then a neutrosophic set $\tau = \langle T_\tau, I_\tau, F_\tau \rangle : \zeta^X \to \zeta$ is said to be a smooth neutrosophic topology on X if satisfies the following conditions:*

C1 $\tau(0_X) = \tau(1_X) = (1, 0, 0)$
C2 $\tau(N \sqcap M) \sqsupseteq \tau(N) \sqcap \tau(M)$, $\forall\ N, M \in \zeta^X$
C3 $\tau(\sqcup_{i \in J} N_i) \sqsupseteq \sqcap_{i \in J} \tau(N_i)$, $\forall\ N_i \in \zeta^X, i \in J$.

The pair (X, τ) is called a neutrosophic topological space.

Definition 2.6. *[3] Let (X, τ) be smooth neutrosophic topological space. For all $x_{r,s,t} \in pt(\zeta^X)$ and $N \in \zeta^X$, the mapping $Q^\tau_{x_{r,s,t}} : \zeta^X \to \zeta$ defined as follows*

$$Q^\tau_{x_{r,s,t}}(N) = \begin{cases} \bigsqcup_{x_{r,s,t} qM \sqsubseteq N} \tau(M); & \text{if } x_{r,s,t} qN, \\ (0, 1, 1), & \text{otherwise.} \end{cases}$$

The set $Q^\tau = \{Q^\tau_{x_{r,s,t}} : x_{r,s,t} \in pt(\zeta^X)\}$ is called neutrosophic Q-neighborhood system.

Definition 2.7. *[5] A neutrosophic topological space (X,τ) is said to be neutrosophic Hausdorff space if every pair of points $x_{r,s,t}$ and $y_{r',s',t'}$ with $x \neq y$, there exists a Q-neighborhood U and V of $x_{r,s,t}$ and $y_{r',s',t'}$ respectively with $x_{r,s,t}\bar{q}V$ and $y_{r',s',t'}\bar{q}U$ and $U\bar{q}V$.*

Definition 2.8. *[15] A mapping $\mathfrak{I} : \zeta^X \to \zeta^Y$ is called a neutrosophic ideal on X if it satisfies the following conditions:*

(1) $\mathfrak{I}(0_X) = (1,0,0)$.
(2) *If $A \sqsubseteq B$, then $\mathfrak{I}(B) \sqsubseteq \mathfrak{I}(A)$ for each $A, B \in \zeta^X$.*
(3) *If $A, B \in \zeta^X$, then $\mathfrak{I}(A \sqcup B) \sqsupset \mathfrak{I}(A) \sqcap \mathfrak{I}(B)$.*

The triple $(X,\tau,\mathfrak{I})$ is called a Neutrosophic Ideal Topological Space.

If $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are two neutrosophic ideals on X, we say that $\mathfrak{I}_1$ is finer than $\mathfrak{I}_2$ (denoted by $\mathfrak{I}_1 \preceq \mathfrak{I}_2$) or $\mathfrak{I}_2$ is coarser than $\mathfrak{I}_1$ if $\mathfrak{I}_2(A) \sqsubseteq \mathfrak{I}_1(A)$.

Definition 2.9. *[15] Let $(X,\tau,\mathfrak{I})$ be a neutrosophic topological space and $A \in \zeta^X$. Then the neutrosophic local function A^* of A is the union of all neutrosophic points $x_{r,s,t}$ such that $Q_{x_{r,s,t}}(U) \sqsupseteq (0,1,1)$ and $\mathfrak{I}(C) \sqsupseteq (0,1,1)$, Then there is at least one $y \in X$ forwhich $U(y) + A(y) - \bar{1}(y) \sqsupseteq C(y)$.*

Theorem 2.10. *[15] Let (X,τ) be neutrosophic topological space and $\mathtt{I}_1$ and $\mathtt{I}_2$ be two neutrosophic Ideals of X. Then for each $\mathtt{A}, \mathtt{B} \in \zeta^X$*

(i) *If $\mathtt{A} \sqsubseteq \mathtt{B}$, then $\mathtt{A}^* \sqsubseteq \mathtt{B}^*$*
(ii) *If $\mathtt{I}_1 \sqsubseteq \mathtt{I}_2$ then $\mathtt{A}^*(\mathtt{I}_1,\tau) \sqsupseteq \mathtt{A}^*(\mathtt{I}_2,\tau)$*
(iii) $\mathtt{A}^* = cl(\mathtt{A}^*) \sqsubseteq cl(\mathtt{A})$
(iv) $(\mathtt{A}^*)^* \sqsubseteq \mathtt{A}^*$
(v) $(\mathtt{A}^* \sqcup \mathtt{B}^*) = (\mathtt{A} \sqcup \mathtt{B})^*$
(vi) *If $\mathtt{I}(\mathtt{B}) \sqsupset (0,1,1)$ then $(\mathtt{A} \sqcup \mathtt{B})^* = \mathtt{A}^* \sqcup \mathtt{B}^* = \mathtt{A}^*$*
(vii) $(\mathtt{A} \sqcap \mathtt{B})^* \sqsubseteq (A^* \sqcap B^*)$

Definition 2.11. *[15] Let $(X,\tau,\mathfrak{I})$ be a neutrosophic ideal topological space and $A \in \zeta^X$. Then $C^*_\tau(A) = A \sqcup A^*$. It is clear, C^*_τ is a neutrosophic closure operator and $\tau_{\mathfrak{I}}$ is the neutrosophic topology generatedd by C^*_τ. Note that, if A is neutrosophic closed in $\tau_{\mathfrak{I}}$, then $A^* \sqsubseteq A$.*

3. **Neutrosophic $\mathfrak{I}$-Hausdorff space in Neutrosophic ideal topological space**

In this section we define $\mathfrak{I}$-Hausdorff space in the context of neutrosophic ideal topological spaces.

Definition 3.1. *Let $(X,\tau,\mathfrak{I})$ be a neutrosophic ideal topological space. Then (X,τ) is said to be neutrosophic $\mathfrak{I}$-Hausdorff with respect to the neutrosophic ideal $\mathfrak{I}$ if for every pair of* *neutrosophic points* $x_{r,s,t}$ *and* $y_{r',s',t'}$ *with* $x_{r,s,t} \neq y_{r',s',t'}$*, there exist* Q*-neighborhoods* U, V *of* $x_{r,s,t}$ *and* $y_{r',s',t'}$ *respectively, there exists* $C_1, C_2 \in \zeta^X$ *with* $\Im(C_1) \sqsupset (0,1,1)$*,* $\Im(C_2) \sqsupset (0,1,1)$ *such that* $x_{r,s,t}\bar{q}V - C_2$ *and* $y_{r',s',t'}\bar{q}U - C_1$*,* $U - C_1\bar{q}V - C_2$*.*

Theorem 3.2. *Let* $(X, \tau, \Im)$ *be a neutrosophic ideal topological space. Then* $(X, \tau, \Im)$ *is neutrosophic* $\Im$*-Hausdorff space if* $(X, \tau_\Im)$ *is neutrosophic Hausdorff space.*

Proof. Assume that $(X, \tau_\Im)$ is Neutrosophic Hausdorff space and $x_{r,s,t} \neq y_{r',s',t'}$. Then there exist Q-neighborhoods U and V of $x_{r,s,t}$ and $y_{r',s',t'}$ with $x_{r,s,t}\bar{q}V$ and $y_{r',s',t'}\bar{q}U$ and $U\bar{q}V$. Since $x_{r,s,t}\bar{q}V$ and $y_{r',s',t'}\bar{q}U$, then we have $x_{r,s,t} \in U^c$ and $y_{r',s',t'} \in V^c$. Also $\tau_\Im(U) \sqsupseteq I(0,1,1)$ and $\tau_\Im(V) \sqsupseteq (0,1,1)$ implies that $C^*_\tau(U^c) = U^c$ and $C^*_\tau(V^c) = V^c$. Therefore U^c and V^c are Neutrosophic $\Im$-closed sets. Clearly we get that $x_{r,s,t} \notin (U^c)^*$ and $y_{r',s',t'} \notin (V^c)^*$.

Since $x_{r,s,t} \notin (U^c)^*$, there exists a Q-neighborhood U_1 of $x_{r,s,t}$ and there exists $C_1 \in \zeta^X$ with $\Im(C_1) \sqsupseteq (0,1,1)$ such that

$$U_1(k) + U^c(k) - \bar{1}(k) \sqsubseteq C_1(k) \quad \text{for every } k \in X \tag{1}$$

Since $y_{r',s',t'} \notin (V^c)^*$, there exists a Q-neighborhood V_1 of $x_{r',s',t'}$ and there exists $C_2 \in \zeta^X$ with $\Im(C_2) \sqsupseteq (0,1,1)$ such that

$$V_1(k) + V^c(k) - \bar{1}(x) \sqsubseteq C_2(k) \quad \text{for every } k \in X \tag{2}$$

By using $x_{r,s,t}\bar{q}V \implies V(x) + (r,s,t) \sqsubseteq (1,1,1)$ and by (1)

$$V_1(x) + (r,s,t) \sqsubseteq V(x) + V^c(x) \sqsubseteq C_2(x) + \bar{1}(x)$$

Hence $(V_1 - C_2)(x) + (r,s,t) \sqsubseteq \bar{1}(x) \implies x_{r,s,t}\bar{q}(V_1 - C_2)$ Similarly, we can get that $y_{r',s',t'}\bar{q}(U_1 - C_1)$. Since $U\bar{q}V$ and by (1) and (2), $(U_1 - C_1)(x) + (V_1 - C_2)(x) \subseteq U(x) + V(x) \subseteq \bar{1}(x)$ Therefore, (X, τ) is $\Im$-Hausdorff space □

The example of $\Im$-hausdorff space is given below.

Example 3.3. *Let* $X = \{x, y\}$*. Define* $U \in \zeta^X$ *as follows:*

$$U(x) = (0.5, 0.2, 0.3), \quad U(y) = (0.1, 0.0, 0.3).$$

Define $\tau = (\tau_T, \tau_I, \tau_F) : \zeta^X \to \zeta$ *as follows:*

$$\tau(K) = \begin{cases} (1,0,0) & \text{if } K = 0_X \text{ or } 1_X \\ \left(\frac{1}{2}, 0, \frac{1}{2}\right) & \text{if } K = U \\ (0,1,1) & \text{otherwise} \end{cases}$$

Clearly, (X, τ) *is a neutrosophic topological space.*

$_T, \Im_I, \Im_F) : \zeta^X \to \zeta$ *as follows:*

$$\Im(S) = \begin{cases} (1,0,0) & \text{if } 0_X \sqsubseteq S \sqsubset U \text{ and } S \sqsubseteq \bar{1} \\ \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) & \text{if } S = U \\ (0,1,1) & \text{otherwise} \end{cases}$$

Hence $\Im$ *is a neutrosophic ideal on* X*.*

Let $x_{r,s,t} \neq y_{r,s,t}$ *in* X*. Then the possible Q-neighborhoods of* $x_{r,s,t}$ *and* $y_{r,s,t}$ *are* U *and* 1_X*.*

Case (i): *Suppose* U *is a Q-neighborhood of* $x_{r,s,t}$ *and* $y_{r,s,t}$*. Let* $C_1 = U = C_2$*. Then,* $\Im(C_1) \sqsupseteq (0,1,1)$ *and* $\Im(C_2) \sqsupseteq (0,1,1)$*. Hence,* $U - C_2 = (0,0,0) = U - C_1$*. Therefore,* $x_{r,s,t}\bar{q}U - C_2$ *and* $y_{r,s,t}\bar{q}U - C_1$*. Also,* $(U - C_2)\bar{q}(V - C_1)$*.*

Case (ii): *Suppose* 1_X *is a Q-neighborhood of* $x_{r,s,t}$ *and* $y_{r,s,t}$*. Let* $C_1 = (1,1,1) = C_2$*. Clearly,* $\Im(C_1) \sqsupseteq (0,1,1)$ *and* $\Im(C_2) \sqsupseteq (0,1,1)$*. Hence* $1_X - C_1 = (0,-1,-1) = 1_X - C_2$*. Therefore we get that* $x_{r,s,t}\bar{q}1_X - C_1$ *and* $y_{r,s,t}\bar{q}1_X - C_2$*. Also,* $(1_X - C_2)\bar{q}(1_X - C_1)$*.*

Case (iii): *Let* U *be a Q-neighborhood of* $x_{r,s,t}$ *and* 1_X *is the Q-neighborhood of* $y_{r,s,t}$*. In this case, choose* $C_1 = U$ *and* $C_2 = \bar{1}$*. Clearly,* $U - C_1 = (0,0,0)$ *and* $1_X - C_2 = (0,-1,-1)$*. Thus* $x_{r,s,t}\bar{q}U - C_1$ *and* $y_{r,s,t}\bar{q}1_X - C_2$*. Also,* $(1_X - C_2)\bar{q}(U - C_1)$*.*

Hence (X,τ) *is a neutrosophic* $\Im$*-Hausdorff space.*

Remark 3.4. *If* $(X,\tau,\Im)$ *is a neutrosophic ideal topolgical space and* $A \in \zeta^X$*, then*

(1) $\tau_A : \zeta^A \to \zeta$ *by*

$$\tau_A(U) = \{\tau(W) : W \in \zeta^X, A \sqcap W = U\}$$

is a neutrosophic subspace topology [1] inherited from τ*. If* $x_{r,s,t} \in A$*, then the Q-neighborhood* U *of* $x_{r,s,t}$ *related to* A *is defined as* $\tau_A(U) \sqsupset (0,1,1)$ *and* $x_{r,s,t}q_A U$*.i.e* $T_U(x) + r > T_A(x)$ *or* $I_U(x) + s < 1$ *or* $F_U(x) + t < 1$*.*

(2) $\Im_A : \zeta^A \to \zeta$ *by* $\Im_A(U) = \{\Im(W) : W \in \zeta^X, A \sqcap W = U\}$ *is neutrosophic ideal on* A*.*

Theorem 3.5. *Let* $(X,\tau,\Im)$ *be neutrosophic* $\Im$*-Hausdorff space and* $A \in \zeta^X$*. Then* (A,τ_A) *is* $\Im_A$*-Hausdorff space, where* τ_A *is the neutrosophic subspace topology inhirited from* τ*.*

Proof. Let $x_{r,s,t} \neq y_{r',s',t'}$ in A. Since X is $\Im$-Hausdorff space, there exist Q-neighborhoods U, V of $x_{r,s,t}$ and $y_{r',s',t'}$ respectively, there exists $C_1, C_2 \in \zeta^X$ with $\Im(C_1) \sqsupseteq (0,1,1)$, $\Im(C_2) \sqsupseteq (0,1,1)$ such that $x_{r,s,t}\bar{q}V - C_2$ and $y_{r',s',t'}\bar{q}U - C_1$, $U - C_1\bar{q}V - C_2$.

Let $U_1 = U \sqcap A$,$V_1 = V \sqcap A$, $D_1 = C_1 \sqcap A$ and $D_2 = C_2 \sqcap A$. Clearly we get that

$$\begin{aligned}\tau_A(U_1) &= \tau_A(U \sqcap A) \sqsupseteq \tau(U) \sqsupset (0,1,1),\\ \tau_A(V_1) &= \tau_A(V \sqcap A) \sqsupseteq \tau(V) \sqsupset (0,1,1),\\ \mathfrak{I}_A(D_1) &= \mathfrak{I}(C_1 \sqcap A) \sqsupseteq \mathfrak{I}(U) \sqsupset (0,1,1)\\ \mathfrak{I}_A(D_1) &= \mathfrak{I}(C_1 \sqcap A) \sqsupseteq \mathfrak{I}(U) \sqsupset (0,1,1).\end{aligned}$$

Since $x_{r,s,t}qU$,

$$\begin{aligned}(r,s,t) + U_1(x) &= (r,s,t) + (A \sqcap U)(x)\\ &= [(r,s,t) + A(x)] \sqcap [(r,s,t) + U(x)]\\ &\sqsupseteq A(x) \sqcap (1,1,1) = (T_A(x),1,1).\end{aligned}$$

Therefore $x_{r,s,t}q_AU_1$. Similarly, $y_{r',s',t'}qV$ implies that$y_{r',s',t'}q_AV_1$.
Since $x_{r,s,t}\bar{q}V - C_1$, $(r,s,t) + (V - C_1)(x) \sqsubseteq \bar{1}(x)$. Now,

$$\begin{aligned}(V_1 - D_1)(x) &= (V \sqcap A)(x) - (C_1 \sqcap A)(x)\\ &= (V - C_1)(x) \sqcup (A - A)(x)\\ &= (V - C_1)(x) \sqsubseteq \bar{1}(x) - (r,s,t)\end{aligned}$$

Therefore $x_{r,s,t}\bar{q}V_1 - D_1$. Similarly, $y_{r',s',t'}\bar{q}U - C_2$ implies that $y_{r',s',t'}q_AV_1$. Also,

$$\begin{aligned}(V_1 - D_1)(k) + (U_1 - D_2)(k) &= [(V \sqcap A)(k) - (C_1 \sqcap A)(k)] + [(U \sqcap A)(k) - (C_2 \sqcap A)(k]\\ &\sqsubseteq (V - C_1)(k) + (U - C_2)(k) \sqsubseteq \bar{1}.\end{aligned}$$

Hence, $(V_1 - D_1)\bar{q}(U_1 - D_2)$. Thus (A, τ_A) is neutrosophic $\mathfrak{I}_A$-Hausdorff space. □

If we define the ideal closure of A by
$\mathfrak{I}Cl(A) = \sqcup\{x_{r,s,t}$: every Q-neighborhood $U$of $x_{r,s,t}$, there exits C_1 with $\mathfrak{I}(C_1) \sqsupset (0,1,1), U - C_1qV\}$,
then the following theorem holds.

Theorem 3.6. *Let $(X,\tau,\mathfrak{I})$ be ideal topological space. If (X,τ) is $\mathfrak{I}$-Hausdorff space, then $\{x_{r,s,t}\} = \sqcap\{\mathfrak{I}Cl(U_x) : U_x$ is neutrosophic Q-neighborhood of $x_{r,s,t}\}$.*

Proof. Let $x_{r,s,t} \in \zeta^X$. For any $y_{r',s',t'} \neq x_{r,s,t}$, there exist Q-neighborhoods U, V of $x_{r,s,t}$ and $y_{r',s',t'}$ respectively, there exists $C_1, C_2 \in \zeta^X$ with $\mathfrak{I}(C_1) \sqsupseteq (0,1,1)$, $\mathfrak{I}(C_2) \sqsupseteq (0,1,1)$ such that $x_{r,s,t}\bar{q}V - C_2$ and $y_{r',s',t'}\bar{q}U - C_1$, $U - C_1\bar{q}V - C_2$. Since U and V is a Q-neighborhoods of $x_{r,s,t}$ and $y_{r',s',t'}$ respectively and $U - C_1\bar{q}V - C_2$, we have $y_{r',s',t'} \notin \mathfrak{I} - Cl(A)$. Therefore $\{x_{r,s,t}\} = \sqcap\{\mathfrak{I}Cl(U_x) : U_x$ is neutrosophic Q-neighborhood of $x_{r,s,t}\}$. □

4. Application

Decision-Making Problem Here's a decision-making problem using the neutrosophic ideal definition:

Problem: Select the best investment option among three alternatives ($A, B,$ and C) using a neutrosophic ideal.

Data:

$$\begin{aligned} A &= (0.7, 0.2, 0.1)(StockMarket) \\ B &= (0.5, 0.3, 0.2)(RealEstate) \\ C &= (0.9, 0.1, 0.0)(GovernmentBonds) \end{aligned}$$

Neutrosophic Ideal (I):

$$\begin{aligned} I(A) &= (0.6, 0.3, 0.1) \\ I(B) &= (0.4, 0.4, 0.2) \\ I(C) &= (0.8, 0.2, 0.0). \end{aligned}$$

Decision-making process:

(1) Compare the neutrosophic ideal values:
$I(A) \sqsubseteq I(C)(\text{ since } 0.6 \leq 0.8, 0.3 \geq 0.2, \text{ and } 0.1 \geq 0.0)$
$I(B) \sqsubseteq I(C)(\text{ since } 0.4 \leq 0.8, 0.4 \geq 0.2, \text{ and } 0.2 \geq 0.0)$
$I(A) \sqcap I(B) = (0.4, 0.4, 0.2)$

(2) Calculate the neutrosophic ideal of the union:
$I(A \sqcup B) = (0.7, 0.2, 0.1)$

(3) Compare the results:
$I(A \sqcup B) \sqsupset I(A) \sqcap I(B)(\text{ since } 0.7 \geq 0.4, 0.2 \leq 0.4, \text{ and } 0.1 \leq 0.2)$

Based on the neutrosophic ideal, the best investment option is C (Government Bonds) since $I(C)$ has the highest truth membership value (0.8) and the lowest falsity membership value (0.0). The combination of A and B (Stock Market and Real Estate) is not better than investing solely in C.

This example illustrates the application of neutrosophic ideals in decision-making under uncertainty. The neutrosophic sets and ideals provide a more comprehensive framework for handling imprecise and uncertain data.

Conclusions

This paper introduces the concept of neutrosophic $\mathfrak{I}$–Hausdorffness within neutrosophic ideal topological spaces, expanding the theoretical framework of neutrosophic set theory. The theorems presented offer deeper insights into the structural properties of these spaces, while the practical example demonstrates the utility of neutrosophic ideals in decision-making processes under uncertainty. This work not only contributes to the mathematical foundation of neutrosophic theory but also highlights its potential for real-world applications in complex, indeterminate environments.

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