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# The Periodicity of Square Fuzzy Neutrosophic Soft MatricesBased on Minimal Strong Components

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**Abstract**. In this paper Minimal Strong Component ($\mathcal{MSC}$) of a Fuzzy Neutrosophic Soft Matrix ($\mathcal{FNSM}$) is suggested. By employing the connection of periodicity behaviours of $\mathcal{FNSM}$ with its cut matrices, the periodicity of power sequence of $\mathcal{FNSM}$ is described. Especially the concepts of $\mathcal{MSC}$ is given and the periodicity of a $\mathcal{FNSM}$ by its (Theorem-4.10) on the basis of the above results, the greatest value $\max \sum_{l_i=n} = [l_i]$ of the periodicity of all $\mathcal{SFNSM}$ for a given positive integer n is obtained. So in a case we have clearly resolved the problem of the greatest value of all periodicity $\mathcal{FNSM}$ for a given positive integer n.

**Keywords:** Fuzzy Neutrosophic Soft Matrix ($\mathcal{FNSM}$), Minimal Strong Component ($\mathcal{MSC}$), Cut Fuzzy Neutrosophic Soft Matrix ($\mathcal{CFNSM}$), Periodicity of Fuzzy Neutrosophic Soft Matrices ($\mathcal{PFNSM}s$)

## 1. Introduction

The models of real-life problems in almost every field of science like mathematics, physics, operations research, medical sciences, engineering, computer science, artificial intelligence, and management sciences are mostly full of complexities. Many theories have been developed to overcome these uncertainties; one among those theories is fuzzy set theory. Zadeh [1] was the first who gave the concept of a Fuzzy Set ($\mathcal{FS}$) are the generalizations or extensions of crisps sets.

In order to add the concept of nonmembership term to the idea of $\mathcal{FS}$, the concept of an Intuitionistic Fuzzy Set ($\mathcal{IFS}$) was introduced by Atanassov in [2], where he added the thought of nonmembership term to the definition of $\mathcal{FS}$. The $\mathcal{IFS}$ is characterized by a membership function $\mu$ and a nonmembership function $\eta$ with ranges [0,1]. The $\mathcal{IFS}$ is the generalization of a FS. An $\mathcal{IFS}$ can be applied in several fields including modeling, medical diagnosis, and decision-making. In [3] Molodtsov introduced the concept of a Soft Set $\mathcal{SS}$ and developed the fundamental results related to this theory. Basic operations including complement, union, and intersection are also defined on this set. Also he used $\mathcal{SS}$s for applications in games, probability, and operational theories. Maji et. al., [4,5] proposed the Fuzzy Soft Sets ($\mathcal{FSS}s$) and Intuitionistic Fuzzy Soft Set ($\mathcal{IFSS}$) by combining $\mathcal{SS}s$ and $\mathcal{FS}s$ and applied them in decision-making problems.

The concept of neutrosophy was introduced by Smarandache in [6]. A Neutrosophic Set ($\mathcal{NS}$) is characterized by a truth membership function $\mathcal{T}$, an indeterminacy function $\mathcal{I}$, and a falsity membership function $\mathcal{F}$. A $\mathcal{FS}$ is a mathematical framework which generalizes the concept of a classical set, $\mathcal{FS}$, $\mathcal{IFS}$, and $\mathcal{IVFS}$. Broumi et,al., [7] proposed the generalized interval neutrosophic soft set and its decision making problem.

Thomason [8] was the first who gave the concept of a Fuzzy Matrix $\mathcal{FM}$. He discussed the convergence of powers of $\mathcal{FM}$, its play a vital role in scientific development. And he also pointed out that the powers of a $\mathcal{FM}$ either converge or oscillate with a finite period. Li [9,10] discussed the periodicity and index of fuzzy matrices in the general case. In [11] Fan proves that the periodicity of a $\mathcal{FM}$ is the least common multiple (l.c.m.) of periodicity of its cut matrices, and the index of a fuzzy matrix is not greater than the maximum index of its cut matrices. It is also shown that the periodicity set of the power sequences of $\mathcal{FM}s$ of order n is not bounded from above by a power of n for all integers n. Liua and Ji [12,13] have discussed the periodicity of Square Fuzzy Matrices $\mathcal{SFM}s$ based on minimal strong components. Atanassov [14,15] has studied intuitionistic fuzzy index matrix and the index matrix representation of the intuitionistic fuzzy graphs has been studied. Murugadas et.al., [16] presented the periodicity of intuitionistic fuzzy matrix. Manoj Bora et.al., [17] introduced the concepts of Intuitionistic Fuzzy Soft Matrix $\mathcal{IFSM}$ theory and its Application in Medical Diagnosis. Arockiarani and Sumathi [18, 19] proposed Fuzzy Neutrosophic Soft Matrix $\mathcal{FNSM}$ and used them in decision making problems. Kavitha et.al., [20–26] introduced some concepts on priodicity of interval values, on powers of matrices and convergence of matrices usig the notion of $\mathcal{FNSM}$. The idea of monotone interval fuzzy neutrosophic soft eigenproblem and convergence of fuzzy neutrosophic soft circulant matrices are proposed by Murugadas et.al., [27,28]. Uma et.al., [29] presented the concepts of $\mathcal{FNSM}s$ of Type-1 and Type-2.

This paper is organized as follows: In section-2, some basic notions related to this topics are recalled. Section-3 we, discuss the properties of periodicity and index of $\mathcal{FNSM}$. In section-4 we, explain the digraph representaion of Strongly Connected $\mathcal{SC}$ and Minimal Strong Components $\mathcal{MSC}$ by using $\mathcal{SFNSM}$. Section-5 we can frame the algorithm to find the $\mathcal{MSC}$ of $\mathcal{FNSM}$. Section-6 is for conculusion.

## 2. Preliminaries

The following definitions is needed to our study.

**Definition 2.1.** [6] A Neutrosophic Set $\mathcal{NS}$ A on the universe of discourse $X$ is defined as $A = \{\langle x,\ \mathcal{T}_A(x),\ \mathcal{I}_A(x),\ \mathcal{F}_A(x)\rangle, x \in X\}$, where $\mathcal{T}, \mathcal{I}, \mathcal{F} : X \to ]^{-}0, 1^{+}[$ and

$$^{-}0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^{+}. \ (2.1)$$

From philosophical point of view the neutrosophic set takes the value from real standard or non-standard subsets of $]^{-}0, 1^{+}[$. But in real life application especially in Scientific and Engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^{-}0, 1^{+}[$. Hence we consider the neutrosophic set which takes the value from the subset of $[0, 1]$. Therefore we can rewrite equation (2.1) as $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$. In short an element $\widetilde{a}$ in the neutrosophic set $A$, can be written as $\widetilde{a} = \langle a^{\mathcal{T}}, a^{\mathcal{I}}, a^{\mathcal{F}}\rangle$, where $a^{\mathcal{T}}$ denotes degree of truth, $a^{\mathcal{I}}$ denotes degree of indeterminacy, $a^{\mathcal{F}}$ denotes degree of falsity such that $0 \leq a^{\mathcal{T}} + a^{\mathcal{I}} + a^{\mathcal{F}} \leq 3$.

**Example 2.2.** Assume that the universe of discourse $X = \{x_1,\ x_2,\ x_3\}$ where $x_1,\ x_2$ and $x_3$ characterize the quality, reliability, and the price of the objects. It may be further assumed that the values of $\{x_1,\ x_2,\ x_3\}$ are in $[0, 1]$ and they are obtained from some investigations of some experts. The experts may impose their opinion in three components viz; the degree of goodness, the degree of indeterminacy and the degree of poorness to explain the characteristics of the objects. Suppose A is a Neutrosophic Set (NS) of $X$, such that
$A = \{\langle x_1,\ 0.4,\ 0.5,\ 0.3\rangle, \langle x_2,\ 0.7,\ 0.2,\ 0.4\rangle, \langle x_3, 0.8,\ 0.3,\ 0.4\rangle\}$ where for $x_1$ the degree of goodness of quality is 0.4, degree of indeterminacy of quality is 0.5 and degree of falsity of quality is 0.3 etc.

**Definition 2.3.** [18] A Fuzzy Neutrosophic Set $\mathcal{FNS}$ $A$ on the universe of discourse $X$ is defined as $A = \{x, \langle \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle, x \in X\}$, where $\mathcal{T}, \mathcal{I}, \mathcal{F} : X \to\ [0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

**Definition 2.4.** [3] Let $U$ be the initial universal set and $E$ be a set of parameter. Consider a non-empty set $A, A \subset E$. Let $P(U)$ denotes the set of all fuzzy neutrosophic sets of $U$. The collection $(F, A)$ is termed to be the fuzzy neutrosophic soft set over $U$, where $F$ is a mapping given by $F : A \to\ P(U)$. Here after we simply consider $A$ as $\mathcal{FNSS}$ over $U$ instead of $(F, A)$.

**Definition 2.5.** [18] Let $U = \{c_1, c_2, ..., c_m\}$ be the universal set and $E$ be the set of parameters given by $E = \{e_1, e_2, ..., e_m\}$. Let $A \subset E$. A pair $(F, A)$ be a $\mathcal{FNSS}$ over $U$. Then the subset of $U \times E$ is defined by $R_A = \{(u, e);\ e \in A,\ u \in F_A(e)\}$
which is called a relation form of $(F_A,\ E)$. The membership function, indeterminacy membership function and non membership function are written by

$\mathcal{T}_{R_A} : U \times E \to [0,1],\ \mathcal{I}_{R_A} : U \times E \to [0,1]$ and $\mathcal{F}_{R_A} : U \times E \to [0,1]$ where $\mathcal{T}_{R_A}(u,e) \in [0,1],\ \mathcal{I}_{R_A}(u,e) \in [0,1]$ and $\mathcal{F}_{R_A}(u,e) \in [0,1]$ are the membership value, indeterminacy value and non membership value respectively of $u \in U$ for each $e \in E$.

If $[(\mathcal{T}_{ij},\ \mathcal{I}_{ij},\ \mathcal{F}_{ij})] = [\mathcal{T}_{ij}(u_i,e_j),\ \mathcal{I}_{ij}(u_i,e_j)\ ,\mathcal{F}_{ij}(u_i,e_j)]$, we define a matrix

$$[\langle \mathcal{T}_{ij}\ ,\mathcal{I}_{ij},\ \mathcal{F}_{ij}\rangle]_{m\times n} = \begin{bmatrix} \langle \mathcal{T}_{11},\ \mathcal{I}_{11},\ \mathcal{F}_{11}\rangle & \cdots & \langle \mathcal{T}_{1n},\ \mathcal{I}_{1n},\ \mathcal{F}_{1n}\rangle \\ \langle \mathcal{T}_{21},\ \mathcal{I}_{21},\ \mathcal{F}_{21}\rangle & \cdots & \langle \mathcal{T}_{2n},\ \mathcal{I}_{2n},\ \mathcal{F}_{2n}\rangle \\ \vdots & \vdots & \vdots \\ \langle \mathcal{T}_{m1},\ \mathcal{I}_{m1},\ \mathcal{F}_{m1}\rangle & \cdots & \langle \mathcal{T}_{mn},\ \mathcal{I}_{mn},\ \mathcal{F}_{mn}\rangle \end{bmatrix}.$$

Which is called an $m \times n$ $\mathcal{FNSM}$ of the $\mathcal{FNSS}(F_A, E)$ over $U$.

**Definition 2.6.** [29] Let $A = (\langle a^{\mathcal{T}}_{ij},\ a^{\mathcal{I}}_{ij},\ a^{\mathcal{F}}_{ij}\rangle),\ B = \langle (b^{\mathcal{T}}_{ij},\ b^{\mathcal{I}}_{ij},\ b^{\mathcal{F}}_{ij}\rangle) \in \mathcal{N}_{m\times n}$. The component wise addition and component wise multiplication is defined as

$A \oplus B = (sup\{a^{\mathcal{T}}_{ij},\ b^{\mathcal{T}}_{ij}\},\ sup\{a^{\mathcal{I}}_{ij},\ b^{\mathcal{I}}_{ij}\},\ inf\{a^{\mathcal{F}}_{ij},\ b^{\mathcal{F}}_{ij}\})$

$A \otimes B = (inf\{a^{\mathcal{T}}_{ij},\ b^{\mathcal{T}}_{ij}\},\ inf\{a^{\mathcal{I}}_{ij},\ b^{\mathcal{I}}_{ij}\},\ sup\{a^{\mathcal{F}}_{ij},\ b^{\mathcal{F}}_{ij}\})$

**Definition 2.7.** Let $A \in \mathcal{N}_{m\times n},\ B \in \mathcal{N}_{n\times p}$, the composition of $A$ and $B$ is defined as

$$A \circ B = \left(\sum_{k=1}^{n}(a^{\mathcal{T}}_{ik} \wedge\ b^{\mathcal{T}}_{kj}),\ \sum_{k=1}^{n}(a^{\mathcal{I}}_{ik} \wedge\ b^{\mathcal{I}}_{kj}),\ \prod_{k=1}^{n}(a^{\mathcal{F}}_{ik} \vee\ b^{\mathcal{F}}_{kj})\right)$$

equivalently we can write the same as

$$= \left(\bigvee_{k=1}^{n}(a^{\mathcal{T}}_{ik} \wedge b^{\mathcal{T}}_{kj}),\ \bigvee_{k=1}^{n}(a^{\mathcal{I}}_{ik} \wedge b^{\mathcal{I}}_{kj}),\ \bigwedge_{k=1}^{n}(a^{\mathcal{F}}_{ik} \vee b^{\mathcal{F}}_{kj})\right).$$

The product $A \circ B$ is defined if and only if the number of columns of $A$ is same as the number of rows of $B$. Then $A$ and $B$ are said to be conformable for multiplication. We shall use $AB$ instead of $A \circ B$.

Where $\sum(a^{\mathcal{T}}_{ik} \wedge\ b^{\mathcal{T}}_{kj})$ means max-min operation and

$\prod_{k=1}^{n}(a^{\mathcal{F}}_{ik} \vee\ b^{\mathcal{F}}_{kj})$ means min-max operation.

**Throught this paper, we are following this notation and notions** [22, 23].

Let $\mathcal{R} = (\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle$ and $\mathcal{P} = (\langle\langle p^{\mathcal{T}}_{ij}, p^{\mathcal{I}}_{ij}, p^{\mathcal{F}}_{ij}\rangle)$ with their elements in the unit interval $I = [\langle 0,0,1\rangle, \langle 1,1,0\rangle]$.

We discuss some definitions and notations.

- $\mathcal{R} \times \mathcal{P} = \langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle \times \langle p^{\mathcal{T}}_{ij}, p^{\mathcal{I}}_{ij}, p^{\mathcal{F}}_{ij}\rangle = (\bigvee_{m=1}^{n}(\langle\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle \wedge \langle p^{\mathcal{T}}_{ij}, p^{\mathcal{I}}_{ij}, p^{\mathcal{F}}_{ij}\rangle))$
- $\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle^{k+1} = \langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle^{k} \times \langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle,\ k = 1, 2, ...$
- $\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle^{0} = \mathcal{E}$, where $\mathcal{E}$ is the unit $\mathcal{FNSM}$.
- $\mathcal{R} \leq \mathcal{P}$ iff $\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle \leq \langle p^{\mathcal{T}}_{ij}, p^{\mathcal{I}}_{ij}, p^{\mathcal{F}}_{ij}\rangle\ \forall i, j \in \{1, 2, ..., n\}$,

- $\mathbb{N}^+ = \{x | x \text{ is a positive integer}\}$.
- In the sequence $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle, \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^2, \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^3, ..., \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^m, ...$, the number of different matrices is at most $l^{n^2}$ [here, l is the number of all the different elements that occure in $\mathcal{FNSM}$ $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle$,] which is neatly finite. Hence, $\exists$ indices $s, t \in \mathbb{N}^+$, $(s \neq t) \ni \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^s = \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^t$.
- Let $\mathcal{H} = \{(s,t) | (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^s = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^t, s \neq t, s, t \in \mathbb{N}^+\}$;
- $\mathcal{D} = \{d | d = |s - t|, (s,t) \in H\}$.
- By the well ordering property (of natural numbers) $\mathcal{D}$ has a least element $d$.
  Clearly, $d \geq 1$.
- Let $K = \{k | \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^k = \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^{k+d}, k \in \mathbb{N}^+\}$.
- Truly, $K$ is a nonempty subset of $\mathbb{N}^+$. By well-ordering property, $K$ has a least elemant $k (k \geq 1)$.
- The following definitions, remarks and lemmas are from [22, 23]
- A path in an ordinary directed graph (digraph) is a sequence of distinct vertices $v_1, v_2, ..., v_n \ni$ for $i = \{1, 2, ..., n-1\}$, there is a directed edge in the graph from $v_i$ to $v_{i+1}$.
- A digraph is Strongly Connected (SC) iff for any two vertices $v_i, v_j$ here $v_j$ is reachable from $v_i$.
- The $\mathcal{S}$trong $\mathcal{C}$omponents ($\mathcal{SC}$) of a digraph $\mathcal{G}$ are those full subgraphs of $\mathcal{G}$ that are SC and are not properly contained in any other SC subgraph of $\mathcal{G}$.
- A cycle in a digraph is a sequence of vertices $v_1, v_2, ..., v_n \ni$ for
  $i = \{1, 2, ..., r-1\}$, there is a directed edge from $v_i$ to $v_{i+1}$ and $v_1 = v_n$ and all the other $v^s$ are distinct.

**Remark 2.8.** An ordinary directed graph is really the same as a Boolean Fuzzy Neutrosophic Soft Matrix $\mathcal{BFNSM}$ and the periodicity of oscillation of a $\mathcal{BFNSM}$ can be determined from its corresponding digraph.

**Lemma 2.9.** The periodicity of a ($\mathcal{SC}$) is the greatest common divisor (g.c.d.) of the lengths of all cycles in its digraph.

**Lemma 2.10.** The periodicity of an ordinary digraph is the l.c.m. of the periods of all ($\mathcal{SC}$) in its graph.

## 3. Properties on Periodicity and Index

In this section, we give an equivalent definition of periodicity and index of a $\mathcal{FNSM}$.

**Definition 3.1.** If $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ is a $\mathcal{FNSM}$, $m, s \in \mathbb{N}^+$ and $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s}$, then we say that $m$ is a periodicity of $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle$, and $s$ starting point of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ corresponding to $m$.

**Proposition 3.2.** If $s$ is a initial point corresponding to $m$; then $n$ is also a first point(pt) corresponding to $m$ $\forall n \in \mathbb{Z}^+, n > s$.

**Proof:** Multiplying $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{n-s}$ on both sides of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s}$, obtains $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{n+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{n}$. Proof Completes. Next part it is known from Property 3.2 that the periodicity $m$ decides a boundary $\mathcal{T}_m$. Every n with $n \geq \mathcal{T}_m$ is start point corresponding to $m$, while every $n$ with $n < \mathcal{T}_m$ is not a first point closed to $m$. We call $\mathcal{T}_m$ an index of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$. Clearly,
$\mathcal{T}_m = \min\{s|(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s}, m \text{ a given positive integer}\}$.

**Definition 3.3.** We define the $d = \min\{m|(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{s}$,
$\forall\ s, m \in \mathbb{N}^+\}$ the least periodicity of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$.
The natural numbers $d$ exists from the well ordering property.

**Proposition 3.4.** Every periodicity m of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ is a multiple of $d$.

**Proof:** Suppose this property does not hold.
Let as take that $m = nd + p,\ 1 \leq p \leq d,\ r = \max\{\mathcal{T}_m, \mathcal{T}_d\}$.
By known Property 3.2, $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{r} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{r+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{r+nd+p} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{r+p}$, it follows that p is periodicity of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$, a contradiction to the definition of $d$.

**Proposition 3.5.** $\mathcal{T}_d$ is an index correponding to every periodicity of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$.

**Proof:** $\mathcal{T}_m$ denote the index corresponding to periodicity $m$ of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$.
By Property 3.4, there exists an integer $l \in \mathbb{N}^+$ such that $m = ld$.
Thus $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_d+m} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_d+ld} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_d}$.
By our definition of $\mathcal{T}_m$, we have $\mathcal{T}_d \geq \mathcal{T}_m$. On the other words, for all $m > 0$, we can find $h \in \mathbb{Z}^+$ suth that $\mathcal{T}_m + mh \geq \mathcal{T}_d$. Thus $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_m+d} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_m+d+mh} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_m+mh} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_m}$, so we have $\mathcal{T}_m \geq \mathcal{T}_d$.

**Definition 3.6.** We said the common index $\mathcal{T}_d$ the index of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$, the least periodicity d the periodicity of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$, denoted by $k, d$, respectively.

**Theorem 3.7.** If $\mathcal{N}, \mathcal{H} \in \mathbb{N}^+$, then $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^{N} = \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle^{N+H} \Leftrightarrow N \geq k, d|H$.

**Proof:** That implies since $H$ is a periodicity of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$, by property 3.4, we get $d|H$. Let $\mathcal{T}_H$ be the index corresponding to $H$. Since $N$ is also an index of $H$, by the definition of $\mathcal{T}_H$, we need $N \geq \mathcal{T}_H$. By Property 3.5, $\mathcal{T}_H = \mathcal{T}_d = k$. So we have $N \geq k$.
$\Leftarrow$ Suppose that $H = md,\ m \in \mathbb{Z}^+,\ N \geq k$, then $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+H} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+md} =$

$(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_d+md} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{T}_d} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_k$. We follows that $H$ is a periodicity of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$. By Property 3.2 and $N \geq k$, we obtain that $N$ is $\mathcal{S}$tarting point ($\mathcal{S}pt$) related to $H$. Thus $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^N = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{N+H}$.

**Corollary 3.8.** In the sequence $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle), (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^2, ..., (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^n, ...$ the number of different $\mathcal{FNSM}$ is $k+d-1$. The set of the $k+d-1$ different $\mathcal{FNSM}$ is $X = \{(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle), (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^2, ..., (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^k, (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+1},$
$..., (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+d-1}$.

**Proof:** If $a \in \mathbb{Z}^+,\ a \geq k+d$, then let $a-k = sd+r,\ (s, r \in \mathbb{Z}^+,\ 0 \leq r \leq d-1)$. By Theorem 3.7 $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^a = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{(k+r)+sd} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+r} \in X$. If $a, b, \in \mathbb{Z}^+,\ 1 \leq a \leq b \leq k+d-1$ and $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^a = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^b$, then $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^a = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{a+(b-a)}$. By Theorem 3.7 $a \geq k,\ d|(b-a)$.

Thus $b-a \geq b \Rightarrow b \geq a+d \geq k+d$, contradiction to the assumption of $b$.

**Lemma 3.9.** If $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ and $(\langle p_{ij}^{\mathcal{T}}, p_{ij}^{\mathcal{I}}, p_{ij}^{\mathcal{F}}\rangle)$ are $\mathcal{FNSM}s$, then $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle) = (\langle p_{ij}^{\mathcal{T}}, p_{ij}^{\mathcal{I}}, p_{ij}^{\mathcal{F}}\rangle) \Leftrightarrow (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda = (\langle p_{ij}^{\mathcal{T}}, p_{ij}^{\mathcal{I}}, p_{ij}^{\mathcal{F}}\rangle)_\lambda,\ \forall \lambda \in I$.

From Lemma 3.9 we find if there is a $\lambda$ and $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda = (\langle p_{ij}^{\mathcal{T}}, p_{ij}^{\mathcal{I}}, p_{ij}^{\mathcal{F}}\rangle)_\lambda$, then $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle) \neq (\langle p_{ij}^{\mathcal{T}}, p_{ij}^{\mathcal{I}}, p_{ij}^{\mathcal{F}}\rangle)$.

Let $d_\lambda, k_\lambda$ denote the periodicity and index of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda$ respectively.

**Theorem 3.10.** $d = [d_\lambda]_{\lambda \in I},\ k = \max_{\lambda \in I}\{k_\lambda\}$. here "[ ]" denotes $l.c.m.$

**Proof:** Set $N = \max_{\lambda \in I}\{k_\lambda\},\ \mathcal{H} = [d_\lambda]_{\lambda \in I}$. Let $N = k_\lambda + r_\lambda,\ \mathcal{H} = l_\lambda d_\lambda$, where $r_\lambda, l_\lambda \in \mathbb{N}^+$, thus $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{(N+\mathcal{H})} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{(N+\mathcal{H})} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{k_\lambda + r_\lambda + l_\lambda d_\lambda} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{k_\lambda + l_\lambda d_\lambda} \times (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{r_\lambda} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{k_\lambda + r_\lambda} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^N = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^N$.

By Lemma 3.9, we get $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{\mathcal{N}+\mathcal{H}} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^N$.

By Lemma 3.9, we have $N \geq k, d|\mathcal{H}$.

We first proof $K = N$. If $k \neq N$, due to the above discussion, we have $k < N$. By the definition of $N, \exists \lambda \in I \ni k$ is not the index of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda$. Thus $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{k+\mathcal{H}} \neq (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^k$. From Lemma 3.9, we get $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+\mathcal{H}} \neq (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^k$, a contradiction to the definition of $k$.

Now we prove that $d = \mathcal{H}$. Since $d|\mathcal{H}$, so $d \leq \mathcal{H}$. If $d < \mathcal{H}$, then by the definition of $\mathcal{H}$, there exists $\lambda$ such that $d_\lambda|d$. By Lemma 3.9, we have $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_\lambda^{k+d} \neq (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_\lambda)^k$. From Lemma 3.9, it follows that $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^{k+d} \neq (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)^k$, a contradiction to the definition of $d$.

The above Theorem 3.10 is points out the relation of periodicity and index between a $\mathcal{FNSM}$ and its $\mathcal{CFNSM}$. This result provides a new approach to the study of periodicity and index of $\mathcal{FNSM}$.

**Corollary 3.11.** A $\mathcal{FNSM}$ converges iff each of its cut fuzzy neutrosophic soft matrices converges.

**Corollary 3.12.** The periodicity of a $\mathcal{FNSM}$ is a prime number $p$ if and only if the periodicity of each of its $\mathcal{CFNSM}s$ are $p$ or 1.

## 4. **Further Description of $\mathcal{PFNSM}$**

Let $\mathcal{S}$ be a strongly connected digraph, we let $d(\mathcal{S})$ be the periodicity of $\mathcal{S}, h(\mathcal{S})$ the number of all the different vertices of $\mathcal{S}, l(C)$ the length of directed cycle $C, \mathcal{G}_\lambda$ the corresponding digraphs of $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_\lambda$. The strong components of $\mathcal{G}_\lambda$ are called the $\mathcal{SC}$.

**Definition 4.1.** $\mathcal{G}_\lambda \subseteq \mathcal{G}_\beta$ is that any point $x, y \in \mathcal{G}_\lambda$, if there is a path between $x$ and $y$ in $\mathcal{G}_\lambda$, then this path is retained in $\mathcal{G}_\beta$, where $\lambda, \beta \in I$.
$\mathcal{G}_\lambda \cap \mathcal{G}_\beta,\ \mathcal{G}_\lambda \cup \mathcal{G}_\beta$, stand for the intersection and combination of paths in digraph $\mathcal{G}_\lambda$ and $\mathcal{G}_\beta$ respectively.

**Definition 4.2.** We say that $\mathcal{S}$ a $\mathcal{SC}$ of a $\mathcal{FNSM}$ $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle$, if there is a $\lambda \in I \ni \mathcal{S}$ is a $\mathcal{SC}$ of $\mathcal{CFNSM}$ $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_\lambda$.

**Proposition 4.3.** If $h(S) = m$, then $l(C) \leq m$, where $C$ is an arbitrary directed cycle of $\mathcal{SC}$ $\mathcal{S}$.
**Proof:** It is trivial from the definition of directed cycle.

**Proposition 4.4.** If $h(S) = m$, then $d(\mathcal{S}) \leq m$.
**Proof:** By referring to Lemma 3.9, Property 4.3 and the properties of $g.c.d$, the proof is clear.

**Proposition 4.5.** If $\mathcal{S}_1, \mathcal{S}_2$ are $\mathcal{SC}s$ and $\mathcal{S}_1 \subseteq \mathcal{S}_2$, then $d(\mathcal{S}_2)|d(\mathcal{S}_1)$ and $h(\mathcal{S}_1) \leq h(\mathcal{S}_2)$.
**Proof:** From $\mathcal{S}_1 \subseteq \mathcal{S}_2$ we obtain that if $C$ is cycle of $\mathcal{S}_2$ then $C$ is also cycle of $\mathcal{S}$. Hence the cycle set of $\mathcal{S}_2$ includes the cycle set of $\mathcal{S}_1$. By Lemma 2.10 and the properties of $g.c.d.$, we get $d(\mathcal{S}_1)|d(\mathcal{S}_2)$. By the definition of $\subseteq$ we need $h(\mathcal{S}_2) \leq h(\mathcal{S}_1)$.

The $l$ different elements of $\mathcal{FNSM}$ $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ are denoted in an increasing order $\lambda_1 < \lambda_2 < ... < \lambda_l$. Then $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ has $l$ different $\mathcal{CFNSM}$ $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_i} (i = 1, 2, ..., l)$. $\mathcal{G}_i$ denote the corresponding digraph of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_i}$.
Let $\mathcal{Q} = \{\mathcal{S} | \mathcal{S}\ is\ a\ \mathcal{SC}\ of \mathcal{G}_i,\ i = 1, 2, ..., l\}$, $\mathcal{M} = \{\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_r | \mathcal{S}_i\ and\ \mathcal{S}_j$ have some common vertices, $\mathcal{S}_i, \mathcal{S}_j \in \mathcal{Q}, i \neq j\}$. Due to the idea of $\mathcal{SC}$ we know that if $S_i, S_j$ are $SCs$ and $S_i, S_j$ have common vertices, then $\mathcal{S}_i, \mathcal{S}_j$ belong to distint $\mathcal{CFNSM}s$. without loss of genarality, we assume that $\mathcal{S}_i \in \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_i}, \mathcal{S}_j \in \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_j}$ and $\lambda_i < \lambda_j$. From the fact that $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_i} \geq \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_j}$ and $\mathcal{S}_i, \mathcal{S}_j$ have common vertices, we conclude that $\mathcal{S}_i \supseteq \mathcal{S}_j$. So $[\mathcal{M}, \subseteq]$ is a partial order set and $\mathcal{M}$ is a subset of $\mathcal{Q}$. The greatest element of $[\mathcal{M}, \subseteq]$. So

$\mathcal{M} \neq \varphi$ and there is a minimal elements in $[\mathcal{M}, \subseteq]$. Next we will explain that the number of set $\mathcal{M}$ is at least one.

**Definition 4.6.** Any $\mathcal{S} \in \mathcal{M}$ is a Minimal Strong Component $\mathcal{MSC}$ if $\mathcal{T} \in \mathcal{M}$ and $\mathcal{T} \subseteq \mathcal{S} \Rightarrow$ that $\mathcal{T} = \mathcal{S},\ \forall \mathcal{T} \in \mathcal{M}$.

**Example 4.7.** This illustration clarify the notion of $\mathcal{MSC}$.

$$(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle) = \begin{bmatrix} \langle 0.9,0.8,0.1\rangle & \langle 0.5,0.4,0.5\rangle & \langle 0,0,1\rangle & \langle 0.5,0.4,0.5\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0.3,0.2,0.7\rangle & \langle 0,0,1\rangle & \langle 0.5,0.4,0.5\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0.5,0.4,0.5\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.1,0.1,0.9\rangle \\ \langle 0.5,0.4,0.5\rangle & \langle 0,0,1\rangle & \langle 0.1,0.1,0.9\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.3,0.2,0.7\rangle & \langle 0.1,0.1,0.9\rangle & \langle 0,0,1\rangle & \langle 0.3,0.2,0.7\rangle \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0.3,0.2,0.7\rangle & \langle 0,0,1\rangle \end{bmatrix}$$

- Then $\lambda_1 = \langle 0,0,1\rangle, \lambda_2 = \langle 0.1,0.1,0.9\rangle, \lambda_3 = \langle 0.3,0.2,0.7\rangle, \lambda_4 = \langle 0.5,0.4,0.5\rangle$, $\lambda_5 = \langle 0.9,0.8,0.1\rangle$,
- $\mathcal{G}_i(i = 1, 2, ..., 4)$ can be represented as follows.
- In $\mathcal{G}_{\langle 0.9,0.8,0.1\rangle}$ there is only one $\mathcal{SC}$ $\mathcal{S}_1 = \{v_1\}$.
- In $\mathcal{G}_{\langle 0.5,0.4,0.5\rangle}$ there are two $\mathcal{SC}s$ $\mathcal{S}_2 = \{v_1, v_4\},\ \mathcal{S}_3 = \{v_2, v_3\}$.
- We notice that $\mathcal{S}_3$ is a $\mathcal{SC}$ which has no common vertices with $\mathcal{S}_1$ and $\mathcal{S}_2$. In this sense we say that $\mathcal{S}_3$ is a newly appeared $\mathcal{SC}$.
- In $\mathcal{G}_{\langle 0.3,0.2,0.7\rangle}$ there are two $\mathcal{SC}s$ $\mathcal{S}_4 = \{v_1, v_2, v_3, v_4\}, \mathcal{S}_5 = \{v_5, v_6\}, \mathcal{S}_5$ has no common vertices with $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ and $\mathcal{S}_4$. So $\mathcal{S}_5$ is a newly appeared $\mathcal{SC}s$ in $\mathcal{G}_{\langle 0.3,0.2,0.7\rangle}$
- In $\mathcal{G}_{\langle 0.1,0.1,0.9\rangle}$ there is a only one component $\mathcal{S}_6 = \{v_1, v_2, x_3, v_4, v_5, v_6\}$ there is a no newly appeared $\mathcal{SC}s$.
- Noticing that each element of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_0$ is 1 we claim that $\mathcal{G}_{\langle 0,0,1\rangle}$ is a $\mathcal{SC}s$ by itself. We denote this $\mathcal{SC}s$ as $\mathcal{S}_7$. Clearly, the number of vertices in $\mathcal{S}_7$ is the same as in $\mathcal{S}_6$ and $\mathcal{S}_6 \subseteq \mathcal{S}_7$. Hence we obtain that
- $\mathcal{Q} = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_5, \mathcal{S}_6, \mathcal{S}_7\}$,
- $\mathcal{M}_1 = \{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_4, \mathcal{S}_6, \mathcal{S}_7\}$, where $\mathcal{S}_1 \subseteq \mathcal{S}_2 \subseteq \mathcal{S}_4 \subseteq \mathcal{S}_6 \subseteq \mathcal{S}_7$,
- $\mathcal{M}_2 = \{\mathcal{S}_3, \mathcal{S}_4, \mathcal{S}_6, \mathcal{S}_7\}$, where $\mathcal{S}_3 \subseteq \mathcal{S}_4 \subseteq \mathcal{S}_6 \subseteq \mathcal{S}_7$,
- $\mathcal{M}_3 = \{\mathcal{S}_5, \mathcal{S}_6, \mathcal{S}_7\}$, where $\mathcal{S}_5 \subseteq \mathcal{S}_6 \subseteq \mathcal{S}_7$.
- The set of all $\mathcal{MSC}$ of $\mathcal{FNSM}(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ is $\Psi = \{\mathcal{S}_1, \mathcal{S}_3, \mathcal{S}_5\}$.

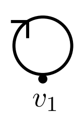



Fig-1. Graph of $G\langle 0.9, 0.8, 0.1\rangle$

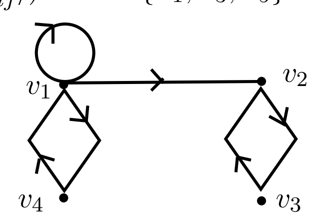



Fig-2.Graph of $G\langle 0.5, 0.4, 0.9\rangle$

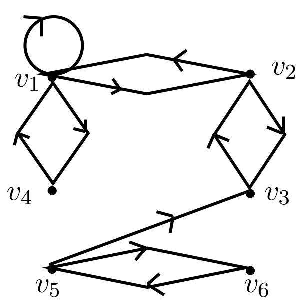



Fig-2.Graph of $G\langle 0.3, 0.2, 0.7\rangle$

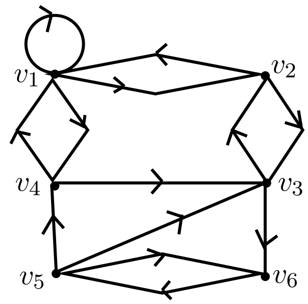



Fig-4 Group of G(0.1, 0.1,0.9)

**Lemma 4.8.** If $\Psi = \{S|S\ is\ a\ MSC\ of\ \mathcal{FNSM}\}$, then $\sum\limits_{s\in\Psi} d(\mathcal{S}) \leq \sum\limits_{s\in\Psi} h(\mathcal{S}) \leq n$.

**Proof:** First we take that if $\mathcal{S}, \mathcal{T} \in \Psi, \mathcal{S} \neq \mathcal{T}$, then $\mathcal{S} \cap \mathcal{T} \neq \phi$.

If $\mathcal{S}, \mathcal{T}$ belong to the same $\mathcal{CFNSM}$, by the definition of $\mathcal{SC}$, we have $\mathcal{S} \cap \mathcal{T} \neq \phi$.

If $\langle \alpha^{\mathcal{T}}_{ij}, \alpha^{\mathcal{I}}_{ij}, \alpha^{\mathcal{F}}_{ij}\rangle > \langle \beta^{\mathcal{T}}_{ij}, \beta^{\mathcal{I}}_{ij}, \beta^{\mathcal{F}}_{ij}\rangle,\ S \subseteq \mathcal{G}_{\langle \alpha^{\mathcal{T}}_{ij}, \alpha^{\mathcal{I}}_{ij}, \alpha^{\mathcal{F}}_{ij}\rangle},\ \mathcal{T} \subseteq \mathcal{G}_{\langle \beta^{\mathcal{T}}_{ij}, \beta^{\mathcal{I}}_{ij}, \beta^{\mathcal{F}}_{ij}\rangle}$ and $\mathcal{S} \cap \mathcal{T} \neq \phi$, then $\lambda_\alpha > \lambda_\beta, \mathcal{R}_{\lambda_\alpha} \leq \mathcal{R}_{\lambda_\beta}$. From the definition of the coresponding matrix of an ordinary digraph, we get $\mathcal{G}_{\langle \alpha^{\mathcal{T}}_{ij}, \alpha^{\mathcal{I}}_{ij}, \alpha^{\mathcal{F}}_{ij}\rangle)} \subseteq \mathcal{G}_{\langle \beta^{\mathcal{T}}_{ij}, \beta^{\mathcal{I}}_{ij}, \beta^{\mathcal{F}}_{ij}\rangle}$. By the definition of $\subseteq, \mathcal{S}$ is also strongly connected in digraph $\mathcal{G}_{\langle \beta^{\mathcal{T}}_{ij}, \beta^{\mathcal{I}}_{ij}, \beta^{\mathcal{F}}_{ij}\rangle}$. If $\mathcal{S} \subseteq \mathcal{S}'$, where $\mathcal{S}'$ is a strong component of $\mathcal{G}_{\langle \beta^{\mathcal{T}}_{ij}, \beta^{\mathcal{I}}_{ij}, \beta^{\mathcal{F}}_{ij}\rangle}$. then $\mathcal{S}' \cap \mathcal{T} \supseteq \mathcal{S} \cap \mathcal{T} \neq \phi$. By our known definition of strong component and from the fact that $\mathcal{S}', \mathcal{T}$ are both strong components of $\mathcal{G}_{\langle \beta^{\mathcal{T}}_{ij}, \beta^{\mathcal{I}}_{ij}, \beta^{\mathcal{F}}_{ij}\rangle}$, we have $\mathcal{S}' = \mathcal{T}$. Hence $\mathcal{S} \subseteq \mathcal{S}' = \mathcal{T}$. However, $\mathcal{S} \neq \mathcal{T}$, so $\mathcal{S} \subset \mathcal{T}$, a contradiction to the fact that $\mathcal{T}$ is a MSC. Thus $\mathcal{S} \cap \mathcal{T} = \phi$.

Let $X = \{x_1, x_2, ..., x_n\}$ stand for the set of n different vertices of corresponding digraph of every $\mathcal{CFNSM}$. From the fact that set of all the different vertices of every MSC is included in X and for $\forall \mathcal{S}, \mathcal{T} \in \Psi$ if $\mathcal{S} \neq \mathcal{T}$, then $\mathcal{S} \cap \mathcal{T} = \phi$, we say that $\sum\limits_{s\in\Psi} h(\mathcal{S}) = h(\bigcup\limits_{s\in\Psi} \mathcal{S}) \leq n$ holds. By property 4.4, we get $\sum\limits_{s\in\Psi} d(\mathcal{S}) \leq \sum\limits_{s\in\Psi} h(\mathcal{S}) \leq n$.

**Corollary 4.9.** The number of $\mathcal{MSC}$ of an arbitrary $\mathcal{SFNSM}$ $(\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle)$ is not greater than n.

In the below content, we first give the description of periodicity of an arbitrary $\mathcal{SFNSM}$ by using the concept of MSC.

**Theorem 4.10.** If $(\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle)$ is a $\mathcal{FNSM}$,
$\Psi = \{\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_w\}$, then $d(\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle) = [d(S_i)]_{s_i} \in \Psi$.

**Proof:** Here $\Psi, \mathcal{Q}$ are the same as above. If the number of elements in $\Psi, \mathcal{Q}$ are $w$ and $u$, respectively, then $w \leq u$. For $\forall \mathcal{T} \in \mathcal{Q}|\Psi$, since $\mathcal{T}$ is not a minimal element, $\exists \mathcal{S} \in \Psi \ni \mathcal{S} \subseteq \mathcal{T}$. By property 4.4, we get

$$d(\mathcal{T})|d(\mathcal{S}).\ (*)$$

From Lemma 2.10 and Theorem 3.10
$d(\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle) = [d(\langle r^{\mathcal{T}}_{ij}, r^{\mathcal{I}}_{ij}, r^{\mathcal{F}}_{ij}\rangle_{\lambda_i})]_{i=1,2,...,l} = [[d(\mathcal{S})]_{s\in\mathcal{G}_i}]_{i=1,2,...,l}$

$= [d(\mathcal{S})]_{\mathcal{S}\in\mathcal{Q}}$
$= [d(\mathcal{S}_1), ..., d(\mathcal{S}_w), d(\mathcal{T}_1), ..., d(\mathcal{T}_h)], \mathcal{T}_j \in \mathcal{Q}|\Psi(j = 1, 2, ..., h).$

By (*), for $\mathcal{T}_j \in \mathcal{Q}\ \Psi$, there exists an $\mathcal{S} \in \{\mathcal{S}_1, \mathcal{S}_2, ..., \mathcal{S}_w\}$ satisfying $d(\mathcal{T}_j)|d(\mathcal{S})$. So $d(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle) = [d(\mathcal{S}_1), ..., d(\mathcal{S}_w)] = [d(\mathcal{S}_i)]_{s_i\in\Psi}$.

**Theorem 4.11.** For given positive integer $n$, the greastest value of periodicity of all square $\mathcal{FNSM}$ is max $\sum\limits_{l_i=n}[l_i]$ $(1 \le i \le n)$.

**Proof:** Let $\mathcal{M} = \{[l_1, ..., l_w]| \sum\limits_{i=1}^{w} l_i \le n,\ l_i \in \mathbb{Z}^+,\ 1 \le w \le n\}$. By Theorem 5.1, for an arbitrary $\mathcal{SFNSM}\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle$,
$d(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle) = [d(S_1), ..., d(\mathcal{S}_w)] \in \mathcal{M}$,
$\mathcal{S}_i \in \Psi, 1 \le d(\mathcal{S}_i) \le n, \sum\limits_{i=1}^{w} d(\mathcal{S}_i) \le n$.

On the other hand, for $\forall[l_1, ..., l_w] \in \mathcal{M}$, $\sum\limits_{i=l}^{w} l_i \le n$, we can find a fuzzy neutrosophic soft matrix $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle$ such that $d(A) = [l_1, ..., l_w]$. In fact, the periodicity of the $l_i \times l_i$ Block Boolean Fuzzy Neutrosophic Soft Matrix ($\mathcal{BBFNSM}$)

$$A = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & E \\ \cdots\cdots & \cdots & \cdots\cdots \\ \langle 1,1,0\rangle & \vdots & \langle 0,0,1\rangle \end{bmatrix}$$

is $l_i$, where $\mathcal{E}$ is the unit matrix. The periodicity of the $\mathcal{SBBFNSM}$

$$A_n = \begin{bmatrix} \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_1 & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \\ \ddots & & & \\ \langle 0,0,1\rangle & \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_W & \vdots & \langle 0,0,1\rangle \\ \cdots\cdots & \cdots\cdots & \cdots & \cdots\cdots \\ \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & E \end{bmatrix}$$

is $[l_1, ..., l_W]$. Hence, the greatest periodicity of square $\mathcal{FNSM}(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ is the greatest value of $\mathcal{M}$.

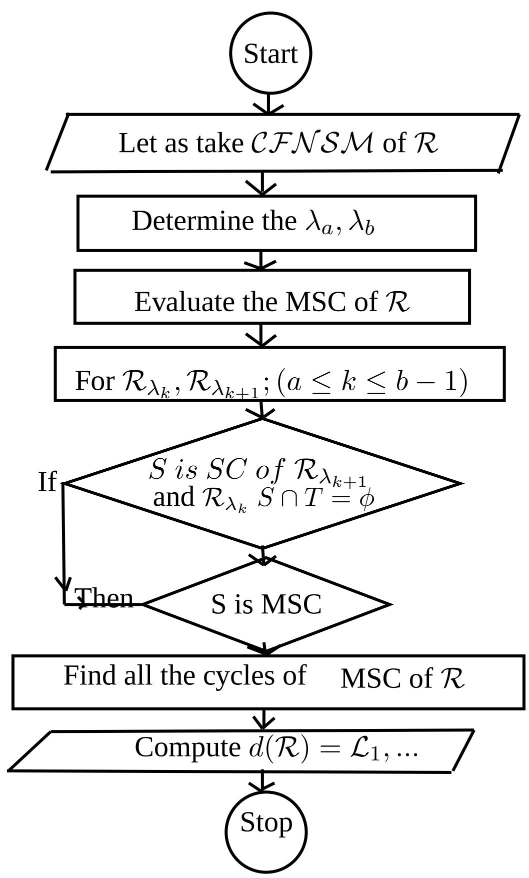
For $\sum\limits_{i=1}^{W} l_i \le n$, we need $[l_1, ..., l_W] \le [l_1, ..., l_W, n - \sum\limits_{i=1}^{W} l_i] \le \max\limits_{\sum l_i=n}[l_1, ..., l_W]$. So $\max\limits_{\sum l_i\le n}[l_1, ..., l_W] = \max\limits_{\sum l_i=n}[l_1, ..., l_W]$.

## 5. **An Algorithm to Find the Period**

- According to Theorem 5.1, we can determine the following algorithm to obtain the periodicity d for an arbitrary $\mathcal{CFNSM}$ $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$.

- **Step 1:** Compute the $\mathcal{CFNSM}$ according to the different elements of $\mathcal{R} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)$.
- Let $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_a}$ be the first $\mathcal{CFNSM}$ such that has at least one directed cycle, and $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle_{\lambda_a}$ the first $\mathcal{CFNSM}$ such that the number of vertices of all its SCs is $n$.
- **Step 2:** Determine $\lambda_a, \lambda_b$.
- **Step 3:** Find MSCs of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_a}, ..., (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_b}$.
- From the definition of MSC and Lemma 3.9, we can find MSCs by the following method.
- SC of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_a}$ are MSC of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)$.
- For $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_k}, (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_{k+1}} (a \leq k \leq b-1)$, if $\mathcal{S}$ is a SC of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_{k+1}}$ and for an arbitary strong component $\mathcal{T}$ if $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)_{\lambda_k}$,
  $S \cap \mathcal{T} = \phi$ holds, then $\mathcal{S}$ is a MSC.
- **Step 4:** Find all the directed cycles of MSC of $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)$.
- **Step 5:** Evaluate $d(\mathcal{R}) = [\mathcal{L}_\infty, \mathcal{L}_\in, ..., \mathcal{L}_{\sqcap_\rangle}]_{i=1,2,...,w}$.

Flowchart to obtain the periodicity $d$ for an arbitrary fuzzy neutrosophic soft matrix $\mathcal{R} = (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}} \rangle)$.



**Theorem 5.1.** The periodicity of square fuzzy neutrosophic soft matrix $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ is $d(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle) = [\mathcal{L}_1, \mathcal{L}_2, ..., \mathcal{L}_{u_i}]_{i=1,2,...,W}$, where () stand for the g.c.d.

**Proof:** All of the different elements and $\mathcal{CFNSM}$ $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)$ are assumed as above. By the definition of $\mathcal{CFNSM}$, we can find $\lambda_a, \lambda_b \in I(\lambda_a \geq \lambda_b)$ satisfying the next conditions.
$(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_a}$ has at least one directed cycle. But if $\lambda_j > \lambda_a \Rightarrow \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_j} \leq \langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_a}$ then $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_j}$ has no cycles. Thus $d(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_j}) = 1$.
(ii) $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_b}$ is the first $\mathcal{CFNSM}$ $\lambda$ that number of different vertices of all its SCs is $n$.
If $\lambda_j < \lambda_b \Rightarrow$ $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_j} \geq (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_a}$, then the number of different vertices of all its SCs for $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_j}$ is also $n$.
From the definition of MSC, for $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle_{\lambda_j})$ mentioned in $(i)$ and $(ii)$ $(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_j}$ has no MSC.
Therefore $\Psi = \{\mathcal{S} | \mathcal{S} \in (\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle)_{\lambda_j}, \lambda_a \geq \lambda_j \geq \lambda_b\}$.
Theorem 5.1 we known that $d(\mathcal{R}) = [\mathcal{L}_1, \mathcal{L}_1, ..., \mathcal{L}_{u_i}]_{i=1,2,...,w}$.

**Example 5.2.** Let $\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle$ be the fuzzy neutrosophic soft matrix mentioned in Example 4.7. Then $\lambda_a = \langle 0.9, 0.8, 0.1\rangle,\ \lambda_b = \langle 0.3, 0.2, 0.7\rangle$
$d(\langle r_{ij}^{\mathcal{T}}, r_{ij}^{\mathcal{I}}, r_{ij}^{\mathcal{F}}\rangle) = [d(\mathcal{S}_1), d(\mathcal{S}_3), , d(\mathcal{S}_5)] = [1, 2, 2] = 2$

## 6. **Conclusion**

We have defined the concept of MSC, and obtained the periodicity of fuzzy neutrosophic soft matrices by the periodicity of its MSC. We have also pointed out that the index of $\mathcal{FNSM}$ is the greatest value of indices of its $\mathcal{CFNSM}$. Future scope of this research work could be to investigate the oscillating period index, strongly connected boolean matrix in the freamework of $\mathcal{FNSM}$. We will applied this results for decision making problems.

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