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Convexity for Interval Valued Neutrosophic Setsand its Application in Decision Making

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Abstract. The notion 'Convexity' is applied in various areas of mathematics particularly in optimization techniques. It is known that this concept is applied in fuzzy sets, which is studied by many authors. This article deals with convexity utilized for neutrosophic and interval valued neutrosophic sets which is a generalization of intuitionistic fuzzy sets. Also some interesting preservation properties of convexity under intersection operators in interval valued neutrosophic sets are discussed. Adding to the discussion, preservation property of digital convexity under intersection using deformation and other techniques are touched upon. Eventually the application of convexity to decision making are illustrated using examples.

Keywords: Convexity; Interval valued neutrosophic sets; Interval valued neutrosophic intersection; Decision making.

1. Introduction

Concepts like adjacency, connectivity, convexity and concavity, level sets, cut sets are useful in many areas of mathematics [7] [8] [9]. In particular the notion convexity is applied in optimization techniques, image processing, decision making etc., [10] [11] [15] [16]. Fuzzy convexity plays a vital role in applications of real time problems which includes decision making.

Neutrosophic sets which is a generalization of intuitionistic fuzzy sets [2] [18] were introduced by Floretin Smarandache [4], interval valued neutrosophic sets were investigated in [17]. These sets serves as the base for solutions of real time problems. On the other hand, key concepts like convexity and level sets for fuzzy sets, interval valued fuzzy sets serves as important notions in decision making [13].

The fundamental definitions of convexity in neutrosophic sets, digital convexity in neutrosophic sets, neutrosophic digital cut sets, neutrosophic digital level sets etc., are defined and their properties are discussed in [14].

In [13] Pedro Huidobro proved many properties of convexity and explained the importance of the concept convexity for Interval Valued Fuzzy Set (IVFS) and its compatibility with intersection operator. Utility of convexity for IVFS in decision making problems especially by the preservation property under intersection operator is illustrated using examples mentioned in [13].

In this article, as an extension of the convexity concept for IVFS in decision making, convexity for Interval Valued Neutrosophic Set (IVNS) is described and using the preservative property of convexity under intersection in IVNS, the concept is applied to decision making problems to find the optimum solution. In addition discussion on digital convexity upon preservation property under intersection operator is investigated.

2. Preliminaries

Definition 2.1. [17] Let X be a space of points (objects), with a generic elements in X, denoted by x. An interval valued neutrosophic set (IVNS) A in X is characterized by truth- membership $T_A(x)$, indeterminacy $I_A(x)$ and falsity- membership function $F_A(x)$. For each point x in X, we have that $T_A(x) = [infT_A(x), supT_A(x)], I_A(x) = [infI_A(x), supI_A(x)], F_A(x) = [infF_A(x), supF_A(x)] \subseteq [0, 1]$ and $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3, x \in X$.
We call it "interval" because it is the subclass of a neutrosophic set, that is, we only consider the subunitary interval of [0, 1]. Therefore, All IVNSs are clearly neutrosophic sets.

Definition 2.2. [17] An IVNS A is empty if and only if its $infT_A(x) = supT_A(x) = 0$, $infI_A(x) = supI_A(x) = 1$, and $infF_A(x) = supF_A(x) = 0$ for any $x \in X$.

Definition 2.3. [17] The complement of an IVNS A is denoted by A^c and is defined as $T^c_A(x) = F_A(x)$, $infI^c_A(x) = 1 - supI_A(x)$, $supI^c_A(x) = 1 - infI_A(x)$, $F^c_A(x) = T_A(x)$ for any x in X.

Definition 2.4. [17] An interval valued neutrosophic set A is contained in an IVNS B, A $\subseteq$ B, if and only if $infT_A(x) \leq infT_B(x)$, $supT_A(x) \leq supT_B(x)$, $infI_A(x) \geq infI_B(x)$, $supI_A(x) \geq supI_B(x)$, $infF_A(x) \geq infF_B(x)$, and $supF_A(x) \geq supF_B(x)$ for any x in X.

Definition 2.5. [17] Two IVNSs A and B are equal, written as A = B, if and only if A $\subseteq$ B and B $\subseteq$ A.

Definition 2.6. [17] The intersection of two interval neutrosophic sets A and B is an IVNS C, written as C = A ∩ B, whose truth-membership, indeterminacy-membership, falsity-membership are related to those of A and B by $infT_C(x) = min(infT_A(x), infT_B(x))$
$supT_C(x) = min(supT_A(x), supT_B(x))$
$infI_C(x) = max(infI_A(x), infI_B(x))$
$supI_C(x) = max(supI_A(x), supI_B(x))$
$infF_C(x) = max(infF_A(x), infF_B(x))$
$supF_C(x) = max(supF_A(x), supF_B(x))$

Definition 2.7. [17] The union of two interval neutrosophic sets A and B is an IVNS C, written as C = A ∪ B, whose truth-membership, indeterminacy-membership, falsity-membership are related to those of A and B by
$infT_C(x) = max(infT_A(x), infT_B(x))$
$supT_C(x) = max(supT_A(x), supT_B(x))$
$infI_C(x) = min(infI_A(x), infI_B(x))$
$supI_C(x) = min(supI_A(x), supI_B(x))$
$infF_C(x) = min(infF_A(x), infF_B(x))$
$supF_C(x) = min(supF_A(x), supF_B(x))$

Definition 2.8. [17] An IVNS A is convex if and only if
$infT_A(\lambda x_1 + (1-\lambda)x_2) \geq min(infT_A(x_1), infT_A(X_2))$,
$supT_A(\lambda x_1 + (1-\lambda)x_2) \geq min(supT_A(x_1), supT_A(X_2))$,
$infI_A(\lambda x_1 + (1-\lambda)x_2) \leq max(infI_A(x_1), infI_A(X_2))$,
$supI_A(\lambda x_1 + (1-\lambda)x_2) \leq max(supI_A(x_1), supI_A(X_2))$,
$infF_A(\lambda x_1 + (1-\lambda)x_2) \leq max(infF_A(x_1), infF_A(X_2))$,
$supF_A(\lambda x_1 + (1-\lambda)x_2) \leq max(supF_A(x_1), supF_A(X_2))$
for all x_1 and X_2 in X and all λ in [0, 1].

Theorem 2.9. *[17] If A and B are convex, so is their intersection.*

Definition 2.10. [17] An IVNS A is strongly convex if and only if
$infT_A(\lambda x_1 + (1-\lambda)x_2) > min(infT_A(x_1), infT_A(X_2))$,
$supT_A(\lambda x_1 + (1-\lambda)x_2) > min(supT_A(x_1), supT_A(X_2))$,
$infI_A(\lambda x_1 + (1-\lambda)x_2) < max(infI_A(x_1), infI_A(X_2))$,
$supI_A(\lambda x_1 + (1-\lambda)x_2) < max(supI_A(x_1), supI_A(X_2))$,
$infF_A(\lambda x_1 + (1-\lambda)x_2) < max(infF_A(x_1), infF_A(X_2))$,
$supF_A(\lambda x_1 + (1-\lambda)x_2) < max(supF_A(x_1), supF_A(X_2))$
for all x_1 and X_2 in X and all λ in [0, 1].

Theorem 2.11. *[17] If A and B are strongly convex, so is their intersection.*

Definition 2.12. [14] Let $A_N = \langle \mu_{A_N}, \sigma_{A_N}, \nu_{A_N} \rangle$ be the neutrosophic subset of E which is DN regular and convex, then $A_N^{'}$ is said to be a digital neutrosophic convex (or DN convex) set, if the digital image of A_N is $A_N^{'}$.
The complement $1 - A_N^{'}$ of the DN convex set $A_N^{'}$ is said to be a digital neutrosophic concave (or DN concave) set.

Definition 2.13. [14] Let $\mathrm{A}_N = \langle \mu_{A_N}, \sigma_{A_N}, \nu_{A_N} \rangle$ be the neutrosophic subset of E. Then the level sets of A_N are defined as $A_N^{\omega} = \{P \in E, \mu_{A_N}(P) \geq \omega, \sigma_{A_N}(P) \geq \omega, \nu_{A_N}(P) \leq \omega, \}, \omega \in I$.

3. Decision making based on Interval valued neutrosophic sets

Decision making problems are proposed in this section. Few theories on interval valued fuzzy sets are used in decision making [7] [18] [13]. Decisions are made based on set of goals and set of constraints with symmetry between them [1].

Symmetry representing "and" connective is used. Here are situations in which uncertainty occurs, this situation can be resolved using membership values, which may be included in some interval through specific point is uncertain.

As proposed by Bellman, Zadeh, Yagu and Basson and by the idea used on Interval value fuzzy set [13] the following is proposed:

Definition 3.1. Let $X = \{x_1, x_2, ..., x_n\}$ be the set of alternatives, $G_1, G_2, ...G_m$ the set of goals that can be expressed as Interval valued neutrosophic set with alternatives, and $C_1, C_2, ...C_n$ be the set of constraints which can also be expressed as Interval valued neutrosophic sets on the space of alternatives. Let $\leq$ be an order on [0, 1]. The goals and constraints combine to form a decision D, which is an Interval valued neutrosophic set resulting from Interval valued neutrosophic intersection of the goals and the constraints. Thus $D = G_1 \cap_N ... \cap_N G_P \cap_N C_1 \cap_N ... \cap_N C_m$.
For the alternatives of x, the decision is denoted as D(x) which satisfies the goals and constraints for any $x \in X$. To make a decision we have to find the best alternative.
It is obvious that D(x) depends on the Interval valued neutrosophic intersection operator which satisfies total ordering property [5]. Therefore the decision D is the Interval valued neutrosophic intersection of all goals and constraints depending on the total ordering property.

Example 3.2. A candidate attempting for business has to choose a location in one of the three alternative x_1, x_2, x_3 . He wants to choose a location that minimizes the land cost G and is located near supplies C_1. Let X = $\{x_1, x_2, x_3\}$. To be more precise we take Internal valued neutrosophic set instead of Neutrosophic set. Let us suppose that the membership, non- membership and indeterminant function of Interval valued neutrosophic goal G is $\{\langle x_1, [0.1, 0.3], [0.2, 0.4], [0.2, 0.4] \rangle\} + \{\langle x_2, [0.4, 0.6], [0, 0.1], [0.1, 0.2] \rangle\} +$

$\{\langle x_3, [0.5, 0.7], [0.1, 0.2], [0.1, 0.2]\rangle\}$ and the $T(x_i)$ and $F(x_i)$ of the interval valued neutrosophic constraints C_1 is $\{\langle x_1, [0.4, 0.6], [0.1, 0.2], [0.1, 0.2]\rangle\} + \{\langle x_2, [0.1, 0.2], [0.2, 0.3], [0.4, 0.7]\rangle\} + \{\langle x_3, [0.3, 0.5], [0, 0.1], [0.2, 0.3]\rangle\}$.

If we consider interval neutrosophic intersection, then the membership, indeterminacy and non- membership values of interval valued neutrosophic decision D_N is

$\{\langle x_1, [0.1, 0.3], [0.2, 0.4], [0.2, 0.4]\rangle\} +$

$\{\langle x_2, [0.1, 0.2], [0.1, 0.3], [0.4, 0.7]\rangle\} +$

$\{\langle x_3, [0.3, 0.5], [0.1, 0.2], [0.2, 0.3]\rangle\}$

and the optimal decision would be x_3, since it is the alternative with a maximum value of D(x) with respect to interval valued neutrosophic intersection which satisfies the total ordering.

4. **Extension principle (Interval valued neutrosophic set)**

With the extension principle on IVNS [12], when the Interval valued neutrosophic constraints or goals are defined in n-different spaces, they can be mapped into the same. When we have n functions which maps $(X_1 \times X_2 \times ...X_n)$ to Y, we would assume that if A $\in$ IVNS$(X_1 \times X_2 \times ...X_n)$, then $A(x_1, x_2, ...x_n) = A(x_1) \cap_N A(x_2) \cap_N ... \cap_N A(x_n)$. $\cap_N$ represents IVN intersection. The following example based is on [9]:

Example 4.1. Suppose the same conditions are as in the previous example, and the space Y means a set of the previous business works done by the financial experts, $Y = \{y_1, y_2, y_3, y_4\}$ with the information that y_1 and y_2 were made by x_1, y_3 was supervised by x_2 and y_4 was produced by x_2 and x_3.

Using the above information we defined the following mapping f : Y $\rightarrow$ X defined as $f(y_1) = x_1, f(y_2) = x_1, f(y_3) = x_2$ and $f(y_4) = \{x_2, x_3\}$.

With Interval valued neutrosophic constraint over Y, the impact of each one of works as:

$C_2(Y) = \{\langle y_1, [0.3, 0.6], [0.4, 0.6], [0.4, 0.7]\rangle,$

$\langle y_2, [0.2, 0.4], [0.5, 0.6], [0.5, 0.7]\rangle,$

$\langle y_3, [0.3, 0.5], [0.4, 0.5], [0.4, 0.6]\rangle,$

$\langle y_4, [0.3, 0.4], [0.4, 0.5], [0.4, 0.6]\rangle\}$

$C_2(Y)$ is denoted as above, it is an Interval valued neutrosophic set over Y. Now the extension principle to have $G_1, G_2, ...G_n$ and $C_1, C_2, ...C_n$ as Interval valued neutrosophic set over Y.

Extension principle [12] can be applied for x_1, $[f(C_2)](x_1) = sup_{y_1,y_2} C_2(x) = sup C_2(y_1), C_2(y_2) = \{\langle [0.3, 0.6], [0.5, 0.6], [0.5, 0.7]\rangle$

Similarly, $[f(C_2)](x_2) = \{\langle [0.3, 0.5], [0.4, 0.5], [0.4, 0.6]\rangle$ and

$[f(C_3)](x_3) = \{\langle [0.3, 0.4], [0.4, 0.5], [0.4, 0.6]\rangle$

$\therefore C_2(X) = \{\langle x_1, [0.3, 0.6], [0.5, 0.6], [0.5, 0.7]\rangle,$

$\langle x_2, [0.3, 0.5], [0.4, 0.5], [0.4, 0.6]\rangle,$

$\langle x_3, [0.3, 0.4], [0.4, 0.5], [0.4, 0.6]\rangle\}$.
The decision is $D = G_i \cap C_1 \cap C_2$. (i.e.) the membership, indeterminancy and non- membership degrees for the alternatives in D are
$\{\langle x_1, [0.1, 0.3], [0.5, 0.6], [0.5, 0.7]\rangle, \langle x_2, [0.1, 0.2], [0.4, 0.5], [0.4, 0.7]\rangle,$
$\langle x_3, [0.3, 0.4], [0.4, 0.5], [0.4, 0.6]\rangle\}$

Corollary 4.2. *Now we combine the decision making problems using the following corollary: Let $G_1, G_2, ...G_p$ be the Interval valued neutrosophic goals, $C_1, C_2, ...C_m$ be the Interval valued neutrosophic constraints and $D = G_1 \cap G_2 ... \cap G_p \cap C_1 \cap C_2 ... \cap C_n$ be the resulting decision. If the Interval valued neutrosophic goals and Interval valued neutrosophic constraints are convex Interval valued neutrosophic set, then the resulting decision D is a convex Interval valued neutrosophic set and the set of maximizing decisions of Interval valued neutrosophic set, D is a convex neutrosophic crisp set [3]. If the Interval valued neutrosophic goals and Interval valued neutrosophic constraints are strictly convex Interval valued neutrosophic set, then the resulting decision D is strictly convex Interval valued neutrosophic set and the set of maximizing decisions of D is a singleton or an empty set.*
The following example explains the importance of the convexity for Interval valued neutrosophic set in decision making

Example 4.3. In the previous example we do not mention the relation among the x_1, x_2 and x_3. Here if $x_1 < x_2 < x_3$, it is clear that G, C_1, C_2 and C_3 are strictly convex IVNS with respect to totally order preserving property of the intersection operator in IVNS. Since,
$D = \{\langle x_1, [0.1, 0.3], [0.5, 0.6], [0.5, 0.7]\rangle,$
$\langle x_2, [0.1, 0.2], [0.4, 0.5], [0.4, 0.7]\rangle,$
$\langle x_3, [0.3, 0.4], [0.4, 0.5], [0.4, 0.6]\rangle\}$
Here x_3 is the optimum solution of the decision problem. Moreover if G, C_1, C_2 are strictly convex IVNS we see that D is not only convex IVNS as $D = \{\langle x_1, [0.1, 0.3], [0.5, 0.6], [0.5, 0.7]\rangle,$
$\langle x_2, [0.1, 0.2], [0.4, 0.5], [0.4, 0.7]\rangle,$
$\langle x_3, [0.3, 0.4], [0.4, 0.5], [0.4, 0.6]\rangle\}$ but also strictly convex.

5. **Remarks on convexity for digital neutrosophic sets**

From the above sections, it is understood that the order preserving property is important for utilizing the convexity property of interval valued neutrosophic sets in decision making problems. While considering the digital neutrosophic convexity property [14], it is predicted that the convexity property is not analogous to that of the neutrosophic convexity property , the order preserving property may not be obtained in digital sets [6]. Also in case of interval valued digital sets the same situation occurs . Hence in order to utilize convex digital sets in decision making problems techniques such as deformation, pixel removing to obtain convexity properties etc., may be used.

6. Conclusions

Decision making using interval valued neutrosophic sets, especially convex sets is illustrated in this article. The major operator called interval valued neutrosophic intersection is utilized in order to obtain the optimal solution for the decision making problem which has the structure or model comprising of maximizing or minimization of goals (objective function) and constraints. The concept intersection of goals and constraints forming the optimum solution is illustrated using examples. Extension principle based on interval valued neutrosophic sets is utilized in case of more than one constrain. A comparison on interval valued neutrosophic sets with the convexity property (satisfying the order preserving property) and the interval valued digital neutrosophic convex sets are discussed. Future work may be done on multi attribute decision making problems.

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