$\mathcal{N}_{\alpha b^* g\alpha}$- Closed Sets in Neutrosophic Topological Spaces

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Abstract. In neutrosophic topological spaces, the notion of neutrosophic $\alpha b^* g\alpha$-closed sets, neutrosophic $\alpha b^* g\alpha$-border, and neutrosophic $\alpha b^* g\alpha$-frontier are described and their properties are discussed. The connection between neutrosophic $\alpha b^* g\alpha$-frontier and neutrosophic $\alpha b^* g\alpha$-border are established

Keywords: $\mathcal{N}_{\alpha b^* g\alpha}$-closed sets, $\mathcal{N}_{\alpha b^* g\alpha}$-border, $\mathcal{N}_{\alpha b^* g\alpha}$-frontier.

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1. Introduction

Neutrosophic sets is a generalisation of Atanassov's [4] intuitionistic fuzzy sets and Zadeh's [20] fuzzy sets, and were first proposed by Smarandache [17] [18]. It also takes into account the membership functions for falsehood, indeterminacy, and truth. In a number of disciplines, including probability, algebra, control theory, topology, etc., Smarandache introduced the neutrosophic idea in response to fuzzy sets and intuitionistic fuzzy sets' inability to handle indeterminacy-membership functions. Neurosophic set based notions were later introduced by Alblowi et al. [1]. In the past 20 years, numerous scholars have utilised these potent ideas to put forth numerous topological hypotheses. A novel idea in neutrosophic topological spaces was proposed by Salama and Alblowi [14]. It gives a brief overview of neutrosophic topology, which is a generalisation of Chang's [6] and Coker's [5] intuitionistic fuzzy topology.

In the subject of neutrosophic topological spaces, Salama et al., [11–13] presented the generalisation of neutrosophic sets, neutrosophic crisp sets, and neutrosophic closed sets. Salama et al., [13] proposed a few neutrosophic continuous functions as an initial set of continuous functions in neutrosophic topology. In addition, a number of scholars have defined a number of closed sets in neutrosophic topology, including generalised neutrosophic closed sets [7] in neutrosophic topological spaces, neutrosophic α-closed sets [3], neutrosophic αg-closed sets [10], and neutrosophic b-closed sets [8]. Neutrosophic b$^*g\alpha$-closed sets were defined by S. Saranya and M. Vigneshwaran [15]. In order to establish a connection between the operators of neutrosophic interior and neutrosophic closure, Iswarya and Bageerathi [9] introduced a novel notion of neutrosophic frontier operator and neutrosophic semi-frontier operator . $\alpha b^*g\alpha$-closed sets are new closed sets defined in topological spaces by Suthi Keerthana K, Vigneshwaran M, and Vidyarani L [19]. These sets have been used to define various topological functions, such as continuous functions, irresolute functions, and homeomorphic functions with certain separable axioms.

This paper presents the idea of neutrosophic $\alpha b^*g\alpha$-closed sets in neutrosophic topological spaces and examines their characteristics as well as how they relate to other known characters. The concepts in neutrosophic $\alpha b^*g\alpha$-interior, neutrosophic $\alpha b^*g\alpha$-closure, neutrosophic $\alpha b^*g\alpha$-border, and neutrosophic $\alpha b^*g\alpha$-frontier are examined. Neutrosophic topological spaces have a relationship between the neutrosophic $\alpha b^*g\alpha$-border and the neutrosophic $\alpha b^*g\alpha$-frontier, along with associated features.

2. **Preliminaries**

The basic definitions which are used in the next section are referred from the references [14], [3], [7], [16], [10], [2], [15].

3. $\mathcal{N}_{\alpha b^* g\alpha}$- **Closed Sets**

Definition 3.1. Let ${}_{\mathcal{N}}E$ be a subset of a ${}_{\mathcal{NTS}}(\mathbb{X}, \tau)$. Then ${}_{\mathcal{N}}E$ is called

- a neutrosophic $\alpha b^*g\alpha$-closed set($\mathcal{N}_{\alpha b^*g\alpha}$-$\mathcal{CS}$) if ${}_{\mathcal{N}\alpha}\text{cl}({}_{\mathcal{N}}E) \subseteq \mathcal{V}$ whenever ${}_{\mathcal{N}}E \subseteq \mathcal{V}$ and $\mathcal{V}$ is a neutrosophic $b^*g\alpha$-open set in $(\mathbb{X}, \tau)$.

Example 3.2. Let $\mathbb{X} = \{{}_na, {}_nb, {}_nc\}$ and the $\mathcal{NS}$, ${}_{\mathcal{N}}L$ and ${}_{\mathcal{N}}M$ are defined as

${}_{\mathcal{N}}L = \{\langle {}_nx, ({}_t0.5, {}_i0.3, {}_f0.7), ({}_t0.4, {}_i0.3, {}_f0.7), ({}_t0.5, {}_i0.2, {}_f0.7)\rangle\, \forall {}_nx \in \mathbb{X}\}$,

${}_{\mathcal{N}}M = \{\langle {}_nx, ({}_t0.7, {}_i0.3, {}_f0.5), ({}_t0.7, {}_i0.3, {}_f0.6), ({}_t0.7, {}_i0.2, {}_f0.5)\rangle\, \forall {}_nx \in \mathbb{X}\}$,

Then the $\mathcal{NT}$, $\tau = \{{}_{\mathcal{N}}0, {}_{\mathcal{N}}L, {}_{\mathcal{N}}M, {}_{\mathcal{N}}1\}$, which are $\mathcal{NOS}$ in the ${}_{\mathcal{NTS}}(\mathbb{X}, \tau)$.

If ${}_{\mathcal{N}}N = \{\langle {}_nx, ({}_t0.5, {}_i0.8, {}_f0.7), ({}_t0.6, {}_i0.8, {}_f0.7), ({}_t0.5, {}_i0.9, {}_f0.7)\rangle\, \forall {}_nx \in \mathbb{X}\}$, and

${}_{\mathcal{N}}E = \{\langle {}_nx, ({}_t0.5, {}_i0.3, {}_f0.7), ({}_t0.6, {}_i0.3, {}_f0.7), ({}_t0.5, {}_i0.3, {}_f0.7)\rangle\, \forall {}_nx \in \mathbb{X}\}$

Then the complements of ${}_{\mathcal{N}}L$, ${}_{\mathcal{N}}M$, ${}_{\mathcal{N}}N$ and ${}_{\mathcal{N}}E$ are

${}_{\mathcal{N}}\overline{L} = \{\langle {}_nx, ({}_t0.7, {}_i0.7, {}_f0.5), ({}_t0.7, {}_i0.7, {}_f0.4), ({}_t0.7, {}_i0.8, {}_f0.5)\rangle\, \forall {}_nx \in \mathbb{X}\}$,

${}_{\mathcal{N}}\overline{M} = \{\langle {}_nx, ({}_t0.5, {}_i0.7, {}_f0.7), ({}_t0.6, {}_i0.7, {}_f0.7), ({}_t0.5, {}_i0.8, {}_f0.7)\rangle\, \forall {}_nx \in \mathbb{X}\}$,

${}_{\mathcal{N}}\overline{N} = \{\langle {}_nx, ({}_t0.7, {}_i0.2, {}_f0.5), ({}_t0.7, {}_i0.2, {}_f0.6), ({}_t0.7, {}_i0.1, {}_f0.5)\rangle\, {}_n\forall x \in \mathbb{X}\}$ and

${}_{\mathcal{N}}\overline{E} = \{\langle {}_n x, ({}_t 0.7, {}_i 0.7, {}_f 0.5), ({}_t 0.7, {}_i 0.7, {}_f 0.6), ({}_t 0.7, {}_i 0.7, {}_f 0.5)\rangle\, \forall_n x \in \mathbb{X}\}$

Hence ${}_{\mathcal{N}}N$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$, ${}_{\mathcal{N}}\overline{N}$ is a $\mathcal{N}_{b^*g\alpha}$- $\mathcal{CS}$, ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$, ${}_{\mathcal{N}}\overline{E}$ is a $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$ of ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$.

$\because {}_{\mathcal{N}\alpha}cl(E) = \{\langle {}_n x, ({}_t 0.5, {}_i 0.3, 0.7), ({}_t 0.6, {}_i 0.3, 0.7), ({}_t 0.5, {}_i 0.3, 0.7)\rangle\, \forall_n x \in \mathbb{X}\}$ which is contained in ${}_{\mathcal{N}}N$. That is ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \subseteq {}_{\mathcal{N}}N$.

Remark 3.3. Let ${}_{\mathcal{N}}A$ be a subset of ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, then $\mathcal{N}_{\alpha b^*g\alpha}$- ${}_{\mathcal{N}}int({}_{\mathcal{N}}A)$ is $\mathcal{N}_{\alpha b^*g\alpha}$-open in $(\mathbb{X}, \tau)$.

Theorem 3.4. *In ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, every $\mathcal{NCS}$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$.*

Proof. *Let ${}_{\mathcal{N}}E \subseteq \mathcal{V}$, where $\mathcal{V}$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$ in $\mathbb{X}$.*

$\because {}_{\mathcal{N}}E$ is $\mathcal{NCS}$, ${}_{\mathcal{N}}cl({}_{\mathcal{N}}E) = {}_{\mathcal{N}}E \subseteq \mathcal{V}$. But ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \subseteq {}_{\mathcal{N}}cl({}_{\mathcal{N}}E) \subseteq \mathcal{V}$, which implies ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \subseteq \mathcal{V}$.

$\therefore {}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$.

The converse is not true.

Example 3.5. Let $\mathbb{X} = \{{}_n a, {}_n b, {}_n c\}$ and the $\mathcal{NS}$, ${}_{\mathcal{N}}L$ and ${}_{\mathcal{N}}M$ are defined as

${}_{\mathcal{N}}L = \{\langle {}_n x, ({}_t 0.5, {}_i 0.3, {}_f 0.7), ({}_t 0.4, {}_i 0.3, {}_f 0.7), ({}_t 0.5, {}_i 0.2, {}_f 0.7)\rangle\, \forall_n x \in \mathbb{X}\}$,

${}_{\mathcal{N}}M = \{\langle {}_n x, ({}_t 0.7, {}_i 0.3, {}_f 0.5), ({}_t 0.7, {}_i 0.3, {}_f 0.6), ({}_t 0.7, {}_i 0.2, {}_f 0.5)\rangle\, \forall_n x \in \mathbb{X}\}$,

Then the $\mathcal{NT}$, $\tau = \{{}_{\mathcal{N}}0, {}_{\mathcal{N}}L, {}_{\mathcal{N}}M, {}_{\mathcal{N}}1\}$ and the complement of $\mathcal{NS}$ of ${}_{\mathcal{N}}L$ and ${}_{\mathcal{N}}M$ are defined as

${}_{\mathcal{N}}\overline{L} = \{\langle {}_n x, ({}_t 0.7, {}_i 0.7, {}_f 0.5), ({}_t 0.7, {}_i 0.7, {}_f 0.4), ({}_t 0.7, {}_i 0.8, {}_f 0.5)\rangle\, \forall_n x \in \mathbb{X}\}$,

${}_{\mathcal{N}}\overline{M} = \{\langle {}_n x, ({}_t 0.5, {}_i 0.7, {}_f 0.7), ({}_t 0.6, {}_i 0.7, {}_f 0.7), ({}_t 0.5, {}_i 0.8, {}_f 0.7)\rangle\, \forall_n x \in \mathbb{X}\}$,

If ${}_{\mathcal{N}}E = \{\langle {}_n x, ({}_t 0.5, {}_i 0.3, {}_f 0.7), ({}_t 0.6, {}_i 0.3, {}_f 0.7), ({}_t 0.5, {}_i 0.3, {}_f 0.7)\rangle\, \forall_n x \in \mathbb{X}\}$,

Then ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ but it is not a $\mathcal{NCS}$ of ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$.

$\because {}_{\mathcal{N}}cl({}_{\mathcal{N}}E) = {}_{\mathcal{N}}\overline{M}$ which is not equal to ${}_{\mathcal{N}}E$.

Theorem 3.6. *In ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, every $\mathcal{N}_{\alpha}$- $\mathcal{CS}$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$.*

Proof. *Let ${}_{\mathcal{N}}E \subseteq \mathcal{V}$, where $\mathcal{V}$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$ in $\mathbb{X}$.*

$\because {}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha}$- $\mathcal{CS}$, ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) = {}_{\mathcal{N}}E$. But ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \subseteq \mathcal{V}$.

$\therefore {}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$.

The converse is not true.

Example 3.7. From the example 3.4, the ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ but it is not a $\mathcal{N}_{\alpha}$- $\mathcal{CS}$ of ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$. $\because {}_{\mathcal{N}}cl({}_{\mathcal{N}}int({}_{\mathcal{N}}cl(E))) = {}_{\mathcal{N}}\overline{M}$ which is not equal to ${}_{\mathcal{N}}E$.

Theorem 3.8. *In ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, every $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{CS}$.*

Proof. *Let ${}_{\mathcal{N}}E \subseteq \mathcal{V}$, where $\mathcal{V}$ is $\mathcal{N}_{*g\alpha}$- $\mathcal{OS}$ in $\mathbb{X}$.*

$\because$ Every $\mathcal{N}_{*g\alpha}$- $\mathcal{OS}$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$, $\mathcal{V}$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$.

$\because {}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$, ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \subseteq \mathcal{V}$. But ${}_{\mathcal{N}b}cl({}_{\mathcal{N}}E) \subseteq_{\mathcal{N}\alpha} cl({}_{\mathcal{N}}E) \subseteq \mathcal{V}$, which implies

$_{\mathcal{N}b}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$. $\therefore$ $_{\mathcal{N}}E$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{CS}$.

The converse is not true.

Example 3.9. Let $\mathbb{X} = \{_na, _nb, _nc\}$ and the $\mathcal{NS}$, $_{\mathcal{N}}L$ and $_{\mathcal{N}}M$ are defined as

$_{\mathcal{N}}L = \{\langle _nx, (_t0.6, _i0.3, _f0.7), (_t0.5, _i0.3, _f0.7), (_t0.5, _i0.2, _f0.7)\rangle \,\forall _nx \in \mathbb{X}\}$,

$_{\mathcal{N}}M = \{\langle _nx, (_t0.7, _i0.3, _f0.5), (_t0.7, _i0.3, _f0.6), (_t0.7, _i0.2, _f0.4)\rangle \,\forall _nx \in \mathbb{X}\}$,

Then $\mathcal{NT}$, $\tau = \{_{\mathcal{N}}0, _{\mathcal{N}}L, _{\mathcal{N}}M, _{\mathcal{N}}1\}$ and the complement of $\mathcal{NS}$ of $_{\mathcal{N}}L$ and $_{\mathcal{N}}M$ are defined as

$_{\mathcal{N}}\overline{L} = \{\langle _nx, (_t0.7, _i0.7, _f0.6), (_t0.7, _i0.7, _f0.5), (_t0.7, _i0.8, _f0.5)\rangle \,\forall _nx \in \mathbb{X}\}$,

$_{\mathcal{N}}\overline{M} = \{\langle _nx, (_t0.5, _i0.7, _f0.7), (_t0.6, _i0.7, _f0.7), (_t0.4, _i0.8, _f0.7)\rangle \,\forall _nx \in \mathbb{X}\}$ and

$_{\mathcal{N}}N = \{\langle _nx, (_t0.7, _i0.7, _f0.6), (_t0.7, _i0.7, _f0.6), (_t0.7, _i0.3, _f0.5)\rangle \,\forall _nx \in \mathbb{X}\}$

If $_{\mathcal{N}}E = \{\langle _nx, (_t0.6, _i0.3, _f0.7), (_t0.6, _i0.3, _f0.7), (_t0.5, _i0.7, _f0.7)\rangle \,\forall _nx \in \mathbb{X}\}$

Then $_{\mathcal{N}}E$ is $\mathcal{N}_{b^*g\alpha}$- $\mathcal{CS}$ but it is not a $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ of $_{\mathcal{NTS}}(\mathbb{X}, \tau)$.

$\because$ $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) = \{\langle _nx, (_t0.7, _i0.3, _f0.6), (_t0.7, _i0.3, _f0.5), (_t0.7, _i0.7, _f0.5)\rangle \,\forall _nx \in \mathbb{X}\}$ which is not contained in $_{\mathcal{N}}N$.

Theorem 3.10. *In* $_{\mathcal{NTS}}(\mathbb{X}, \tau)$, *every* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ *is* $\mathcal{N}_{gs}$- $\mathcal{CS}$.

Proof. *Let* $_{\mathcal{N}}E \subseteq \mathcal{V}$, *where* $\mathcal{V}$ *is* $\mathcal{NOS}$ *in* $\mathbb{X}$.

$\because$ *Every* $\mathcal{NOS}$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$, $\mathcal{V}$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$.

$\because$ $_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$, $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$. *But* $_{\mathcal{N}s}cl(_{\mathcal{N}}E) \subseteq {_{\mathcal{N}\alpha}cl}(_{\mathcal{N}}E) \subseteq \mathcal{V}$, *which implies* $_{\mathcal{N}s}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$. $\therefore$ $_{\mathcal{N}}E$ *is* $\mathcal{N}_{gs}$- $\mathcal{CS}$.

The converse is not true.

Example 3.11. Let $\mathbb{X} = \{_na, _nb, _nc\}$ and the $\mathcal{NS}$, $_{\mathcal{N}}L$ and $_{\mathcal{N}}M$ are defined as

$_{\mathcal{N}}L = \{\langle _nx, (_t0.6, _i0.3, _f0.7), (_t0.5, _i0.3, _f0.7), (_t0.5, _i0.2, _f0.7)\rangle \,\forall _nx \in \mathbb{X}\}$,

$_{\mathcal{N}}M = \{\langle _nx, (_t0.7, _i0.3, _f0.5), (_t0.7, _i0.3, _f0.6), (_t0.7, _i0.7, _f0.4)\rangle \,\forall _nx \in \mathbb{X}\}$,

Then $\mathcal{NT}$, $\tau = \{_{\mathcal{N}}0, _{\mathcal{N}}L, _{\mathcal{N}}M, _{\mathcal{N}}1\}$ and the complement of $\mathcal{NS}$ of $_{\mathcal{N}}L$ and $_{\mathcal{N}}M$ are defined as

$_{\mathcal{N}}\overline{L} = \{\langle _nx, (_t0.7, _i0.7, _f0.6), (_t0.7, _i0.7, _f0.5), (_t0.7, _i0.8, _f0.5)\rangle \,\forall _nx \in \mathbb{X}\}$,

$_{\mathcal{N}}\overline{M} = \{\langle _nx, (_t0.5, _i0.7, _f0.7), (_t0.6, _i0.7, _f0.7), (_t0.4, _i0.3, _f0.7)\rangle \,\forall _nx \in \mathbb{X}\}$ and

$_{\mathcal{N}}N = \{\langle _nx, (_t0.7, _i0.7, _f0.6), (_t0.7, _i0.7, _f0.6), (_t0.7, _i0.3, _f0.5)\rangle \,\forall _nx \in \mathbb{X}\}$

If $_{\mathcal{N}}E = \{\langle _nx, (_t0.6, _i0.3, _f0.7), (_t0.6, _i0.3, _f0.7), (_t0.5, _i0.7, _f0.7)\rangle \,\forall _nx \in \mathbb{X}\}$

Here $_{\mathcal{N}}E$ is $\mathcal{N}_{gs}$- $\mathcal{CS}$ but it is not a $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ of $_{\mathcal{NTS}}(\mathbb{X}, \tau)$.

$\because$ $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) = \{\langle _nx, (_t0.7, _i0.3, _f0.6), (_t0.7, _i0.3, _f0.5), (_t0.7, _i0.7, _f0.5)\rangle \,\forall _nx \in \mathbb{X}\}$ which is not contained in $_{\mathcal{N}}N$.

Theorem 3.12. *In* $_{\mathcal{NTS}}(\mathbb{X}, \tau)$, *every* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ *is* $\mathcal{N}_{gp}$- $\mathcal{CS}$.

Proof. *Let* $_{\mathcal{N}}E \subseteq \mathcal{V}$, *where* $\mathcal{V}$ *is* $\mathcal{NOS}$ *in* $\mathbb{X}$.

$\because$ *Every* $\mathcal{NOS}$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$, $\mathcal{V}$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$. $\because$ $_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$, $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$. *But* $_{\mathcal{N}p}cl(_{\mathcal{N}}E) \subseteq {_{\mathcal{N}\alpha}cl}(_{\mathcal{N}}E) \subseteq \mathcal{V}$, *which implies* $_{\mathcal{N}p}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$.

$\therefore\ _{\mathcal{N}}E$ *is* $\mathcal{N}_{gp}$*-* $\mathcal{CS}$*.*

The converse is not true.

Example 3.13. Let $\mathbb{X} = \{_na, _nb, _nc\}$ and the $\mathcal{NS}$, $_{\mathcal{N}}L$ and $_{\mathcal{N}}M$ are defined as

$_{\mathcal{N}}L = \{\langle _nx, (_t0.5, _i0.3, _f0.7), (_t0.4, _i0.3, _f0.7), (_t0.5, _i0.2, _f0.7)\rangle\ \forall_n x \in \mathbb{X}\}$,

$_{\mathcal{N}}M = \{\langle _nx, (_t0.7, _i0.3, _f0.5), (_t0.7, _i0.3, _f0.6), (_t0.7, _i0.2, _f0.5)\rangle\ \forall_n x \in \mathbb{X}\}$,

Then the $\mathcal{NT}$, $\tau = \{_{\mathcal{N}}0, _{\mathcal{N}}L, _{\mathcal{N}}M, _{\mathcal{N}}1\}$ and the complement of $\mathcal{NS}$ of $_{\mathcal{N}}L$ and $_{\mathcal{N}}M$ are defined as

$_{\mathcal{N}}\overline{L} = \{\langle _nx, (_t0.7, _i0.7, _f0.5), (_t0.7, _i0.7, _f0.4), (_t0.7, _i0.8, _f0.5)\rangle\ \forall_n x \in \mathbb{X}\}$,

$_{\mathcal{N}}\overline{M} = \{\langle _nx, (_t0.5, _i0.7, _f0.7), (_t0.6, _i0.7, _f0.7), (_t0.5, _i0.8, _f0.7)\rangle\ \forall_n x \in \mathbb{X}\}$ and

$_{\mathcal{N}}N = \{\langle _nx, (_t0.4, _i0.8, _f0.7), (_t0.6, _i0.5, _f0.7), (_t0.5, _i0.9, _f0.7)\rangle\ \forall_n x \in \mathbb{X}\}$

If $_{\mathcal{N}}E = \{\langle _nx, (_t0.5, _i0.3, _f0.7), (_t0.6, _i0.3, _f0.7), (_t0.5, _i0.2, _f0.7)\rangle\ \forall_n x \in \mathbb{X}\}$

Here $_{\mathcal{N}}E$ is $\mathcal{N}_{gp}$- $\mathcal{CS}$ but it is not a $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ of $_{\mathcal{NTS}}(\mathbb{X}, \tau)$.

$\because\ _{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) = _{\mathcal{N}}\overline{M}$ which is not contained in $_{\mathcal{N}}N$.

Theorem 3.14. *The union of any two* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$ *is also a* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*.*

Proof. *Let* $_{\mathcal{N}}E$ *and* $_{\mathcal{N}}F$ *be two* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*. Let* $\mathcal{V}$ *be a* $\mathcal{N}_{b^*g\alpha}$*-* $\mathcal{OS}$ *in* $\mathbb{X}$ *s.t* $_{\mathcal{N}}E \subseteq \mathcal{V}$ *and* $_{\mathcal{N}}F \subseteq \mathcal{V}$*. Then,* $_{\mathcal{N}}E \cup _{\mathcal{N}}F \subseteq \mathcal{V}$*.*

$\because\ _{\mathcal{N}}E$ *and* $_{\mathcal{N}}F$ *are* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*, implies* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$ *and* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}F) \subseteq \mathcal{V}$*. Now,* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E \cup _{\mathcal{N}}F) = _{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \cup _{\mathcal{N}\alpha}cl(_{\mathcal{N}}F) \subseteq \mathcal{V}$*. Thus,* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E \cup _{\mathcal{N}}F) \subseteq \mathcal{V}$ *whenever* $_{\mathcal{N}}E \cup _{\mathcal{N}}F \subseteq \mathcal{V}$*,* $\mathcal{V}$ *is* $\mathcal{N}_{b^*g\alpha}$*-* $\mathcal{OS}$ *in* $(\mathbb{X}, \tau)$ *implies* $_{\mathcal{N}}E \cup _{\mathcal{N}}F$ *is a* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*.*

Theorem 3.15. *The intersection of any two* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$ *is also a* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*.*

Proof. *Let* $_{\mathcal{N}}E$ *and* $_{\mathcal{N}}F$ *be two* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*. Let* $\mathcal{V}$ *be a* $\mathcal{N}_{b^*g\alpha}$*-* $\mathcal{OS}$ *in* $(\mathbb{X}, \tau)$ *s.t* $_{\mathcal{N}}E \subseteq \mathcal{V}$ *and* $_{\mathcal{N}}F \subseteq \mathcal{V}$*. Then,* $_{\mathcal{N}}E \cap _{\mathcal{N}}F \subseteq \mathcal{V}$*.* $\because\ _{\mathcal{N}}E$ *and* $_{\mathcal{N}}F$ *are* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*, implies* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$ *and* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}F) \subseteq \mathcal{V}$*. Now,* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E \cap _{\mathcal{N}}F) = _{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \cap _{\mathcal{N}\alpha}cl(_{\mathcal{N}}F) \subseteq \mathcal{V}$*. Thus,*

$_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E \cap _{\mathcal{N}}F) \subseteq \mathcal{V}$ *whenever* $_{\mathcal{N}}E \cap _{\mathcal{N}}F \subseteq \mathcal{V}$*,* $\mathcal{V}$ *is* $\mathcal{N}_{b^*g\alpha}$*-* $\mathcal{OS}$ *in* $(\mathbb{X}, \tau)$ *implies* $_{\mathcal{N}}E \cap _{\mathcal{N}}F$ *is a* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$*.*

Theorem 3.16. *Let* $_{\mathcal{N}}E$ *be a* $\mathcal{N}_{\alpha b^*g\alpha}$*-closed subset of* $(\mathbb{X}, \tau)$*. If* $_{\mathcal{N}}E \subseteq _{\mathcal{N}}F \subseteq _{\mathcal{N}\alpha}cl(_{\mathcal{N}}E)$*, then* $_{\mathcal{N}}F$ *is also a* $\mathcal{N}_{\alpha b^*g\alpha}$*-closed subset of* $(\mathbb{X}, \tau)$*.*

Proof. *Let* $_{\mathcal{N}}F \subseteq \mathcal{V}$*, where* $\mathcal{V}$ *is* $\mathcal{N}_{b^*g\alpha}$*-* $\mathcal{OS}$ *in* $(\mathbb{X}, \tau)$*. Then* $_{\mathcal{N}}E \subseteq _{\mathcal{N}}F$ *implies* $_{\mathcal{N}}E \subseteq \mathcal{V}$*.* $\because\ _{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$*,* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) \subseteq \mathcal{V}$*. Also* $_{\mathcal{N}}F \subseteq _{\mathcal{N}\alpha}cl(_{\mathcal{N}}E)$ *implies* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}F) \subseteq _{\mathcal{N}\alpha}cl(_{\mathcal{N}}E)$*. Thus,* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}F) \subseteq \mathcal{V}$ *and* $_{\mathcal{N}}F$ *is* $\mathcal{N}\alpha b^*g\alpha$*-* $\mathcal{CS}$*.*

Theorem 3.17. *If a set* $_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$*-* $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$ *iff* $_{\mathcal{N}\alpha}cl(_{\mathcal{N}}E) - _{\mathcal{N}}E$ *contains no non-empty* $\mathcal{N}_{b^*g\alpha}$*-* $\mathcal{CS}$*.*

Proof. Necessity: *Let* ${}_{\mathcal{N}}F$ *be a* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{CS}$ *in* $(\mathbb{X}, \tau)$ *such that* ${}_{\mathcal{N}}F \subseteq {}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) - {}_{\mathcal{N}}E$. *Then* ${}_{\mathcal{N}}F \subseteq \mathbb{X} - {}_{\mathcal{N}}E$. *This implies* ${}_{\mathcal{N}}E \subseteq \mathbb{X} - {}_{\mathcal{N}}F$. *Now* $\mathbb{X} - {}_{\mathcal{N}}F$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$ *of* $(\mathbb{X}, \tau)$ *such that* ${}_{\mathcal{N}}E \subseteq \mathbb{X} - {}_{\mathcal{N}}F$. $\because {}_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$ *then* ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \subseteq \mathbb{X} - {}_{\mathcal{N}}F$. *Thus* ${}_{\mathcal{N}}F \subseteq \mathbb{X} - {}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E)$. *Now* ${}_{\mathcal{N}}F \subseteq_{\mathcal{N}\alpha} cl({}_{\mathcal{N}}E)$ $(\mathbb{X} -_{\mathcal{N}\alpha} cl({}_{\mathcal{N}}E)) = {}_{\mathcal{N}}0$.

Sufficiency: *Assume* ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) - {}_{\mathcal{N}}E$ *contains no non-empty* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$. *Let* ${}_{\mathcal{N}}E \subseteq \mathcal{V}$, $\mathcal{V}$ *is* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{OS}$. *Suppose* ${}_{\mathcal{N}\alpha}cl({}_{\mathcal{N}}E) \nsubseteq \mathcal{V}$, *then* ${}_{\mathcal{N}\alpha}cl(E) \cap \mathcal{V}^c$ *is a non-empty* $\mathcal{N}_{b^*g\alpha}$- $\mathcal{CS}$ *of* ${}_{\mathcal{N}\alpha}cl(E) - {}_{\mathcal{N}}E$, *which is a contradiction.*

$\therefore N_{\alpha}cl({}_{\mathcal{N}}E) \subseteq \mathcal{V}$. *Hence* ${}_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$- $\mathcal{CS}$.

4. $\mathcal{N}_{\alpha b^*g\alpha}$-Border

Definition 4.1. For any subset ${}_{\mathcal{N}}E$ of $\mathbb{X}$, the neutrosophic $\alpha b^*g\alpha$-border of ${}_{\mathcal{N}}E$ is defined by $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = {}_{\mathcal{N}}E \setminus \mathcal{N}_{\alpha b^*g\alpha}\text{-}\ {}_{\mathcal{N}}int({}_{\mathcal{N}}E)$

Theorem 4.2. *In* ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, *for any subset* ${}_{\mathcal{N}}E$ *of* $\mathbb{X}$, *the following statements are hold.*

(i) $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd(\phi)] = N_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd(X)] = \phi$

(ii) ${}_{\mathcal{N}}E = \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) \cup \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$

(iii) $\mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) \cap \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = \phi$

(iv) $\mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) = {}_{\mathcal{N}}E \setminus \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$

(v) $\mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int(\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]) = \phi$

(vi) ${}_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$*-open iff* $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = \phi$

(vii) $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd(\mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E))] = \phi$

(viii) $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd(\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)])] = \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$

(ix) $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = {}_{\mathcal{N}}E \cap \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}cl(X \setminus {}_{\mathcal{N}}E)$

Proof. *Statements (i) to (iv) are obvious by the definition of* $\mathcal{N}_{\alpha b^*g\alpha}$*-border of* ${}_{\mathcal{N}}E$. *If possible, let* ${}_{n}x \in \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int(\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)])$.

Then ${}_{n}x \in \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$, *since* $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] \subseteq {}_{\mathcal{N}}E$, ${}_{n}x \in \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int(\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]) \subseteq \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E)$.

$\therefore {}_{n}x \in \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) \cap \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$, *which is the contradiction to (iii). Hence (v) is proved.* ${}_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$*-open iff* $\mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) = {}_{\mathcal{N}}E$. *But* $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = {}_{\mathcal{N}}E \setminus \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E)$ *implies* $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = \phi$.

This proves (vi) & (vii). When ${}_{\mathcal{N}}E = \mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$, *then definition of* $\mathcal{N}_{\alpha b^*g\alpha}$*-border of* ${}_{\mathcal{N}}E$ *becomes* $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd(\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)])] =$

$\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] \setminus \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int(\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)])$.

By using (vii), we get the proof of (viii).

Now, $\mathcal{N}_{\alpha b^*g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = {}_{\mathcal{N}}E \setminus \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) =$

${}_{\mathcal{N}}E \cap (\mathbb{X} \setminus \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E)) = {}_{\mathcal{N}}E \cap \mathcal{N}_{\alpha b^*g\alpha} - {}_{\mathcal{N}}cl(\mathbb{X} \setminus {}_{\mathcal{N}}E)$.

5. $\mathcal{N}_{\alpha b^* g\alpha}$-Frontier

Definition 5.1. For any subset ${}_{\mathcal{N}}E$ of $\mathbb{X}$, the neutrosophic $\alpha b^* g\alpha$-frontier of ${}_{\mathcal{N}}E$ is defined by
$\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] = \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}cl({}_{\mathcal{N}}E) \setminus \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E)$

Theorem 5.2. *In ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, for any subset ${}_{\mathcal{N}}E$ of $\mathbb{X}$, the following statements are hold.*
(i) $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(\phi)] = \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(X)] = \phi$
(ii) $\mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) \cap \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] = \phi$
(iii) $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] \subseteq \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}cl({}_{\mathcal{N}}E)$
(iv) $\mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) \cup \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] = \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}cl({}_{\mathcal{N}}E)$
(v) $\mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) = {}_{\mathcal{N}}E \setminus \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$
(vi) If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$-closed iff ${}_{\mathcal{N}}E = \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) \cup \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$
(vii) ${}_{\mathcal{N}}fr({}_{\mathcal{N}}E) = {}_{\mathcal{N}}fr(\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)])$
(viii) If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$-open, then
${}_{\mathcal{N}}E \cap \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] = \phi$
(ix) $\mathbb{X} = \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}cl({}_{\mathcal{N}}E) \cup \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}cl(\mathbb{X} \setminus {}_{\mathcal{N}}E)$
(x) If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$-open, then
$\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(N_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E))] \subseteq \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$
(xi) If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$-closed, then
$\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(N_{\alpha b^* g\alpha} - {}_{\mathcal{N}}cl({}_{\mathcal{N}}E))] \subseteq \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$
(xii) If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$-open iff then
$\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(N_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E))] \cap \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) = \phi$
Proof. *Statements (i) to (vii) are true by the definition of $\mathcal{N}_{\alpha b^* g\alpha}$- frontier of ${}_{\mathcal{N}}E$. By Remark (3.3), If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$-open, ${}_{\mathcal{N}}E = \mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E)$ and by statement (ii), ${}_{\mathcal{N}}E \cap \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] = \phi$. Hence (viii) is proved. (ix) is obvious. Since $\mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E)$ is $\mathcal{N}_{\alpha b^* g\alpha}$-open, then $\mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E) = {}_{\mathcal{N}}E$, which implies $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(\mathcal{N}_{\alpha b^* g\alpha} - {}_{\mathcal{N}}int({}_{\mathcal{N}}E))] \subseteq \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$. Similarly, (xi) can be proved. By Remark (3.3) and by statement (ii), (xii) is straight forward.*

6. Relationship Between $\mathcal{N}_{\alpha b^* g\alpha}$-Frontier and $\mathcal{N}_{\alpha b^* g\alpha}$-Border

Theorem 6.1. *In ${}_{\mathcal{N}}\mathcal{TS}(\mathbb{X}, \tau)$, for any subset ${}_{\mathcal{N}}E$ of $\mathbb{X}$, the following statements are hold.*
(i) $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] \setminus \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] = \phi$
(ii) $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] \subseteq \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$
(iii) $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr(\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)])] = \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)]$
(iv) $\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}bd(\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)])] = \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$
(v) If ${}_{\mathcal{N}}E$ is $\mathcal{N}_{\alpha b^* g\alpha}$- open, then
$\mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)] \cup \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}bd({}_{\mathcal{N}}E)] = \mathcal{N}_{\alpha b^* g\alpha}[{}_{\mathcal{N}}fr({}_{\mathcal{N}}E)]$

(vi) $\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)] \cap \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)] = \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]$
(vii) $\overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)]} \cup \overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]} = \overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]}$
(viii) $\overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)]} \cap \overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]} = \overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)]}$

Proof. *Statement (i) to (iv) are obvious by the definitions of* $\mathcal{N}_{\alpha b^*g\alpha}$*- Frontier and* $\mathcal{N}_{\alpha b^*g\alpha}$*-border of a set.* $\because$ $_{\mathcal{N}}E$ *is* $\mathcal{N}_{\alpha b^*g\alpha}$*- open, then we have a statement from* $\mathcal{N}_{\alpha b^*g\alpha}$*-border of a set,* $\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(E)] = \phi$*, which implies* $\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)] \cup \phi = \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)]$*. Hence (v) is proved. We know from statement (ii),* $\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)] \subseteq \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)]$ *which implies* $\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)] \cap \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)] = \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]$*. It gives the proof of (vi). By the above statement,*
$\overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]} = \overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)] \cap \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]}$*, and by using De Morgan's law,*
$\overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)] \cap \mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]} =$
$\overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}fr(_{\mathcal{N}}E)]} \cup \overline{\mathcal{N}_{\alpha b^*g\alpha}[_{\mathcal{N}}bd(_{\mathcal{N}}E)]}$*, it gives the proof of (vii).*
Similarly we can prove the statement (viii).

7. Conclusions

The $\mathcal{N}_{\alpha b^*g\alpha}$-closed set in $_{\mathcal{N}}\mathcal{TS}$ was defined in this article, and its relationship to other known $_{\mathcal{N}}\mathcal{S}$ in $_{\mathcal{N}}\mathcal{TS}$ was examined. We also introduced and investigated the properties of $\mathcal{N}_{\alpha b^*g\alpha}$-frontier and $\mathcal{N}_{\alpha b^*g\alpha}$-border of a set. $\mathcal{N}_{\alpha b^*g\alpha}$-frontier of a set in $_{\mathcal{N}}\mathcal{TS}$ and found to be connected. A few more functions, including $\mathcal{N}_{\alpha b^*g\alpha}$-continuous, irresolute functions, can be derived from this set. Furthermore, it can be expanded to include the homeomorphism of $_{\mathcal{N}}\mathcal{TS}$.

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