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On Supra Neutrosophic Multiset Topological Spaces

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Abstract: In this article, we present a novel concept of the supra neutrosophic multiset topological space. We describe the behaviour of neutrosophic sets and multiset with in this framework. Additionally, we define the supra neutrosophic open multiset (SNOMS), supra neutrosohic closed multiset (SNCMS), supra neutrosophic interior multiset, and supra neutrosophic closure multiset, and provide examples to illustrate their properties.

Keywords: Supra Neutrosophic Open Multiset, Supra Neutrosophic Closed Multiset, Supra Neutrosophic Interior Multiset, Supra Neutrosophic Closure Multiset, Supra Neutrosophic Multiset Topological sapce.

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1. Introduction

In recent years, multiset and neutrosophic sets have become a subject of great interest for researches. Mathematicans always like to solve a complicated problem in a simple way and to find out the most feasible solution. Neutrosophy has been introduced and studied by Smarandache [11] and developed the neutrsphic topological sapce for and introduced the multiset topological space their properties open bms, closed bms , closure bms, interior and their theorem and their properties are discussed in [8] [7]. The properties of neutrosophic multiset group are establised [9],[10],[3][2]. Then [6] supra topological space establised the [4] and their derived the properties and examples,[5] the pre open set in supra topological sapce. [12] a group of multiset of power whole set in multiset topological space. The application of the neutrosophic of decision making in disease diagnosis in [1].

In this paper we intoduced the new concept for the supra neutrosophic multiset topological space and their properties for supra neutrosophic open multiset, supra neutrosophic closed multiset, supra neutrosophic interior multiset, supra neutrosophic closure multiset, supra neutrosophic multiset topological sapce their discussed properties , theorem and examples.

2. Preliminaries

We define functions T, F and I from X to $[0, 1]$, where T is membership value, F fails membership value, and I is the indeterminacy value. The definition of a neutrosophic multiset was first defined by Smarandache [11] as follows:

Definition 2.1: [11] A neutrosophic multiset is a neutrosophic set where one or more elements are repeated with the same neutrosophic components, or with different neutrosophic components.

Definition 2.2: [11]The empty neutrosophic multiset is denoted by N_θ and defined by $N_\theta = \{< x, (0, 1, 1) >: \forall x \in X\}$ where x can be repeated.

Definition 2.3:[11] Let $A = \{(x, < T_A(x), I_A(x), F_A(x) >) : x \in X\}$ be a neutrosophic multiset on X. Then the complement of A is denoted by A^c and defined by
$A^c = \{(x, < F_{A(x)}, 1 - I_{A(x)}, T_{A(X)} >) : x \in X\}$. Where x can be repeated based on its multiplicity and the corresponding T, F, I values may or may not be equal.

Definition 2.4:[11] Let X be a non-empty set and neutrosophic multisets A and B in the form $A = \{(x, < T_A(x), I_A(x), F_A(x) >) : x \in X\}$ and $B = \{(x, < T_B(x), I_B(x), F_B(x) >) : x \in X\}$, then the operations of maximal union and minimal intersection of NM set relation are defined as follows:

1. $(A \cup B)_{max} = \{(x, < T_{(A\cup B)_{max}}(x), I_{(A\cup B)_{max}}(x), F_{(A\cup B)_{max}}(x) >) : x \in X\}$, where $T_{(A\cup B)max}(x) = \max\{T_A(x), T_B(x)\}, F_{(A\cup B)}(x) = \min\{F_A(x), F_B(x)\}$ and $I_{(A\cup B)})(x) = \min\{I_A(x), I_B(x)\}$.
2. $(A \cap B)_{min} = \{(x, < T_{(A\cap B)_{min}}(x), I_{(A\cap B)_{min}}(x), F_{(A\cap B)_{min}}(x) >) : x \in X\}$,where $T_{(A\cap B)min}(x) = \min\{T_A(x), T_B(x)\}$, $F_{(A\cap B)}(x) = \max\{F_A(x), F_B(x)\}$ and $I_{(A\cup B)}(x) = \max\{I_A(x), I_B(x)\}$.

Definition 2.5:[11] Let X be neutrosophic multiset and a non-empty family $\mathcal{T}$ subsets of W_X is said to be neutrosophic multiset topological space if the following axioms hold:

(1) $N_\emptyset, W_X \in \mathcal{T}$.

(2) $A \cap B \in \mathcal{T}$, for $A, B \in \mathcal{T}$.

(3) $\bigcup_{i\in\Lambda} A_i \in \mathcal{T}$, $\forall\{A_i : i \in \Lambda\} \subseteq \mathcal{T}$.

In this case, the pair $(W_X, \mathcal{T})$ is called a neutrosophic multiset topological space (NMTS in short) and any neutrosophic multiset in $\mathcal{T}$ is known as an open neutrosophic multiset (ONMS in short) in W_X. The elements of $\mathcal{T}^c$ are called closed neutrosophic multisets, otherwise, a neutrosophic set F is closed if and only if its complement F^c is an open neutrosophic multiset.

Definition 2.6:[11] Let $(W_X, \mathcal{T}_1)$ and $(W_X, \mathcal{T}_2)$ be two neutrosophic multiset topological spaces on W_X. Then $\mathcal{T}_1$ is said to be contained in $\mathcal{T}_2$ that is if $\mathcal{T}_1 \subseteq \mathcal{T}_2$, i.e., $A \in \mathcal{T}_2$ for each $A \in \mathcal{T}_1$. In this case, we also say that $\mathcal{T}_1$ is coarser than $\mathcal{T}_2$.
Definition 2.7. [8] Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a multiset topological space of M if τ satisfies the following properties.
(i) The mset M and the empty mset ϕ are in τ.
(ii) The mset union of the elements of any sub collection of τ is τ.
(iii) The mset intersection of the elements of any finite sub collection of τ is in τ".
Definition 2.8.[8] "A sub mset N of M-topological space M in $[X]^w$ is said to be closed if the mset $M \ominus N$ is open. In discrete M-topological space every mset is an open mset as well as a closed mset. In the M-topological space PF(M)$\cup\phi$, every mset is an open mset as well as a closed mset".
Definition 2.9.[8] "Given a suBMS et A of an M-topological space M in $[X]^w$, the Interior of

A is defined as the mset union of all open mset contained in A and its denoted by $Int(A)$. i.e., Int(A)=$\cup\{G \subseteq M : G$is an open mset and $G \subseteq$ A$\}$ and $C_{Int(A)}(x) = max\{C_G(x) : G \subseteq A\}$".

Definition 2.10.[8] "Given a subset A of an M-topological space M in $[X]^w$, the closure of A is defined as the mset intersection of all closed mset containing A and its denoted by $Cl(A)$. i.e., $Cl(A) = \cap\{K \subseteq M : K$is a closed mset and A $\subseteq$ K$\}$ and $C_{Cl(A)}(x) = min\{C_K(x) : A \subseteq K\}$".

Definition 2.11[6] A subfamily τ^* of X is said to be a supra topology on X if ,

(i) X, $\phi \in \tau^*$.

(ii) If $A_i \in \tau^*$ for all $i \in J$, then $\cup A_i \in \tau^*$.

(X, τ^*) is called a supra topological space. The elements of τ^* are called supra open sets in (X, τ^*) and complement of a supra open set is called a supra closed set.

Definition 2.12[6] The supra closure of a set A is denoted by supra cl(A) and defined as supra as supra cl(A)= $\cap\{B : B$ is a supra closed and $A \subseteq B\}$.

The supra interior of a set A is denoted by supra int(A), and defined as supra int(A)= $\cup\{B : B$ is a supra open and $A \supseteq B\}$.

Definition 2.13[6] Let (X, τ) be a topological space and τ^* be a supra topology on X. We call τ^* a supra topology associated with τ if $\tau \subseteq \tau^*$.

3. On Supra Neutrosophic Multiset Topological Spaces

In this section, we define supra neutrosophic multiset topological spaces and their properties are discussed. Throghout this paper we represent supra neutrosophic multiset topological space by SNMTS, neutrosophic closed multiset by SNCMS, supra neutrosophic open multiset by SNOMS,

Definition 3.1: Let X be neutrosophic multiset and a non empty family τ^*_μ neutrosophic submultiset of X is said to be supra neutrosophic multiset topological space if the following axioms hold.

(i) $0_{\mathscr{N}\mathscr{M}}$, $1_{\mathscr{N}\mathscr{M}} \in \tau^*_\mu$

(ii) $\bigcup_{i\in\wedge} \mathscr{A}_i \in \tau^*_\mu$ for all $\{\mathscr{A}_i : i \in \wedge\} \in \tau^*_\mu$.

The pair $(X^*_\mu \tau$) is called a supra neutrosophic multiset topological space (SNMSTS in short) and any supra neutrosophic multiset in τ^*_μ is known as supra neutrosophic open multiset (SNOMS in short) . The elements of τ^{*c}_μ are called supra neutrosophic closed multiset (SNCMS in short).

Example 3.2: Let $X = \{a, b, c\}$ and $A = \{(a, < 0.3, 0.4, 0.5 >), (a, < 0.3, 0.4, 0.5 >), (b, < 1, 0, 0.2 >), (b, < 1, 0, 0.2 >), (c, < 0.7, 1, 0 >)\}$ is a neutrosophic multiset.
$\tau^*_\mu = \{0_{\mathscr{N}\mathscr{M}}, 1_{\mathscr{N}\mathscr{M}}, (a, < 0.3, 0.4, 0.5 >), (b, < 1, 0, 0.2 >), (c, < 0.7, 1, 0 >)\}$ is a SNMSTS.

Definition 3.3: Let τ^*_μ be SNMSTS and let $A = \{< x, \mu_{\mathscr{N}\mathscr{M}}, \sigma_{\mathscr{N}\mathscr{M}}, \delta_{\mathscr{N}\mathscr{M}} >: x \in X\}$ be neutrosophic multiset, then the supra neutrosophic interior multiset of A is the union of all supra neutrosophic open multiset (SNOMS) of X contains in A and is defined as
$Sint_{\mathscr{N}\mathscr{M}}(A) = \{< x, \cup_{(max)}\mu_{\mathscr{N}\mathscr{M}}, \cap_{(min)}\sigma_{\mathscr{N}\mathscr{M}}, \cap_{(min)}\delta_{\mathscr{N}\mathscr{M}} >: x \in X\}$

Example 3.4: Let $X = \{a, b\}$ and $A = \{(a, < 0.3, 0.4, 0.5 >), (a, < 0.3, 0.4, 0.5 >), (b, < 1, 0, 0.2 >), (b, < 1, 0, 0.2 >), (c, < 0.7, 1, 0 >)\}$. Then $\tau^*_\mu = \{(a, < 0.3, 0.4, 0.5 >), (0_{\mathscr{N}\mathscr{M}}, 1_{NM}, b, < 0.2, 0.3, 0.4 >), (b, < 0.2, 0.3, 0.4 >)\}$ be a supra neutrosophic multiset topological space.
Let $B = \{a, < 0.1, 0.2, 0.3 >\}$ then $B \subseteq A$, $Sint_{\mathscr{N}\mathscr{M}}(B) = 0_{NM}$.

Theorem 3.5: Let (X, τ_μ^*) be a supra neutrosophic multiset topological space. Let A, B be two NMS on X, then the following property hold:

(i) $Sint_{\mathscr{N}\mathscr{M}}(0_{\mathscr{N}\mathscr{M}}) = 0_{\mathscr{N}\mathscr{M}}$, $Sint_{NM}(1_{\mathscr{N}\mathscr{M}}) = 1_{\mathscr{N}\mathscr{M}}$

(ii) $Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq A$.

(iii) $A \subseteq B \Rightarrow Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq Sint_{\mathscr{N}\mathscr{M}}(B)$.

Proof : (i) Since $0_{\mathscr{N}\mathscr{M}}$ and $1_{\mathscr{N}\mathscr{M}}$ are supra neutrosophic open multiset. We have
$Sint_{\mathscr{N}\mathscr{M}}(0_{\mathscr{N}\mathscr{M}}) = 0_{\mathscr{N}\mathscr{M}}$
$Sint_{\mathscr{N}\mathscr{M}}(1_{\mathscr{N}\mathscr{M}}) = 1_{NM}$
(ii) Let $x \in Sint_{\mathscr{N}\mathscr{M}}(A)$. Since x is an interior element of A, which implies that A is an neighbourhood of x. Thus $x \in A$. Hence $Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq A$.
(iii) Let $x \in Sint_{\mathscr{N}\mathscr{M}}(B)$. Since x is an interior point of A, so A is a neighbourhood of x. Since $A \subseteq B$, so B is also neighbourhood of x, which implies that $x \in Sint_{\mathscr{N}\mathscr{M}}(B)$. Thus $x \in Sint_{\mathscr{N}\mathscr{M}}(A) \Rightarrow x \in Sint_{\mathscr{N}\mathscr{M}}(B)$. Hence $A \subseteq B \Rightarrow Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq Sint_{\mathscr{N}\mathscr{M}}(B)$.

Proposition 3.6: Let (X, τ_μ^*) be a SNMTS. Then

(a) $Sint_{\mathscr{N}\mathscr{M}}(A \cap B) = Sint_{\mathscr{N}\mathscr{M}}(A) \cap Sint_{\mathscr{N}\mathscr{M}}(B)$.

(b) $Sint_{\mathscr{N}\mathscr{M}}(A) \cup Sint_{\mathscr{N}\mathscr{M}}(B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A \cup B)$.

Proof : (a) Since $A \cap B \subseteq A$ and $A \cap B \subseteq B$ then we have from (iii) $A \subseteq B \Rightarrow Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq Sint_{\mathscr{N}\mathscr{M}}(B)$. $Sint_{\mathscr{N}\mathscr{M}}(A \cap B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A)$ and $Sint_{\mathscr{N}\mathscr{M}}(A \cap B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(B)$. $\Rightarrow$ $Sint_{\mathscr{N}\mathscr{M}}(A \cap B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A) \cap Sint_{\mathscr{N}\mathscr{M}}(B)$. (1)
Now, let $x \in Sint_{\mathscr{N}\mathscr{M}}(A) \cap Sint_{\mathscr{N}\mathscr{M}}(B)$. Hence x is an interior point of each of the sets A and B. It gives the A and B are neigbourhood of x, so their intersection $A \cap B$ is also a neighbourhood of x. Therefore, $x \in Sint_{\mathscr{N}\mathscr{M}}(A \cap B)$.
Let $x \in Sint_{\mathscr{N}\mathscr{M}}(A) \cap Sint_{\mathscr{N}\mathscr{M}}(B)$.
$\Rightarrow x \in Sint_{\mathscr{N}\mathscr{M}}(A \cap B)$ which $\Rightarrow Sint_{\mathscr{N}\mathscr{M}}(A) \cap Sint_{\mathscr{N}\mathscr{M}}(B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A \cap B)$. (2) From (1) and (2) $Sint_{\mathscr{N}\mathscr{M}}(A \cap B) = Sint_{\mathscr{N}\mathscr{M}}(A) \cap Sint_{\mathscr{N}\mathscr{M}}(B)$. **(b)** From $A \subseteq B \Rightarrow Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq Sint_{\mathscr{N}\mathscr{M}}(B)$. We have:
$A \subseteq (A \cup B) \Rightarrow Sint_{\mathscr{N}\mathscr{M}}(A) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A \cup B)$ and
$B \subseteq (A \cup B) \Rightarrow Sint_{\mathscr{N}\mathscr{M}}(B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A \cup B)$ Hence $Sint_{\mathscr{N}\mathscr{M}}(A) \cup Sint_{\mathscr{N}\mathscr{M}}(B) \subseteq Sint_{\mathscr{N}\mathscr{M}}(A \cup B)$.

Definition 3.7: Let (X, τ_μ^*) be a SNMSTS and let $A = \{< x, \mu_{\mathscr{N}\mathscr{M}}, S_{\mathscr{N}\mathscr{M}}, \delta_{\mathscr{N}\mathscr{M}} >: x \in X\}$ be NMS. Then the supra neutrosophic closure multiset of A is the intersection of all supra neutrosophic closed multisets (SNCMS) of X containing in A and is defined as $SCl_{\mathscr{N}\mathscr{M}}(A) = \{< x, \cap\mu_{\mathscr{N}\mathscr{M}}, \cup\sigma_{\mathscr{N}\mathscr{M}}, \cup\delta_{\mathscr{N}\mathscr{M}} >: x \in X\}$.

Proposition 3.8: Let (X, τ_μ^*) be a SNMSTS. Then

(i) $SCl_{\mathscr{N}\mathscr{M}}(0_{\mathscr{N}\mathscr{M}}) = 0_{\mathscr{N}\mathscr{M}}$, $SCl_{\mathscr{N}\mathscr{M}}(1_{\mathscr{N}\mathscr{M}}) = 1_{\mathscr{N}\mathscr{M}}$

(ii) $P \subseteq SCl_{\mathscr{N}\mathscr{M}}(P)$

(iii) P is SNCMS if and only if $P \subseteq SCl_{\mathscr{N}\mathscr{M}}(P)$

(iv) $P \subseteq Q \Rightarrow SCl_{\mathscr{N}\mathscr{M}}(P) \subseteq SCl_{\mathscr{N}\mathscr{M}}(Q)$

Proof : (i) Since $0_{\mathscr{N}\mathscr{M}}$ and $1_{\mathscr{N}\mathscr{M}}$ are supra neutrosophic closed sets. We have $SCl_{\mathscr{N}\mathscr{M}}(0_{\mathscr{N}\mathscr{M}}) = 0_{\mathscr{N}\mathscr{M}}$,$SCl_{\mathscr{N}\mathscr{M}}(1_{\mathscr{N}\mathscr{M}}) = 1_{\mathscr{N}\mathscr{M}}$.
(ii) Since $SCl_{\mathscr{N}\mathscr{M}}(P)$ is the smallest SNCMS containing P, so $P \subseteq SCl_{\mathscr{N}\mathscr{M}}(P)$.
(iii) Since P is SNCMS then P itself is the smallest SNCMS containing P and so $SCl_{\mathscr{N}\mathscr{M}}(P) = P$. Conversely, let $SCl_{\mathscr{N}\mathscr{M}}(P) = P$. Then $SCl_{\mathscr{N}\mathscr{M}}(P)$ is SNCMS and hence P is also SNCMS.
(iv) From (ii), we have $Q \subset SCl_{\mathscr{N}\mathscr{M}}(Q)$. Since $P \subseteq Q$, so we have $P \subseteq SCl_{\mathscr{N}\mathscr{M}}(Q)$. But $SCl_{\mathscr{N}\mathscr{M}}(Q)$ is a SNCMS. So $SCl_{\mathscr{N}\mathscr{M}}(Q)$ is a SNCMS containing P. Since $SCl_{\mathscr{N}\mathscr{M}}(P)$ is the smallest SNCMS containing P so we have $SCl_{\mathscr{N}\mathscr{M}}(P) \subseteq SCl_{\mathscr{N}\mathscr{M}}(Q)$. Hence $P \subseteq Q \Rightarrow SCl_{\mathscr{N}\mathscr{M}}(P) \subseteq SCl_{\mathscr{N}\mathscr{M}}(Q)$.

4. On Supra Neutrosophic Multi Semi-Open Set Topological Spaces

Definition 4.1: Let $\mathcal{A}$ be an NM of an SNMSTS (X, τ^*_μ), then $\mathcal{A}$ is called a *supra neutrosophic multi semi-open set (SNMSOS)* of NMS if $\exists \mathcal{B} \in \tau^*_\mu$, such that $\mathcal{A} \leq MN \sim Sint_{\mathscr{N}\mathscr{M}}(MN \sim SCl_{\mathscr{N}\mathscr{M}}(B)))$.

Example 4.2:. Let $X = \{a, b\}$:

$$\mathcal{A} = \left\{ \begin{array}{l} \langle a, 0.8, 0.1, 0.2\rangle, \langle a, 0.7, 0.1, 0.3\rangle, \langle a, 0.6, 0.2, 0.4\rangle, \\ \langle b, 0.7, 0.2, 0.3\rangle, \langle b, 0.6, 0.3, 0.4\rangle, \langle b, 0.4, 0.2, 0.5\rangle \end{array} \right\}$$

$$\mathcal{B} = \left\{ \begin{array}{l} \langle a, 0.9, 0.1, 0.1\rangle, \langle a, 0.8, 0.1, 0.2\rangle, \langle a, 0.7, 0.2, 0.3\rangle, \\ \langle b, 0.8, 0.2, 0.2\rangle, \langle b, 0.7, 0.2, 0.3\rangle, \langle b, 0.5, 0.2, 0.4\rangle \end{array} \right\}$$

Then $\tau^*_\mu = \{0_{\mathscr{N}\mathscr{M}}, 1_{\mathscr{N}\mathscr{M}}, \mathcal{B}\}$ is a supra neutrosophic multiset topological space.
Then $Sint_{\mathscr{N}\mathscr{M}}(\mathcal{B}) = 1_{\mathscr{N}\mathscr{M}}$, $SInt_{\mathscr{N}\mathscr{M}}(SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B})) = 1_{\mathscr{N}\mathscr{M}}$.
Hence, $\mathcal{B}$ is SNMSOS.

Definition 4.3: Let $\mathcal{A}$ be an NMS of an SNMSTS (X, τ^*_μ), then $\mathcal{A}$ is called a *supra neutrosophic multi semi-closed set (SNMSCoS)* of X if $\exists \mathcal{B}^c \in \tau^*_\mu$, such that $MN \sim SCl_{\mathscr{N}\mathscr{M}}(MN \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{B})) \subseteq \mathcal{A}$.

Theorem 4.4: The statements below are equivalent:

(i) $\mathcal{A}$ is an SNMCoS;

(ii) $\mathcal{A}^c$ is an SNMOS;

(iii) $NM \sim SInt_{\mathscr{N}\mathscr{M}}(NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})) \subseteq \mathcal{A}$;

(iv) $NM \sim SCl_{\mathscr{N}\mathscr{M}}(NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{A}^c)) \supseteq \mathcal{A}^c$.

Proof: (i) and (ii) are equivalent, since for an SNMS, $\mathcal{A}$ of an SNMTS (X, τ^*_μ) such that

$$1_{NM} - NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{A}) = NM \sim SCl_{\mathscr{N}\mathscr{M}}(1_{NM} - \mathcal{A})$$

and

$$1_{NM} - NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}) = NM \sim SInt_{\mathscr{N}\mathscr{M}}(1_{NM} - \mathcal{A}).$$

(i) $\Rightarrow$ (iii): By definition 4.3, $\exists$ an SNMCoS, $\mathcal{B}$ such that $NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{B}) \subseteq \mathcal{A} \subseteq \mathcal{B}$; hence,

$$NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{B}) \subseteq \mathcal{A} \subseteq NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}) \subseteq \mathcal{B}.$$

Since $NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{B})$ is the largest SNMOS contained in $\mathcal{B}$, we have

$$NM \sim Int_{\mathscr{N}\mathscr{M}}(NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B})) \subseteq NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{B}) \subseteq \mathcal{A};$$

(iii) $\Rightarrow$ (i) follows by taking $\mathcal{B} = NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})$;
(ii) $\Leftrightarrow$ (iv) can similarly be proved. □

Theorem 4.5:
Arbitrary union of SNMSOSs is an SNMSOS;
Proof: Let $\{\mathcal{A}_\alpha\}$ be a collection of SNMSOSs of an SNMSTS (X, τ_μ^*). Then $\exists$ a $\mathcal{B}_\alpha \in \tau_\mu^*$ such that $\mathcal{B}_\alpha \subseteq \mathcal{A}_\alpha \subseteq NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B}_\alpha)$ for each α. Thus,

$$\bigcup_\alpha \mathcal{B}_\alpha \subseteq \bigcup_\alpha \mathcal{A}_\alpha \subseteq \bigcup_\alpha (NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B}_\alpha)) \subseteq NM \sim SCl_{\mathscr{N}\mathscr{M}}\left(\bigcup_\alpha \mathcal{B}_\alpha\right)$$

and $\bigcup_\alpha \mathcal{B}_\alpha \in \tau_\mu^*$, this shows that $\bigcup_\alpha \mathcal{A}_\alpha$ is an SNMSOS;
Definition 4.6. An NMS $\mathcal{A}$ of an SNMSTS (X, τ_μ^*) is called a *supra neutrosophic multi regularly open set (SNMROS)* of (X, τ_μ^*) if

$$NM \sim SInt_{\mathscr{N}\mathscr{M}}(NM \sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})) = \mathcal{A}.$$

Then *supra neutrosophic multi regularly closed set (SNMRCoS)* of (X, τ_μ^*) if

$$NM \sim SCl_{\mathscr{N}\mathscr{M}}(NM \sim SInt_{\mathscr{N}\mathscr{M}}(\mathcal{A})) = \mathcal{A}.$$

Theorem 4.7. An NMS, $\mathcal{A}$ of SNMSTS (X, τ_μ^*) is an SNMRO if $\mathcal{A}^c$ is SNMRCo.
Proof: It follows from theorem 4.4. □
Remark 2.8. It is obvious that every SNMROS (SNMRCoS) is an SNMOS (SNMCoS). The converse need not be true.
Example 4.9. Let $X = \{a, b\}$ and

$$\mathcal{A} = \left\{ \begin{array}{l} \langle a, 0.8, 0.1, 0.2\rangle, \langle a, 0.7, 0.1, 0.3\rangle, \langle a, 0.6, 0.2, 0.4\rangle, \\ \langle b, 0.7, 0.2, 0.3\rangle, \langle b, 0.6, 0.3, 0.4\rangle, \langle b, 0.4, 0.2, 0.5\rangle \end{array} \right\}$$

$$\mathcal{B} = \left\{ \begin{array}{l} \langle a, 0.9, 0.1, 0.1\rangle, \langle a, 0.8, 0.1, 0.2\rangle, \langle a, 0.7, 0.2, 0.3\rangle, \\ \langle b, 0.8, 0.2, 0.2\rangle, \langle b, 0.7, 0.2, 0.3\rangle, \langle b, 0.5, 0.2, 0.4\rangle \end{array} \right\}$$

Then $\tau_\mu^* = \{0_{\mathscr{N}\mathscr{M}}, 1_{\mathscr{N}\mathscr{M}}, \mathcal{B}\}$ is a supra neutrosophic multiset topological space.
Then $SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B}) = 1_{\mathscr{N}\mathscr{M}}$, $SInt_{\mathscr{N}\mathscr{M}}(SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B})) = 1_{\mathscr{N}\mathscr{M}}$, which is not SNMROS.
Remark 4.10. The union of any two SNMROSs (SNMRCoS) need not be an SNMROS (SNMRCoS).

Example 4.11. Let $X = \{a, b\}$ and

$$\tau_\mu^* = \{0_{\mathscr{N}\mathscr{M}}, 1_{\mathscr{N}\mathscr{M}}, \mathcal{A}, \mathcal{B}, \mathcal{A} \cup \mathcal{B}\}$$

be a supra neutrosophic multiset topological space, where

$$\mathcal{A} = \left\{ \begin{array}{l} \langle a, 0.4, 0.5, 0.6\rangle, \langle a, 0.3, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.8\rangle, \\ \langle b, 0.7, 0.5, 0.3\rangle, \langle b, 0.6, 0.5, 0.4\rangle, \langle b, 0.4, 0.5, 0.6\rangle \end{array} \right\}$$

$$\mathcal{B} = \left\{ \begin{array}{l} \langle a, 0.6, 0.5, 0.4\rangle, \langle a, 0.7, 0.5, 0.3\rangle, \langle a, 0.8, 0.4, 0.2\rangle, \\ \langle b, 0.3, 0.5, 0.7\rangle, \langle b, 0.4, 0.5, 0.6\rangle, \langle b, 0.6, 0.5, 0.4\rangle \end{array} \right\}$$

$$\mathcal{A}\cup\mathcal{B}=\left\{\begin{array}{l}\langle a,0.6,0.5,0.4\rangle,\langle a,0.7,0.5,0.3\rangle,\langle a,0.8,0.4,0.2\rangle,\\ \langle b,0.7,0.5,0.3\rangle,\langle b,0.6,0.5,0.4\rangle,\langle b,0.4,0.5,0.6\rangle\end{array}\right\}$$

Here, $SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})=\mathcal{B}$, $SInt_{\mathscr{N}\mathscr{M}}(SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}))=\mathcal{A}$, and $SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B})=\mathcal{A}^C$, $SInt_{\mathscr{N}\mathscr{M}}(SCl_{\mathscr{N}\mathscr{M}}(\mathcal{B}))=\mathcal{B}$. Then $SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}\cup\mathcal{B})=1_{\mathscr{N}\mathscr{M}}$. Thus, $SInt_{\mathscr{N}\mathscr{M}}(SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}\cup\mathcal{B}))=1_{\mathscr{N}\mathscr{M}}$. Hence, $\mathcal{A}$ and $\mathcal{B}$ are SNMROS, but $\mathcal{A}\cup\mathcal{B}$ is not SNMROS.

Theorem 4.12.

The union of any two SNMRCoSs is an SNMRCoS.

Proof:

Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be any two SNMROSs of an SNMTS (X,τ_μ^*). Since $\mathcal{A}_1\cup\mathcal{A}_2$ is SNMOS (from Remark 3), we have $\mathcal{A}_1\cup\mathcal{A}_2\supseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\subseteq SInt_{\mathscr{N}\mathscr{M}}(\mathcal{A}_1\cup\mathcal{A}_2))$. Now,

$$NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\subseteq SInt_{\mathscr{N}\mathscr{M}}(\mathcal{A}_1\cup\mathcal{A}_2))\supseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\sim Int_{\mathscr{N}\mathscr{M}}(\mathcal{A}_1))=\mathcal{A}_1$$

and

$$NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\sim Int_{\mathscr{N}\mathscr{M}}(\mathcal{A}_1\cup\mathcal{A}_2))\supseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\subseteq SInt(\mathcal{A}_2))=\mathcal{A}_2$$

implies that $\mathcal{A}_1\cup\mathcal{A}_2\subseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\subseteq SInt_{\mathscr{N}\mathscr{M}}(\mathcal{A}_1\cup\mathcal{A}_2))$, hence the theorem. □

Theorem 4.13.

(i) The closure of an SNMOS is an SNMRCoS;

(ii) The interior of an SNMCoS is an SNMROS.

Proof:

(i) Let $\mathcal{A}$ be an SNMOS of an SNMTS $(X,\tau^{\mu}_{\mathscr{N}\mathscr{M}})$. Clearly, $NM\sim Int_{\mathscr{N}\mathscr{M}}(NM\sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}))\subseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})$. Now, $\mathcal{A}$ is SNMOS implies that $\mathcal{A}\subseteq NM\sim Int_{\mathscr{N}\mathscr{M}}(NM\sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A}))$, and hence, $NM\sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})\subseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\sim Int_{\mathscr{N}\mathscr{M}}(NM\sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})))$. Thus, $NM\sim SCl_{\mathscr{N}\mathscr{M}}(\mathcal{A})$ is SNMRCoS;

(ii) Let $\mathcal{A}$ be an SNMCoS of an SNMTS $(X,\tau^{\mu}_{\mathscr{N}\mathscr{M}})$. Clearly, $NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\sim Int_{\mathscr{N}\mathscr{M}}(\mathcal{A}))\supseteq NM\sim Int_{\mathscr{N}\mathscr{M}}(\mathcal{A})$. Now, $\mathcal{A}$ is SNMCoS implies that $\mathcal{A}\supseteq NM\sim SCl_{\mathscr{N}\mathscr{M}}(NM\sim Int_{\mathscr{N}\mathscr{M}}(\mathcal{A}))$, and hence, $NM\sim Int_{\mathscr{N}\mathscr{M}}(\mathcal{A})\supseteq NM\sim Int_{\mathscr{N}\mathscr{M}}(NM\sim SCl_{\mathscr{N}\mathscr{M}}$

Definition 4.14. Let $\phi:(X,\tau_\mu^*)\to(Y,\tau_{\mu1}^*)$ be a mapping from an SNMSTS (X,τ_μ^*) to another SNMSTS $(Y,\tau_{\mu1}^*)$. Then ϕ is known as a supra neutrosophic multiset continuous mapping (SNMCM) if $\phi^{-1}(\mathcal{A})\in\tau_\mu^*$ for each $\mathcal{A}\in\tau_{\mu1}^*$, or equivalently $\phi^{-1}(\mathcal{B})$ is an SNMCoS of X for each SNMCoS $\mathcal{B}$ of Y.

Example 4.15. Let $X=Y=\{a,b,c\}$ and
$\mathcal{A}=\{\langle a,0.4,0.5,0.6\rangle,\langle a,0.3,0.5,0.7\rangle,\langle a,0.2,0.6,0.8\rangle,\langle b,0.3,0.5,0.4\rangle,\langle b,0.2,0.5,0.6\rangle,\langle b,0.1,0.5,0.7\rangle,\langle c,0.4,0.5,0.6\rangle,\langle c,0.3,0.5,0.7\rangle,\langle c,0.2,0.6,0.8\rangle\}$,
$\mathcal{B}=\{\langle a,0.6,0.1,0.2\rangle,\langle a,0.5,0.1,0.3\rangle,\langle a,0.4,0.2,0.4\rangle,\langle b,0.3,0.5,0.4\rangle,\langle b,0.2,0.5,0.6\rangle,\langle b,0.1,0.5,0.7\rangle,\langle c,0.4,0.5,0.6\rangle,\langle c,0.3,0.5,0.7\rangle,\langle c,0.2,0.6,0.8\rangle\}$.

Then $\tau_\mu^*=\{0_{\mathscr{N}\mathscr{M}},1_{\mathscr{N}\mathscr{M}},\mathcal{A}\}$ and $\tau_{\mu1}^*=\{0_{\mathscr{N}\mathscr{M}},1_{\mathscr{N}\mathscr{M}},\mathcal{B}\}$ are supra neutrosophic multiset topological spaces. Now, define a mapping $f:(X,\tau_\mu^*)\to(Y,\tau_{\mu1}^*)$ by $f(a)=f(c)=c$ and $f(b)=b$. Thus, f is SNMCM.

Definition 4.16. Let $\phi:(X,\tau_\mu^*)\to(Y,\tau_{\mu1}^*)$ be a mapping from an SNMSTS (X,τ_μ^*) to another SNMSTS $(Y,\tau_{\mu1}^*)$. Then ϕ is called a supra neutrosophic multiset open mapping (SNMoM) if $\phi(\mathcal{A})\in\tau_{\mu1}^*$ for each $\mathcal{A}\in\tau_\mu^*$.

Definition 4.17. Let $\phi : (X, \tau_\mu^*) \to (Y, \tau_{\mu 1}^*)$ be a mapping from an SNMTS (X, τ_μ^*) to another SNMTS $(Y, \tau_{\mu 1}^*)$. Then ϕ is said to be a supra neutrosophic multiset closed mapping (SNMCoM) if $\phi(\mathcal{B})$ is an SNMCoS of Y for each SNMCoS $\mathcal{B}$ of X.

Definition 4.18. Let $\phi : (X, \tau_\mu^*) \to (Y, \tau_{\mu 1}^*)$ be a mapping from an SNMTS (X, τ_μ^*) to another SNMTS $(Y, \tau_{\mu 1}^*)$. Then ϕ is called a supra neutrosophic multi semi-continuous mapping (SNMSCM) if $\phi^{-1}(\mathcal{A})$ is the SNMSOS of X, for each $\mathcal{A} \in \tau_{\mu 1}^*$.

Definition 4.19. Let $\phi : (X, \tau_\mu^*) \to (Y, \tau_{\mu 1}^*)$ be a mapping from an SNMTS (X, τ_μ^*) to another SNMTS $(Y, \tau_{\mu 1}^*)$. Then ϕ is called a supra neutrosophic multi semi-open mapping (SNMSOM) if $\phi(\mathcal{A})$ is a SOSNMS for each $\mathcal{A} \in \tau_\mu^*$.

Example 4.20. Let $X = Y = \{a, b, c\}$ and

$\mathcal{A} = \{\langle a, 0.6, 0.1, 0.2\rangle, \langle a, 0.5, 0.1, 0.3\rangle, \langle a, 0.4, 0.2, 0.4\rangle, \langle b, 0.3, 0.5, 0.4\rangle, \langle b, 0.2, 0.5, 0.6\rangle, \langle b, 0.1, 0.5, 0.7\rangle, \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.3, 0.5, 0.7\rangle, \langle c, 0.2, 0.6, 0.8\rangle\}$,

$\mathcal{B} = \{\langle a, 0.3, 0.5, 0.4\rangle, \langle a, 0.2, 0.5, 0.6\rangle, \langle a, 0.1, 0.5, 0.7\rangle, \langle b, 0.6, 0.1, 0.2\rangle, \langle b, 0.5, 0.1, 0.3\rangle, \langle b, 0.4, 0.2, 0.4\rangle, \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.3, 0.5, 0.7\rangle, \langle c, 0.2, 0.6, 0.8\rangle\}$.

Then $\tau_{\mu 1}^* = \{0_{\mathcal{NM}}, 1_{\mathcal{NM}}, \mathcal{A}\}$ and $\tau_{\mu 1}^* = \{0_{\mathcal{NM}}, 1_{\mathcal{NM}}, \mathcal{B}\}$ are supra neutrosophic multiset topological spaces. Clearly, $\mathcal{A}$ is a semi-open set. Then a mapping $f : (X, \tau_\mu^*) \to (Y, \tau_{\mu 1}^*)$ defined by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Hence, f is SNMSOM.

Definition 4.21. Let $\phi : (X, \tau_\mu^*) \to (Y, \tau_{\mu 1}^*)$ be a mapping from an SNMSTS (X, τ_μ^*) to another SNMTS (Y, τ_μ^*). Then ϕ is called a supra neutrosophic multiset semi-closed mapping (SNMSCoM) if $\phi(\mathcal{B})$ is an SNMSCoS for each SNMCoS $\mathcal{B}$ of X.

Remark 4.22. From Remark 1, an SNMCM (SNMOM, SNMCoM) is also an SNMSCM (SNMSOM, SNMSCoM).

Example 4.23. Let $X = Y = \{a, b, c\}$ and

$\mathcal{A} = \{\langle a, 0.4, 0.5, 0.6\rangle, \langle a, 0.3, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.8\rangle, \langle b, 0.3, 0.5, 0.4\rangle, \langle b, 0.2, 0.5, 0.6\rangle, \langle b, 0.1, 0.5, 0.7\rangle, \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.3, 0.5, 0.7\rangle, \langle c, 0.2, 0.6, 0.8\rangle\}$

$\mathcal{B} = \{\langle a, 0.4, 0.5, 0.6\rangle, \langle a, 0.3, 0.5, 0.7\rangle, \langle a, 0.2, 0.6, 0.8\rangle, \langle b, 0.4, 0.6, 0.4\rangle, \langle b, 0.3, 0.5, 0.5\rangle, \langle b, 0.2, 0.5, 0.6\rangle, \langle c, 0.6, 0.5, 0.5\rangle, \langle c, 0.4, 0.5, 0.6\rangle, \langle c, 0.2, 0.6, 0.9\rangle\}$.

Then $\tau_\mu^* = \{0_{\mathcal{NM}}, 1_{\mathcal{NM}}, \mathcal{A}\}$ and $\tau_{\mu 1}^* = \{0_{\mathcal{NM}}, 1_{\mathcal{NM}}, \mathcal{B}\}$ are supra neutrosophic multiset topological spaces. Let us define a mapping $f : (X, \tau_\mu^*) \to (Y, \tau_{\mu 1}^*)$ by $f(a) = f(c) = c$ and $f(b) = b$. Thus, f is SNMSCM, which is not an SNMCM.

Conclusion

In this paper we established some properties of the supra neutrosophic multi- set topological space such as supra neutrosocphic multiset topological space, supra neutrosophic open multiset,supra neutrosophic closed multiset, supra neutrosophic interior multiset, supra neutrosophic closure multiset and their theorem and properties. Also we have introduced the notion of the supra neutrosophic multiset and examined some properties.

References

[1] Md. Abdel-Basset, A.Gamal, G. Manogaaran, and H.V.Long, *A novel group decision making model based on neutrosophic disease diagnosis*, Multimedia Tools and Applications, (2019),1-26.

[2] B. Basumatary, N. Wary, D.D. Mwchahary, A.K.Brahma, J. Basumatary, and J. Basumatary, *A study on some properties of neutrosophic multi topological group*, 13(9),(2021),1689.

[3] B. Basumatary, N. Wary, and S. Broumi, *On some properties of interior, closure, boundary and exterior of neturosophic multi topological space*, International Journals of Neutrosophic Science, 17(1),(2021),30-40.

[4] S.A.El-Sheikh, R.A-K. Omar, M.Raafat, *Supra M-topological space and decompositions of some types of supra msets*, International Journals of Mathematics Trends and Techonlogy., 20 (1), 2015.

[5] M.E.El-Shafei, A.H.Zakari and T.M.Al-shami, *Some Applications of supra Preopen sets*, Journals of Mathematics., ID 9634206, 2020.

[6] Fathi Hesham Khedr, *On Supra Topological Space*, Indian Journal Pure and Applied Mathematics, 14(4), (1983), 502-510.

[7] K. P. Girish, Sunil Jacob John, *Relation and function in multiset context.*, Information of science, 179, (2009), 758-768.

[8] K. P. Girish, Sunil Jacob John, *Multiset topologies induced by multiset relation.*, Information of science, 188, (2012), 298-313.

[9] F.G. Lupianez, *On Neurtosophic topologies* , The International Journal of system and cybernetics, 37(6), (2008),797-800.

[10] A. Salama, S.Alblowi, *Neutrosophic set and Neutrosophic Topological spaces*, IOSR Journal of Mathematics,3(4),(2012), 31-35.

[11] F.Smarandache, *Introduction to Neutrosophic Statistics*, Ed-1, Sitech and Education Publishing, Columbus, Ohio, USA,(2014).

[12] Y. Tella, S.Daniel, *Anote on power whole multiset of a multiset topologies*, Journal of Computer and Mathematical Sciences, 5(1), (2014),1-122.

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