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Comparative Analysis of Neutrosophic, Pythagorean neutrosophic, and Fermatean neutrosophic Soft Matrices in the context of Industrial Accidents: A Case Study

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Abstract: Industrial accidents pose significant risks to human lives, the environment, and economic stability. This study presents a comparative analysis of three plithogenic frameworks—Neutrosophic soft sets, Pythagorean neutrosophic soft sets, and Fermatean neutrosophic soft sets—aiming to model and analyze industrial accidents. The primary objective is to evaluate the effectiveness of these soft sets in handling the uncertainties, complexities, and imprecisions inherent in accident data. Utilizing Dhar's (2021) algorithm for Neutrosophic soft matrices, we introduce a new parameter called a risk score, which consolidates various values for each industry or parameter into a singular magnitude. Our findings indicate that neutrosophic soft sets are more accurate than Pythagorean neutrosophic soft sets and Fermatean neutrosophic soft sets. The study concludes that adopting these advanced frameworks can enhance decision-making processes and safety protocols, thereby mitigating the adverse impacts of industrial accidents.

Keywords: Industrial Accidents; Pythagorean Neutrosophic Soft Sets; Fermatean Neutrosophic Soft Sets; Uncertainty Handling; Safety Protocols.

1. Introduction

Plithogeny is a theory developed by Florentin Smarandache that introduces the notion of a "plithogenic set," which is a union of multiple sets. This allows for the representation of complex ideas by combining simpler ones. Neutrosophic sets, also introduced by Florentin Smarandache, extend the classical notion of sets to accommodate indeterminate or uncertain information. Unlike classical sets, where an element either belongs or does not belong to the set, neutrosophic sets allow for elements to have degrees of membership, non-membership, and indeterminacy simultaneously. This makes them suitable for modeling and analyzing situations where the boundaries between categories are fuzzy or where the information is incomplete or contradictory [1].

The idea of fuzzy sets was introduced by Zadeh in "Fuzzy sets. Information and control (1965)" [2]. Fuzzy sets are required as classical sets do not provide a good representation of the uncertainties encountered in real-life scenarios. Fuzzy sets contain a membership value and a non-membership value ranging from 0 to 1, pertaining to the belongingness and non-belongingness of an element to the universal set. It is not always that the value of the summation of membership and non- membership values is 1; hence a degree of uncertainty was introduced by Atanassov who generalized fuzzy sets and named the new set, intuitionistic fuzzy set [3].

Intuitionistic fuzzy sets contain a hesitation margin to quantify the ambiguity of human nature in different situations. The hesitation degree can take values from 0 to 1. A null hesitation degree indicates that the values of membership and non-membership of an element are certain and are agreed upon by everyone present. But in most cases, a degree of uncertainty exists as in decision- making problems, sales analysis, new product marketing, financial services, etc. Modeling and analyzing real-life problems in various areas such as disaster management, marketing, environment, medicine, economics, social sciences, etc., requires a hesitation margin, but most of the modeling and computation tools use classical sets. Hence theories namely the theory of probability, evidence, fuzzy set, intuitionistic fuzzy set, rough set, etc., were introduced for dealing with uncertainties. Expanding upon these theories by addressing their difficulties, Molodtsov introduced the concept of soft sets [4].

Soft sets have a good potential in solving practical problems in various areas of difficulty. Maji and colleagues, in their works, extensively explored the theory of fuzzy soft sets [5]. Additionally, subsequent research expanded the theory to include intuitionistic fuzzy soft sets [6]. Smarandache further generalized soft sets into hypersoft sets, applying them in decision-making processes [7]. Furthermore, Vellapandi and Gunasekaran investigated a novel decision-making approach utilizing multi soft set logic [8]. These studies collectively contribute to the advancement and application of soft set theories in various domains.

Smarandache introduced the concepts of neutrosophic sets to deal with uncertain and conflicting information present in belief systems [9]. To further extend the capabilities of neutrosophic sets, Smarandache introduced Pythagorean neutrosophic sets, and Senapati & Yager introduced Fermatean neutrosophic sets [10]. It is assumed that these neutrosophic sets produce more accurate results. An application of neutrosophic sets was seen in the paper "Neutrosophic soft matrices and its application in medical diagnosis" by Mamoni Dhar, which uses neutrosophic sets in diagnostic sciences [11]. This paper extends the approach by doing a comparative analysis of neutrosophic sets, Pythagorean neutrosophic sets, and Fermatean neutrosophic sets in analyzing industrial accidents.

1.1 Preliminaries

Some vital definitions for terms and concepts used throughout the paper are listed below:

Classical Set: A classical set is a collection of distinct elements where each element either belongs to the set or does not. Membership is binary and unambiguous.

Example: The set of natural numbers less than 5, $A = \{1,2,3,4\}$.

Fuzzy set: A fuzzy set consists of elements whose membership values range from 0 to 1. If þ(x) represents the degree of membership of x in a set, then þ(x) = 1 implies x is fully in the set, þ(x) = 0 implies x is not in the set at all and 0<þ(x)<1 implies x is partially in the set.

Example: Consider a fuzzy set A consisting of how tall the people are in a universal set of discourse U={P, Q, R, S}. The membership function þ(x) assigns a degree of membership based on their height, let þ (P)=0.9 (P is very tall), þ(Q) = 0.6 (Q is moderately tall), þ(R)=0.7 (R is also moderately tall) and þ(S) = 0.3 (S is less tall). The fuzzy set A can be represented as, A={(P,0.9),(Q,0.6),(R,0.7),(S,0.3)}.

Intuitionistic Fuzzy Set: An intuitionistic fuzzy consists of a membership function θ: U → [0,1] and a non-membership function φ: U → [0,1] to illustrate how much each element x in U belongs and does not belong to a set. The condition is that 0£ θ(x)+ φ(x) £1.

Example: Consider an intuitionistic fuzzy set A representing tall people in a universe U = {P, Q, R, S} where, {θ (P)=0.9, φ(P)=0.05}, {θ(Q)=0.6, φ(Q)=0.3}, {θ(R)=0.7, φ(R)=0.2},{θ(S)=0.3, φ(S)=0.6}. Then intuitionistic fuzzy set A can be represented as, A={(P,(0.9,0.05)),(Q,(0.6,0.3)),(R,(0.7,0.2)),(S,(0.3,0.6))}.

Neutrosophic set: A neutrosophic set consists of a membership function θ: U → [0,1], a non-membership function φ: U → [0,1], and an indeterminacy membership function ϕ: U → [0,1]. These three functions characterize each element x of a universal set U on how much they belong, do not belong and uncertainty of each element in a set. The condition is that 0£ θ(x)+ φ(x)+ ϕ(x) £3.

Example: Consider an intuitionistic fuzzy set A representing tall people in a universe U = {P, Q, R, S} where, {θ (P)=0.9, ϕ(P)=0.1, φ(P)=0.05}, {θ(Q)=0.6, ϕ(Q)=0,2, φ(Q)=0.3}, {θ(R)=0.7, ϕ(R)=0.1, φ(R)=0.2},{θ(S)=0.3, ϕ(S)=0.4, φ(S)=0.6}. Then intuitionistic fuzzy set A can be represented as, A={(P,(0.9,0.1,0.05)),(Q,(0.6,0.2,0.3)),(R,(0.7,0.1,0.2)),(S,(0.3,0.4,0.6))}.

Pythagorean neutrosophic set: A Pythagorean neutrosophic set consists of a membership function θ: U → [0,1], a non-membership function φ: U → [0,1] , and an indeterminacy membership function ϕ: U → [0,1] where each element x in the universal set U must satisfy 0£ $θ(x)^2$+ $φ(x)^2$+ $ϕ(x)^2$ £1.

Fermatean neutrosophic set: A fermatean neutrosophic set consists of a membership function θ: U → [0,1], a non-membership function φ: U → [0,1] and an indeterminacy membership function ϕ: U → [0,1] where each element x in the universal set U must satisfy 0£ $θ(x)^3$+ $φ(x)^3$+ $ϕ(x)^3$ £1.

Soft set: A soft set is a mapping from a certain set of parameters to a subset under the set of universal discourse. If Z is the mapping, then the soft set is represented by, Z: X→ P(U) where X is the set consisting of the parameters and P is the power set of the Universe of discourse, U. (X, Z) is called a soft set over U.

Example: Let the universe of discourse U be a set of cars, U = {car1, car2, car3}. Let the set of parameters consist of the fuel on which the cars run, represented by X, X = {biodiesel(x1), diesel(x2), electric(x3)}. The mapping Z can be defined as follows, Z(x1) = {car1, car3}, Z(x2) = {car2, car3}, and Z(x3) = {car2}. The soft set (X, Z) can be represented as, (X, Z) = {(x1, {car1, car3}), (x2, {car2, car3}), (x3, {car2})}

Soft Matrix: A soft matrix is a representation of soft sets in matrix form, where rows and columns represent different parameters and their corresponding values.

Example: Let the universe of discourse U be a set of cars, U = {car1, car2, car3}. Let the set of parameters consist of the fuel on which the cars run, represented by X, X = {biodiesel(x1), diesel(x2), electric(x3)}. The mapping Z can be defined as follows, Z(x1) = {car1, car3}, Z(x2) = {car2, car3}, and Z(x3) = {car2}. The soft matrix can be represented by,

Table (i)

X/U	**Car1**	**Car2**	**Car3**
Biodiesel(x1)	1	0	1
Diesel(x2)	0	1	1
Electric(x3)	0	1	0

Fuzzy Soft Set: A fuzzy soft set combines the concepts of fuzzy sets and soft sets, where the mapping involves fuzzy sets instead of classical sets, allowing for partial membership.

Example: Let the universe of discourse U be a set of cars, U = {car1, car2, car3}. Let the set of parameters consist of features of the cars, represented by X, X = {comfort(x1), fuel efficiency(x2), safety(x3)}. The fuzzy soft set (X,Z) can be represented in matrix form as,

Table (ii)

X/U	**Car1**	**Car2**	**Car3**
Comfort(x1)	0.8	0.4	0.6
Fuel efficiency(x2)	0.7	0.9	0.5
Safety(x3)	0.5	0.6	0.9

Neutrosophic soft matrix: A neutrosophic soft matrix combines the principle of neutrosophic sets and soft matrices hence allowing a mapping between parameters and subsets to be characterized with a membership value θ: U → [0,1], a non-membership value φ: U → [0,1], and an indeterminacy value ϕ: U → [0,1].

Example: Let M be a neutrosophic soft matrix of a set of cars, U = {car1, car2, car3} based on the parameters X = {comfort(x1), fuel efficiency(x2)}. Each element m_{ij} of M is represented as $m_{ij} = (\theta_{ij}(x), \phi_{ij}(x), \varphi_{ij}(x))$

Table (iii)

X/U	**Car1**	**Car2**	**Car3**
Comfort(x1)	(0.9, 0.05, 0)	(0.7, 0.2, 0)	(0.8, 0.1, 0)
Fuel efficiency(x2)	(0.8, 0, 0.1)	(0.6, 0, 0.3)	(0.7, 0, 0.2)

Pythagorean neutrosophic soft matrix: A pythagorean neutrosophic soft matrix combines the principle of pythagorean neutrosophic sets and soft matrices hence allowing mapping between parameters and subsets to be characterized with a membership value θ: U → [0,1], a non-membership value φ: U → [0,1], and an indeterminacy value ϕ: U → [0,1] where each element x in the universal set U must satisfy 0£ $\theta(x)^2+ \varphi(x)^2+ \phi(x)^2$ £1.

Fermatean neutrosophic soft matrix: A fermatean neutrosophic soft matrix combines the principle of fearmatean neutrosophic sets and soft matrices hence allowing mapping between parameters and subsets to be characterized with a membership value θ: U → [0,1], a non-membership value φ: U → [0,1], and an indeterminacy value ϕ: U → [0,1] where each element x in the universal set U must satisfy 0£ $\theta(x)^3+ \varphi(x)^3+ \phi(x)^3$ £1.

Complement of Neutrosophic soft matrix, Pythagorean neutrosophic soft matrix and Fermatean neutrosophic soft matrix: Let M be a soft matrix such that M Î NSM_{ij} or M Î $PNSM_{ij}$ or M Î $FNSM_{ij}$. An element of M denoted by m_{ij} for all i and j values is described as $m_{ij} = (\theta_{ij}(x), \phi_{ij}(x), \varphi_{ij}(x))$ then the complement of M is denoted by M^c where the elements of M^c is given by $m_{ij}^c = (\varphi_{ij}(x), \phi_{ij}(x), \theta_{ij}(x))$ for all i and j values.

Example: Let M be a neutrosophic soft matrix of a set of cars, U = {car1, car2, car3} based on the parameters X = {comfort(x1), fuel efficiency(x2)}.

Table (iv)

X/U	**Car1**	**Car2**	**Car3**
Comfort(x1)	(0.9, 0.05, 0)	(0.7, 0.2, 0)	(0.8, 0.1, 0)
Fuel efficiency(x2)	(0.8, 0, 0.1)	(0.6, 0, 0.3)	(0.7, 0, 0.2)

The complement of M is M^c.

Table (v)

X/U	**Car1**	**Car2**	**Car3**
Comfort(x1)	(0, 0.05, 0.9)	(0, 0.2, 0.7)	(0, 0.1, 0.8)
Fuel efficiency(x2)	(0.1, 0, 0.8)	(0.3, 0, 0.6)	(0.2, 0, 0.7)

Max-min Product of NSM, PNSM, FNSM: Let M and N be two neutrosophic soft matrices, represented as, $M = [\theta_{ij}^M, \phi_{ij}^M, \varphi_{ij}^M]$ and $N = [\theta_{ij}^N, \phi_{ij}^N, \varphi_{ij}^N]$. The max-min product of the two neutrosophic soft matrices M and N, denoted as $M*N$ is defined as follows:

$(M*N)_{ij} =[\max(\min(\theta_{ij}^M,\theta_{ij}^N)),\min(\max(\phi_{ij}^M,\phi_{ij}^N)),\min(\max(\varphi_{ij}^M,\varphi_{ij}^N))]$

1.2 Motivation

In the realm of mathematical computations involving soft matrices, current research focuses only on neutrosophic soft matrices. This paper seeks to expand this by introducing novel methodologies for soft matrix calculations specifically for Pythagorean neutrosophic and Fermatean neutrosophic matrices. By doing so, it not only enhances the understanding of these matrix types but also compares their results, offering a comprehensive discussion on the reasons behind the observed differences. This comparative analysis is a novel contribution, providing deeper insights into the properties and applications of these matrices.

In the field of chemical engineering, ensuring industrial safety is paramount to preventing accidents and safeguarding human lives and the environment. This paper addresses this critical concern by identifying the most prevalent types of accidents in various industries and the primary causes behind them. Utilizing the innovative concept of a risk score, it quantifies these causes, thereby providing a practical framework for risk assessment and management. This approach not only highlights potential hazards but also offers a systematic method for mitigating risks, ultimately contributing to safer industrial practices.

1.3 Theory

1.3.1. Types of industries:

1. *Petrochemical Industry*

   The petroleum industry manufactures chemicals from crude oil and natural gas. The industry's products are plastics, fertilizers, solvents, and many other substances used as raw materials in different branches of the economy. This industry has many different types of disasters that might take place at any given time. Explosions might happen when dealing with explosive liquids or gases and their leakages or accidental ignition. Because of the presence of highly inflammable materials, these fires become particularly hazardous [16]. Unintentional chemical releases may occur during processing or transportation with considerable health and environmental impacts [36]. Earthquakes or floods, among other natural phenomena, can damage infrastructure, exposing chemicals to humans and the environment [37]. Mistakes in operation together with oversights are the major causes of accidents due to human error in running equipment that uses chemicals like in industries [40]. Design factors, such as faulty equipment and poor design, can lead to breakdowns [28]. Simultaneously, improper maintenance and management failures, also known as maintenance factors, might enhance the probability of accidents [43].

2. *Oil and Gas Industry*

   Oil and gas is the industry that involves the exploration, extraction, refining, and transportation of oil and natural gas to produce fuels, lubricants, and feedstock for petrochemicals. One risk is the rise of accidents in this field. Many of those happen due to high pressure for gas storage or any substances which cause explosions when mixed and exposed to fire. Thus, fire outbreaks might occur once there is oil spillage or any gaseous compound leaks out [47]. One of the primary concerns about pollution is that it causes spills and leaks, leading to vast environmental damage [50]. The safety of the structure is at risk during weather events and natural disasters that are natural hazards [19]. Major accidents can be caused by human error or neglect in operation [51]. Catastrophic failures can happen when there is a failure in the design or equipment of a system [42]. Unsafe conditions leading to an increase in the probability of accidents may be brought about by inadequate maintenance practices referred to as maintenance factors [45].

3. *Power Generation Industry*

   Various sources are used to generate electricity within the power generation industry; for example, fossil fuels, nuclear power, as well as renewable resources like wind power or solar energy among others. Gas explosions or boiler explosions are some of the common accidents that happen in this industry [29]. Fires caused by electrical faults or overheating are also common occurrences within this sector [20]. However, runaway reactions, especially in nuclear power plants, may be catastrophic [22]. Infrastructure suffers damage due to natural hazards like extreme weather, which in turn causes failures [21]. Accidents happen because of monitoring errors or control failures caused by human mistakes [39]. Also causing accidents is the failure of vital parts of a design or faults with some elements of it [41]. Another cause of accidents is poor maintenance practices that compromise safety systems, increasing the chances of their occurrence [46].

4. *Chemicals and Solvents Industry*

   In the field of chemistry, researchers work on creating a myriad of chemicals as well as solvents which are used in multiple ways from industry to home goods. Chemical releases may occur unintentionally during processing procedures or their transportation, thereby risking life and the environment [30]. Due to the flammability of a lot of chemicals, fires can be especially massive [48]. Environmental pollution easily takes place whenever disposal is done incorrectly or in case leakages happen on any type of chemical [31]. Natural crises can generate a failure in the container that will create an incident, just as human mistakes can lead to accidents when handling chemicals incorrectly [39]. Problems with poorly designed storage or handling equipment that have been identified as design factors may trigger equipment failure [28]. Furthermore, lack of prevention or management defaults, also referred to as maintenance factors, improve risks from accidents [44].

5. *Food and Water Industry*

   The food and water sector includes manufacturing, processing, and distribution. These sectors encompass agricultural companies, food processors, and water works among them. Pollution exposes us to great risk because our sources may be poisoned either in the form of food crops or drinking water [33]. Even mishandling induces diseases if not hazardous substances that contaminate these sources directly [24]. Sometimes chemicals are mistakenly discharged while dealing with them during their treatments [35]. Natural disasters, for example, can interrupt supply routes or pollute resources, while wrong processing due to human errors such as handling causes accidents [37]. Malfunctions arise from machinery designs as a type of processing or treatment fault, and lastly, risks can be ramped up due to incompetence in managing resources, otherwise known as maintenance-related factors [38].

1.3.2. Types of accidents

1. *Explosions*

   Violent energy bursts happening when highly pressurized combustible gases, vapors, or dust encounter flames can result in catastrophic damage to property and human fatalities as well. Explosions in this sector mainly arise from equipment failure, which leads to spillage of these dangerous substances. When these escape and combine with oxygen in the atmosphere and encounter ignition sources, vast explosions occur [47]. Accessible gases or dust can make eruptions in power generation or particularly in gas and ordinary fuel plants [29]. Such events are majorly influenced by design factors, for example, ineffective ventilation, mechanical defaults like damaged control valves or pipes, as well as non-examination during repairs and maintenance [28]. Additionally, infrastructure damages might come up because of natural occurrences like earthquakes, which can trigger outflows and then explosions [37].

2. *Fires*

   Many fires are caused every now and then because of flammable substances getting lit. In the petrochemical, oil and gas, chemicals, and solvents industry, there is a possibility of experiencing leaks or spillages of gases or liquids that ignite because they come into contact with hot surfaces, sparks, or static electricity [16]. Fires start in power generation when machines become too hot or electrical systems fail [20]. Human error such as improper handling of materials, design flaws like inadequate fire suppression systems, and maintenance issues like neglected electrical systems are some of the factors that lead to these incidents [41]. It is important to mention that natural hazards, such as lightning strikes, also play a role in setting off fires [19].

3. *Chemical Release*

   The negligent release of harmful substances is dangerous for human health as well as environmental safety. Chemical release may be experienced while producing or storing items within oil, gas, petrochemicals, chemical industries, and solvents alike. This mainly results from equipment breaking down just like when pipes give way, and tanks rupture, among other things such as human errors through mishandling chemicals [30]. Design factors, including inadequate containment measures, and maintenance errors, such as not replacing worn-out seals or gaskets, play significant roles in these incidents [44].

4. *Pollution*

   Pollution is when harmful substances contaminate air, water, or soil, for example, because of industrial activities. In the petroleum and gas exploration, production, and distribution industries, as well as in the chemical and solvent, food, and water industries, these wastes could be spilled, leaked, or dumped carelessly [33]. There are many things that can lead to this, including equipment breakdowns, for example, tanks that spill out their contents, or errors by people who handle materials poorly; but these factors are equated with natural disasters resulting in contaminants being carried away during rainstorms as well [36]. Moreover, one must also consider other contributing aspects like poor equipment design or improper use due to lack of proper attention on examining these pipes during regular intervals over subsequent weeks or months [28].

5. *Disease*

A petrochemical plant may release chemicals into the atmosphere causing pollution as well as illnesses due to poor maintenance practices or mechanical defects [35]. Due to incompetence in design or human error and chemical release at an oil and gas facility leading to emissions which harm people breathing [51]. The food and water industry can have pandemics as a result of harmful microorganisms contaminating foods or water products [24]. Common causes include poor washing habits, poorly processed foods & mismanaged water sources [34]. In order to mitigate dangers and protect the health of the general public, there is a necessity for proper comprehension of the interconnection between these industries, various accidents encountered within them, and the causes of such accidents [32].

1.3.3. Type of factors

1. *Natural Hazards*

Natural catastrophes take the form of disasters resulting from the interaction between the earth (geology) and its life forms (environment), whether human or physical, which cause harm to human projects like buildings, roads, etc., leading to accidents (unintentional harmful events) [36]. For example, there are calamities like floods in Singapore, while lightning has been found responsible for the greatest number of injuries as well as deaths after thunderstorms [37]. During bursts in petrochemical and oil gas industries, earthquakes may lead to breakages of pipelines, tank explosions, among other things due to vibration or breaking apart some vital facilities [17]. Facilities must be protected from floods due to chemical releases or contamination in the chemicals and solvents industry [30]. Power lines and infrastructure in power generation can be damaged by hurricanes, leading to fire outbreaks and power outages [29]. In the food and water industry, natural calamities can invade water supplies and crops leading to pollution and disease outbreaks [24]. Such unwanted determinants always take place unexpectedly, thus requiring strong design plans and preparedness approaches against them [38].

2. *Human Error/Operation Factors*

The term human error and operational factors are used to talk about mistakes or slips in judgment that are made by industrial processes because of workers [39]. Any individual can make a mistake in different ways, including touching materials wrongly, using the wrong methods with machines, or not taking care of protective clothing seriously when working at a construction site [40]. For instance, if someone who works at a petrochemical plant makes a mistake while regulating temperature levels or pressures in the processing plant, it can result in runaway reactions [16]. Proper shutdown procedures must be adhered to in the oil and gas industry, or else risk fires and the release of chemicals [19]. Similarly, errors in monitoring systems can result in overheating of the equipment, hence fires or explosions in the power generation industry [22]. The food and water industry can cause outbreaks of diseases and contamination due to poor hygiene practices or poor handling of raw materials [34]. Avoiding improper training, following safety protocols, and proper supervision will reduce human errors [43].

3. *Design Factor/Equipment Fault*

Industrial systems and equipment suffer from design factors and equipment faults [41]. Inadequate safety margins, poor material selections, or purposes without enough substitution systems can be a list of design mishandling [28]. Conversely, equipment problems can arise from assembly errors, normal wear and tear, or inadequate testing [42]. Poorly designed storage tanks used in the petrochemical industry may not be able to withstand very high pressures, causing explosions or releases of harmful substances [44]. Faulty valves or pipelines can cause leaks and fires in the oil and gas industry [19]. There can be explosions or runaway reactions in cases of turbine or boiler failures that are defective [22]. In the chemicals and solvents sector, uncontrolled reactions might occur from poorly designed reaction vessels [28]. The food and water industry might also experience contamination and pollution because of equipment failures in processing plants [33]. Regular inspections, robust design standards, and testing are crucial to prevent these issues [46].

4. *Maintenance Factor*

Problems with maintenance could happen due to the oversight of regular maintenance procedures, not replacing worn parts, or the use of low-quality repair methods [45]. Lapses in maintenance control systems could occur because of inappropriate preventive maintenance schedules, absence of skilled personnel, or poor documentation practices [43]. Leaks and explosions are consequences of failing to carry out regular inspections and maintain pipelines in the sector that deals with oil chemicals [19]. Furthermore, fires may result from poor servicing of drilling rigs in oil and gas industries, causing either loss or blowout [18]. Insufficient power will be experienced in case there is an interruption within electric systems that are not often maintained during the generation process [20]. This is also related to chemical releases from solvents, but more specifically if storage tanks are not considered under the chemicals category [31]. Contamination and disease outbreaks can result from inadequate upkeep of processing equipment in the food and water sector [24]. It is crucial to implement a thorough maintenance program and effective management strategies for reliability and safety [38].

2. Materials and Methods

2.1. Algorithm

The algorithm proposed in this study is fundamentally based on the formulae developed by Mamoni Dhar in their work on Neutrosophic Soft Sets, as detailed in reference [11]. Their pioneering methods provided a robust foundation for our case study and allowed us to expand these same concepts and algorithms to Pythagorean Neutrosophic Soft Sets and Fermatean Neutrosophic Soft Sets. The algorithm is detailed as a flowchart below:

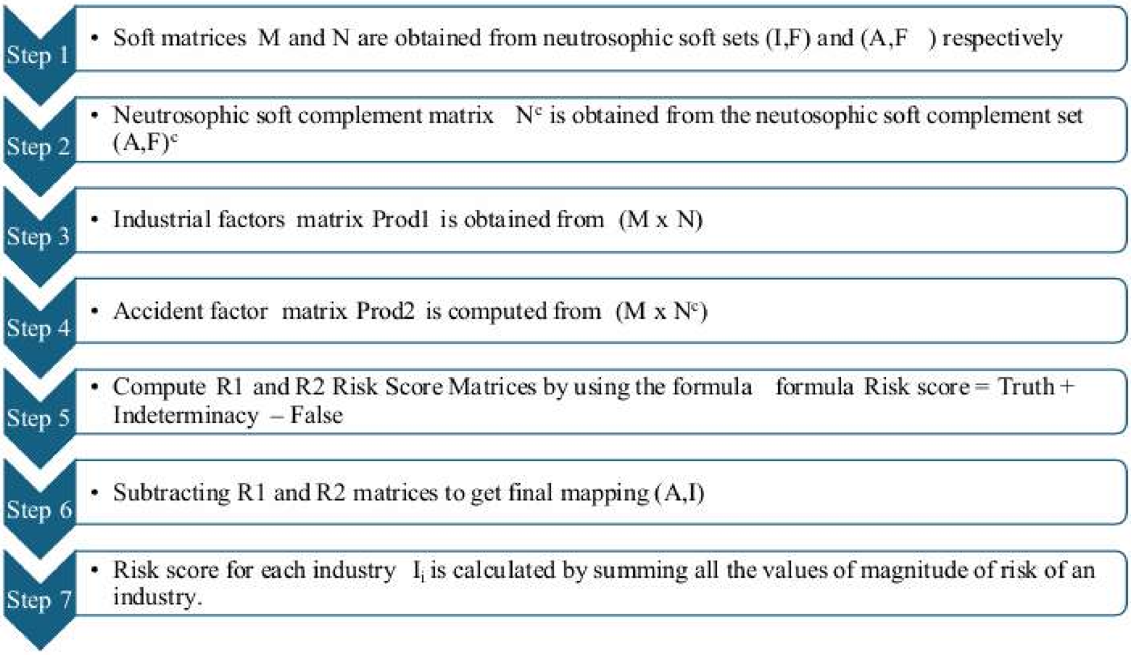


Figure (i)

2.2. Methodology

2.2.1 Neutrosophic Soft Set

Let the overall set of industries be the Universal set 'I' where I = $\{I_1, I_2, I_3, I_4, I_5\}$ such that:

I_1 is the Petrochemical Industry

I_2 is the Oil and Gas Extraction Industry

I_3 is the Energy Generation Industry

I_4 are the Fertilizer and Pharmaceutical Industries

I_5 are the Food and Water Industries

Let the overall type of accident be represented by matrix 'A' where A= $\{A_1, A_2, A_3, A_4, A_5\}$ where:

A_1 is Explosion

A_2 is Fire
A_3 is Chemical release
A_4 is Pollution
A_5 is Disease

The possible factors causing these accidents in the above industries can be represented by matrix 'F' such that F = $\{F_1, F_2, F_3, F_4\}$ where:

F_1 is Natural Disasters
F_2 is Human Error
F_3 is Design Error
F_4 is Maintenance Error

Consider a neutrosophic soft mapping factoring in the types of industries and the factors that can potentially contribute to their accidents. M is a Mapping such that M_N : F $\rightarrow$ M_N^I

(M_N,F)={{ M_N (F_1)={(I_1, 0.35,0.75,0.4),(I_2, 0.4,0.7,0.3), (I_3, 0.35,0.7,0.5), (I_4, 0.1,0.8,0.2) , (I_5, 0.1,0.8,0.25)},

{M_N (F_2)={(I_1, 0.65,0.4,0.3),(I_2, 0.55,0.3,0.7), (I_3, 0.75,0.3,0.4), (I_4, 0.25,0.8,0.2), (I_5, 0.3,0.6,0.4) },

M_N (F_3)={(I_1, 0.4,0.65,0.65),(I_2, 0.8,0.2,0.5), (I_3, 0.6,0.45,0.6), (I_4, 0.15,0.85,0.25), (I_5, 0.15,0.7,0.5) },

{M_N (F_4)={(I_1, 0.55,0.45,0.3),(I_2, 0.65,0.25,0.6), (I_3, 0.7,0.3,0.45), (I_4, 0.35,0.7,0.15), (I_5, 0.15,0.7,0.5)}}.

Collating the mapping M into a Neutrosophic soft matrix 'M_N' between Universal sets I and F.

$$M_N = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{cccc} F_1 & F_2 & F_3 & F_4 \\ \end{array}$$

$$M_N = \begin{bmatrix} (0.35,0.75,0.4) & (0.65,0.4,0.3) & (0.4,0.65,0.65) & (0.55,0.45,0.3) \\ (0.4,0.7,0.3) & (0.55,0.3,0.7) & (0.8,0.2,0.5) & (0.65,0.25,0.6) \\ (0.35,0.7,0.5) & (0.75,0.3,0.4) & (0.6,0.45,0.6) & (0.7,0.3,0.45) \\ (0.1,0.8,0.2) & (0.25,0.8,0.2) & (0.15,0.85,0.25) & (0.35,0.7,0.15) \\ (0.1,0.8,0.25) & (0.3,0.6,0.4) & (0.15,0.7,0.5) & (0.15,0.7,0.5) \end{bmatrix}$$

Consider another neutrosophic soft mapping factoring in the types of accidents and the factors that can potentially contribute them. N is a Mapping such that N_N : A $\rightarrow$ N_N^F

(N_N, A) ={ N_N (A_1)={(F_1, 0.2,0.7,0.2),(F_2, 0.7,0.2,0.6), (F_3, 0.6,0.25,0.75), (F_4, 0.8,0.1,0.5)},
{N_N (A_2)={(F_1, 0.5,0.5,0.6), (F_2, 0.75,0.25,0.4), (F_3, 0.55,0.4,0.55), (F_4, 0.7,0.3,0.45)},
{N_N (A_3)={(F_1, 0.1,0.8,0.8), (F_2, 0.4,0.6,0.5), (F_3, 0.55,0.5,0.45), (F_4, 0.7,0.4,0.35)},
{N_N (A_4)={(F_1, 0.1,0.8,0.7), (F_2, 0.05,0.75,0.4), (F_3, 0.2,0.7,0.3), (F_4, 0.3,0.65,0.2)},
{N_N (A_5)={(F_1, 0.1,0.8,0.7), (F_2, 0.05,0.75,0.4), (F_3, 0.1,0.7,0.4), (F_4, 0.3,0.65,0.2)},

Collating the mapping N_N into a Neutrosophic soft matrix 'N_N' between Universal sets F and A:

$$N_N = \begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{array}\begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} (0.2,0.7,0.2) & (0.5,0.5,0.6) & (0.1,0.8,0.8) & (0.1,0.8,0.7) & (0.1,0.8,0.7) \\ (0.7,0.2,0.6) & (0.75,0.25,0.4) & (0.4,0.6,0.5) & (0.05,0.75,0.4) & (0.05,0.75,0.4) \\ (0.6,0.25,0.75) & (0.55,0.4,0.55) & (0.55,0.5,0.45) & (0.2,0.7,0.3) & (0.1,0.7,0.4) \\ (0.8,0.1,0.5) & (0.7,0.3,0.45) & (0.7,0.4,0.35) & (0.3,0.65,0.2) & (0.3,0.65,0.2) \end{bmatrix} \end{array}$$

Using Max-Min formulae to calculate product matrix $Prod1_N$

Some examples of the function are given below:

$Prod1_{11}$ = (Max (0.2,0.65,0.4,0.55) , Min(0.75,0.4,0.65,0.45) , Min(0.4,0.6,0.75,0.5)) = (0.65,0.4,0.4)

$Prod1_{12}$ = (Max (0.35,0.65,0.4,0.3) , Min(0.75,0.4,0.65,0.45) , Min(0.6,0.4,0.65,0.45)) = (0.65,0.4,0.4)

$Prod1_{21}$ = (Max (0.2,0.55,0.6,0.65) , Min(0.7,0.3,0.25,0.25) , Min(0.3,0.7,0.75,0.6)) = (0.65,0.25,0.3)

This was continued for all 25 entries…

When combining all the above values, we get $Prod1_N$ matrix

$$Prod1_N = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array}\begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.65,0.4,0.4 & 0.65,0.4,0.4 & 0.55,0.45,0.35 & 0.3,0.65,0.3 & 0.3,0.65,0.3 \\ 0.65,0.25,0.3 & 0.65,0.3,0.55 & 0.65,0.4,0.5 & 0.3,0.65,0.5 & 0.3,0.65,0.5 \\ 0.7,0.3,0.5 & 0.75,0.3,0.4 & 0.7,0.4,0.45 & 0.3,0.65,0.4 & 0.3,0.65,0.4 \\ 0.35,0.7,0.2 & 0.35,0.7,0.4 & 0.35,0.7,0.35 & 0.3,0.7,0.2 & 0.35,0.7,0.2 \\ 0.3,0.6,0.25 & 0.3,0.6,0.4 & 0.3,0.6,0.5 & 0.3,0.7,0.4 & 0.3,0.7,0.4 \end{bmatrix} \end{array}$$

And so, applying the formula Risk score = Truth + Indeterminacy – False, we get risk score matrix R_{N1}

$$R_{N1} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array}\begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.65 & 0.65 & 0.65 & 0.65 & 0.65 \\ 0.6 & 0.4 & 0.55 & 0.45 & 0.45 \\ 0.5 & 0.65 & 0.65 & 0.55 & 0.55 \\ 0.85 & 0.65 & 0.7 & 0.8 & 0.85 \\ 0.65 & 0.5 & 0.4 & 0.6 & 0.6 \end{bmatrix} \end{array}$$

Next. the complement of Neutrosophic soft matrix N_N is N_N^C.

$$N_N^C = \begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{array}\begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} (0.2,0.7,0.2) & (0.6,0.5,0.5) & (0.8,0.8,0.1) & (0.7,0.8,0.1) & (0.7,0.8,0.1) \\ (0.6,0.2,0.7) & (0.4,0.25,0.75) & (0.5,0.6,0.4) & (0.4,0.75,0.05) & (0.4,0.75,0.05) \\ (0.75,0.25,0.6) & (0.55,0.4,0.55) & (0.45,0.5,0.55) & (0.3,0.7,0.2) & (0.4,0.7,0.1) \\ (0.5,0.1,0.8) & (0.45,0.3,0.7) & (0.35,0.4,0.7) & (0.2,0.65,0.3) & (0.2,0.65,0.3) \end{bmatrix} \end{array}$$

Using Max-Min formulae to calculate product matrix $Prod2_N$

I_1 I_2 I_3 I_4 I_5 A_1 A_2 A_3 A_4 A_5

$$Prod2_N = \begin{bmatrix} 0.6,0.4,0.4 & 0.35,0.4,0.5 & 0.5,0.45,0.4 & 0.4,0.65,0.3 & 0.4,0.65,0.3 \\ 0.75,0.25,0.3 & 0.55,0.3,0.5 & 0.5,0.4,0.3 & 0.4,0.65,0.5 & 0.4,0.65,0.5 \\ 0.75,0.3,0.5 & 0.55,0.3,0.5 & 0.5,0.4,0.4 & 0.2,0.65,0.4 & 0.2,0.65,0.4 \\ 0.35,0.7,0.2 & 0.35,0.7,0.5 & 0.35,0.7,0.2 & 0.25,0.7,0.2 & 0.25,0.7,0.2 \\ 0.3,0.6,0.25 & 0.3,0.6,0.5 & 0.3,0.6,0.25 & 0.3,0.7,0.25 & 0.3,0.7,0.25 \end{bmatrix}$$

Again, applying the formula Risk score = Truth + Indeterminacy – False, we get risk score matrix R_{N2}

$$R_{N2} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 \\ I_1 & 0.6 & 0.25 & 0.55 & 0.75 & 0.75 \\ I_2 & 0.7 & 0.35 & 0.6 & 0.25 & 0.25 \\ I_3 & 0.55 & 0.35 & 0.5 & 0.45 & 0.45 \\ I_4 & 0.85 & 0.55 & 0.85 & 0.75 & 0.75 \\ I_5 & 0.65 & 0.4 & 0.65 & 0.75 & 0.75 \end{matrix}$$

Subtracting R_{N1} and R_{N2} to find magnitude of risk for every specific type of industry gives the final mapping.

$$R_{N1} - R_{N2} = \begin{matrix} & A_1 & A_2 & A_3 & A_4 & A_5 \\ I_1 & 0.05 & 0.4 & 0.1 & -0.1 & -0.1 \\ I_2 & -0.1 & 0.05 & -0.05 & 0.2 & 0.2 \\ I_3 & -0.05 & 0.3 & 0.15 & 0.1 & 0.1 \\ I_4 & 0 & 0.1 & -0.15 & 0.05 & 0.1 \\ I_5 & 0 & 0.1 & -0.25 & -0.15 & -0.15 \end{matrix}$$

The calculation of the final risk score is done by using the following formula and this yields:

$$\text{Risk Score } (I_n) = \sum_{k=1}^{5} (R_{N1} - R_{N2})_{nk}$$

Table (vi)

Industry	Risk score for Neutrosophic Soft Values
I_1	0.35
I_2	0.3
I_3	0.6
I_4	0.1
I_5	-0.45

2.2.2 Pythagorean neutrosophic Soft Set

Repeating the same procedure for Pythagorean neutrosophic soft mapping and creating mappings M_P : F → M_P^I and N_P : A → N_P^F

I_1
I_2
I_3
I_4 F_1 F_2 F_3 F_4
I_5

$$M_P = \begin{bmatrix} (0.4,0.8,0.45) & (0.7,0.45,0.35) & (0.45,0.7,0.7) & (0.6,0.5,0.35) \\ (0.45,0.75,0.35) & (0.6,0.35,0.75) & (0.85,0.25,0.55) & (0.7,0.3,0.65) \\ (0.4,0.75,0.55) & (0.8,0.35,0.45) & (0.65,0.5,0.65) & (0.75,0.35,0.5) \\ (0.15,0.85,0.25) & (0.3,0.85,0.25) & (0.2,0.9,0.3) & (0.4,0.75,0.2) \\ (0.15,0.85,0.3) & (0.35,0.65,0.45) & (0.2,0.75,0.55) & (0.2,0.75,0.55) \end{bmatrix}$$

$$N_P = \begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} (0.25,0.75,0.25) & (0.55,0.55,0.65) & (0.15,0.85,0.85) & (0.15,0.85,0.75) & (0.15,0.85,0.75) \\ (0.75,0.25,0.65) & (0.8,0.3,0.45) & (0.45,0.65,0.55) & (0.1,0.8,0.45) & (0.1,0.8,0.45) \\ (0.65,0.3,0.8) & (0.6,0.45,0.6) & (0.6,0.55,0.5) & (0.25,0.75,0.35) & (0.15,0.75,0.45) \\ (0.85,0.15,0.55) & (0.75,0.35,0.5) & (0.75,0.45,0.4) & (0.35,0.7,0.25) & (0.35,0.7,0.25) \end{bmatrix} \end{array}$$

Once again, applying the max-min formula and finding Risk score = Truth + Indeterminacy – False, we get risk score matrices R_{P1} and R_{P2} respectively.

$$R_{P1} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.6 & 0.65 & 0.7 & 0.7 & 0.7 \\ 0.65 & 0.45 & 0.6 & 0.5 & 0.5 \\ 0.55 & 0.65 & 0.7 & 0.55 & 0.55 \\ 0.85 & 0.65 & 0.7 & 0.8 & 0.8 \\ 0.8 & 0.55 & 0.55 & 0.4 & 0.4 \end{bmatrix} \end{array}$$

$$R_{P2} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.75 & 0.5 & 0.65 & 0.55 & 0.55 \\ 1 & 0.45 & 0.4 & 0.2 & 0.2 \\ 0.65 & 0.6 & 0.45 & 0.35 & 0.35 \\ 0.85 & 0.65 & 0.7 & 0.75 & 0.75 \\ 0.7 & 0.55 & 0.45 & 0.55 & 0.55 \end{bmatrix} \end{array}$$

Then subtracting R_{P1} and R_{P2} to find magnitude of risk for every specific type of industry gives the final mapping.

$$R_{P1} - R_{P2} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} -0.15 & 0.15 & 0.05 & 0.15 & 0.15 \\ -0.35 & 0 & 0.2 & 0.3 & 0.3 \\ -0.1 & 0.05 & 0.25 & 0.2 & 0.2 \\ 0 & 0 & 0 & 0.05 & 0.05 \\ 0.1 & 0 & 0.1 & -0.15 & -0.15 \end{bmatrix} \end{array}$$

Calculating Risk Scores:

Table (vii)

Industry	Risk score for Pythagorean neutrosophic soft values
I_1	0.35
I_2	0.45
I_3	0.6
I_4	0.1
I_5	-0.1

4.3 Fermatean neutrosophic Soft Set

Repeating the same procedure for Fermatean neutrosophic soft mapping and creating mappings M_F : F → M_F^I and N_F : A → N_F^F

$$M_F = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{cccc} F_1 & F_2 & F_3 & F_4 \end{array} \\ \begin{bmatrix} (0.45,0.85,0.5) & (0.75,0.5,0.4) & (0.5,0.75,0.75) & (0.65,0.55,0.4) \\ (0.5,0.8,0.4) & (0.65,0.4,0.8) & (0.9,0.3,0.6) & (0.75,0.35,0.7) \\ (0.45,0.8,0.6) & (0.85,0.4,0.5) & (0.7,0.55,0.7) & (0.8,0.4,0.55) \\ (0.2,0.9,0.3) & (0.35,0.9,0.3) & (0.25,0.95,0.35) & (0.45,0.8,0.25) \\ (0.2,0.9,0.35) & (0.4,0.7,0.5) & (0.25,0.8,0.6) & (0.25,0.8,0.6) \end{bmatrix} \end{array}$$

$$N_F = \begin{array}{c} \\ F_1 \\ F_2 \\ F_3 \\ F_4 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} (0.3,0.8,0.3) & (0.6,0.6,0.7) & (0.2,0.9,0.9) & (0.2,0.9,0.8) & (0.2,0.9,0.8) \\ (0.8,0.3,0.7) & (0.85,0.35,0.5) & (0.5,0.7,0.6) & (0.15,0.85,0.5) & (0.15,0.85,0.5) \\ (0.7,0.35,0.85) & (0.65,0.5,0.65) & (0.65,0.6,0.56) & (0.3,0.8,0.4) & (0.2,0.8,0.5) \\ (0.9,0.2,0.6) & (0.8,0.4,0.55) & (0.8,0.5,0.45) & (0.4,0.75,0.3) & (0.4,0.75,0.3) \end{bmatrix} \end{array}$$

Again, applying the max-min formula and finding Risk score = Truth + Indeterminacy – False, we get risk score matrices R_{F1} and R_{F2} respectively.

$$R_{F1} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.75 & 0.75 & 0.75 & 0.75 & 0.75 \\ 0.7 & 0.5 & 0.65 & 0.55 & 0.55 \\ 0.6 & 0.75 & 0.75 & 0.65 & 0.65 \\ 0.95 & 0.75 & 0.8 & 0.9 & 0.9 \\ 0.77 & 0.6 & 0.5 & 0.55 & 0.55 \end{bmatrix} \end{array}$$

$$R_{P2} = \begin{array}{c} \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \begin{array}{c} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \end{array} \\ \begin{bmatrix} 0.7 & 0.45 & 0.65 & 0.85 & 0.85 \\ 0.8 & 0.45 & 0.7 & 0.85 & 0.85 \\ 0.5 & 0.45 & 0.6 & 0.75 & 0.75 \\ 0.95 & 0.65 & 0.95 & 0.85 & 0.85 \\ 0.75 & 0.5 & 0.75 & 0.85 & 0.85 \end{bmatrix} \end{array}$$

Then subtracting R_{P1} and R_{P2} to find magnitude of risk for every specific type of industry gives the final mapping.

$$\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{array} \quad A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5$$

$$R_{F1} - R_{F2} = \begin{bmatrix} 0.05 & 0.3 & 0.1 & -0.1 & -0.1 \\ -0.1 & 0.05 & -0.05 & -0.3 & -0.3 \\ 0.1 & 0.3 & 0.15 & -0.1 & -0.1 \\ 0 & 0.1 & -0.15 & 0.05 & 0.05 \\ 0 & 0.1 & -0.25 & -0.3 & -0.3 \end{bmatrix}$$

Calculating risk scores

Table (viii)

Industry	**Risk score for Fermatean neutrosophic soft values**
I_1	0.25
I_2	-0.7
I_3	0.35
I_4	0.05
I_5	-0.75

3. Results

3.1. Analysis

3.1.1. Comparative Evaluation of Neutrosophic Frameworks

The values taken for the matrices are arbitrary, based on theoretical knowledge of accidents and the IChemE accident report. From the analysis of the soft matrices, we can see that Neutrosophic analysis is more accurate than Pythagorean neutrosophic and Fermatean neutrosophic.

- The most common accident is fire, and the power generation industry has the highest risk score in all these analyses, with fire being the most likely accident in the power generation industry.
- In the Fermatean neutrosophic analysis, the most likely accident for all industries is fire. Perhaps the reason for this can be theorized that Fermatean neutrosophic analysis considers the severity of the accident over a short period of time, whereas Neutrosophic and Pythagorean neutrosophic analyses consider longer periods of time.
- For example, for the oil and gas industry, only fire is listed as the probable accident in the Fermatean neutrosophic analysis. However, in the Pythagorean neutrosophic analysis, pollution and disease are listed as the most probable accidents. While fire damages more infrastructure and causes severe implications, it can be argued that pollution and disease have long-lasting effects, which are irreversible and have a higher mortality rate.

3.1.2. Risk Scores Across Industries

The food, water and fertilizer, and pharmaceutical industries have the lowest risk scores.

- This might be because these industries work on batch operations, while the power generation industry, oil and gas sector, and petrochemical industry operate on a continuous basis.

- A batch operation is where an industrial plant has a startup and shutdown sequence and does not work for a continuous 24 hours.
- Plants that run continuously do not shut down daily, only when required, such as in emergencies or during maintenance. Plants working continuously tend to have less time for maintenance, hence giving rise to more safety risks.
- Shift changes can cause confusion between day shift and night shift workers. Many accidents occur during the night due to the tiredness of working during the twilight hours.

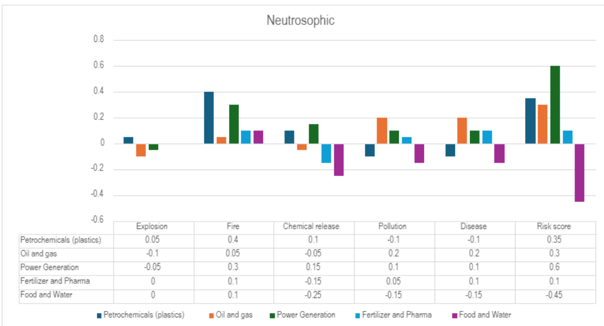
3.1.3. Human Factors and Operational Risks

Additionally, it is crucial to consider the impact of human factors in the analysis.

- Continuous operations demand high levels of vigilance and can lead to fatigue among workers, thereby increasing the likelihood of accidents.
- Proper training and adequate rest periods are essential to mitigate these risks.
- The inherent nature of industries like power generation and petrochemicals, which handle hazardous materials and high-pressure systems, adds to the complexity and potential danger.
- On the other hand, industries with batch operations can implement thorough inspections and maintenance during downtime, significantly reducing the risk of accidents.

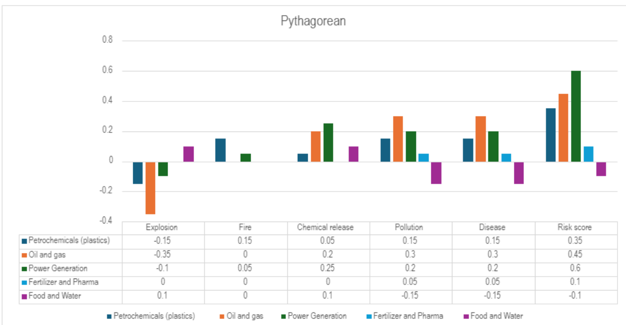
This comprehensive approach highlights the need for industry-specific safety protocols and regular risk assessments to ensure a safer working environment across different sectors.

3.2. *Graphical Representation of Results*



	Explosion	Fire	Chemical release	Pollution	Disease	Risk score
Petrochemicals (plastics)	0.05	0.4	0.1	-0.1	-0.1	0.35
Oil and gas	-0.1	0.05	-0.05	0.2	0.2	0.3
Power Generation	-0.05	0.3	0.15	0.1	0.1	0.6
Fertilizer and Pharma	0	0.1	-0.15	0.05	0.1	0.1
Food and Water	0	0.1	-0.25	-0.15	-0.15	-0.45

Figure (ii)



	Explosion	Fire	Chemical release	Pollution	Disease	Risk score
Petrochemicals (plastics)	-0.15	0.15	0.05	0.15	0.15	0.35
Oil and gas	-0.35	0	0.2	0.3	0.3	0.45
Power Generation	-0.1	0.05	0.25	0.2	0.2	0.6
Fertilizer and Pharma	0	0	0	0.05	0.05	0.1
Food and Water	0.1	0	0.1	-0.15	-0.15	-0.1

Figure (iii)

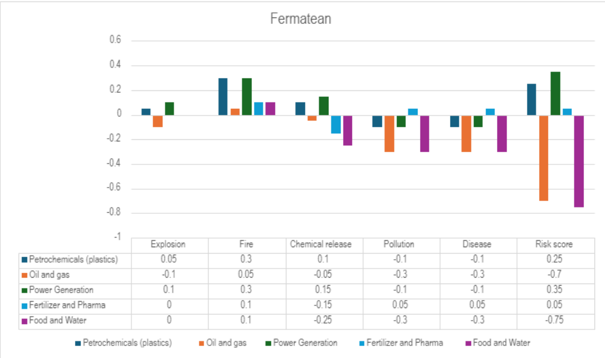


Figure (iv)

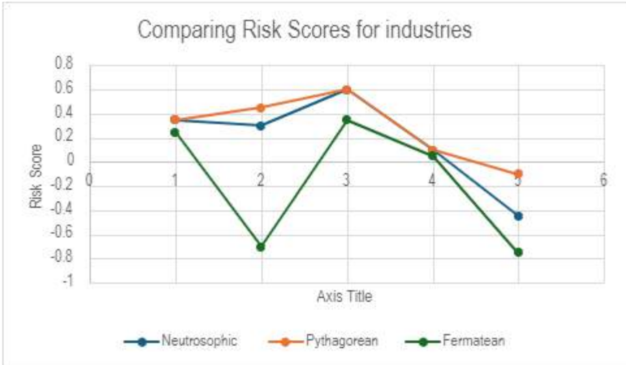


Figure (v)

4. Applications

The findings of this study have significant potential applications in various industrial sectors prone to industrial accidents. By implementing the advanced neutrosophic soft set frameworks, industries can improve their risk assessment and decision-making processes, leading to enhanced safety protocols and reduced accident rates. Specifically, the newly introduced risk score parameter can be utilized by safety engineers and managers to consolidate and analyze complex accident data, providing a more accurate measure of potential hazards. This, in turn, can facilitate better emergency response planning, targeted safety training programs, and optimized resource allocation for accident prevention measures.

Conclusions

The comparative analysis of Neutrosophic soft matrix, Pythagorean neutrosophic soft matrix analysis, and Fermatean neutrosophic soft matrix analysis has given rise to the conclusion that the power generation industry is the most dangerous workplace with fire being the most common accident in any industry. The findings of the paper are summarized in the following points.

- Neutrosophic analysis is deemed more accurate than Pythagorean and Fermatean neutrosophic analyses.
- The power generation industry has the highest risk score.
- Fermatean Neutrosophic Analysis emphasizing short-term severity, consistently identifying fire as the most likely accident across industries.
- Pythagorean analysis considers long-term impacts, showing pollution and disease as major concerns, which have irreversible and high mortality rates.
- Power generation, oil and gas, and petrochemical industries operate continuously, leading to higher safety risks due to limited maintenance opportunities and potential worker fatigue.
- Industries like food, water, fertilizer, and pharmaceuticals have lower risk scores due to batch operations allowing for regular maintenance and inspections.
- Continuous operations contribute to worker fatigue and increasing accident likelihood.
- Industries dealing with hazardous materials and high-pressure systems, such as power generation and petrochemicals, face greater complexity and potential dangers.

Using the soft matrix analysis as a basis for various other industries we can identify the most probable accidents and take steps to mitigate the likelihood of the accident.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

The values for the neutrosophic, Pythagorean neutrosophic and Fermatean neutrosophic soft matrices are based on the theoretical knowledge and the incident report of the IChemE accident database taken from their e-book named "Learning lessons from major incidents - Improving process safety by sharing experience".

Provided are the references for the e-book

https://www.icheme.org/media/24872/learning-lessons-from-major-incidents.pdf

https://www.icheme.org/media/20722/icheme-lessons-learned-database-rev-11.pdf

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