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Neutrosophic semi δ-preopen sets and neutrosophic semi δ-pre continuity

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Abstract. The purpose of this paper is to introduce the concepts of neutrosophic semi δ-preopen sets and neutrosophic semi δ-precontinuous mappings in neutrosophic topological spaces and obtain some of their properties and characterizations.

Keywords: Neutrosophic set, neutrosophic topology, neutrosophic semi δ-preopen sets and neutrosophic semi δ-precontinuity.

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1. Introduction

As an elaboration of Zadeh's fuzzy sets [19] from 1965 and Atanassav's intuitionistic fuzzy sets [4] from 1983, Smarandache has proposed and described neutrosophic sets.He [13] defined neutrosophic set on a non empty set by considering three components, namely membership, Indeterminacy and non-membership whose sum lies between 0 and 3. Some more properties of neutrosophic sets are presented by Smarandache [13–15], Salama and Alblowi [11] , Lupiáñez [9]. Smarandache's Neutrosophic concepts have wide range of real time applications for the fields of Information systems, Computer science, Artificial Intelligence, Applied Mathematics and Decision making. In 2008, Lupiáñez [9] introduced the neutrosophic topology as a extension of intuitionistic fuzzy topology. Since 2008 many authors such as Lupiáñez [9, 10], Salama et.al. [11, 12] Karatas and Cemil [8], Acikgoz and his coworkers [1], Dhavaseelan et.al. [5], Al-Musaw [2], Dey and Ray [?] and others contributed in neutrosophic topological spaces. Recently, many weak and strong forms of neurosophic open sets such as neutrosophic regular open [3], neutrosophic α-open [3], neutrosophic semi open [3, 7],neutrosophic pre open [3,18],neutrosophic semi pre open [3,16],neutrosophic b-open [6], neutrosophic δ-open sets [17], neutrosophic δ-pre open [17] and weak and strong forms of neurosophic continuity such as neutrosophic semi continuity,and neurosophic precontinuity [3, 18] and neurosophic α-continuity [3], and neurosophic semi pre continuity [3, 16], neutrosophic b-continuity [6] have been investigated by different authors. In this paper we introduce a super class of above classes of neutrosophic open sets and a super class above mentioned neutrosophic continuity of mappings and studied their characterizations and properties.

2. Preliminaries

This section contains some basic definitions and preliminary results which will be needed in the sequel.

Definition 2.1. [13] A Neutrosophic set (**NS**) in X is a structure

$$A = \{< x, \mu_A(x), \varpi_A(x), \gamma_A(x) >: x \in X\}$$

where $\mu_A : X \to]^-0, 1^+[$, $\varpi A : X \to]^-0, 1^+[$, and $\gamma_A : X \to]^-0, 1^+[$ denotes the membership, indeterminacy, and non-membership of A satisfies the condition if $^-0 \leq \mu_A(x) + \varpi_A(x) + \gamma_A(x) \leq 3^+, \ \forall\ x \in X$.

In the real life applications in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$. Hence we consider the neutrosophic set which takes the value from the closed interval [0,1] and sum of membership, indeterminacy, and non-membership degrees of each element of universe of discourse lies between 0 and 3.

Definition 2.2. [12] Let X be a non empty set and the neutrosophic sets A and neutrosophic set B be in the form $A = \{< x, \mu_A(x), \varpi_A(x), \gamma_A(x) >: x \in X\}$, $B = \{< x, \mu_B(x), \varpi_B(x), \gamma_B(x) >: x \in X\}$ and let $\{A_i : i \in J\}$ be a arbitrary family of neutrosophic sets in X. Then:

(a) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$, $\varpi_A(x) \leq \varpi_B(x)$, and $\gamma_A(x) \geq \gamma_B(x)$.
(b) $A = B$ if $A \subseteq B$ and $B \subseteq A$.
(c) $A^c = \{< x, \gamma_A(x), \varpi_A(x), \mu_A(x) >: x \in X\}$.
(d) $\cap A_i = \{< x, \wedge\mu_{A_i}(x), \wedge\varpi_{A_i}(x), \vee\gamma_{A_i}(x) >: x \in X\}$.
(e) $\cup A_i = \{< x, \vee\mu_{A_i}(x), \vee\varpi_{A_i}(x), \wedge\gamma_{A_i}(x) >: x \in X\}$.
(f) $\tilde{\mathbf{0}} = \{< x, 0, 0, 1 >: x \in X\}$ and $\tilde{\mathbf{1}} = \{< x, 1, 1, 0 >: x \in X\}$

Definition 2.3. [9, 11] A neutrosophic topology on a non empty set X is a family τ of neutrosophic sets in X, satisfying the following axioms:

(T_1) $\tilde{\mathbf{0}}$ and $\tilde{\mathbf{1}} \in \tau$

(T_2) $G_1 \bigcap G_2 \in \tau$

(T_3) $G_1 \bigcup G_2 \in \tau$

In this case the pair (X, τ) is called a neutrosophic topological space and each neutrosophic set in τ is known as a neutrosophic open set in X. The complement A^c of a neutrosophic open set A is called a neutrosophic closed set in X.

Definition 2.4. [11] Let (X, τ) be a neutrosophic topological space and A be a neutrosophic set in X. Then the neutrosophic interior and neutrosophic closure of A are defined by:

Cl(A) = ∩ {K: K is a neutrosophic closed set such that A ⊆ K }

Int(A) = ∪ {K: K is a neutrosophic open set such that K ⊆ A }

Definition 2.5. [5] Let $\alpha, \eta, \beta \in [0, 1]$ and $0 \leq \alpha + \eta + \beta \leq 3$. A neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X is a neutrosophic set in X defined by

$$x_{(\alpha,\eta,\beta)}(y) = \begin{cases} (\alpha, \eta, \beta) & \text{if } y = x \\ (0, 0, 1) & \text{if } y \neq x \end{cases} \tag{1}$$

Definition 2.6. [1] Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point in X and $A = \{< x, \mu_A, \varpi_A, \gamma_A >: x \in X\}$ is a neutrosophic set in X. Then $x_{(\alpha,\eta,\beta)} \subseteq A$ if and only if $\alpha \subseteq \mu_A(x)$, $\eta \subseteq \varpi_A$, and $\beta \supseteq \nu_A(x)$.

Definition 2.7. [1] A nuetrosophic point $x_{(\alpha,\eta,\beta)}$ is said to be quasi-coincident (q-coincident, for short) with A, denoted by $x_{(\alpha,\eta,\beta)}qA$ iff $x_{(\alpha,\eta,\beta)} \nsubseteq A^c$. If $x_{(\alpha,\eta,\beta)}$ is not quasi-coincident with A, we denote by $\rceil(x_{(\alpha,\eta,\beta)}qA)$.

Definition 2.8. [1] Two neutrosophic set A and B of X said to be q-coincident (denoted by A_qB) if $A \nsubseteq B^c$.

Lemma 2.9. *[1] For any two neutrosophic sets A and B of X , $\rceil(A_qB) \Leftrightarrow A \subset B^c$. where $\rceil(A_qB)$ A is not q-coincident with B.*

Definition 2.10. [3] A neutrosophic set A of a NTS (X, τ) is called neutrosophic regular open set (resp. neutrosophic regular closed) if $A = int(cl(A))$ (resp. $A = cl(int(A))$).

Remark 2.11. [3] Every neutrosophic regular open (resp. neutrosophic regular closed) set is neutrosophic open (resp. neutrosophic closed), But the converse may not be true.

Definition 2.12. [17] The δ-interior (denoted by δint) of a neutrosophic set A of a NTS (X, τ) is the union of all neutrosophic regular open sets contained in A.

Definition 2.13. [17] The δ-closure (denoted by δcl) of a neutrosophic set A of a NTS (X, τ) is the intersection of all neutrosophic regular closed sets containing A.

Definition 2.14. [3, 6, 17] A neutrosophic set A of a NTS (X, τ) is called neutrosophic semiopen (resp. neutrosophic preopen ,, neutrosophic α-open, neutrosophic semi preopen ,]neutrosophic δ-open , neutrosophic δ-preopen, neutrosophic δ-semiopen , neutrosophic b-open if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(A))$, $A \subseteq int(cl(int(A)))$,$A \subseteq cl(int(cl(A)))$, $A = \delta int(A)$, $A \subseteq int(\delta cl(A))$, $A \subseteq cl(\delta int(A))$, $A \subseteq cl(int(A)) \cup int(cl(A)))$.

The family of all neutrosophic semiopen (resp. neutrosophic preopen, neutrosophic α-open, neutrosophic semi preopen, neutrosophic δ-open, δ-preopen, neutrosophic δ-semiopen, neutrosophic γ-open) sets of a NTS (X, τ) is denoted by $NSO(X)$ (resp. $NPO(X), N\alpha O(X), NSPO(X), N\delta O(X), N\delta PO(X), N\delta SO(X), NbO(X))$.

Definition 2.15. [3,6,17] A neutrosophic set A in a NTS (X, τ) is called neutrosophic neutrosophic semiclosed (resp. neutrosophic preclosed, neutrosophic α-closed, neutrosophic semi preclosed, neutrosophic δ-preclosed, neutrosophic δ-semiclosed, neutrosophic γ-closed) if $A^c \in NSO(X)$ (resp. $NPO(X), N\alpha O(X), NSPO(X), N\delta O(X), N\delta PO(X), N\delta SO(X), NbO(X))$.

Remark 2.16. [3, 16–18] Every neutrosophic δ-open (resp. neutrosophic δ-closed) set is neutrosophic open (resp. neutrosophic closed), every neutrosophic open (resp. neutrosophic closed) set is neutrosophic α-open (resp. neutrosophic α-closed), every neutrosophic α-open (resp. neutrosophic α-closed) set is neutrosophic semiopen (resp. neutrosophic semiclosed) as well as neutrosophic preopen (resp. neutrosophic preclosed) and every neutrosophic semiopen (resp. neutrosophic semiclosed) set and every neutrosophic preopen (resp. neutrosophic preclosed) set is neutrosophic semi-preopen (resp. neutrosophic semi-preclosed). But the separate converses may not be true.

Remark 2.17. [17] Every neutrosophic preopen (resp. neutrosophic preclosed) set is neutrosophic δ-preopen (resp. neutrosophic δ-preclosed) but the converse may not be true.

Remark 2.18. [6] Every neutrosophic semiopen (resp. neutrosophic semiclosed) and neutrosophic preopen (resp. neutrosophic preclosed) set is neutrosophic b-open (resp.neutrosophic b-closed), and every neutrosophic b-open (resp.neutrosophic b-closed)set is neutrosophic semi-preopen (resp.neutrosophic semi-preclosed)but the separate converses may not be true.

Definition 2.19. [1] Consider that f is a mapping from X to Y.

(a) Let A be a neutrosophic set in X with membership function $\mu_A(x)$, indeterminacy function $\varpi_A(x)$ and non-membership function $\sigma_A(x)$. The image of A under f, written as f(A), is a neutrosophic set of Y whose membership function, indeterminacy function and non-membership function are defined as

$$\mu_{f(A)}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\mu_A(x)\}, & \text{if } f^{-1}(y) \neq \phi, \\ 0, & \text{if } f^{-1}(y) = \phi, \end{cases}$$

$$\varpi_{f(A)}(y) = \begin{cases} \sup\limits_{x \in f^{-1}(y)} \{\varpi_A(x)\}, & \text{if } f^{-1}(y) \neq \phi, \\ 0, & iff^{-1}(y) = \phi, \end{cases}$$

$$\gamma_{f(A)}(y) = \begin{cases} \inf\limits_{x \in f^{-1}(y)} \{\gamma_A(x)\}, & \text{if } f^{-1}(y) \neq \phi, \\ 0, & iff^{-1}(y) = \phi, \end{cases}$$

$\forall\ , y \in Y$. Where $f^{-1}(y) = \{x : f(x) = y\}$.

(b) Let B be a neutrosophic set in Y with membership function $\mu_B(y)$, indeterminacy function $\varpi_B(y)$ and non-membership function $\gamma_B(y)$. Then, the inverse image of B under f, written as $f^{-1}(B)$ is a neutrosophic set of X whose membership function, indeterminacy function and non-membership function are defined as $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$, $\varpi_{f^{-1}(B)}(x) = \varpi_B(f(x))$, and $\gamma_{f^{-1}(B)}(x) = \gamma_B(f(x))$, respectively $\forall\ x \in X$.

Definition 2.20. [3,6,17] A mapping $f : (X, \tau) \to (Y, \sigma)$ is called neutrosophic semi continuous (resp. neutrosophic pre continuous , neutrosophic α-continuous,neutrosophic semi pre continuous, neutrosophic δ-pre continuous, neutrosophic b-continuous if $f^{-1}(A) \in NSO(X)$ (resp. $f^{-1}(A) \in NPO(X)$, $f^{-1}(A) \in N\alpha O(X)$, $f^{-1}(A) \in NSPO(X)$, $f^{-1}(A) \in N\delta PO(X)$, $f^{-1}(A) \in NbO(X)$) for each neutrosophic set $A \in \sigma$.

Remark 2.21. [3, 16, 17] Every neutrosophic continuous mappings is neutrosophic α-continuous, every neutrosophic α-continuous mapping is neutrosophic semi continuous and neutrosophic pre continuous , every neutrosophic semi continuous (resp. neutrosophic pre continuous) mapping is neutrosophic b-continuous and neutrosophic b-continuous mapping is neutrosophic semi pre continuous but the separate converses may not be true.The concepts of neutrosophic semi continuous and neutrosophic pre continuous mappings are independent.

Remark 2.22. [17] Every neutrosophic pre continuous mapping is neutrosophic δ-pre continuous but the converse may not be true.

3. Neutrosophic semi δ-preopen sets

In this section, we introduce the concept of neutrosophic semi δ-preopen set and study some of their properties in neutrosophic topological spaces.

Definition 3.1. A neutrosophic set A in a NTS (X, τ) is called:

(a) neutrosophic semi δ-preopen if there exists a neutrosophic δ-preopen set O such that $O \subseteq A \subseteq \delta cl(O)$.

(b) neutrosophic semi δ-preclosed if there exists a neutrosophic δ-preclosed set F such that $\delta int(F) \subseteq A \subseteq F$.

The family of all neutrosophic semi δ-preopen (resp. neutrosophic semi δ-preclosed) sets of a NTS (X, τ) is denoted by $NS\delta PO(X)$ (resp. $NS\delta PC(X)$).

Remark 3.2. Every neutrosophic δ-semiopen (resp. neutrosophic δ-semiclosed)set is neutrosophic semiopen (resp. neutrosophic semiclosed) but the converse may not be true.

Example 3.3. Let $X = \{a, b\}$ and neutrosophic sets A, B, O are defined as follows:

$$A = \{< a, 0.3, 0.5, 0.7 >, < b, 0.4, 0.5, 0.6 >\}$$

$$B = \{< a, 0.4, 0.5, 0.6 >, < b, 0.3, 0.5, 0.7 >\}$$

$$O = \{< a, 0.5, 0.5, 0.5 >, < b, 0.6, 0.5, 0.4 >\}$$

let $\tau = \{\tilde{0}, A, B, A \cup B, A \cap B, \tilde{1}\}$ be the neutrosophic topology on (X, τ). Then O is neutrosophic semiopen but not neutrosophic δ-semiopen.

Remark 3.4. Every neutrosophic δ-open (resp. neutrosophic δ-closed) set is neutrosophic δ-semiopen (resp. neutrosophic δ-semiclosed) but the converse may not be true. For, in the NTS (X, τ) of example (3.3), the neutrosophic set A is neutrosophic δ-semiopen but not neutrosophic δ-open.

Remark 3.5. The concepts of neutrosophic δ-semiopen and neutrosophic open sets are independent. For, in the NTS (X, τ) of example (3.3), the neutrosophic set O is neutrosophic δ-semiopen but not neutrosophic open and neutrosophic A is neutrosophic open but not neutrosophic δ-semiopen.

Theorem 3.6. *An neutrosophic set $A \in NS\delta PC(X)$ if and only if $A^c \in NS\delta PO(X)$.*

Remark 3.7. Every neutrosophic semi preopen (resp. neutrosophic semi preclosed) set and Every neutrosophic δ-preopen (resp. neutrosophic δ-preclosed) set is neutrosophic semi δ-preopen (resp. neutrosophic semi δ-preclosed). But the separate converse may not be true.

Example 3.8. Let $X = \{a, b\}$ and neutrosophic sets A, B, O, F are neutrosophic sets defined as follows:

$$A = \{< a, 0.5, 0.5, 0.5 >, < b, 0.3, 0.5, 0.7 >\}$$

$$B = \{< a, 0.5, 0.5, 0.5 >, < b, 0.1, 0.5, 0.9 >\}$$

$$O = \{< a, 0.5, 0.5, 0.5 >, < b, 0.9, 0.5, 0.1 >\}$$

$$F = \{< a, 0.5, 0.5, 0.5 >, < b, 0.6, 0.5, 0.4 >\}$$

let $\tau = \{\tilde{0}, A, B, \tilde{1}\}$ be a neutrosophic topology on (X, τ), Then

(a) O is neutrosophic semi δ-preopen (resp. O^c neutrosophic semi preclosed) but not neutrosophic semi preopen (resp. neutrosophic semi preclosed).

(b) F is neutrosophic semi δ-preopen (resp. F^c is neutrosophic semi preclosed) but not neutrosophic δ-preopen (resp. neutrosophic δ-preclosed).

Remark 3.9. It is clear that from remark (2.11), (2.16), (2.17), (2.18), (3.2), (3.4), (3.5) and (3.7) that the following figure of implications is true.

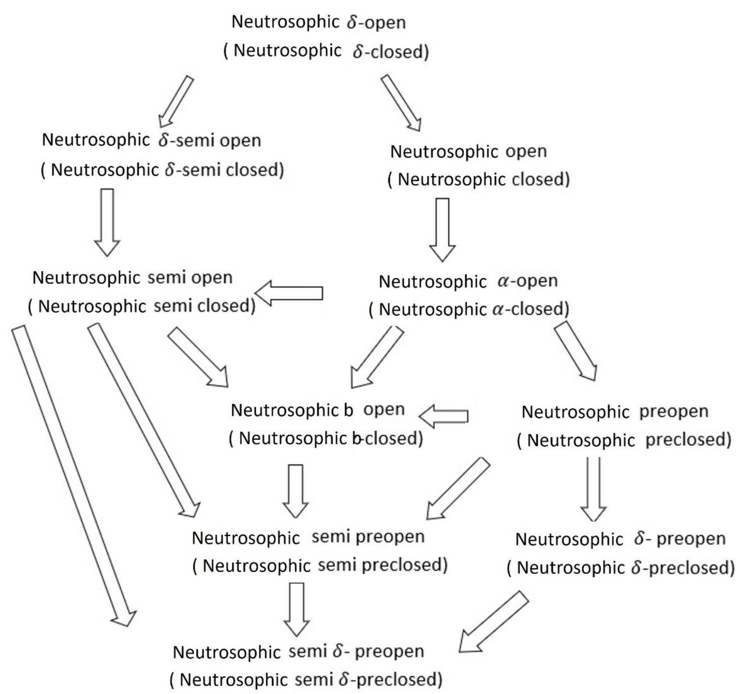

Figure 1

Theorem 3.10. *Let (X, τ) be a NTS . Then*

(a) *Any union of neutrosophic semi δ-preopen sets is neutrosophic semi δ-preopen.*

(b) *Any intersection of neutrosophic semi δ-preclosed sets is neutrosophic semi δ-preclosed.*

Theorem 3.11. *A neutrosophic set $A \in NS\delta PO(X)$ if and only if for every neutrosophic point $x_{(\alpha,\eta,\beta)} \in A$ there exists a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O \subseteq A$.*

Proof. If $A \in NS\delta PO(X)$ then we may take $O = A$ for every $x_{(\alpha,\eta,\beta)} \in A$.

Conversely. We have $A =_{x_{(\alpha,\eta,\beta)} \in A} \cup\{x_{(\alpha,\eta,\beta)}\} \subseteq_{x_{(\alpha,\eta,\beta)} \in A} \cup O \subseteq A$.The result now follows from the fact that any union of neutrosophic δ-preopen sets is neutrosophic δ-preopen. □

Theorem 3.12. *Let (X, τ) be a NTS .*

(a) *If $A \subseteq O \subseteq \delta cl(A)$ and $A \in NS\delta PO(X)$ then $O \in NS\delta PO(X)$,*

(b) *If $\delta int(B) \subseteq F \subseteq B$ and $B \in NS\delta PC(X)$ then $F \in NS\delta PC(X)$*

Proof. (a) Let $O_1 \in N\delta PO(X)$ such that $O_1 \subseteq A \subseteq \delta cl(O_1)$.
Clearly $O_1 \subseteq O$ and $A \subseteq \delta cl(O_1)$ implies that $\delta cl(A) \subseteq \delta cl(O_1)$. Consequently,
$O_1 \subseteq O \subseteq \delta cl(O_1)$. Hence $O \in NS\delta PO(X)$.
(b) Follows from (a). □

Lemma 3.13. *An neutrosophic set $A \in N\delta PO(X)$ if and only if there exists a neutrosophic set O such that $A \subseteq O \subseteq \delta cl(A)$.*

Proof. **Necessity** If $A \in N\delta PO(X)$ then $A \subseteq int(\delta cl(A))$.
Put $O = int(\delta cl(A))$ then O is a neutrosophic open and $A \subseteq O \subseteq \delta cl(A)$.
Sufficiency Let O be a neutrosophic open set such that $A \subseteq O \subseteq \delta cl(A)$, then $A \subseteq int(O) \subseteq int(\delta cl(A))$. Hence $A \in N\delta PO(X)$. □

Lemma 3.14. *Let Y be a neutrosophic subspace of NTS (X, τ) and A be a neutrosophic set in Y. If $A \in N\delta PO(X)$ then $A \in N\delta PO(Y)$.*

Proof. Since $A \in N\delta PO(X)$, By Lemma (3.13), there exists a neutrosophic set O in (X, τ) such that $A \subseteq O \subseteq \delta cl(A)$. Therefore $A \cap Y \subseteq O \cap Y \subseteq \delta cl(A) \cap Y = \delta cl_Y(A)$. It follows that $A \subseteq O \subseteq \delta cl_Y(A)$. Hence by Lemma (3.13), $A \in N\delta PO(Y)$. □

Theorem 3.15. *Let Y be a neutrosophic subspace of NTS (X, τ) and A be a neutrosophic set in Y. If $A \in NS\delta PO(X)$ then $A \in NS\delta PO(Y)$.*

Proof. Let $O \in N\delta PO(X)$ such that $O \subseteq A \subseteq cl(O)$. Then $O \cap Y \subseteq A \cap Y \subseteq cl(O) \cap Y$. It follows that $O \subseteq A \subseteq cl_Y(O)$. Now by Lemma (3.14), $O \in N\delta PO(Y)$ and hence $A \in NS\delta PO(Y)$. □

Theorem 3.16. *Let X and Y be NTS , such that X is product related to Y.*

(a) *If $A \in N\delta PO(X)$ and $O \in N\delta PO(Y)$, then $A \times O \in N\delta PO(X \times Y)$.*

(b) *N $A \in NS\delta PO(X)$ and $O \in NS\delta PO(Y)$, then $A \times O \in NS\delta PO(X \times Y)$.*

Proof. **(a)** N $A \in N\delta PO(X)$ and $O \in N\delta PO(Y)$. Then $A \times O \subseteq int(\delta cl(A)) \times int(\delta cl(O)) = int(\delta cl(A \times O))$.
(b) Let $F \subseteq A \subseteq \delta cl(F)$ and $\psi \subseteq O \subseteq \delta cl(\psi)$, $F \in N\delta PO(X)$ and $\psi \in N\delta PO(Y)$. Then $F \times \psi \subseteq A \times O \subseteq \delta cl(F) \times \delta cl(\psi) = \delta cl(A \times \psi)$. Now the result follows from (a). □

Definition 3.17. Let (X, τ) be a NTS and A be a neutrosophic set of X. Then the neutrosophic semi δ-preinterior (denoted by $s\delta pint$) and neutrosophic semi δ-preclosure (denoted by $s\delta pcl$) of A respectively defined as follows:
$s\delta pint(A) = \cup\{O : O \subseteq A; O \in NS\delta PO(X)\}$,
$s\delta pcl(A) = \cap\{O : O \supseteq A; O \in NS\delta PC(X)\}$.

The following theorem can be easily verified.

Theorem 3.18. *Let A and O be neutrosophic sets in a NTS (X, τ). Then:*

(a) $s\delta pcl(A) \subseteq cl(A)$
(b) $s\delta pcl(A)$ *is a neutrosophic semi δ-preclosed.*
(c) $A \in NS\delta PC(X) \Leftrightarrow A = s\delta pcl(A)$.
(d) $A \subseteq O \Rightarrow s\delta pcl(A) \subseteq s\delta pcl(O)$.
(e) $int(A) \subseteq s\delta pint(A)$.
(f) $s\delta pint(A)$ *is a neutrosophic semi preopen.*
(g) $A \in NS\delta PO(X) \Leftrightarrow A = s\delta pint(A)$.
(h) $A \subseteq O \Rightarrow s\delta pint(A) \subseteq s\delta pint(O)$.
(i) $s\delta pint(A^c) = (s\delta pcl(A))^c$.

Definition 3.19. Let A be a neutrosophic sets in a NTS (X, τ) and $x_{(\alpha,\eta,\beta)}$ is a neutrosophic point of X. Then A is called:

(a) neutrosophic semi δ-preneighborhood of $x_{(\alpha,\eta,\beta)}$ if there exists a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O \subseteq A$.
(b) neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$ if there exists a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)q}O \subseteq A$.

Theorem 3.20. *An neutrosophic set $A \in NS\delta PO(X)$ if and only if for each neutrosophic point $x_{(\alpha,\eta,\beta)} \in A$, A is a neutrosophic semi δ-preneighborhood of $x_{(\alpha,\eta,\beta)}$.*

Theorem 3.21. *Let A be a neutrosophic sets in a NTS (X, τ). Then a neutrosophic point $x_{(\alpha,\eta,\beta)} \in s\delta pcl(A)$, if and only if every neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$ is quasi-coincident with A.*

Proof. **Necessity** Suppose $x_{(\alpha,\eta,\beta)} \in s\delta pcl(A)$ and if possible let there exists a neutrosophic semi δ-pre Q-neighborhood O of $x_{(\alpha,\eta,\beta)}$ such that $\rceil(O_q A)$. Then there exists a neutrosophic set

$O_1 \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)q}O_1 \subseteq O_1 \subseteq O$ which show that $\rceil(O_{1q}A)$ and hence $A \subseteq O_1^c$. As $O_1^c \in NS\delta PC(X)$, $s\delta pcl(A) \subseteq O_1^c$. Since $x_{(\alpha,\eta,\beta)} \in O_1^c$, we obtain that $x_{(\alpha,\eta,\beta)} \notin s\delta pcl(A)$ which is a contradiction.

Sufficiency Suppose every neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$ is quasi-coincident with A. If $x_{(\alpha,\eta,\beta)} \notin s\delta pcl(A)$ then there exists a neutrosophic semi δ-preclosed set $O \supseteq A$ such that $x_{(\alpha,\eta,\beta)} \notin O$. So $O^c \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)q}O^c$ and $\rceil(O^c_q A)$ a contradiction. □

Definition 3.22. A mappings $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be neutrosophic δ-preirresolute if $f^{-1}(A) \in NS\delta PO(X)$ for every neutrosophic set $A \in N\delta PO(Y)$.

Theorem 3.23. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a neutrosophic δ-pre irresolute and neutrosophic open mapping, then $f^{-1}(A) \in NS\delta PO(X)$, for every $A \in NS\delta PO(Y)$.*

Proof. Let $A \in NS\delta PO(Y)$. Then there exists a neutrosophic set $O \in N\delta PO(X)$ such that $O \subseteq A \subseteq \delta cl(O)$. Therefore $f^{-1}(O) \subseteq f^{-1}(A) \subseteq f^{-1}(\delta cl(O))$ since f is neutrosophic open and δ-pre irresolute. $f^{-1}(O) \subseteq f^{-1}(A) \subseteq f^{-1}(\delta cl(O)) \subseteq \delta cl(f^{-1}(O))$ and $f^{-1}(O) \in NS\delta PO(X)$. Hence $f^{-1}(A) \in NS\delta PO(X)$. □

4. Neutrosophic semi δ-precontinuous mappings

Definition 4.1. A mappings $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be neutrosophic semi δ-precontinuous if $f^{-1}(A) \in NS\delta PO(X)$ for every neutrosophic set $A \in \sigma$.

Remark 4.2. Every neutrosophic δ-pre continuous (resp. neutrosophic semi precontinuous) mappings is neutrosophic semi δ-precontinuous but the converse may not be true.

Example 4.3. Let $X = \{a, b\}$ and $Y = \{p, q\}$ and neutrosophic sets A, B, O, F are defined as follows:

$$A = \{< a, 0.5, 0.5, 0.5 >, < b, 0.3, 0.5, 0.7 >\}$$

$$B = \{< a, 0.5, 0.5, 0.5 >, < b, 0.1, 0.5, 0.9 >\}$$

$$O = \{< p, 0.5, 0.5, 0.5 >, < q, 0.9, 0.5, 0.1 >\}$$

$$F = \{< p, 0.5, 0.5, 0.5 >, < q, 0.6, 0.5, 0.4 >\}$$

let $\tau_1 = \{\tilde{0}, A, B, \tilde{1}\}$, $\tau_2 = \{\tilde{0}, O, \tilde{1}\}$ and $\tau_3 = \{\tilde{0}, F, \tilde{1}\}$. Then the mapping $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ defined by $f(a) = p$, $f(b) = q$ is a neutrosophic semi δ-pre continuous but not neutrosophic semi precontinuous and the mapping $g : (X, \tau_1) \rightarrow (Y, \tau_3)$ defined by $g(a) = p$, $g(b) = q$ is a neutrosophic semi δ-precontinuous but not neutrosophic δ-pre continuous.

Remark 4.4. Remark (2.21), (2.22) and (4.2) reveals that the following figure of implications is true.

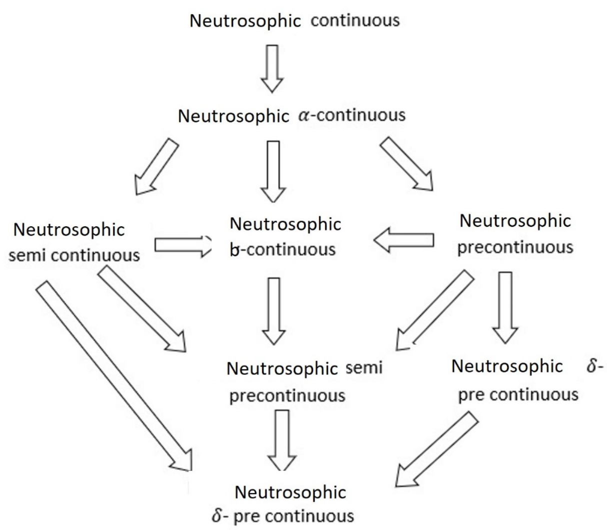

Figure 2

Theorem 4.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping from a NTS (X, τ) to NTS (Y, σ). Then the following statements are equivalent:*

(a) *f is neutrosophic semi δ-precontinuous.*

(b) *for every neutrosophic closed set A in Y, $f^{-1}(A) \in NS\delta PC(X)$.*

(c) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ in X and every neutrosophic open set A such that $f(x_{(\alpha,\eta,\beta)}) \in NS\delta PO(X)$ there is a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O$ and $f(O) \subseteq A$.*

(d) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neighborhood A of $f(x_{(\alpha,\eta,\beta)})$, $f^{-1}(A)$ is a neutrosophic semi δ-pre neighborhood of $x_{(\alpha,\eta,\beta)}$.*

(e) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neighborhood A of $f(x_{(\alpha,\eta,\beta)})$, there is a neutrosophic semi δ-pre neighborhood U of $x_{(\alpha,\eta,\beta)}$ such that $f(U) \subseteq A$.*

(f) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every neutrosophic open set A such that $f(x_{(\alpha,\eta,\beta)})_q A$, there is a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)_q} O$ and $f(O) \subseteq A$.*

(g) *for every neutrosophic point $x_{(\alpha,\eta,\beta)}$ of X and every Q-neighborhood A of $f(x_{(\alpha,\eta,\beta)})$, $f^{-1}(A)$ is a neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$.*

(h) *for every neutrosophic point* $x_{(\alpha,\eta,\beta)}$ *of* X *and every* Q*-neighborhood* A *of* $f(x_{(\alpha,\eta,\beta)})$, *there is a neutrosophic semi pre* Q*-neighborhood* U *of* $x_{(\alpha,\eta,\beta)}$ *such that* $f(U) \subseteq A$.

(i) $f(s\delta pcl(A)) \subseteq cl(f(A))$, *for every neutrosophic set* A *of* X.

(j) $s\delta pcl(f^{-1}(O)) \subseteq f^{-1}(cl(O))$, *for every neutrosophic set* O *of* Y.

(k) $f^{-1}(int(0)) \subseteq s\delta pint(f^{-1}(O))$, *for every neutrosophic set* O *of* Y.

Proof. $(a) \Rightarrow (b)$ Obvious.

$(a) \Rightarrow (c)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a neutrosophic open set in Y such that $f(x_{(\alpha,\eta,\beta)}) \in A$. Put $O = f^{-1}(A)$, then by (a), $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O$ and $f(O) \subseteq A$.

$(c) \Rightarrow (a)$ Let A be a neutrosophic open set in Y and $x_{(\alpha,\eta,\beta)} \in f^{-1}(A)$. Then $f(x_{(\alpha,\eta,\beta)}) \in A$. Now by (c) there is a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O$ and $f(O) \subseteq A$. Then $x_{(\alpha,\eta,\beta)} \in O \subseteq f^{-1}(A)$. Hence by theorem (3.11), $f^{-1}(A) \in NS\delta PO(X)$.

$(a) \Rightarrow (d)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a neighborhood of $f(x_{(\alpha,\eta,\beta)})$. Then there is a neutrosophic open set U such that $f(x_{(\alpha,\eta,\beta)}) \in U \subseteq A$. Now $F^{-1}(U) \in NS\delta PO(X)$ and $x_{(\alpha,\eta,\beta)} \in f^{-1}(U) \subseteq f^{-1}(A)$. Thus $f^{-1}(A)$ is a neutrosophic semi pre neighborhood of $x_{(\alpha,\eta,\beta)}$ in X.

$(d) \Rightarrow (e)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a neighborhood of $f(x_{(\alpha,\eta,\beta)})$. Then $U = f^{-1}(A)$ is a neutrosophic semi δ-pre neighborhood of $x_{(\alpha,\eta,\beta)}$ and $f(U) = f(f^{-1}(A)) \subseteq A$.

$(e) \Rightarrow (c)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a neutrosophic open set such that $f(x_{(\alpha,\eta,\beta)}) \in A$. Then A is a neighborhood of $f(x_{(\alpha,\eta,\beta)})$. So there is neutrosophic semi δ-pre neighborhood U of $x_{(\alpha,\eta,\beta)}$ in X such that $x_{(\alpha,\eta,\beta)} \in U$ and $f(U) \subseteq A$. Hence there is a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)} \in O \subseteq U$ and so $f(O) \subseteq f(U) \subseteq A$.

$(a) \Rightarrow (f)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a neutrosophic open set in Y such that $f(x_{(\alpha,\eta,\beta)})_q \in A$. Let $O = f^{-1}(A)$. Then $O \in NS\delta PO(X)$, $x_{(\alpha,\eta,\beta)q}O$ and $f(O) = f(f^{-1}(A)) \subseteq A$.

$(f) \Rightarrow (a)$ Let A be a neutrosophic open set in Y and $x_{(\alpha,\eta,\beta)} \in f^{-1}(A)$ clearly $f(x_{(\alpha,\eta,\beta)}) \in A$, choose the neutrosophic point $x^c_{(\alpha,\eta,\beta)}$ defined as

$$x^c_{(\alpha,\eta,\beta)}(z) = \begin{cases} (\beta,\eta,\alpha) & \text{if } z = x \\ (1,0,0) & \text{if } z \neq x \end{cases} \tag{2}$$

Then $f(x_{(\alpha,\eta,\beta)})_q A$ and so by (f), there exists a neutrosophic set $O \in NS\delta PO(X)$, such that $x_{(\alpha,\eta,\beta)q}O$ and $f(O) \subseteq A$.

Now $x^c_{(\alpha,\eta,\beta)q}O$ implies $x_{(\alpha,\eta,\beta)} \in O$.

Thus $x_{(\alpha,\eta,\beta)} \in O \subseteq f^{-1}(A)$. Hence by theorem (3.11) $f^{-1} \in NS\delta PO(X)$.

$(f) \Rightarrow (g)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a Q-neighborhood of $f(x_{(\alpha,\eta,\beta)})$. Then there is a neutrosophic open set A_1 in Y such that $f(x_{(\alpha,\eta,\beta)})_q A_1 \subseteq A$. By hypothesis there is a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)q} O$ and $f(O) \subseteq A_1$. Thus $x_{(\alpha,\eta,\beta)q} O \subseteq f^{-1}(A_1) \subseteq f^{-1}(A)$.
Hence $f^{-1}(A)$ is a neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$.

$(g) \Rightarrow (h)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a Q-neighborhood of $f(x_{(\alpha,\eta,\beta)})$. Then $U = f^{-1}(A)$ is a neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$ and $f(U) = f(f^{-1}(A)) \subseteq A$.

$(g) \Rightarrow (f)$ Let $x_{(\alpha,\eta,\beta)}$ be a neutrosophic point of X and A be a neutrosophic open set such that $f(x_{(\alpha,\eta,\beta)})_q \in A$. Then A is Q-neighborhood of $f(x_{(\alpha,\eta,\beta)})$. So there is a neutrosophic semi δ-pre Q-neighborhood δ of $x_{(\alpha,\eta,\beta)}$ such that $f(U) \subseteq A$. Now U being a neutrosophic semi δ-pre Q-neighborhood of $x_{(\alpha,\eta,\beta)}$. Then there exists a neutrosophic set $O \in NS\delta PO(X)$ such that $x_{(\alpha,\eta,\beta)q} O \subseteq U$. Hence $x_{(\alpha,\eta,\beta)q} O$ and $f(O) \subseteq f(U) \subseteq A$.

$(b) \Leftrightarrow (i)$ Obvious.
$(h) \Leftrightarrow (j)$ Obvious.
$(j) \Leftrightarrow (k)$ Obvious. □

Theorem 4.6. *Let X, X_1, X_2 be a neutrosophic topological spaces and $p_i : X_1 \times X_2 \to X_i$ $(i = 1, 2)$ be the projection of $X_1 \times X_2$ into X_i. Then if $f : X_1 \times X_2$ is a neutrosophic semi δ- pre continuous mapping, it follows that $p_i o f$ is also a neutrosophic semi δ- pre continuous mapping.*

Theorem 4.7. *Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. If the graph mapping $g : X \to X \times Y$ of f is a neutrosophic semi δ-pre continuous, then f is a neutrosophic semi δ-pre continuous.*

Proof. Let O be a neutrosophic open set of Y. Then $\tilde{1} \times O$ is a neutrosophic open in $X \times Y$. Since g is a neutrosophic semi δ-pre continuous, $g^{-1}(\tilde{1} \times O) \in NS\delta PO(X)$. But $f^{-1}(O) = \tilde{1} \cap f^{-1}(O) = g^{-1}(\tilde{1} \times O)$, $f^{-1}(O) \in NS\delta PO(X)$. Hence f is a neutrosophic semi δ-pre continuous. □

Theorem 4.8. *Let X_i and X_i^* $(i = 1, 2)$ be a neutrosophic topological spaces such that X_1 is product related to X_2. If $f_i : X_i \to X_i^*$ $(i = 1, 2)$ is a neutrosophic semi δ-pre continuous, then $f_1 \times f_2 : X_1 \times X_2 \to X_i^* \times X_2^*$ is a neutrosophic semi δ-pre continuous.*

Proof. Let $A = \cup(A_\alpha \times O_\beta)$ where $A_\alpha^{'}s$ and $O_\beta^{'}s$ are neutrosophic open sets of X_1^* and X_2^* respectively, be a neutrosophic open sets of $X_1^* \times X_2^*$. We obtain $(f_1 \times f_2)^{-1}(A) = \cup\{f_1^{-1}(A_\alpha) \times f_2^{-1}(O_\beta)\}$. Since f_1 and f_2 are neutrosophic semi δ-pre continuous, $f_1^{-1}(A_\alpha) \in NS\delta PO(X_1)$ and $f_2^{-1}(O_\beta) \in NS\delta PO(X_2)$. Therefore by theorem (3.16)(b), $f_1^{-1}(A_\alpha) \times f_2^{-1}(O_\beta) \in NS\delta PO(X_1 \times X_2)$. Hence by theorem (3.10), $(f_1 \times f_2)^{-1}(A) \in NS\delta PO(X_1 \times X_2)$. □

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