NSS
Neutrosophic Sets and Systems, Vol. 73, 2024
University of New Mexico

Schweizer-Sklar power aggregation operators based on complexsingle-valued neutrosophic information using SMART and their application in green supply chain management

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Abstract. A complex single-valued neutrosophic (CSVN) set is a very useful tool to handle the uncertainty, inconsistency, and ambiguity of knowledge in a periodic framework. The Schweizer-Sklar (SS) t-norm (TN) and t-conorm (TCN) enhance the flexibility of a procedure for aggregation depending on its parameters, whereas the power aggregation(PA) operator prevents insufficient impact of excessively high or low arguments on the results. Based on the idea of Schweizer-Sklar power aggregation operators, this paper aims to propose a number of aggregation operators such as complex single-valued neutrosophic Schweizer-Sklar power weighted averaging (CSVNSSPWA), complex single-valued neutrosophic Schweizer-Sklar power ordered weighted averaging (CSVNSSPOWA). Weights determination to solve green supply chain management problems in the single valued neutrosophic context is accomplished through the Simple Multi-Attribute Rating Technique (SMART). Furthermore, an example is given to show the uniqueness of the proposed operator and the difference between the proposed work and previous research.

Keywords: Complex single valued neutrosophic set; Power aggregation operator; Schweizer- Sklar operator; SMART; Green supply chain management.

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1. **Introduction**

Uncertainty is one of the conditions where certainty of knowledge is low or unclear, or when results and patterns are less identifiable or precise. It is an inherent feature of different areas of expertise such as sciences, mathematics, economy, and decision-making. In general, the multi attribute group decision making (MAGDM) helps to handle decision-making problems as it considers the expert's opinion and available choices. MAGDM problems deal with the identification of the most appropriate choice or set of alternatives regarding some criteria that reflect the views of the decision maker, which are expressed by certain attribute values. To address the uncertainty information, Zadeh [1] introduced the extension of the classical set theory in the form of fuzzy set (FS) theory with the aid of membership degree μ, which lies between 0 and 1.

Atanassov [2] introduced the concept of intuitionistic fuzzy sets (IFS), which defines membership and non- membership functions with the restriction that the total of membership and non-membership grades must be ≤ 1. However, it is not able to solve all kinds of uncertain problems or issues and other real life problems especially when dealing with indeterminate data, Smarandache [3] defined neutrosophic set (NS) to deal with the information that is indeterminate or inconsistent in a real environment independently. Mariming Wang et al. [4] proposed the concept of single valued neutrosophic(SVN) based on the real standard interval [0,1] for solving the decision-making problems. This set is a generalization of the following sets; crisp set, fuzzy set(FS), interval-valued fuzzy set(IVFS), intuitionistic fuzzy set(IFS), interval-valued intuitionistic fuzzy set(IVIFS), etc. Table 1 consists of the comparison of CNS with existing methods.

Table 1. Analysis of CNS against existing sets

Models	Truth	Indeterminacy	Falsity	Periodicity
FS	✓	×	×	×
IFS	✓	×	✓	×
NS	✓	✓	✓	×
CFS	✓	×	×	✓
CIFS	✓	×	✓	✓
CNS	✓	✓	✓	✓

Recently, an emerging trend is the use of big data which is often associated with uncertainty and periodicity. However, FS, IFS, IVIFS, and SVN sets are applied to handle uncertain data but cannot address periodical data. To handle this problem, Ramot et al. [5] put forward a groundbreaking idea and named it complex fuzzy set (CFS) as a synthesis of the other two concepts: fuzzy set and complex number. The complex fuzzy set still retains the representation of the uncertainty of information by the amplitude term having a value belonging to the set [0,1] with addition of phase term. The developed complex fuzzy set is a fundamentally different from the concept of fuzzy complex number developed by Buckley [6], Nguyen et al [7] and Zhang et al [8]. Membership functions of complex fuzzy sets have amplitude and phase terms which exhibit wave characteristics such as interference and periodicity. The interference can be constructive or destructive based on the phase term. The concept of the complex intuitionistic fuzzy set (CIFS) was introduced by Alkouri and Saleh [9]. In order to confer even more flexibility to decision-makers, it is recommended that the experts provide their preferences in intervals. In this context, Garg and Rani [10] have presented the concept of complex interval-valued intuitionistic fuzzy set (CIVIFS), its algebraic operators and aggregation, along with the development of a model for handling MAGDM. Ali and Smarandache [11] further put forward the complex neutrosophic set (CNS) improving the prior CFS, CIFS as well as CIVIFS, and used CNS for signal processing. The CNS is defined from the concepts of complex-valued truth, indeterminate and falsehood membership functions.

1.1. *Literature study*

MAGDM is mainly utilized for selecting problems and corresponds to decision space with limited choice and preference order. A aggregation operator(AO) is one of the most used operators in solving multi- attribute decision making problems where its main purpose to combine multiple values into a single numeric value. Grabisch et al. [12] in the "Encyclopedia of Mathematics and its Applications" and Beliakov et at. [13], who has taken work related to aggregation functions who defined averaging operators for Atanassov's IFS. A few years ago, several geometric aggregation operators have been considered by Xu and Yager [14], Wang and Liu [15], and Xu with Da [16]. Aggregation operators have been studied by various authors; in particular, Garg and Arora [17] as well as Xia et al. [18] have focused on Archimedean t-norm and t-conorm. The single-valued neutrosophic set was later widely used in operational research to develop multi- attribute decision-making techniques such as TOPSIS [19], COPRAS [20], VIKOR [21] , WASPAS [22] , EDAS [23] , TODIM [24] and others.

Finally, after some attempts, Ramot et al. [5] discover the new theory called CFS by incorporating the phase term into the supporting grade. Rani and Garg [10]established a MADM method by describing the basic algorithm of CIFS with different power operators. Moreover, Mahmood and Ali [42] derived the complex single valued neutrosophic set. Deschrijver et al. [25] and Klement et al. [26] have explored various types of t-norms and t-conorms. In 2010, Xu and Yager [27] first proposed and defined the abstraction idea of power aggregation. Since the range of the power aggregation operators has been successfully derived, many people has applied it in many fields. The Schweizer-Sklar (SS) t-norm and t-conorm theory was developed by Schweizer and Sklar [28]. This was achieved by adding a parameter p, which is more flexible and helpful in handling complex situations. The Hamacher and Lukasiewicz t-norm information can be obtained easily by using the parameter $p = -1$. Hence, the theory of TODIM and Schweizer-Sklar power aggregation operators can be derived respectively by Zindani et al [29]. T-norms, also known as triangular norms, along with their dual counterparts, t-conorms, play a crucial role in fuzzy systems, particularly in fuzzy sets and its derivatives. Notably, some of the most well-known t-norms and t-conorms, such as Frank's, Archimedean, Einstein, Dombi, and Algebraic t-norms and t-conorms, as well as the Aczel-Alsina (AA) t-norm, Schweizer-Sklar, Yager, and others, have been thoroughly investigated by Klir [30]. In addition, given the interdependency of criteria, it is appropriate to aggregate with a mean function that is either power, Bonferroni or Heronian. The weights of the attribute have also been another concern particularly in cases where they are completely unknown. In these instances, the SMART method can be applied to derive the attribute weights based on the subjective insights of decision experts. Keshavarz-Ghorabaee et al. [44] employed the SMART method for the computation of the weights of criteria.

The main motive of Green supply chain management(GSCM) is to improve the supply chain's environmental capacity sustainability, which has been substantiated in empirical research by a recent review on operations management [33]. GSCM has been widely used as a preventive technique in enhancing the environmental quality of products and policies that meet set environmental standards. The choices on both supply management and environmental issues are shifting towards increasing systematic processes globally. This indicates the need to come up with ways of integrating environmental aspects of supply chain management analyses. Analyzing the literature where the topic of GSCM was discussed, the author stated that the amount of research in this area remains rather limited.

The need to adopt the complex neutrosophic model in the area is to accurately solve and handle uncertainty and ambiguity information within the decision-making of GSCM. The more inclusive classification also helps to explain to investigators, interpreters and educationalists the relevance of stating that a more useful methodology is connected to GSCM. Table 2 reveals a gap in the existing literature on Schweizer-Sklar power aggregation operators.

1.2. *Motivation of the paper*

Here are the motivations behind the research presented in the Schweizer- Sklar power operators under CSVN information:

1. Enhancing the accuracy of decision-making processes by introducing Schweizer- Sklar power operators for handling complex information.
2. Contributing to the advancement of decision analytics theory by developing and analyzing new operators within the CSVN framework.
3. Comparing newly proposed operators with existing ones to validate their effectiveness and demonstrate their superiority.
4. Providing practical tools and insights for decision-makers in various domains by showcasing the utility of Schweizer- Sklar power operators under CSVN sets.

TABLE 2. Comparison with the existing Schweizer-Sklar power aggregation operator with different sets

Author	Set	Averaging Operators	DM Solution Method	Application	Weighting Vector	Periodicity Involved
H. Zhang et al (2019) [36]	Neutrosophic Set	SVNSSMM WSVNSSMM	Ranking	Application in Company Investment	Known	No
D. Zindawi et al (2020) [37]	Interval-valued Intuitionistic Fuzzy Set	IVIFSSPWA IVIFSSPWG	TODIM	Material, Personnel, Supplier Selection	Known	No
A. Biwas, N. Deb (2021) [38]	Pythagorean Fuzzy Set	PFSSPA PFSSPWA PFSSPG PFSSPWG	Ranking	Selection of Best Emerging Technology Enterprise	Known	No
Q. Khan et al (2021) [39]	T-Spherical Fuzzy Set	T-SPHFSSPHEM T-SPHFSSGHEM T-SPHFSSPWHEM T-SPHFSSPWGHEM	Ranking	Water Reuse Application	Known	No
U. Khalid (2023) [40]	Interval-valued Pythagorean Fuzzy Set	IVPFSSPG IVPFSSPA IVPFSSPWA	Ranking	Rice Quality Management	Known	No
U. Kalsoom et al (2023) [41]	Complex Interval-valued Intuitionistic Fuzzy Set	CIVIFSSPA CIVIFSSPOA CIVIFSSPG CIVIFSSPOG	Ranking	Selection of Green Suppliers	Unknown	Yes
P. Liu et al (2024) [42]	Complex Intuitionistic Fuzzy Set	CA-IFSSPA CA-IFSSPOA CA-IFSSPG CA-IFSSPOG	Ranking	Application of an Electronic Commerce	Unknown Distributor	Yes
Bai Chungsong (2024) [43]	Cubic Intuitionistic Fuzzy Set	CIFSSPWA CIFSSPWG	Ranking	Diabetes Care	Known	Yes
Proposed	Complex Neutrosophic Set	CSVNSSPWA CSVNSSOWA	Ranking	Green Supply Chain Management	Unknown (SMART)	Yes

1.3. *Contribution of the paper*

(1) To develop a complex single-valued neutrosophic set model utilizing an application of the Schweizer-Sklar t-norm and t-conorm.

(2) To derive complex single-valued neutrosophic Schweizer-Sklar power weighted average (CSVNSSPWA) and complex single-valued neutrosophic Schweizer-Sklar power ordered weighted average (CSVNSSPOWA) operators.

(3) To examine the key properties of these newly proposed operators.

(4) To formulate a multi attribute group decision making (MADM) method for evaluating green supply chain management problem using proposed operators complex neutrosophic information.

(5) To illustrate the effectiveness of the proposed methodology by comparing the newly developed operators with known operators through application example.

1.4. *Layout of the paper*

This paper is organized as follows: some important definitions are recalled in Section 2, namely CSVNs, PA operators and several of Schweizer-Sklar operational laws. Section 3 also presents and discusses the CSVNSSPWA, CSVNSSPOWA operators. In Section 4, a decision-making approach based on CSVNs is suggested and the procedure to evaluate MADM problems is described. Section 5 gives a numerical example to illustrate the decision-making process using the obtained operators and makes a comparison with other operators in order to reveal the advantages and efficiency of the developed methods. Finally, the concluding summary is provided in Section 6.

2. **Preliminary**

This section introduces fundamental concepts of CSVN set within the universal set $\bar{\bar{\mathcal{U}}}$.

Definition 2.1. *[11] A complex single valued neutrosophic (CSVN) $\mathcal{I}$, on the universal set $\bar{\bar{\mathcal{U}}}$ is of the form :*

$$\mathcal{I} = \{\langle \dot{æ}, \mathbb{T}_{\mathcal{I}}(\dot{æ}), \mathbb{I}_{\mathcal{I}}(\dot{æ}), \mathbb{F}_{\mathcal{I}}(\dot{æ}) | \dot{æ} \in \bar{\bar{\mathcal{U}}} \rangle\}$$

where, complex valued truth membership function expressed by $\mathbb{T}_{\mathcal{I}}(\dot{æ}) = \mathcal{P}_{\mathcal{I}}.e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}}}(\dot{æ})}$, complex valued indeterminacy membership function expressed by $\mathbb{I}_{\mathcal{I}}(\dot{æ}) = \mathcal{S}_{\mathcal{I}}.e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}}}(\dot{æ})}$, complex valued falsity membership function expressed by $\mathbb{F}_{\mathcal{I}}(\dot{æ}) = \mathcal{R}_{\mathcal{I}}.e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}}}(\dot{æ})}$. Here, $\mathcal{P}_{\mathcal{I}}$, $\mathcal{S}_{\mathcal{I}}$, $\mathcal{R}_{\mathcal{I}} \in [0,1]$ and $\omega_{\mathcal{P}_{\mathcal{I}}}$, $\omega_{\mathcal{S}_{\mathcal{I}}}$, $\omega_{\mathcal{R}_{\mathcal{I}}} \in [0,1]$ and $i = \sqrt{-1}$ such that $0 \leq \mathcal{P}_{\mathcal{I}} + \mathcal{S}_{\mathcal{I}} + \mathcal{R}_{\mathcal{I}} \leq 3$. If $\bar{\bar{\mathcal{U}}}$ contain single element then, for CSVN set $\mathcal{I}$ on $\bar{\bar{\mathcal{U}}}$, we write $X = ((\mathcal{P}_{\mathcal{I}}, \omega_{\mathcal{P}_{\mathcal{I}}}), (\mathcal{S}_{\mathcal{I}}, \omega_{\mathcal{S}_{\mathcal{I}}}), (\mathcal{R}_{\mathcal{I}}, \omega_{\mathcal{R}_{\mathcal{I}}}))$ and is termed as complex single value neutrosophic number.

Definition 2.2. *Let $\bar{\bar{X}} = ((\mathcal{P}_x, \omega_{\mathcal{P}_x}), (\mathcal{S}_x, \omega_{\mathcal{S}_x}), (\mathcal{R}_x, \omega_{\mathcal{R}_x}))$ and $\bar{\bar{Y}} = ((\mathcal{P}_y, \omega_{\mathcal{P}_y}), (\mathcal{S}_y, \omega_{\mathcal{S}_y}), (\mathcal{R}_y, \omega_{\mathcal{R}_y}))$ be the two complex single valued neutrosophic number (CSVNN) and $\lambda \geq 0$ be a real number. Then we have,*

(1) $\bar{\bar{X}} \subseteq \bar{\bar{Y}}$ *if and only if* $\mathcal{P}_x \leq \mathcal{P}_y$, $\mathcal{S}_x \geq \mathcal{S}_y$, $\mathcal{R}_x \geq \mathcal{R}_y$ *and* $\omega_{\mathcal{P}_x} \leq \omega_{\mathcal{P}_y}$, $\omega_{\mathcal{S}_x} \geq \omega_{\mathcal{S}_y}$, $\omega_{\mathcal{R}_x} \geq \omega_{\mathcal{R}_y}$.
(2) $\bar{\bar{X}} = \bar{\bar{Y}}$ *if* $\bar{\bar{X}} \subseteq \bar{\bar{Y}}$ *and* $\bar{\bar{Y}} \subseteq \bar{\bar{X}}$.
(3) $\bar{\bar{X}}^c = ((\mathcal{R}_x, \omega_{\mathcal{R}_x}), (\mathcal{S}_x, \omega_{\mathcal{S}_x}), (\mathcal{P}_x, \omega_{\mathcal{P}_x}))$.
(4) $\bar{\bar{X}} \cup \bar{\bar{Y}} = \langle max(\mathcal{P}_x, \mathcal{P}_y).e^{i2\pi(max(\omega_{\mathcal{P}_x}, \omega_{\mathcal{P}_y}))}, min(\mathcal{S}_x, \mathcal{S}_y).e^{i2\pi(min(\omega_{\mathcal{S}_x}, \omega_{\mathcal{S}_y}))}, min(\mathcal{R}_x, \mathcal{R}_y).e^{i2\pi(min(\omega_{\mathcal{R}_x}, \omega_{\mathcal{R}_y}))}\rangle$.

(5) $\bar{\bar{X}} \cap \bar{\bar{Y}} = \langle min(\mathcal{P}_x, \mathcal{P}_y).e^{i2\pi(min(\omega_{\mathcal{P}_x}, \omega_{\mathcal{P}_y}))}, \quad max(\mathcal{S}_x, \mathcal{S}_y).e^{i2\pi(max(\omega_{\mathcal{S}_x}, \omega_{\mathcal{S}_y}))}, max(\mathcal{R}_x, \mathcal{R}_y).e^{i2\pi(max(\omega_{\mathcal{R}_x}, \omega_{\mathcal{R}_y}))}\rangle.$

Definition 2.3. *The basic operational laws on these numbers are given by,*

(1) $\bar{\bar{X}} \oplus \bar{\bar{Y}} = \langle(\mathcal{P}_x + \mathcal{P}_y - \mathcal{P}_x\mathcal{P}_y).e^{i2\pi(\omega_{\mathcal{P}_x} + \omega_{\mathcal{P}_y} - \omega_{\mathcal{P}_x}\omega_{\mathcal{P}_y})}, \mathcal{S}_x.\mathcal{S}_y e^{i2\pi(\omega_{\mathcal{S}_x}.\omega_{\mathcal{S}_y})}, \mathcal{R}_x.\mathcal{R}_y e^{i2\pi(\omega_{\mathcal{R}_x}.\omega_{\mathcal{R}_y})}\rangle,$

(2) $\bar{\bar{X}} \otimes \bar{\bar{Y}} = \langle(\mathcal{P}_x.\mathcal{P}_y e^{i2\pi(\omega_{\mathcal{P}_x}.\omega_{\mathcal{P}_y})}, \mathcal{S}_x + \mathcal{S}_y - \mathcal{S}_x\mathcal{S}_y.e^{i2\pi(\omega_{\mathcal{S}_x} + \omega_{\mathcal{S}_y} - \omega_{\mathcal{S}_x}\omega_{\mathcal{S}_y})}, \mathcal{R}_x + \mathcal{R}_y - \mathcal{R}_x\mathcal{R}_y.e^{i2\pi(\omega_{\mathcal{R}_x} + \omega_{\mathcal{R}_y} - \omega_{\mathcal{R}_x}\omega_{\mathcal{R}_y})}\rangle,$

(3) $\lambda\bar{\bar{X}} = \langle(1-(1-\mathcal{P}_x)^\lambda)e^{i2\pi(1-(1-\mathcal{P}_x)^\lambda)}, (\mathcal{S}_x)^\lambda e^{i2\pi(\omega_{\mathcal{S}_x})^\lambda}, (\mathcal{R}_x)^\lambda e^{i2\pi(\omega_{\mathcal{R}_x})^\lambda}\rangle,$

(4) $\bar{\bar{X}}^\lambda = \langle(\mathcal{P}_x)^\lambda e^{i2\pi(\omega_{\mathcal{P}_x})^\lambda}, (1-(1-\mathcal{S}_x)^\lambda)e^{i2\pi(1-(1-\mathcal{S}_x)^\lambda)}, (1-(1-\mathcal{R}_x)^\lambda)e^{i2\pi(1-(1-\mathcal{R}_x)^\lambda)}\rangle.$

Definition 2.4. *[34] For any CSVN $\mathcal{I} = ((\mathcal{P}_\mathcal{I}, \omega_{\mathcal{P}_\mathcal{I}}), (\mathcal{S}_\mathcal{I}, \omega_{\mathcal{S}_\mathcal{I}}), (\mathcal{R}_\mathcal{I}, \omega_{\mathcal{R}_\mathcal{I}}))$, its score and accuracy functions can be explained as follows,*

$$\mathfrak{S}_\mathcal{I} = \frac{|\mathcal{P}_\mathcal{I} - \mathcal{S}_\mathcal{I} - \mathcal{R}_\mathcal{I} + \omega_{\mathcal{P}_\mathcal{I}} - \omega_{\mathcal{S}_\mathcal{I}} - \omega_{\mathcal{R}_\mathcal{I}}|}{3} \tag{1}$$

$$\mathfrak{A}_\mathcal{I} = \frac{\mathcal{P}_\mathcal{I} + \mathcal{S}_\mathcal{I} + \mathcal{R}_\mathcal{I} + \omega_{\mathcal{P}_\mathcal{I}} + \omega_{\mathcal{S}_\mathcal{I}} + \omega_{\mathcal{R}_\mathcal{I}}}{3} \tag{2}$$

Definition 2.5. *Let $\mathcal{I}_1 = ((\mathcal{P}_{\mathcal{I}_1}, \omega_{\mathcal{P}_{\mathcal{I}_1}}), (\mathcal{S}_{\mathcal{I}_1}, \omega_{\mathcal{S}_{\mathcal{I}_1}}), (\mathcal{R}_{\mathcal{I}_1}, \omega_{\mathcal{R}_{\mathcal{I}_1}}))$ and $\mathcal{I}_2 = ((\mathcal{P}_{\mathcal{I}_2}, \omega_{\mathcal{P}_{\mathcal{I}_2}}), (\mathcal{S}_{\mathcal{I}_2}, \omega_{\mathcal{S}_{\mathcal{I}_2}}), (\mathcal{R}_{\mathcal{I}_2}, \omega_{\mathcal{R}_{\mathcal{I}_2}}))$ be any two CSVNs and $\mathfrak{S}_{\mathcal{I}_j}$ and $\mathfrak{A}_{\mathcal{I}_j}$ for j=1,2, representing their respective score and accuracy values, we obtain the following results:*

(1) *If $\mathfrak{S}_{\mathcal{I}_1} > \mathfrak{S}_{\mathcal{I}_2}$, then $\mathcal{I}_1 > \mathcal{I}_2$.*
(2) *If $\mathfrak{S}_{\mathcal{I}_1} < \mathfrak{S}_{\mathcal{I}_2}$, then $\mathcal{I}_1 < \mathcal{I}_2$.*
(3) *If $\mathfrak{S}_{\mathcal{I}_1} > \mathfrak{S}_{\mathcal{I}_2}$, then*
  a. *If $\mathfrak{A}_{\mathcal{I}_1} > \mathfrak{A}_{\mathcal{I}_2}$, then $\mathcal{I}_1 > \mathcal{I}_2$.*
  b. *If $\mathfrak{A}_{\mathcal{I}_1} < \mathfrak{A}_{\mathcal{I}_2}$, then $\mathcal{I}_1 < \mathcal{I}_2$.*

Definition 2.6. *[32] Let $\mathcal{I}_1 = ((\mathcal{P}_{\mathcal{I}_1}, \omega_{\mathcal{P}_{\mathcal{I}_1}}), (\mathcal{S}_{\mathcal{I}_1}, \omega_{\mathcal{S}_{\mathcal{I}_1}}), (\mathcal{R}_{\mathcal{I}_1}, \omega_{\mathcal{R}_{\mathcal{I}_1}}))$ and $\mathcal{I}_2 = ((\mathcal{P}_{\mathcal{I}_2}, \omega_{\mathcal{P}_{\mathcal{I}_2}}), (\mathcal{S}_{\mathcal{I}_2}, \omega_{\mathcal{S}_{\mathcal{I}_2}}), (\mathcal{R}_{\mathcal{I}_2}, \omega_{\mathcal{R}_{\mathcal{I}_2}}))$ be any two CSVN numbers, then the Euclidean distance between them is determined by:*

$$\mathfrak{D}(\mathcal{I}_1, \mathcal{I}_2) = \sqrt{\begin{array}{l}\frac{1}{6}\left(|\mathcal{P}_{\mathcal{I}_1} - \mathcal{P}_{\mathcal{I}_2}|^2 + |\mathcal{S}_{\mathcal{I}_1} - \mathcal{S}_{\mathcal{I}_2}|^2 + |\mathcal{R}_{\mathcal{I}_1} - \mathcal{R}_{\mathcal{I}_2}|^2\right. \\ \left. + |\omega_{\mathcal{P}_{\mathcal{I}_1}} - \omega_{\mathcal{P}_{\mathcal{I}_2}}|^2 + |\omega_{\mathcal{S}_{\mathcal{I}_1}} - \omega_{\mathcal{S}_{\mathcal{I}_2}}|^2 + |\omega_{\mathcal{R}_{\mathcal{I}_1}} - \omega_{\mathcal{R}_{\mathcal{I}_2}}|^2\right)\end{array}} \tag{3}$$

Definition 2.7. *[31] For any set of positive numbers $\mathcal{I}_i$, where i=1,2,..n, the final forms of the power average(PA) operator is given by:*

$$PA(\breve{\mathcal{Y}}_1, \breve{\mathcal{Y}}_2, \ldots, \breve{\mathcal{Y}}_n) = \frac{\sum_{i=1}^{n}(1+\acute{U}(\breve{\mathcal{Y}}_i))\breve{\mathcal{Y}}_i}{\sum_{i=1}^{n} 1+\acute{U}(\breve{\mathcal{Y}}_i)} \tag{4}$$

The value of $\acute{U}(\breve{\mathcal{Y}}_i) = \sum_{j=1, j\neq k}^{n}$ sup $(\breve{\mathcal{Y}}_i, \breve{\mathcal{Y}}_j)$ the support of the $\breve{\mathcal{Y}}_i$ & $\breve{\mathcal{Y}}_j$ under the consideration of some variable properties:

(1) *sup* $(\breve{\mathcal{Y}}_i, \breve{\mathcal{Y}}_j) \in [0,1]$;
(2) *sup* $(\breve{\mathcal{Y}}_i, \breve{\mathcal{Y}}_j) =$ *sup* $(\breve{\mathcal{Y}}_j, \breve{\mathcal{Y}}_i)$;
(3) *sup* $(\breve{\mathcal{Y}}_i, \breve{\mathcal{Y}}_j) \geq$ *sup* $(\breve{\mathcal{Y}}_s, \breve{\mathcal{Y}}_t)$ *if* $|\breve{\mathcal{Y}}_i, \breve{\mathcal{Y}}_j| < |\breve{\mathcal{Y}}_s, \breve{\mathcal{Y}}_t|$.

Inspired by Yager (2001), Xu and Yager (2010) developed the power geometric (PG) operator, which is based on the PA operator and the geometric mean.

Definition 2.8. *[28] Let $\mathcal{I}_1 = ((\mathcal{P}_{\mathcal{I}_1}, \omega_{\mathcal{P}_{\mathcal{I}_1}}), (\mathcal{S}_{\mathcal{I}_1}, \omega_{\mathcal{S}_{\mathcal{I}_1}}), (\mathcal{R}_{\mathcal{I}_1}, \omega_{\mathcal{R}_{\mathcal{I}_1}}))$ and $\mathcal{I}_2 = ((\mathcal{P}_{\mathcal{I}_2}, \omega_{\mathcal{P}_{\mathcal{I}_2}}), (\mathcal{S}_{\mathcal{I}_2}, \omega_{\mathcal{S}_{\mathcal{I}_2}}), (\mathcal{R}_{\mathcal{I}_2}, \omega_{\mathcal{R}_{\mathcal{I}_2}}))$ be any two CSVN numbers.The generalized union and intersection are specified as follows:*

$\mathcal{I}_1 \cup_{\mathfrak{T},\mathfrak{T}^*} \mathcal{I}_2 = \{\langle \dot{æ}, \{\mathfrak{T}^*\{\mathbb{T}_{\mathcal{I}_1}(\dot{æ}), \mathbb{T}_{\mathcal{I}_2}(\dot{æ})\}\}, \{\mathfrak{T}\{\mathbb{I}_{\mathcal{I}_1}(\dot{æ}), \mathbb{I}_{\mathcal{I}_2}(\dot{æ})\}\}, \{\mathfrak{T}\{\mathbb{F}_{\mathcal{I}_1}(\dot{æ}), \mathbb{F}_{\mathcal{I}_2}\}(\dot{æ})\} | \dot{æ} \in X\rangle\}$;
$\mathcal{I}_1 \cap_{\mathfrak{T},\mathfrak{T}^*} \mathcal{I}_2 = \{\langle \dot{æ}, \{\mathfrak{T}\{\mathbb{T}_{\mathcal{I}_1}(\dot{æ}), \mathbb{T}_{\mathcal{I}_2}(\dot{æ})\}\}, \{\mathfrak{T}^*\{\mathbb{I}_{\mathcal{I}_1}(\dot{æ}), \mathbb{I}_{\mathcal{I}_2}(\dot{æ})\}\}, \{\mathfrak{T}^*\{\mathbb{F}_{\mathcal{I}_1}(\dot{æ}), \mathbb{F}_{\mathcal{I}_2}(\dot{æ})\}\} | \dot{æ} \in X\}\rangle\}$; *where $\mathfrak{T}$ and $\mathfrak{T}^*$ denote the t-norm and t-conorm respectively. However, Schwweizer-Sklar t-norm and t-conorm are defined as follows:*

$$\mathfrak{T}(\mathfrak{x}, \mathfrak{y}) = (\mathfrak{x}^\tau + \mathfrak{y}^\tau - 1)^{\frac{1}{\tau}}$$
$$\mathfrak{T}^*(\mathfrak{x}, \mathfrak{y}) = 1 - ((1-\mathfrak{x})^\tau + (1-\mathfrak{y})^\tau - 1)^{\frac{1}{\tau}}$$

where $\tau \leq 0$ and $(\mathfrak{x}, \mathfrak{y}) \in [0,1]$. Furthermore, when $\tau = 0$, then $\mathfrak{T}(\mathfrak{x}, \mathfrak{y}) = \mathfrak{x}\mathfrak{y}$ and $\mathfrak{T}^*(\mathfrak{x}, \mathfrak{y}) = \mathfrak{x} + \mathfrak{y} - \mathfrak{x}\mathfrak{y}$. These are algebraic t-norm and t-conorm respectively.

3. Formulation of CSVN Schweizer-Sklar Power Aggregation Operators

This section focuses on determining the Schweizer-Sklar operating rules for CSVN data. We propose a theory for power aggregation operators, including CSVNSSPWA, CSVNSSPOWA operators based on Schweizer-Sklar operational law. Additionally, properties and results for the derived work are outlined.

3.1. *CSVN Schweizer-Sklar power averaging operators*

Definition 3.1. *Let $\mathcal{I}_i = ((\mathcal{P}_{\mathcal{I}_i}, \omega_{\mathcal{P}_{\mathcal{I}_i}}), (\mathcal{S}_{\mathcal{I}_i}, \omega_{\mathcal{S}_{\mathcal{I}_i}}), (\mathcal{R}_{\mathcal{I}_i}, \omega_{\mathcal{R}_{\mathcal{I}_i}}))$,where $i = 1,2$ be a collection of CSVNs. Then the CSVNSSPA operator of dimension n is a mapping CSVNSSPA:$\mathcal{I}^n \rightarrow \mathcal{I}$ such that*

$$CSVNSSPA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \frac{\oplus_{i=1}^{n}((1+\mathfrak{U}(\mathcal{I}_i))\mathcal{I}_i)}{\sum_{i=1}^{n}(1+\mathfrak{U}(\mathcal{I}_i))} \tag{5}$$

where $\mathcal{I}$ is the set of all CSVN numbers and $\mathfrak{U}(\mathcal{I}_j) = \sum_{k=1,k\neq i}^{n} Sup(\mathcal{I}_i, \mathcal{I}_k)$.

Theorem 3.1. *For a group of CSVNs $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$,where $i = 1, 2, ..., n$, the value aggregated by the developed CSVNSSPA operator is still a CSVNN and specified by:*

$$CSVNSSPA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n)$$

$$= \begin{pmatrix} \left[1 - (\sum_{i=1}^{n} \Xi_i (1-\mathcal{P}_{\mathcal{I}_i})^{\alpha})^{\frac{1}{\alpha}}\right] \cdot e^{2\pi i\left[1-\left(\sum_{i=1}^{n} \Xi_i (1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^{n} \Xi_i \mathcal{S}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}} \cdot e^{2\pi i\left[\sum_{i=1}^{n} \Xi_i \omega_{\mathcal{S}_{\mathcal{I}_i}}^{\alpha}\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^{n} \Xi_i \mathcal{R}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}} \cdot e^{2\pi i\left[\sum_{i=1}^{n} \Xi_i \omega_{\mathcal{R}_{\mathcal{I}_i}}^{\alpha}\right]^{\frac{1}{\alpha}}} \end{pmatrix} \tag{6}$$

where $(i = 1, 2, ..., n)$ is a set of integrated weights, $\Xi_i = \frac{1+\mathfrak{U}(\mathcal{I}_i)}{\sum_{i=1}^{n}(1+\mathfrak{U}(\mathcal{I}_i))}$.

Proof. Equation 6 will be proven through the method of mathematical induction.

$$\begin{aligned} CSVNSSPA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) &= \frac{\oplus_{i=1}^{n}((1+\mathfrak{U}(\mathcal{I}_i))\mathcal{I}_i)}{\sum_{i=1}^{n}(1+\mathfrak{U}(\mathcal{I}_i))} \\ &= \oplus_{i=1}^{n}\left[\frac{((1+\mathfrak{U}(\mathcal{I}_i))\mathcal{I}_i)}{\sum_{i=1}^{n}(1+\mathfrak{U}(\mathcal{I}_i))}\right] \\ &= \oplus_{i=1}^{n}\left[\Xi_i \mathcal{I}_i\right] \end{aligned}$$

where, $\Xi_i = \frac{((1+\mathfrak{U}(\mathcal{I}_i))\mathcal{I}_i)}{\sum_{i=1}^{n}(1+\mathfrak{U}(\mathcal{I}_i))}$.

Then, based on the operational rules of CSVN numbers using Schweizer- Skalr operations, we have

$$\Xi_i \mathcal{I}_i = \begin{pmatrix} \left[1 - (\Xi_i (1-\mathcal{P}_{\mathcal{I}_i})^{\alpha} - \Xi_i + 1)^{\frac{1}{\alpha}}\right] \cdot e^{2\pi i\left[1-\left(\Xi_i (1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha} - \Xi_i + 1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\Xi_i \mathcal{S}_{\mathcal{I}_i}^{\alpha} - \Xi_i + 1\right]^{\frac{1}{\alpha}} \cdot e^{2\pi i\left[\Xi_i \omega_{\mathcal{S}_{\mathcal{I}_i}}^{\alpha} - \Xi_i + 1\right]^{\frac{1}{\alpha}}}, \\ \left[\Xi_i \mathcal{R}_{\mathcal{I}_i}^{\alpha} - \Xi_i + 1\right]^{\frac{1}{\alpha}} \cdot e^{2\pi i\left[\Xi_i \omega_{\mathcal{R}_{\mathcal{I}_i}}^{\alpha} - \Xi_i + 1\right]^{\frac{1}{\alpha}}} \end{pmatrix}.$$

When $n = 2$, we have

$$\Xi_1 \mathcal{I}_1 = \begin{pmatrix} \left[1 - (\Xi_1 (1-\mathcal{P}_{\mathcal{I}_1})^{\alpha} - \Xi_1 + 1)^{\frac{1}{\alpha}}\right] \cdot e^{2\pi i\left[1-\left(\Xi_1 (1-\omega_{\mathcal{P}_{\mathcal{I}_1}})^{\alpha} - \Xi_1 + 1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\Xi_1 \mathcal{S}_{\mathcal{I}_1}^{\alpha} - \Xi_1 + 1\right]^{\frac{1}{\alpha}} \cdot e^{2\pi i\left[\Xi_1 \omega_{\mathcal{S}_{\mathcal{I}_1}}^{\alpha} - \Xi_1 + 1\right]^{\frac{1}{\alpha}}}, \\ \left[\Xi_1 \mathcal{R}_{\mathcal{I}_1}^{\alpha} - \Xi_1 + 1\right]^{\frac{1}{\alpha}} \cdot e^{2\pi i\left[\Xi_1 \omega_{\mathcal{R}_{\mathcal{I}_1}}^{\alpha} - \Xi_1 + 1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

$$\Xi_2\mathcal{I}_2 = \begin{pmatrix} \left[1-(\Xi_2(1-\mathcal{P}_{\mathcal{I}_2})^\alpha - \Xi_2+1)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\Xi_2(1-\omega_{\mathcal{P}_{\mathcal{I}_2}})^\alpha-\Xi_2+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\Xi_2\mathcal{S}^\alpha_{\mathcal{I}_2}-\Xi_2+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_2\omega^\alpha_{\mathcal{S}_{\mathcal{I}_2}}-\Xi_2+1\right]^{\frac{1}{\alpha}}}, \\ \left[\Xi_2\mathcal{R}^\alpha_{\mathcal{I}_2}-\Xi_2+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_2\omega^\alpha_{\mathcal{R}_{\mathcal{I}_2}}-\Xi_2+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

then,

$$CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2) = \Xi_1\mathcal{I}_1 \oplus \Xi_2\mathcal{I}_2$$

$$= \begin{pmatrix} \left[1-(\Xi_1(1-\mathcal{P}_{\mathcal{I}_1})^\alpha - \Xi_1+1)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\Xi_1(1-\omega_{\mathcal{P}_{\mathcal{I}_1}})^\alpha-\Xi_1+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\Xi_1\mathcal{S}^\alpha_{\mathcal{I}_1}-\Xi_1+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_1\omega^\alpha_{\mathcal{S}_{\mathcal{I}_1}}-\Xi_1+1\right]^{\frac{1}{\alpha}}}, \\ \left[\Xi_1\mathcal{R}^\alpha_{\mathcal{I}_1}-\Xi_1+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_1\omega^\alpha_{\mathcal{R}_{\mathcal{I}_1}}-\Xi_1+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

$$\oplus \begin{pmatrix} \left[1-(\Xi_2(1-\mathcal{P}_{\mathcal{I}_2})^\alpha - \Xi_2+1)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\Xi_2(1-\omega_{\mathcal{P}_{\mathcal{I}_2}})^\alpha-\Xi_2+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\Xi_2\mathcal{S}^\alpha_{\mathcal{I}_2}-\Xi_2+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_2\omega^\alpha_{\mathcal{S}_{\mathcal{I}_2}}-\Xi_2+1\right]^{\frac{1}{\alpha}}}, \\ \left[\Xi_2\mathcal{R}^\alpha_{\mathcal{I}_2}-\Xi_2+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_2\omega^\alpha_{\mathcal{R}_{\mathcal{I}_2}}-\Xi_2+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

$$= \begin{pmatrix} \left[1-\left(\sum_{i=1}^2\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^\alpha - \sum_{i=1}^2\Xi_i+1\right)^{\frac{1}{\alpha}}\right] \cdot \\ e^{2\pi i\left[1-\left(\sum_{i=1}^2\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^\alpha-\sum_{i=1}^2\Xi_i+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^2\Xi_i\mathcal{S}^\alpha_{\mathcal{I}_i}-\sum_{i=1}^2\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^2\Xi_i\omega^\alpha_{\mathcal{S}_{\mathcal{I}_i}}-\sum_{i=1}^2\Xi_i+1\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^2\Xi_i\mathcal{R}^\alpha_{\mathcal{I}_i}-\sum_{i=1}^2\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^2\Xi_i\omega^\alpha_{\mathcal{R}_{\mathcal{I}_i}}-\sum_{i=1}^2\Xi_i+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

Suppose n=m,

$$CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_m)$$

$$= \begin{pmatrix} \left[1-(\sum_{i=1}^m\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^\alpha - \sum_{i=1}^m\Xi_i+1)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\sum_{i=1}^m\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^\alpha-\sum_{i=1}^m\Xi_i+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^m\Xi_i\mathcal{S}^\alpha_{\mathcal{I}_i}-\sum_{i=1}^m\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^m\Xi_i\omega^\alpha_{\mathcal{S}_{\mathcal{I}_i}}-\sum_{i=1}^m\Xi_i+1\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^m\Xi_i\mathcal{R}^\alpha_{\mathcal{I}_i}-\sum_{i=1}^m\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^m\Xi_i\omega^\alpha_{\mathcal{R}_{\mathcal{I}_i}}-\sum_{i=1}^m\Xi_i+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

When n= m+1, according to the operational rules of CSVN numbers, we have $\Xi_{m+1}\mathcal{I}_{m+1}$

$$= \begin{pmatrix} \left[1-\left(\Xi_{m+1}(1-\mathcal{P}_{\mathcal{I}_{m+1}})^{\alpha}-\Xi_{m+1}+1\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\Xi_{m+1}(1-\omega_{\mathcal{P}_{\mathcal{I}_{m+1}}})^{\alpha}-\Xi_{m+1}+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\Xi_{m+1}\mathcal{S}^{\alpha}_{\mathcal{I}_{m+1}}-\Xi_{m+1}+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_{m+1}\omega^{\alpha}_{\mathcal{S}_{\mathcal{I}_{m+1}}}-\Xi_{m+1}+1\right]^{\frac{1}{\alpha}}}, \\ \left[\Xi_{m+1}\mathcal{R}^{\alpha}_{\mathcal{I}_{m+1}}-\Xi_{m+1}+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\Xi_{m+1}\omega^{\alpha}_{\mathcal{R}_{\mathcal{I}_{m+1}}}-\Xi_{m+1}+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

and

$$CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\dots,\mathcal{I}_m,\mathcal{I}_{m+1}) = CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\dots,\mathcal{I}_m)\oplus(\Xi_{m+1}\mathcal{I}_{m+1})$$

$$= \begin{pmatrix} \left[1-\left(\sum_{i=1}^{m+1}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{m+1}\Xi_i+1\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\sum_{i=1}^{m+1}\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{m+1}\Xi_i+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^{m+1}\Xi_i\mathcal{S}^{\alpha}_{\mathcal{I}_i}-\sum_{i=1}^{m+1}\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{m+1}\Xi_i\omega^{\alpha}_{\mathcal{S}_{\mathcal{I}_i}}-\sum_{i=1}^{m+1}\Xi_i+1\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^{m+1}\Xi_i\mathcal{R}^{\alpha}_{\mathcal{I}_i}-\sum_{i=1}^{m+1}\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{m+1}\Xi_i\omega^{\alpha}_{\mathcal{R}_{\mathcal{I}_i}}-\sum_{i=1}^{m+1}\Xi_i+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

□ Following this, we will investigate the principles of idempotency, monotonicity, and boundedness based on the information given in Equation 6.

Proposition 1. (Idempotency)
For any collection of CSVNs, $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$, where (i=1,2,...,n), if $\mathcal{I}_i = \mathcal{I} = (\mathcal{P}_{\mathcal{I}}e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}}}}, \mathcal{S}_{\mathcal{I}}e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}}}}, \mathcal{R}_{\mathcal{I}}e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}}}})$, then $CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\dots,\mathcal{I}_n) = \mathcal{I}$.

Proof.

$$CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_n)=\begin{pmatrix} \left[1-(\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1)^{\frac{1}{\alpha}}\right]\cdot \\ e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^{n}\Xi_i\mathcal{S}_{\mathcal{I}_i}^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{n}\Xi_i\omega_{\mathcal{S}_{\mathcal{I}_i}}^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^{n}\Xi_i\mathcal{R}_{\mathcal{I}_i}^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{n}\Xi_i\omega_{\mathcal{R}_{\mathcal{I}_i}}^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

$$=\begin{pmatrix} \left[1-((1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}-1+1)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left((1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}-1+1\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\mathcal{S}_{\mathcal{I}_i}^{\alpha}-1+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\omega_{\mathcal{S}_{\mathcal{I}_i}}^{\alpha}-1+1\right]^{\frac{1}{\alpha}}}, \\ \left[\mathcal{R}_{\mathcal{I}_i}^{\alpha}-1+1\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\omega_{\mathcal{R}_{\mathcal{I}_i}}^{\alpha}-1+1\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

$$=\begin{pmatrix} [1-((1-\mathcal{P}_{\mathcal{I}_i})^{\alpha})^{\frac{1}{\alpha}}]\cdot e^{2\pi i\left[1-\left((1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\mathcal{S}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\omega_{\mathcal{S}_{\mathcal{I}_i}^{\alpha}}\right]^{\frac{1}{\alpha}}}, \\ \left[\mathcal{R}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\omega_{\mathcal{R}_{\mathcal{I}_i}^{\alpha}}\right]^{\frac{1}{\alpha}}} \end{pmatrix}$$

$$=\left([1-(1-\mathcal{P}_{\mathcal{I}_i})].e^{2\pi i[1-(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})]},[\mathcal{S}_{\mathcal{I}_i}].e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}},[\mathcal{R}_{\mathcal{I}_i}].e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right)$$

$$=\left(\mathcal{P}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i}}},\mathcal{S}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}},\mathcal{R}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right)$$

$$=\mathcal{I}.$$

□**Proposition 2.** (Monotonicity)

For any collection of CSVNs $\mathcal{I}_i=\left\langle\mathcal{P}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i}}},\mathcal{S}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}},\mathcal{R}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right\rangle$ which satisfies $CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_n)\leq CSVNSSPA(\mathcal{I}'_1,\mathcal{I}'_2,\ldots,\mathcal{I}'_n)$, i=1,2,...,n if $\mathcal{I}_i=\mathcal{I}'_i$.

Proof. Let if $\mathcal{I}_i=\mathcal{I}'_i$, then $\mathcal{P}_{\mathcal{I}_i}\leq\mathcal{P}'_{\mathcal{I}_i}$, $\mathcal{S}_{\mathcal{I}_i}\leq\mathcal{S}'_{\mathcal{I}_i}$, $\mathcal{R}_{\mathcal{I}_i}\leq\mathcal{R}'_{\mathcal{I}_i}$ and $\omega_{\mathcal{P}_{\mathcal{I}_i}}\leq\omega_{\mathcal{P}'_{\mathcal{I}_i}}$, $\omega_{\mathcal{S}_{\mathcal{I}_i}}\leq\omega_{\mathcal{S}'_{\mathcal{I}_i}}$, $\omega_{\mathcal{R}_{\mathcal{I}_i}}\leq\omega_{\mathcal{R}'_{\mathcal{I}_i}}$, thus

$$
\begin{aligned}
&\mathcal{P}_{\mathcal{I}_i} \leq \mathcal{P}'_{\mathcal{I}_i}\\
&\quad\Rightarrow \quad 1-\mathcal{P}_{\mathcal{I}_i} \geq 1-\mathcal{P}'_{\mathcal{I}_i}\\
&\quad\Rightarrow \quad (1-\mathcal{P}_{\mathcal{I}_i})^{\alpha} \geq (1-\mathcal{P}'_{\mathcal{I}_i})^{\alpha}\\
&\quad\Rightarrow \quad \sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha} \geq \sum_{i=1}^{n}\Xi_i(1-\mathcal{P}'_{\mathcal{I}_i})^{\alpha}\\
&\quad\Rightarrow \quad \left[\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}\\
&\quad\Rightarrow \quad -\left[\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq -\left[\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}\\
&\quad\Rightarrow \quad \left[1-\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[1-\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}
\end{aligned}
$$

also

$$
\Rightarrow \quad \left[1-\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[1-\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{P}'_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}
$$

Further, we derive that,

$$
\begin{aligned}
&\mathcal{S}_{\mathcal{I}_i} \leq \mathcal{S}'_{\mathcal{I}_i}\\
&\quad\Rightarrow \quad \mathcal{S}_{\mathcal{I}_i} \geq \mathcal{S}'_{\mathcal{I}_i}\\
&\quad\Rightarrow \quad (\mathcal{S}_{\mathcal{I}_i})^{\alpha} \geq (\mathcal{S}'_{\mathcal{I}_i})^{\alpha}\\
&\quad\Rightarrow \quad \sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i})^{\alpha} \geq \sum_{i=1}^{n}\Xi_i(\mathcal{S}'_{\mathcal{I}_i})^{\alpha}\\
&\quad\Rightarrow \quad \left[\sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[\sum_{i=1}^{n}\Xi_i(\mathcal{S}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}\\
&\quad\Rightarrow \quad -\left[\sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq -\left[\sum_{i=1}^{n}\Xi_i(\mathcal{S}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}\\
&\quad\Rightarrow \quad \left[1-\sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[1-\sum_{i=1}^{n}\Xi_i(\mathcal{S}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}
\end{aligned}
$$

also

$$
\Rightarrow \quad \left[1-\sum_{i=1}^{n}\Xi_i(\omega_{\mathcal{S}_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[1-\sum_{i=1}^{n}\Xi_i(\omega_{\mathcal{S}'_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}
$$

Similarly,

$\mathcal{R}_{\mathcal{I}_i} \leq \mathcal{R}'_{\mathcal{I}_i}$

$$\Rightarrow \quad \left[1-\sum_{i=1}^{n}\Xi_i(\omega_{\mathcal{R}_{\mathcal{I}_i}})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}} \geq \left[1-\sum_{i=1}^{n}\Xi_i(\omega\mathcal{R}'_{\mathcal{I}_i})^{\alpha}-\sum_{i=1}^{n}\Xi_i+1\right]^{\frac{1}{\alpha}}$$

By this, the final results can be expressed as follows:

$$CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_n) \leq CSVNSSPA(\mathcal{I}'_1,\mathcal{I}'_2,\ldots,\mathcal{I}'_n).$$

□ **Proposition 3.** (Boundedness)

For any collection of CSVNs $\mathcal{I}_i = \left\langle \mathcal{P}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right\rangle$, i=1,2,...,n if $\mathcal{I}_i^- = \langle[\min \mathcal{P}_{\mathcal{I}_i}].e^{2\pi i[\min \omega_{\mathcal{P}_{\mathcal{I}_i}}]}, [\max \mathcal{S}_{\mathcal{I}_i}].e^{2\pi i[\max \omega_{\mathcal{S}_{\mathcal{I}_i}}]}, [\max \mathcal{R}_{\mathcal{I}_i}].e^{2\pi i[\max \omega_{\mathcal{R}_{\mathcal{I}_i}}]}\rangle$ and $\mathcal{I}_i^+ = \langle[\max \mathcal{P}_{\mathcal{I}_i}].e^{2\pi i[\max \omega_{\mathcal{P}_{\mathcal{I}_i}}]}, [\min \mathcal{S}_{\mathcal{I}_i}].e^{2\pi i[\min \omega_{\mathcal{S}_{\mathcal{I}_i}}]}, [\min \mathcal{R}_{\mathcal{I}_i}].e^{2\pi i[\min \omega_{\mathcal{R}_{\mathcal{I}_i}}]}\rangle$ then, $\mathcal{I}_i^- \leq CSVNSSPA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_n) \leq \mathcal{I}_i^+$.

Proof. Suppose that $\mathcal{I}_i = \left\langle \mathcal{P}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right\rangle$, i=1,2,...,n be the collection of CSVNs. Let $\mathcal{I}_i^- = min\{\mathcal{I}_1,\mathcal{I}_2,\ldots\mathcal{I}_n\} = \left\langle \mathcal{P}_{\mathcal{I}_i^-}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i^-}}}, \mathcal{S}_{\mathcal{I}_i^-}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i^-}}}, \mathcal{R}_{\mathcal{I}_i^-}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i^-}}}\right\rangle$ and $\mathcal{I}_i^- = max\{\mathcal{I}_1,\mathcal{I}_2,\ldots\mathcal{I}_n\} = \left\langle \mathcal{P}_{\mathcal{I}_i^+}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i^+}}}, \mathcal{S}_{\mathcal{I}_i^+}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i^+}}}, \mathcal{R}_{\mathcal{I}_i^+}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i^+}}}\right\rangle$. Then, we have: $\mathcal{P}_{\mathcal{I}_i^-} = \{min\, \mathcal{P}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i}}}\}$, $\mathcal{S}_{\mathcal{I}_i^-} = \{max\, \mathcal{S}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}}\}$, $\mathcal{R}_{\mathcal{I}_i^-} = \{max\, \mathcal{R}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\}$ and $\mathcal{P}_{\mathcal{I}_i^+} = \{max\, \mathcal{P}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{P}_{\mathcal{I}_i}}}\}$, $\mathcal{S}_{\mathcal{I}_i^+} = \{min\, \mathcal{S}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{S}_{\mathcal{I}_i}}}\}$, $\mathcal{R}_{\mathcal{I}_i^+} = \{min\, \mathcal{R}_{\mathcal{I}_i}.e^{2\pi i\omega_{\mathcal{R}_{\mathcal{I}_i}}}\}$. Now,

$$\left(\begin{array}{c}
\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i^-})^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i^-}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]} \leq \\
\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]} \leq \\
\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\mathcal{P}_{\mathcal{I}_i^+})^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i^+}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]}
\end{array}\right)$$

$$\left(\begin{array}{c}
\left(\sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i^-})^{\alpha}\right)^{\frac{1}{\alpha}}\cdot e^{2\pi i\left(\sum_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{S}_{\mathcal{I}_i^-}})^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \\
\left(\sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i})^{\alpha}\right)^{\frac{1}{\alpha}}\cdot e^{2\pi i\left(\sum_{i=1}^{n}\Xi_i(\omega_{\mathcal{S}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \\
\left(\sum_{i=1}^{n}\Xi_i(\mathcal{S}_{\mathcal{I}_i^+})^{\alpha}\right)^{\frac{1}{\alpha}}\cdot e^{2\pi i\left(\sum_{i=1}^{n}\Xi_i(\omega_{\mathcal{S}_{\mathcal{I}_i^+}})^{\alpha}\right)^{\frac{1}{\alpha}}}
\end{array}\right)$$

$$\begin{pmatrix} \left(\sum\limits_{i=1}^{n}\Xi_i(\mathcal{R}_{\mathcal{I}_i^-})^{\alpha}\right)^{\frac{1}{\alpha}}\cdot e^{2\pi i\left(\sum\limits_{i=1}^{n}\Xi_i(1-\omega_{\mathcal{R}_{\mathcal{I}_i^-}})^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \\ \left(\sum\limits_{i=1}^{n}\Xi_i(\mathcal{R}_{\mathcal{I}_i})^{\alpha}\right)^{\frac{1}{\alpha}}\cdot e^{2\pi i\left(\sum\limits_{i=1}^{n}\Xi_i(\omega_{\mathcal{R}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}} \leq \\ \left(\sum\limits_{i=1}^{n}\Xi_i(\mathcal{R}_{\mathcal{I}_i^+})^{\alpha}\right)^{\frac{1}{\alpha}}\cdot e^{2\pi i\left(\sum\limits_{i=1}^{n}\Xi_i(\omega_{\mathcal{R}_{\mathcal{I}_i^+}})^{\alpha}\right)^{\frac{1}{\alpha}}} \end{pmatrix}$$

From this, it is concluded that $\mathcal{I}_i^- \leq CSVNSSPA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) \leq \mathcal{I}_i^+$. □

3.2. *CSVN Schweizer- Sklar power weighted averaging operator*

Definition 3.2. *For any collection of CSVNs $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$, $(i = 1, 2)$. Then the CSVNSSPWA operator of dimension n is a mapping CSVNSSPA:$\mathcal{I}^n \rightarrow \mathcal{I}$ such that*

$$CSVNSSPWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \frac{\oplus_{i=1}^{n}(\breve{\mathbb{W}}_i(1+\mathfrak{U}(\mathcal{I}_i))\mathcal{I}_i)}{\sum_{i=1}^{n}\breve{\mathbb{W}}_i(1+\mathfrak{U}(\mathcal{I}_i))} \tag{7}$$

where $\mathcal{I}$ is the set of all CSVN numbers and $\mathfrak{U}(\mathcal{I}_i) = \sum_{k=1,k\neq i}^{n} Sup(\mathcal{I}_i, \mathcal{I}_k)$ and $\breve{\mathbb{W}}_i = (\breve{\mathbb{W}}_1, \breve{\mathbb{W}}_2, \ldots, \breve{\mathbb{W}}_n)^T$ is the weight vector of $\mathcal{I}_i(i = 1, 2, ..., n), \breve{\mathbb{W}}_i \in [0, 1], \sum_{i=1}^{n}\breve{\mathbb{W}}_i = 1$. Suppose when $\breve{\mathbb{W}}_i = \{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\}^T$, CSVNSSPWA operator will be reduced to CSVNSSPA operator.

Theorem 3.2. *Let $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i}e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$, $(i = 1, 2)$ be a set of CSVN numbers and $\alpha < 0$ and $\breve{\mathbb{W}}_i = (\breve{\mathbb{W}}_1, \breve{\mathbb{W}}_2, \ldots, \breve{\mathbb{W}}_n)^T$ is the weight vector of $\mathcal{I}_i(i = 1, 2, ..., n), \sum_{i=1}^{n}\breve{\mathbb{W}}_i = 1, \breve{\mathbb{W}}_i \in [0, 1]$, then the aggregated value obtained using CSVNSSPWA operator is also a CSVN number and can be expressed as follows:*

$$CSVNSSPWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n)$$

$$= \begin{pmatrix} \left[1-\left(\sum_{i=1}^{n}\mathbb{W}_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\mathbb{W}_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^{n}\mathbb{W}_i\mathcal{S}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{n}\mathbb{W}_i\omega_{\mathcal{S}_{\mathcal{I}_i}^{\alpha}}\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^{n}\mathbb{W}_i\mathcal{R}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{n}\mathbb{W}_i\omega_{\mathcal{R}_{\mathcal{I}_i}^{\alpha}}\right]^{\frac{1}{\alpha}}} \end{pmatrix} \tag{8}$$

where $\mathbb{W}_i(i = 1, 2, ..., n)$ is a set of integrated weights, $\mathbb{W}_i = \frac{\breve{\mathbb{W}}_i(1+\mathfrak{U}(\mathcal{I}_i))}{\sum_{i=1}^{n}\breve{\mathbb{W}}_i(1+\mathfrak{U}(\mathcal{I}_i))}$.

Proof. As the proof of Theorem 3.2 parallels that of Theorem 3.1, it is omitted. The following properties hold: □

Proposition 4. (idempotency)
Let $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$,where i=1,2,...,n be a set of CSVN numbers, if $\mathcal{I}_i = \mathcal{I} = (\mathcal{P}_{\mathcal{I}} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}}}}, \mathcal{S}_{\mathcal{I}} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}}}}, \mathcal{R}_{\mathcal{I}} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}}}})$, then $CSVNSSPWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \mathcal{I}$.

Proposition 5. (Boundedness)
Let $\mathcal{I}_i = \left(\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right)$, $i = 1, 2, \ldots, n$ be a set of CSVN numbers. Then, $\mathcal{I}_i^+ = \left\langle \ [\max \mathcal{P}_{\mathcal{I}_i}] e^{2\pi i[\max \omega_{\mathcal{P}_{\mathcal{I}_i}}]}, [\min \mathcal{S}_{\mathcal{I}_i}] e^{2\pi i[\min \omega_{\mathcal{S}_{\mathcal{I}_i}}]}, [\min \mathcal{R}_{\mathcal{I}_i}] e^{2\pi i[\min \omega_{\mathcal{R}_{\mathcal{I}_i}}]}, \ \right\rangle$ and $\mathcal{I}_i^- = \left\langle \ [\min \mathcal{P}_{\mathcal{I}_i}] e^{2\pi i[\min \omega_{\mathcal{P}_{\mathcal{I}_i}}]}, [\max \mathcal{S}_{\mathcal{I}_i}] e^{2\pi i[\max \omega_{\mathcal{S}_{\mathcal{I}_i}}]}, [\max \mathcal{R}_{\mathcal{I}_i}] e^{2\pi i[\max \omega_{\mathcal{R}_{\mathcal{I}_i}}]}, \ \right\rangle$ then,

$$\mathcal{I}_i^- \leq CSVNSSPWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) \leq \mathcal{I}_i^+$$

Proposition 6. (Monotonicity)
Let $\mathcal{I}_i(i = 1, 2, ...n)$ be any permutation of $\mathcal{I}_i(i = 1, 2, ...n)$. Then, $CSVNSSPWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) \leq CSVNSSPWA(\mathcal{I}'_1, \mathcal{I}'_2, \ldots, \mathcal{I}'_n)$.

3.3. *CSVN Schweizer - Sklar power ordered weighted averaging operator*

Definition 3.3. *For any collection of CSVNs* $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$, *$(i = 1, 2)$. Then the CSVNSSPOWA operator of dimension n is a mapping CSVNSSPOWA:$\mathcal{I}^n \to \mathcal{I}$ such that*

$$CSVNSSPOWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \frac{\oplus_{i=1}^n (\breve{\mathbb{W}}_i (1 + \mathfrak{U}(\mathcal{I}_{\sigma(i)}))\mathcal{I}_{\sigma(i)})}{\sum_{i=1}^n \breve{\mathbb{W}}_i (1 + \mathfrak{U}(\mathcal{I}_{\sigma(i)}))} \tag{9}$$

where $\sigma(i)$ is the permutation such that $\mathcal{I}_{\sigma(i-1)} \geq \mathcal{I}_{\sigma(i)}$ for any i=1,2,..,n and $\breve{\mathbb{W}}(\mathcal{I}_{\sigma(i)}) = \sum_{h=1,h\neq i}^n Sup(\mathcal{I}_{\sigma(i)}, \mathcal{I}_{\sigma(h)})$ and $\breve{\mathbb{W}}_i = (\breve{\mathbb{W}}_1, \breve{\mathbb{W}}_2, \ldots, \breve{\mathbb{W}}_n)^T$ is the weight vector of $\mathcal{I}_i(i = 1, 2, ..., n), \breve{\mathbb{W}}_i \in [0, 1], \sum_{i=1}^n \breve{\mathbb{W}}_i = 1$. Suppose when $\breve{\mathbb{W}}_i = \{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\}^T$, CSVNSSPOWA operator will be reduced to CSVNSSPA operator.

Theorem 3.3. *Let* $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$, *$(i = 1, 2)$ be a set of CSVN numbers and $\alpha < 0$ and $\breve{\mathbb{W}}_i = (\breve{\mathbb{W}}_1, \breve{\mathbb{W}}_2, \ldots, \breve{\mathbb{W}}_n)^T$ is the weight vector of $\mathcal{I}_i(i = 1, 2, ..., n), \sum_{i=1}^n \breve{\mathbb{W}}_i = 1, \breve{\mathbb{W}}_i \in [0, 1]$, then the aggregated value obtained using*

$$CSVNSSPOWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n)$$
$$= \begin{pmatrix} \left[1 - (\sum_{i=1}^n \mathfrak{E}_i (1 - \mathcal{P}_{\mathcal{I}_i})^\alpha)^{\frac{1}{\alpha}}\right] e^{2\pi i\left[1 - \left(\sum_{i=1}^n \mathfrak{E}_i (1 - \omega_{\mathcal{P}_{\mathcal{I}_i}})^\alpha\right)^{\frac{1}{\alpha}}\right]}, \\ \left[\sum_{i=1}^n \mathfrak{E}_i \mathcal{S}_{\mathcal{I}_i}^\alpha\right]^{\frac{1}{\alpha}} e^{2\pi i\left[\sum_{i=1}^n \mathfrak{E}_i \omega_{\mathcal{S}_{\mathcal{I}_i}^\alpha}\right]^{\frac{1}{\alpha}}}, \\ \left[\sum_{i=1}^n \mathfrak{E}_i \mathcal{R}_{\mathcal{I}_i}^\alpha\right]^{\frac{1}{\alpha}} e^{2\pi i\left[\sum_{i=1}^n \mathfrak{E}_i \omega_{\mathcal{R}_{\mathcal{I}_i}^\alpha}\right]^{\frac{1}{\alpha}}} \end{pmatrix} \tag{10}$$

where $\mathfrak{E}_i(i = 1, 2, ..., n)$ is a set of integrated weights, $\mathfrak{E}_i = \frac{\breve{\mathbb{W}}_i(1+\mathfrak{U}(\mathcal{I}_{\sigma(i)}))}{\sum_{i=1}^n \breve{\mathbb{W}}_i(1+\mathfrak{U}(\mathcal{I}_{\sigma(i)}))}$.

Proposition 7. (idempotency)
Let $\mathcal{I}_i = (\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}})$,where i=1,2,...,n be a set of CSVN numbers, if $\mathcal{I}_i = \mathcal{I} = (\mathcal{P}_{\mathcal{I}} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}}}}, \mathcal{S}_{\mathcal{I}} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}}}}, \mathcal{R}_{\mathcal{I}} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}}}})$, then $CSVNSSPOWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) = \mathcal{I}$.

Proposition 8. (Boundedness)
Let $\mathcal{I}_i = \left(\mathcal{P}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{P}_{\mathcal{I}_i}}}, \mathcal{S}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{S}_{\mathcal{I}_i}}}, \mathcal{R}_{\mathcal{I}_i} e^{i2\pi\omega_{\mathcal{R}_{\mathcal{I}_i}}}\right)$, $i = 1, 2, \ldots, n$ be a set of CSVN numbers. Then, $\mathcal{I}_i^+ = \left\langle\ [\max \mathcal{P}_{\mathcal{I}_i}] e^{2\pi i [\max \omega_{\mathcal{P}_{\mathcal{I}_i}}]}, [\min \mathcal{S}_{\mathcal{I}_i}] e^{2\pi i [\min \omega_{\mathcal{S}_{\mathcal{I}_i}}]}, [\min \mathcal{R}_{\mathcal{I}_i}] e^{2\pi i [\min \omega_{\mathcal{R}_{\mathcal{I}_i}}]},\ \right\rangle$ and $\mathcal{I}_i^- = \left\langle\ [\min \mathcal{P}_{\mathcal{I}_i}] e^{2\pi i [\min \omega_{\mathcal{P}_{\mathcal{I}_i}}]}, [\max \mathcal{S}_{\mathcal{I}_i}] e^{2\pi i [\max \omega_{\mathcal{S}_{\mathcal{I}_i}}]}, [\max \mathcal{R}_{\mathcal{I}_i}] e^{2\pi i [\max \omega_{\mathcal{R}_{\mathcal{I}_i}}]},\ \right\rangle$ then,

$$\mathcal{I}_i^- \leq CSVNSSPOWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) \leq \mathcal{I}_i^+$$

Proposition 9. (Monotonicity)
Let $\mathcal{I}_i(i = 1, 2, ...n)$ be any permutation of $\mathcal{I}_i(i = 1, 2, ...n)$. Then, $CSVNSSPOWA(\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_n) \leq CSVNSSPOWA(\mathcal{I}'_1, \mathcal{I}'_2, \ldots, \mathcal{I}'_n)$.

4. Decision-making procedures

In this section, we present a procedure for addressing group decision-making within a complex neutrosophic environment. Suppose $\ddot{\mathfrak{G}} = \{\ddot{\mathfrak{G}}_1, \ddot{\mathfrak{G}}_2, ..., \ddot{\mathfrak{G}}_n\}$ be a set of n alternatives, $\mathfrak{R} = \{\mathfrak{R}_1, \mathfrak{R}_2, ..., \mathfrak{R}_n\}$ be a set of n attributes with weight vectors $\mathbb{W} = (w_1, w_2, ..., w_n)^t$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, and $\dot{\dot{E}} = \{\dot{e}_1, \dot{e}_2, ..., \dot{e}_n\}$ with weight vectors $\mathbb{W} = (w_1, w_2, ..., w_n)^t$ such that $w_p \in [0, 1]$ and $\sum_{p=1}^{n} w_p = 1$. Thus, the decision matrix can be expressed as:

$$H^a = [\mathcal{I}_{ij}]_{m\times n} = \begin{array}{c} \\ \ddot{\mathfrak{G}}_1 \\ \ddot{\mathfrak{G}}_2 \\ \cdots \\ \ddot{\mathfrak{G}}_m \end{array} \begin{array}{c} \begin{matrix} \mathfrak{R}_1 & \mathfrak{R}_2 & \ldots & \mathfrak{R}_n \end{matrix} \\ \begin{bmatrix} \mathcal{I}_{11}^a & \mathcal{I}_{12}^a & \ldots & \mathcal{I}_{1n}^a \\ \mathcal{I}_{21}^a & \mathcal{I}_{22}^a & \ldots & \mathcal{I}_{2n}^a \\ \ldots & \ldots & \ldots & \ldots \\ \mathcal{I}_{m1}^a & \mathcal{I}_{m2}^a & \ldots & \mathcal{I}_{mn}^a \end{bmatrix} \end{array}$$

The following steps should be taken to identify the best alternative.

Step 1: Normalize the CSVN decision matrix by the following equation in case, the MAGDM problem contains cost and benefit factors as shown in Equation 11.

$$\mathcal{I}_{ij} = \begin{cases} \langle \mathbb{T}_{ij}^a, \mathbb{I}_{ij}^a, \mathbb{F}_{ij}^a \rangle & \text{if benefit type} \\ \langle \mathbb{F}_{ij}^a, \mathbb{I}_{ij}^a, \mathbb{T}_{ij}^a \rangle & \text{if cost type} \end{cases} \tag{11}$$

Step 2: Determine the weight $\mathfrak{D}j$ for each criterion $\mathfrak{R}j$. This process involves using the SMART technique to establish the criteria weights according to the subjective assessments of decision experts. In this approach, the decision maker ranks the criteria from least to most important. There are 10 points for the least important criterion and the most important criterion gets 100 points. The rest of the criteria are assigned point scores in an order based on their relative weights. The weight of each criterion is then obtained by normalizing total points so that their sum is equal to 1.

Step 3: Calculate the overall evaluation score for each alternative as follows:

$$CSVNSSPWA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_n)=\begin{pmatrix}\left[1-\left(\sum_{i=1}^{n}\mathbb{W}_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot\\ e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\mathbb{W}_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]},\\ \left[\left(\sum_{i=1}^{n}\mathbb{W}_i\mathcal{S}_{\mathcal{I}_i}^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[\left(\sum_{i=1}^{n}\mathbb{W}_i\omega_{\mathcal{S}_{\mathcal{I}_i}^{\alpha}}\right)^{\frac{1}{\alpha}}\right]},\\ \left[\left(\sum_{i=1}^{n}\mathbb{W}_i\mathcal{R}_{\mathcal{I}_i}^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot e^{2\pi i\left[\left(\sum_{i=1}^{n}\mathbb{W}_i\omega_{\mathcal{R}_{\mathcal{I}_i}^{\alpha}}\right)^{\frac{1}{\alpha}}\right]}\end{pmatrix}\tag{12}$$

$$CSVNSSPOWA(\mathcal{I}_1,\mathcal{I}_2,\ldots,\mathcal{I}_n)=\begin{pmatrix}\left[1-\left(\sum_{i=1}^{n}\mathfrak{E}_i(1-\mathcal{P}_{\mathcal{I}_i})^{\alpha}\right)^{\frac{1}{\alpha}}\right]\cdot\\ e^{2\pi i\left[1-\left(\sum_{i=1}^{n}\mathfrak{E}_i(1-\omega_{\mathcal{P}_{\mathcal{I}_i}})^{\alpha}\right)^{\frac{1}{\alpha}}\right]},\\ \left[\sum_{i=1}^{n}\mathfrak{E}_i\mathcal{S}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{n}\mathfrak{E}_i\omega_{\mathcal{S}_{\mathcal{I}_i}^{\alpha}}\right]^{\frac{1}{\alpha}}},\\ \left[\sum_{i=1}^{n}\mathfrak{E}_i\mathcal{R}_{\mathcal{I}_i}^{\alpha}\right]^{\frac{1}{\alpha}}\cdot e^{2\pi i\left[\sum_{i=1}^{n}\mathfrak{E}_i\omega_{\mathcal{R}_{\mathcal{I}_i}^{\alpha}}\right]^{\frac{1}{\alpha}}}\end{pmatrix}\tag{13}$$

where $i=1,2,\ldots,m; j=1,2,\ldots,n; a=1,2,\ldots,l$.

Step 4: Determine the aggregate score values for the overall evaluations $\mathcal{I}_i(i=1,2,\ldots,m)$.

Step 5: Rank the alternatives.

5. Application to Green Supply Chain Management

In this section, we form a multi attribute group decision making (MAGDM) framework using the proposed CSVNSSPWA, CSVNSSPOWA operators and employed it to assess green suppliers from the perspective of GSCM. In this context, objective is about green suppliers which are known as sustainable or eco-friendly suppliers who provides products and services are less damaging to the environment as compared to the conventional suppliers.

We choose the five green suppliers $\ddot{\mathfrak{G}}_1,\ddot{\mathfrak{G}}_2,\ddot{\mathfrak{G}}_3,\ddot{\mathfrak{G}}_4,\ddot{\mathfrak{G}}_5$ and four attributes as

- $\mathfrak{R}_1$ - product quality factor ,
- $\mathfrak{R}_2$ - environmental factor,
- $\mathfrak{R}_3$ - Delivery factor,

- $\mathfrak{R}_4$ - price factor.

The weight of the decision makers is given by $\mathbb{W} = \{0.35, 0.25, 0.40\}$. Obtain the decision makers input by utilizing the linguistic terms and their corresponding CSVNs outlined in Table 3. Table 4, 5, 6 represents the CSVN information.

TABLE 3. Linguistic terms for CSVNs

Linguistic terms	**CSVNs**
VH - Very high	[(0.6,0.5), (0.2,0.3), (0.4,0.1)]
H - High	[(0.8,0.3), (0.6,0.2), (0.3,0.4)]
MH - Moderately high	[(0.5,0.1), (0.4,0.6), (0.9,0.3)]
M - Medium	[(0.7,0.2), (0.1,0.4), (0.2,0.5)]
ML - Moderately low	[(0.3,0.1), (0.8,0.8), (0.5,0.3)]
L - Low	[(0.4,0.3), (0.6,0.4), (0.2,0.5)]
VL - Very low	[(0.5,0.2), (0.9,0.1), (0.7,0.4)]

TABLE 4. CSVN information in the decision matrix DM_1

	$\mathfrak{R}_1$	$\mathfrak{R}_2$
$\ddot{\mathfrak{G}}_1$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$
$\ddot{\mathfrak{G}}_2$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_3$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_4$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$
$\ddot{\mathfrak{G}}_5$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$
	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\ddot{\mathfrak{G}}_1$	$[(0.5, 0.1), (0.4, 0.6), (0.9, 0.3)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_2$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$
$\ddot{\mathfrak{G}}_3$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$
$\ddot{\mathfrak{G}}_4$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$
$\ddot{\mathfrak{G}}_5$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$

TABLE 5. CSVN information in the decision matrix DM_2

	$\mathfrak{R}_1$	$\mathfrak{R}_2$
$\ddot{\mathfrak{G}}_1$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_2$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$
$\ddot{\mathfrak{G}}_3$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$	$[(0.5, 0.1), (0.4, 0.6), (0.9, 0.3)]$
$\ddot{\mathfrak{G}}_4$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$
$\ddot{\mathfrak{G}}_5$	$[(0.5, 0.1), (0.4, 0.6), (0.9, 0.3)]$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$
	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\ddot{\mathfrak{G}}_1$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$
$\ddot{\mathfrak{G}}_2$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_3$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$
$\ddot{\mathfrak{G}}_4$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_5$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$

TABLE 6. CSVN information in the decision matrix DM_3

	$\mathfrak{R}_1$	$\mathfrak{R}_2$
$\ddot{\mathfrak{G}}_1$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$
$\ddot{\mathfrak{G}}_2$	$[(0.5, 0.1), (0.4, 0.6), (0.9, 0.3)]$	$[(0.5, 0.1), (0.4, 0.6), (0.9, 0.3)]$
$\ddot{\mathfrak{G}}_3$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$	$[(0.8, 0.3), (0.6, 0.2), (0.3, 0.4)]$
$\ddot{\mathfrak{G}}_4$	$[(0.5, 0.1), (0.4, 0.6), (0.9, 0.3)]$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$
$\ddot{\mathfrak{G}}_5$	$[(0.3, 0.1), (0.8, 0.8), (0.5, 0.3)]$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$
	$\mathfrak{R}_3$	$\mathfrak{R}_4$
$\ddot{\mathfrak{G}}_1$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$
$\ddot{\mathfrak{G}}_2$	$[(0.5, 0.2), (0.9, 0.1), (0.7, 0.4)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_3$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$	$[(0.6, 0.5), (0.2, 0.3), (0.4, 0.1)]$
$\ddot{\mathfrak{G}}_4$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$	$[(0.4, 0.3), (0.6, 0.4), (0.2, 0.5)]$
$\ddot{\mathfrak{G}}_5$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$	$[(0.7, 0.2), (0.1, 0.4), (0.2, 0.5)]$

Step 1: Construct a matrix (see Table 4, 5, 6) and standardise it based on the varying information types.

$$\mathcal{I}_{ij} = \begin{cases} \langle(\mathcal{P}_{\mathcal{I}}, \omega_{\mathcal{P}_{\mathcal{I}}}), (\mathcal{S}_{\mathcal{I}}, \omega_{\mathcal{S}_{\mathcal{I}}}), (\mathcal{R}_{\mathcal{I}}, \omega_{\mathcal{R}_{\mathcal{I}}})\rangle & \text{if benefit type} \\ \langle(\mathcal{R}_{\mathcal{I}}, \omega_{\mathcal{R}_{\mathcal{I}}}), (\mathcal{S}_{\mathcal{I}}, \omega_{\mathcal{S}_{\mathcal{I}}}), (\mathcal{P}_{\mathcal{I}}, \omega_{\mathcal{P}_{\mathcal{I}}})\rangle & \text{if cost type} \end{cases}$$

Normalization is not required since all the given attributes are of the benefit type.

Step 2: In this step, we use the SMART method to calculate attribute weights. Experts assign scores to each criterion, ranging from 10 for the least important to 100 for the most important. These scores are then normalized by dividing each criterion's score by the total sum of all scores, determining the final attribute weights. Table 7 displays the points allocated by each expert and the corresponding normalized attribute weights.

TABLE 7. Attribute weights

Attribute	Points assigned by			Sum	Normalized
	DM_1	DM_2	DM_3	of points	weights $\mathfrak{D}j$
$\mathfrak{R}_1$	80	70	90	240	0.26
$\mathfrak{R}_2$	90	60	80	230	0.25
$\mathfrak{R}_3$	70	80	60	210	0.22
$\mathfrak{R}_4$	90	80	80	250	0.27
		Total		**930**	**1**

Step 3: Using the theory of CSVNSSPWA, CSVNSSPOWA operators using Equations 12 and 13, we aggregate our specified decision matrix for $\alpha = -2$. The results are shown in Table 8.

TABLE 8. Aggregated matrix for CSVNSSPWA and CSVNSSPOWA operators

	CSVNSSPWA	**CSVNSSPOWA**
$\ddot{\mathfrak{G}}_1$	$[(0.6706, 0.3533), (0.2018, 0.216), (0.3005, 0.1698)]$	$[(0.6725, 0.3533), (0.1963, 0.2192), (0.3059, 0.1689)]$
$\ddot{\mathfrak{G}}_2$	$[(0.6514, 0.2487), (0.2261, 0.1975), (0.3045, 0.2715)]$	$[(0.6525, 0.2498), (0.2212, 0.2), (0.3034, 0.267)]$
$\ddot{\mathfrak{G}}_3$	$[(0.6261, 0.2487), (0.1959, 0.2039), (0.2713, 0.2043)]$	$[(0.6331, 0.3249), (0.2047, 0.2161), (0.2694, 0.1996)]$
$\ddot{\mathfrak{G}}_4$	$[(0.6273, 0.2581), (0.1992, 0.2019), (0.2656, 0.292)]$	$[(0.6166, 0.2594), (0.2037, 0.1984), (0.269, 0.2839)]$
$\ddot{\mathfrak{G}}_5$	$[(0.6611, 0.1861), (0.1846, 0.2084), (0.309, 0.364)]$	$[(0.6685, 0.1891), (0.1864, 0.2111), (0.3083, 0.3641)]$

Step 4: The score values for the CSVNSSPWA, CSVNSSPOWA operators are calculated and presented in Table 9.

Table 9. Score values

	CSVNSSPWA	CSVNSSOWA
$\ddot{\mathfrak{G}}_1$	0.0452	0.0452
$\ddot{\mathfrak{G}}_2$	0.0331	0.0298
$\ddot{\mathfrak{G}}_3$	0.0226	0.0228
$\ddot{\mathfrak{G}}_4$	0.0245	0.0263
$\ddot{\mathfrak{G}}_5$	0.0729	0.0708

Step 5: Rank the alternatives are showed in Table 10.

Table 10. Ranking of the alternatives

Operator	Ranking
CSVNSSPWA	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3$
CSVNSSOWA	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3$

5.1. *Parameter analysis on decision results*

This section presents the results of our proposed approaches by varying parametric values. We perform all the proposed methodologies with several values of the parameters to investigate the effects of the parametric values on the score values of the result obtained through the use of the CSVNSSPWA, CSVNSSPOWA operators. Table 11 summarize all the score values that have been obtained for the proposed operators respectively. Integrated results may be different at different parametric values α. The choice $\ddot{\mathfrak{G}}_5$ becomes the top preference for the operators. Figure 1 illustrate the computed score values for the CSVNSSPWA, CSVNSSPOWA operators respectively, as detailed in Table 11.

TABLE 11. Results of score values by the CSVNSSPWA and CSVNSSPOWA operators

Parameters	**CSVNSSPWA**	**CSVNSSPOWA**
$\alpha = -1$	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_1$	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_1$
$\alpha = -2$	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3$	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3$
$\alpha = -5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_5$
$\alpha = -10$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$
$\alpha = -20$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$
$\alpha = -30$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$
$\alpha = -50$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$
$\alpha = -100$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$
$\alpha = -200$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$	$\ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_5$

5.2. *Comparative Analysis*

To highlight the proficiency and prevalence of the proposed operators, it is essential to compare them with existing methods. For this, we choose an idea of Aczel - Alsina aggregation operators for CSVNS was exposed by Areeba Naseem et al. [35]. Based on the preceding discussion, the comparison results are tabulated in Table 12. Figure 2 shows the graphical structure of all computed score values from Table 12.

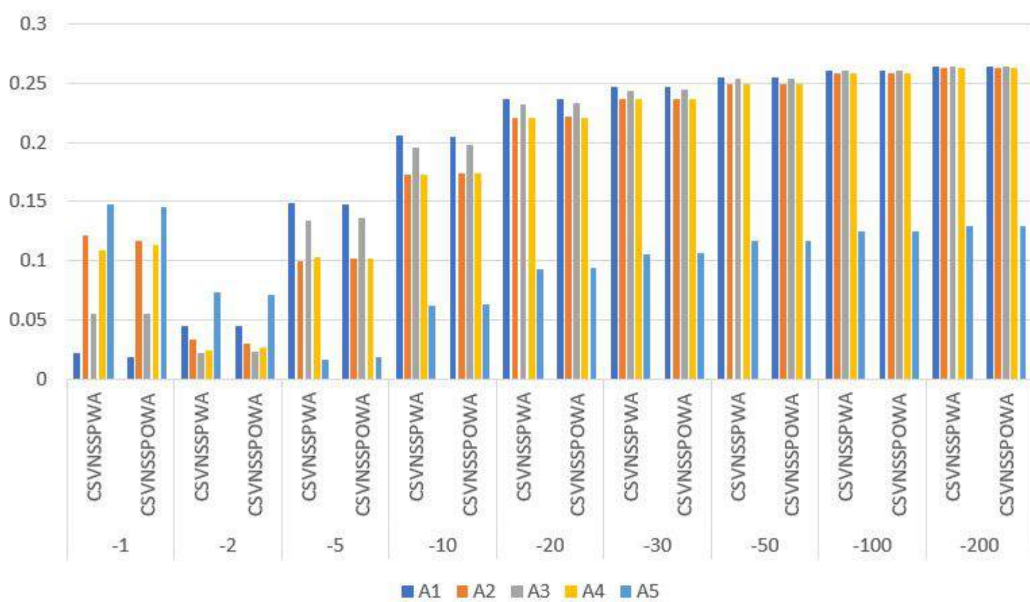

FIGURE 1. The results of the CSVNSSPWA and CSVNSSPOWA operator

Table 12. Comparative study

Aggregation operators	Score values	Ranking
CSVNAAWA ($\alpha = 1$) [35]	$\ddot{\mathfrak{G}}_1$= 0.1104, $\ddot{\mathfrak{G}}_2$= 0.2304, $\ddot{\mathfrak{G}}_3$= 0.1569, $\ddot{\mathfrak{G}}_4$= 0.2118, $\ddot{\mathfrak{G}}_5$= 0.2682.	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_1$
CSVNAAOWA ($\alpha = 1$) [35]	$\ddot{\mathfrak{G}}_1$= 0.1127, $\ddot{\mathfrak{G}}_2$= 0.2264, $\ddot{\mathfrak{G}}_3$= 0.1539, $\ddot{\mathfrak{G}}_4$= 0.2092, $\ddot{\mathfrak{G}}_5$= 0.2658.	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3 > \ddot{\mathfrak{G}}_1$
CSVNSSPWA ($\alpha = -2$)	$\ddot{\mathfrak{G}}_1$= 0.0452, $\ddot{\mathfrak{G}}_2$= 0.0331, $\ddot{\mathfrak{G}}_3$= 0.0226, $\ddot{\mathfrak{G}}_4$= 0.0245, $\ddot{\mathfrak{G}}_5$= 0.0729.	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3$
CSVNSSPOWA ($\alpha = -2$)	$\ddot{\mathfrak{G}}_1$= 0.0452, $\ddot{\mathfrak{G}}_2$= 0.0298, $\ddot{\mathfrak{G}}_3$= 0.0228, $\ddot{\mathfrak{G}}_4$= 0.0263, $\ddot{\mathfrak{G}}_5$= 0.0708.	$\ddot{\mathfrak{G}}_5 > \ddot{\mathfrak{G}}_1 > \ddot{\mathfrak{G}}_2 > \ddot{\mathfrak{G}}_4 > \ddot{\mathfrak{G}}_3$

5.3. *Stability Analysis*

In this section, Spearman's rank correlation is used to find out the stability of the proposed operators of different parameters and along with them. Table 13 shows the relationship between all the proposed operators and other parameters.

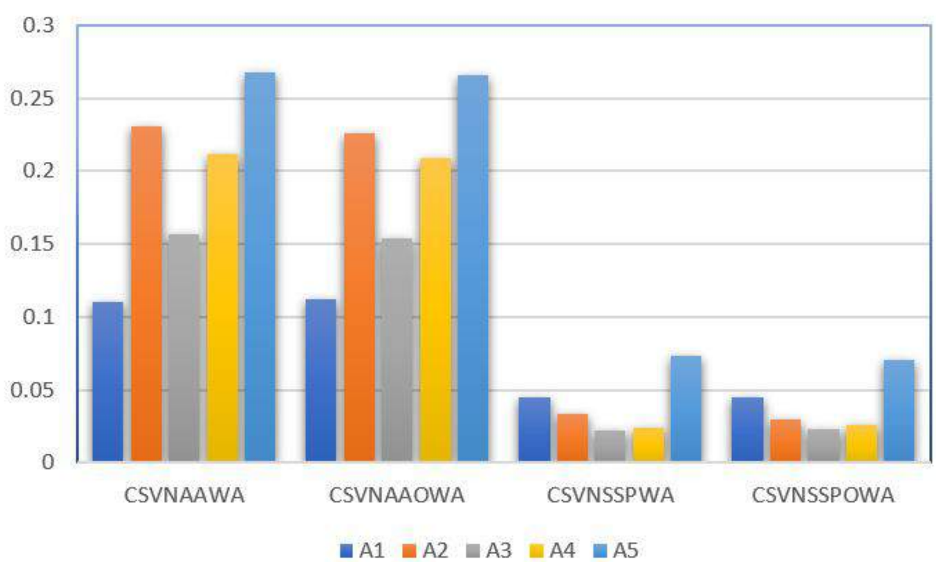

Figure 2. Aggregated results by the comparative study

TABLE 13. Comparative study

Parameter	**Aggregation Operator**	**Spearman's Rank Correlation**
$\alpha = -1$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -2$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -5$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -10$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -20$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -30$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -50$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -100$	CSVNSSPWA CSVNSSPOWA	1
$\alpha = -200$	CSVNSSPWA CSVNSSPOWA	1

6. **Conclusion**

MAGDM is a group decision-making approach that takes multiple criteria or attributes into account. The technique involves evaluating and comparing alternative choices based on these criteria to reach a collective decision. The process is facilitated by using a variety of analytical methods, including decision matrices, pairwise comparison and group decision techniques. Here we present the basic operations for the Schweizer- Sklar t-norms and t-conorms involving CNS information, which is better to fuzzify uncertainty in different fuzzy domains and provide an accurate estimation based on the decision analysis. We examined $\ddot{\mathfrak{G}}_5$ as a flexible supplier of GSCM, considering our operators. In addition, we also explore in detail how different parametric values influence the results of the proposed methodologies. Finally, a comparative technique is used to present the results of existing strategies with the developed methodologies. In the future, we intend to use novel aggregation operators, similarity measures, and new MADM methods for more complex environments. These advancements are expected to address challenging real-world problems effectively.

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Received: July 03, 2024. Accepted: August 28, 2024