NSS
Neutrosophic Sets and Systems, Vol. 73, 2024
University of New Mexico

On Diminishing Fuzzy Neutrosophic Topological Spaces over Nano Topology

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Abstract. As an extension and also, a necessity to compute the indeterminacy, the concept of diminishing fuzzy sets is protracted to introduce the conception of diminishing fuzzy (single-valued) neutrosophic sets in the standard unit interval. In addition, the axioms of neutrosophic topological spaces are extended to configure the collection, diminishing fuzzy neutrosophic topology, and also, the basic theorems of nano topology is incorporated to the case of diminishing fuzzy neutrosophic set in terms of the approximation space and the boundary region by acquiring the same operators for the quantities, non-membership and indeterminacy are also discussed. Moreover, a nano continuous function from one diminishing fuzzy neutrosophic nano topological space to another is defined with an example. Furthermore, the approximation space for the neighborhood of the diminishing cells in terms of the idea of pixel neighborhood by means of the diminishing fuzzy neutrosophic membership function and also, approximation space for the NFA of the layer word are defined with an illustration.

Keywords: Diminishing fuzzy sets, Fuzzy neutrosophic topology, Nano continuous, Neighborhood, NFA.

1. Introduction

The imprecision in nature, which is formulated as fuzzy set theory that contemplates both vagueness and uncertainty in terms of grades of membership by addressing the classical notion of general set theory, and the domains of mathematics such as general topology, algebra, operations research, image processing, and so on, had been fuzzified to model various real life applications in a different aspect [21]. Intuitionistic fuzzy set theory generalizes both fuzzy sets and classical sets which consist of membership functions along with non-membership functions [9]. As an extension of intuitionistic fuzzy set theory, neutrosophic set theory is coined employing truth-membership, falsity-membership and indeterminacy-membership constructed over]-0, 1 +[instead of [0, 1] to handle the impreciseness, incompleteness, uncertainty, and indeterminacy that arise in the real-life scenario, which was introduced in 1999 [19]. In 2010, the set-theoretic operations for the neutrosophic sets are defined over the standard unit interval called single valued neutrosophic sets (fuzzy neutrosophic sets) [4] and some of the researchers gave prominence to this study in other domains [1, 16, 18].

In 2011, an infinite array with diminishing cells is termed and denoted by D_r in which each layer is identified with the help of binary strings [7] and application of D_r array can be found in [15]. In 2012, the term nano topology based on the rough set theory concept in terms of lower approximation and upper approximation which represents interior and closure operations of a topology respectively was initiated [10, 11] and the applications in real life scenarios [2, 6, 12, 14, 20]. In 2018, a new hybrid by combining both nano topology and neutrosophic topology was called Neutrosophic nano topology based on the approximation space and the boundary region over the neutrosophic membership grades [13]. In 2020, the D_r array is generalized to the case of diminishing fuzzy sets (DFS) by defining a function based on the positions of the D_r cells and also, proved some of the fundamental theorems of fuzzy topological space for DFS [8].

This paper organizes its objectives as follows: Section 2 presents the basic definitions of diminishing cells with infinite array, single valued neutrosophic sets, fuzzy neutrosophic topology, nano topology and nano neutrosophic topology and some of the basic operations of the fuzzy neutrosophic sets. Section 3 discusses the basic theorems, lemma and proposition of nano neutrosophic topology and neutrosophic topology for diminishing fuzzy neutrosophic sets as well as a nano continuity on diminishing fuzzy neutrosophic topological space and the approximation space for the diminishing cells and the NFA for the layer word. Section 4 ends with conclusion remarks.

2. **Preliminaries**

This section presents the construction of diminishing cells with infinite array, the definitions of single valued neutrosophic set with its operations, fuzzy neutrosophic topology, nano topology and nano neutrosophic topology.

The diminishing cells in an infinite array is constructed by reducing the length and the breadth of the rectangular array recursively by a common ratio r from the source cell in the Euclidean plane along row wise and column wise respectively where $r = \frac{1}{n}, n \geq 2$ and $n \in \mathbb{N}$ and this special type of array is denoted by D_r [8]. A non-deterministic finite accepter (NFA) $\mathcal{M} = \{S, \sigma_n, \Gamma, q_0, q_n\}$ where S is the finite set of internal states, Σ_n is an alphabet, $\Gamma : S \times \Sigma_n^* \to 2^S$ [17]. Let $\mathcal{U} = \{u\}$ be a domain of discourse with a collection of $u \in \mathcal{U}$. A fuzzy set A in $\mathcal{U}$ is defined by $\mu_A : \mathcal{U} \to [0, 1]$, with the grades of membership $\mu_A(u)$ for each $u \in \mathcal{U}$ [21].

Let $U \neq \emptyset$ be a domain of finite set and R be an indiscernible relation over U. Then,

U is divided into disjoint equivalence classes and members belonging to the same equivalence class are indiscernible with one another. The pair (U, R) is called the approximation space corresponding to $X \subset U$. Then, the collection $\mathcal{T}_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ forms a nano topology, where the lower approximation $L_R(X)$ is the union of all equivalence classes $R(x)$ such that $R(x) \subseteq X$, the upper approximation $U_R(X)$ is the union of all equivalence classes intersecting with X is non empty and the boundary region $B_R(X)$ is the difference between $U_R(X)$ and $L_R(X)$ of X and its complement is called the dual nano topology, denoted by $[\mathcal{T}_R(X)]^c$. Every member belonging to $\mathcal{T}_R(X)$ is known as the nano-open sets whereas the members belonging to $[\mathcal{T}_R(X)]^c$ are referred to as nano closed sets in the nano topological space $(U, \mathcal{T}_R(X))$. The basis for $\mathcal{T}_R(X)$ with respect to X is given by $\beta_R(X) = \{U, L_R(X), B_R(X)\}$ [10]. Let $(U, \mathcal{T}_R(X))$ and $(V, \mathcal{T}_R(Y))$ be two nano topological spaces with respect to the subsets of X and Y respectively. Then, a function $f : (U, \mathcal{T}_R(X)) \to (V, \mathcal{T}_R(Y))$ is nano continuous on U if the inverse image of every nano-open set in V is nano-open in U [11]. Let $\mathcal{G}$ be a graph with a sub-graph $\mathcal{H}$ and $\mathcal{N}(p)$ be a neighborhood of the point of $\mathcal{G}$. Then, the lower approximation $L_R(\mathcal{H})$ on points of $\mathcal{H}$ is defined as the union of all neighborhood points of $\mathcal{G}$ which is contained in $\mathcal{H}$, the upper approximation $U_R(\mathcal{H})$ is the set of all neighborhood points of $\mathcal{H}$ and the boundary region is the difference between $L_R(\mathcal{H})$ and $U_R(\mathcal{H})$ [14]. A single valued neutrosophic set (SVNS) also known as fuzzy neutrosophic set A in $\mathcal{U}$ is defined in terms of the following components: truth-membership function μ_{T_A}, indeterminacy-membership function μ_{I_A} and falsity-membership function μ_{F_A} denoted by $(\mu_{T_A}, \mu_{I_A}, \mu_{F_A})$ where $\mu_{T_A}(x), \mu_{I_A}(x), \mu_{F_A}(x) \in [0, 1]$ for each point $x \in X$. The operations on SVNSs are given by: Complement: $\mu_c(A)(x) = (\mu_{F_A}(x), 1 - \mu_{I_A}(x), \mu_{T_A}(x))$, Union: $\mu_{A\cup B}(x) = (\vee(\mu_{T_A}(x), \mu_{T_B}(x)), \vee(\mu_{I_A}(x), \mu_{I_B}(x)), \wedge(\mu_{F_A}(x), \mu_{F_B}(x)))$, Intersection: $\mu_{A\cap B}(x) = (\wedge(\mu_{T_A}(x), \mu_{T_B}(x)), \wedge(\mu_{I_A}(x), \mu_{I_B}(x)), \vee(\mu_{F_A}(x), \mu_{F_B}(x)))$, Difference: $\mu_{A\ B}(x) = (\wedge(\mu_{T_A}(x), \mu_{F_B}(x)), \wedge(\mu_{I_A}(x), 1 - \mu_{I_B}(x)), \vee(\mu_{F_A}(x), \mu_{T_B}(x)))$, Containment: $\mu_{A\subset B}(x) = (\mu_{T_A}(x) \leq \mu_{T_B}(x), \mu_{F_A}(x) \leq \mu_{F_B}(x), \mu_{F_A}(x) \geq \mu_{F_B}(x))$ for each $x \in X$, where A and B are the two single valued neutrosophic sets [4]. A neutrosophic topology (NT for short) on a $X \neq \emptyset$ is a family $\mathcal{T}_N$ of neutrosophic subsets in X satisfying the following axioms: $(NT_1) 0_N, 1_N \in \mathcal{T}_N$, $(NT_2) G_1 \cap G_2 \in \mathcal{T}_N$, for any $G_1, G_2 \in \mathcal{T}_N$, $(NT_3) \cup G_i \in \mathcal{T}_N \forall \{G_i : i \in J\} \subset \mathcal{T}_N$ and we call $(X, \mathcal{T}_N)$ is called a neutrosophic topological space (NTS for short) and its elements are known as neutrosophic open set (NOS for short) in X. A neutrosophic set F is closed $\Leftrightarrow$ its complement C(F) is neutrosophic open [1].

Let $U \neq \emptyset$ be a set with an equivalence relation R on U. Let F be a neutrosophic set in U with the membership function μ_F, the indeterminacy function σ_F and the non-membership function $\sqsubseteq_F$. The neutrosophic nano lower approximation, neutrosophic nano upper approximation and neutrosophic nano boundary of F in the approximation space (U, R) denoted by $\underline{N}(F), \overline{N}(F)$

and $BN(F)$ are respectively defined as follows: $\underline{N}(F) = \{< x, \mu_{\underline{R}(A)}(x), \sigma_{\underline{R}(A)}(x), \sqsubseteq_{\underline{R}(A)}(x) > |y \in [x]_R, x \in U\}$, $\overline{N}(F) = \{< x, \mu_{\overline{R}(A)}(x), \sigma_{\overline{R}(A)}(x), \sqsubseteq_{\overline{R}(A)}(x) > |y \in [x]_R, x \in U\}$, $BN(F) = \overline{N}(F) - \underline{N}(F)$ where, $\mu_{\underline{R}(A)}(x) = \wedge_{y\in[x]_R}\mu_A(y)$, $\sigma_{\underline{R}(A)}(x) = \wedge_{y\in[x]_R}\sigma_A(y)$, $\sqsubseteq_{\underline{R}(A)}(x) = \vee_{y\in[x]_R}\sqsubseteq_A(y)$, $\mu_{\overline{R}(A)}(x) = \vee_{y\in[x]_R}\mu_A(y)$, $\sigma_{\overline{R}(A)}(x) = \vee_{y\in[x]_R}\sigma_A(y)$, $\sqsubseteq_{\overline{R}(A)}(x) = \wedge_{y\in[x]_R}\sqsubseteq_A(y)$ and if the collection $\mathcal{T}_{N_R}(F) = \{0_N, 1_N, \underline{N}(F), \overline{N}(F), BN(F)\}$ forms a topology then it is said to be a neutrosophic nano topology. We call $(U, \mathcal{T}_{N_R}(F))$ as the neutrosophic nano topological space. Every element in $\mathcal{T}_{N_R}(F)$ is called a neutrosophic nano open set [13].

3. Approximation Space and Diminishing Fuzzy Neutrosophic Sets (DFNS)

In this section, the concept of pixel neighborhood and approximation space induced by neighborhood of a vertex in graph thoery are incorporated in the conceptualization of the diminishing cells in an infinite array and also, the pictorial representation of diminishing fuzzy neutrosophic membership grades for the diminishing cells is presented and the approximation space for the NFA of the layer word is achieved. Furthermore, the definition of diminishing fuzzy set is presented and based on that, diminishing fuzzy neutrosophic set (DFNS) is introduced. Some basic theorems, lemma and propositions of nano neutrosophic topology and neutrosophic topology are extended in terms of diminishing fuzzy neutrosophic sets as well as the notion of nano continuity on diminishing fuzzy neutrosophic topological space (DFNTS), which is studied with an example.

Definition 3.1. Let D_r be a diminishing array and X_{N_r} be a sub-array of D_r array. Let Y_{N_r} be a sub-array of X_{N_r} and $N_{X_{N_r}}$ be a neighborhood of cells in X_{N_r}. Then, we define the following:

- The lower approximation of the neighborhood of the cells in X_{N_r} is defined as $L_{N_{Y_{N_r}}} : X_{N_r} \to X_{N_r}$ such that $L_{N_{Y_{N_r}}} = \bigcup_{(x,y)\in X_{N_r}} \{(x, y)|N_{X_{N_r}} \in N_{Y_{N_r}}\}$
- The upper approximation of the neighborhood of the cells in X_{N_r} is defined as $U_{N_{Y_{N_r}}} : X_{N_r} \to X_{N_r}$ such that $U_{N_{Y_{N_r}}} = \{N_{(x,y)}|(x, y) \in Y_{N_r}\}$
- The boundary region of the neighborhood of the cells in X_{N_r} is defined as $U_{N_{Y_{N_r}}} - L_{N_{Y_{N_r}}}$.

Example 3.2. Without loss of generality, Let $X_{N_r} = \{(2, 3), (3, 3), (4, 3), (2, 4), (2, 5), (3, 4), (4, 4), (3, 5), (4, 5)\}$ be a sub-array of D_r array Then, the neighborhood cells of $(2, 3)$ are $\{(3, 3), (3, 4), (2, 4)\}$, $N_{(3,3)} = \{(4, 3), (4, 4), (3, 4), (2, 4), (2, 3)\}$, $N_{(4,3)} = \{(4, 4), (3, 4), (3, 3)\}$, $N_{(2,4)} = \{(2, 3), (3, 3), (3, 4), (3, 5), (2, 5)\}$, $N_{(2,5)} = \{(2, 4), (3, 4), (3, 5)\}$, $N_{(3,4)} = \{(2, 3), (3, 3), (4, 3), (4, 4), (4, 5), (3, 5), (2, 5), (2, 4)\}$, $N_{(4,4)} = \{(4, 3), (3, 3), (3, 4), (3, 5), (4, 5)\}$, $N_{(3,5)} = \{(2, 5), (2, 4), (3, 4), (4, 4), (4, 5)\}$, and $N_{(4,5)} = \{(4, 4), (3, 4), (3, 5)\}$. Then, $L_{N_{Y_{N_r}}} = U_{N_{Y_{N_r}}} = Y_{N_r}$ which implies $B_{N_{Y_{N_r}}} = \emptyset$. This is true for all the sub-arrays of sub-array.

Remark 3.3. The lower and upper approximations for the sub-array are equal for every sub-arrays. Thus, the collection $\{X_{N_r}, \emptyset, L_{N_{Y_{N_r}}}\}$ forms a nano topological space for all the sub-arrays of sub-arrays in D_r.

Definition 3.4. The language on diminishing cells is defined as

$$\mathcal{D}_r^{\mathcal{L}}(X_{ij}^r) = \begin{cases} 0^i 1^{j-i}, & i < j \\ 1^j 0^{i-j}, & i > j \\ 1^j, & i = j \end{cases}$$

The right angle path from the cell $X_{k,1}$ to the cell $X_{1,k}$ along the cells $X_{k,2}, X_{k,3}, ...X_{k,k}, X_{k-1,k}, X_{k-2,k}, ...X_{2,k}, k \geq 2, k \in \mathbb{N}$ is called a layer. The concatenation of words on a layer is called a layer word. For example, the cells with words, $X_{21}^{10}, X_{22}^{11}, X_{12}^{01}$ form a layer word 101101.

Definition 3.5. Let D_r be a diminishing array and X_{N_r} be a sub-array of D_r array. $D_r^{L_n}$ be a n^{th} layer of the D_r array and Q_r' be a subset of Q_r, a finite set of internal states. Then, we define the approximation space for NFA as follows:

- The lower approximation of L_{Q_r} is defined as $L_{Q_r} : Q_r \to Q_r$ such that $L_{Q_r}(Q_r') = \cup_{q_i \in Q_r}\{q_i | \Gamma(q_i, w) \cap Q_r'\}$,
- The lower approximation of U_{Q_r} is defined as $U_{Q_r} : Q_r \to Q_r$ such that $U_{Q_r}(Q_r') = \cup_{q_i \in Q_r}\{q_i | \Gamma(q_i, w) \subseteq Q_r' \neq \emptyset\}$,
- The boundary region, $B_{Q_r}(Q_r') = U_{Q_r}(Q_r') - L_{Q_r}(Q_r')$.

Example 3.6. Let X_{N_r} be a sub-array of D_r array with a layer 1 and then, the respective layer word is 101101. Let $\mathcal{M} = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}, \Gamma, q_0, \{q_1, q_2, q_3\})$ with the transitions $\Gamma(q_0, 0) = \{q_1\}$, $\Gamma(q_0, 1) = \{q_0\}$, $\Gamma(q_1, 0) = \emptyset$, $\Gamma(q_1, 1) = \{q_2\}$, $\Gamma(q_2, 0) = \{q_3\}$, $\Gamma(q_2, 1) = \{q_2\}$, $\Gamma(q_3, 0) = \emptyset$, and $\Gamma(q_3, 1) = \{q_3\}$ and $Q_r' = \{q_1, q_3\}$

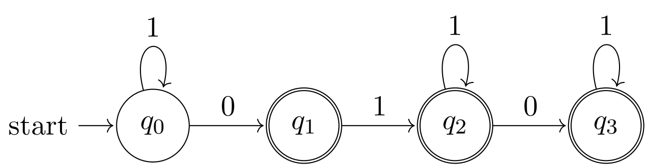

FIGURE 1. Non-deterministic finite automata for layer word with the layer 1

The approximation space of this NFA is obtained as follows

- $L_{Q_r}(Q_r') = \{q_3\}$
- $U_{Q_r}(Q_r') = \{q_0, q_2, q_3\}$
- $B_{Q_r}(Q_r') = \{q_0, q_2\}$

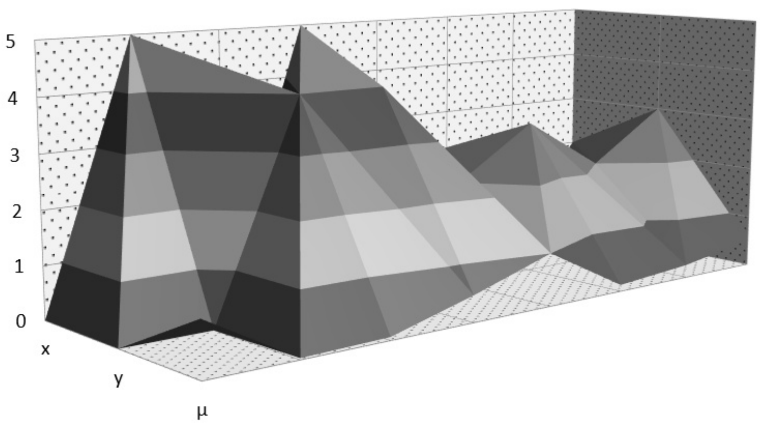

FIGURE 2. Surface plotting of diminishing fuzzy sets

Proposition 3.7. *Let D_r be a diminishing array and X_{N_r} be a sub-array of D_r array. $D_r^{L_n}$ be a n^{th} layer of the D_r array and Q'_r be a subset of Q_r, a finite set of internal states. Then,*

- $Q'_r \subset L_{Q_r}(Q'_r) \subset U_{Q_r}(Q'_r)$
- $B_{Q_r}(Q'_r) \subset U_{Q_r}(Q'_r) \subset Q_r$
- $L_{Q_r}(Q'_r) \subset B_{Q_r}(Q'_r) \subset U_{Q_r}(Q'_r)$

Remark 3.8. The collection $\mathcal{T}_r = \{\emptyset, Q_r, L_{Q_r}(Q'_r), U_{Q_r}(Q'_r), B_{Q_r}(Q'_r)\}$ forms a topology for the NFA of the layer word with respect to the finite subset of internal states Q'_r.

Definition 3.9. We define a function $\mu_{D_r} : D_r \to [0,1]$ by

$$\mu_{D_r}(x,y) = \begin{cases} 1 & x = y \\ \frac{1}{n^k} & x \neq y, k = |x-y| \end{cases}$$

where $n \in \mathbb{N}, (x,y) \in D_r$ and $n \geq 2$. We call this set as diminishing fuzzy set (DFS) and it is denoted by D_r. The class of all diminishing fuzzy sets (DFSs) is denoted by $\mathcal{D}_r$ which may be a denumerable set. The graphical representation of the diminishing fuzzy set is given in the below figure.

Definition 3.10. The grades of truth-membership, indeterminacy-membership and falsity –membership for diminishing fuzzy neutrosophic (single valued neutrosophic) set based on the measure function of DFS defined as

$$\mu_{T_{D_r}}(x,y) = \begin{cases} 1 & x = y \\ \frac{1}{n^k} & x \neq y, k = |x-y| \end{cases}$$

$$\mu_{I_{D_r}}(x,y) = \begin{cases} 0(\text{or})1 & x = y \\ \frac{n^k}{n^k+1} & x \neq y, k = |x-y| \end{cases}$$

$$\mu_{F_{D_r}}(x,y) = \begin{cases} 0 & x = y \\ \frac{n^k-1}{n^k} & x \neq y, k = |x-y| \end{cases}$$

Then, the diminishing fuzzy neutrosophic set (DFNS) can be written as

$$\mu_{N_{D_r}}(x,y) = \begin{cases} (1,0,0) & x = y \\ (\frac{1}{n^k}, \frac{n^k}{n^k+1}, \frac{n^k-1}{n^k}) & x \neq y, k = |x-y| \end{cases}$$

(or)

$$\mu_{N_{D_r}}(x,y) = \begin{cases} (1,1,0) & x = y \\ (\frac{1}{n^k}, \frac{n^k}{n^k+1}, \frac{n^k-1}{n^k}) & x \neq y, k = |x-y| \end{cases}$$

where $n \in \mathbb{N}, (x,y) \in D_r, n \geq 2$ and $0 \leq \mu_{T_{D_r}}, \mu_{I_{D_r}}, \mu_{F_{D_r}} \leq 1, 0 \leq \mu_{T_{D_r}} + \mu_{F_{D_r}} \leq 1$, $0 \leq \mu_{T_{D_r}} + \mu_{I_{D_r}} + \mu_{F_{D_r}} \leq 2$ (i.e.,) $0 \leq \frac{2n^k+1}{n^k+1} / \leq 2$.

Note 3.11. In the above definition, truth membership values are independent followed by both falsity and indeterminacy membership values that are dependent. The indeterminacy and the falsehood quantities would share the same operator throughout this paper. For example, for the arithmetic operation union, truth membership would have **max** operator whereas indeterminacy and falsehood membership will take **min** operator.

The 4-neighborhood and 8-neighborhood of the cells in the D_r array with their diminishing fuzzy neutrosophic membership grades are pictorially represented as follows:

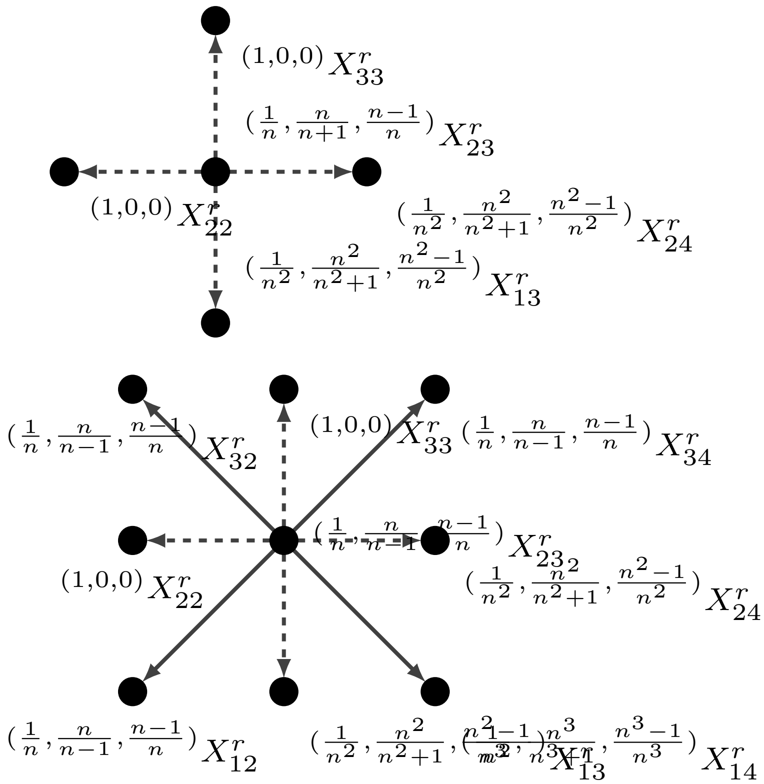

FIGURE 3. 4–Connectivity and 8–connectivity of the diminishing cells in terms of diminishing fuzzy neutosophic sets

Proposition 3.12. *Let X_{N_r} be a sub-array of D_r array with the collection $\mathcal{T}_{N_r}$. Then, $(\mu_{N_{D_r}}, \mathcal{T}_{N_r})$ is a diminishing fuzzy neutrosophic topological space.*

Proof. Without loss of generality, let us take $X_{N_r} = \{(x,y)\}$ be a finite collection of points $(x,y) \in D_r$ with its following diminishing single valued neutrosophic subsets $\mu_{N_{A_r}}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_2,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_3,y_3)}{(0,1,1)}\}$ with $n \geq 2, k = 1,2$ for (x_1,y_1) and (x_2,y_2), respectively, whereas $(x_3,y_3) = 0_{N_r}, \mu_{N_{B_r}}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_2,y_2)}{(0,1,1)}, \frac{(x_3,y_3)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}\}$ with $n \geq 2, k = 1,3$ for (x_1,y_1) and (x_3,y_3), respectively, whereas $(x_2,y_2) = 0_{N_r}, \mu_{N_{C_r}}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_2,y_2)}{(0,1,1)}, \frac{(x_3,y_3)}{(0,1,1)}\}$ with $n \geq 2, k = 1$ for (x_1,y_1) and $(x_3,y_3) = (x_2,y_2) = 0_{N_r}, \mu_{N_{D_r}}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_2,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_3,y_3)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}\}$ with $n \geq 2, k = 1,2,3$ for $(x_1,y_1), (x_2,y_2)$ and (x_3,y_3), respectively, and we claim that the topology $\mathcal{T}_{N_r}(x,y) = \{0_{N_r}, 1_{N_r}, \mu_{N_{A_r}}(x,y), \mu_{N_{B_r}}(x,y), \mu_{N_{C_r}}(x,y), \mu_{N_{D_r}}(x,y)\}$ is a diminishing fuzzy neutrosophic topology on X_{N_r} where $0_{N_r} = \{(0,1,1)\}$ and $1_{N_r} = \{(1,0,0)\}$. Then, $\mu_{N_{A_r}} \cap \mu_{N_{B_r}}(x,y) = \mu_{N_{C_r}}(x,y) \in \mathcal{T}_{N_r}, \mu_{N_{A_r}} \cap \mu_{N_{C_r}}(x,y) = \mu_{N_{C_r}}(x,y) \in \mathcal{T}_{N_r}, \mu_{N_{B_r}} \cap \mu_{N_{C_r}}(x,y) = \mu_{N_{C_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{A_r}} \cap \mu_{N_{D_r}}(x,y) = \mu_{N_{A_r}}(x,y) \in \mathcal{T}_{N_r}, \mu_{N_{B_r}} \cap \mu_{N_{D_r}}(x,y) = \mu_{N_{B_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{C_r}} \cap \mu_{N_{D_r}}(x,y) = \mu_{N_{C_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{A_r}} \cup \mu_{N_{B_r}}(x,y) = \mu_{N_{D_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{A_r}} \cup \mu_{N_{C_r}}(x,y) =$

$\mu_{N_{A_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{B_r}} \cup \mu_{N_{C_r}}(x,y) = \mu_{N_{B_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{A_r}} \cup \mu_{N_{D_r}}(x,y) = \mu_{N_{D_r}}(x,y) \in \mathcal{T}_{N_r}$, $\mu_{N_{B_r}} \cup \mu_{N_{D_r}}(x,y) = \mu_{N_{D_r}}(x,y) \in \mathcal{T}_{N_r}, \mu_{N_{C_r}} \cup \mu_{N_{D_r}}(x,y) = \mu_{N_{D_r}}(x,y) \in \mathcal{T}_{N_r}$ and so on. Thus, $\mathcal{T}_{N_r}$ is a diminishing fuzzy neutrosophic topology on X_{N_r}. Hence proved. □

Remark 3.13. Any diminishing fuzzy neutrosophic set in $\mathcal{T}_{N_r}$ is known as the diminishing fuzzy neutrosophic open set (DFNOS) in X_{N_r} and its complement is called as diminishing fuzzy neutrosophic closed set (DFNCS).

Lemma 3.14. *Let X_{N_r} be a subarray of D_r array and R_{N_r} be an equivalence relation on it. Then, (X_{N_r}, R_{N_r}) forms an approximation space for diminishing nano neutrosophic sets.*

Proof. We prove this theorem by considering the grades of membership arbitrarily for the components $\mu_{T_{D_r}}, \mu_{F_{D_r}}, \mu_{I_{D_r}}$. Let us consider the domain of discourse, $X_{N_r} = \{(x,y)\}$. Suppose that the relation X_{N_r}/R_{N_r} is defined as $X_{N_r}/R_{N_r}(x,y) = \{(x_1,y_1),(x_2,y_2)\},(x_1,y_2),(x_2,y_1)\}$. Let $\mu_{N_{A_r}}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3}7/8)}, \frac{(x_2,y_2)}{(1,0,0)}\}$ with $n \geq 2, k = 1$ for $(x_1,y_1), k = 2$ for $(x_1,y_2), k = 3$ for (x_2,y_1) and (x_2,y_2) whereas $(x_2,y_2) = 1_{N_r}$. Then, we have $\underline{N_r(\mu_{N_{A_r}})}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}\}$, $\overline{N_r(\mu_{N_{A_r}})}(x,y) = \{\frac{(x_1,y_1)}{(1,0,0)}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(1,0,0)}\}$. If $B_{N_r}(\mu_{N_{A_r}})(x,y) = \overline{N_r(\mu_{N_{A_r}})}(x,y) - \underline{N_r(\mu_{N_{A_r}})}(x,y)$, then $B_{N_r}(\mu_{N_{A_r}})(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(\frac{1}{n},\frac{n}{n-1},\frac{n-1}{n})}\}$. Thus, we obtained the approximation space with respect to the lower approximation space, the upper approximation space and the boundary region using the basic operations of the neutrosophic set. □

Remark 3.15. The fuzzy neutrosophic elements present in the boundary region B_{N_r} may not satisfy the basic definition of DFNS. Since, it involves the complement of the component $\mu_{I_{D_r}}$ yet it satisfies the general neutrosophic set property. Hence, we can call it, the elements of a fuzzy neutrosophic set.

Theorem 3.16. *Let X_{N_r} be a sub-array of D_r array and R_{N_r} be an equivalence relation on it with the collection of diminishing nano fuzzy neutrosophic subsets $\mathcal{T}_{N_{R_r}}$. Then, $(X_{N_r}, \mathcal{T}_{N_{R_r}})$ is a nano topological space for the diminishing fuzzy neutrosophic set.*

Proof. We prove this theorem with the help of lemma 3.5. Let us consider the universe of discourse as the pixel values of the image domain, $X_{N_r} = \{(x,y)\}$. Suppose that the relation X_{N_r}/R_{N_r} is defined as $X_{N_r}/R_{N_r}(x,y) = \{\{(x_1,y_1),(x_2,y_2)\},(x_1,y_2),(x_2,y_1)\}$ and $\mu_{N_{A_r}}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(1,0,0)}\}$ where $n \geq 2, k = 1$ for

$(x_1, y_1), k = 2$ for $(x_1, y_2), k = 3$ for (x_2, y_1) and $(x_2, y_2) = 1_{N_r}$ with the approximation space and the boundary region $\underline{N_r(\mu_{N_{A_r}})}(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n}1/2)}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}\}$, $\overline{N_r(\mu_{N_{A_r}})}(x,y) = \{\frac{(x_1,y_1)}{(1,0,0)}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(1,0,0)}\}$ and $B_{N_r}(\mu_{N_{A_r}})(x,y) = \{\frac{(x_1,y_1)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}, \frac{(x_1,y_2)}{(\frac{1}{n^2},\frac{n^2}{n^2+1},\frac{n^2-1}{n^2})}, \frac{(x_2,y_1)}{(\frac{1}{n^3},\frac{n^3}{n^3+1},\frac{n^3-1}{n^3})}, \frac{(x_2,y_2)}{(\frac{1}{n},\frac{n}{n+1},\frac{n-1}{n})}\}$ respectively. Then, $\mathcal{T}_{N_{R_r}}(x,y) = \{0_{N_r}, 1_{N_r}, \underline{N_r(\mu_{N_{A_r}})}(x,y), \overline{N_r(\mu_{N_{A_r}})}(x,y), B_{N_r}(\mu_{N_{A_r}})(x,y)\}$ is a nano topology on X_{N_r}. Since, $\underline{N_r(\mu_{N_{A_r}})} \cap \overline{N_r(\mu_{N_{A_r}})}(x,y) = \underline{N_r(\mu_{N_{A_r}})}(x,y) \in \mathcal{T}_{N_{R_r}}, \underline{N_r(\mu_{N_{A_r}})}m \cap B_{N_r}(\mu_{N_{A_r}})(x,y) = \underline{N_r(\mu_{N_{A_r}})}(x,y) \in \mathcal{T}_{N_{R_r}}, \underline{N_r(\mu_{N_{A_r}})} \cup \overline{N_r(\mu_{N_{A_r}})}(x,y) = \overline{N_r(\mu_{N_{A_r}})}(x,y) \in \mathcal{T}_{N_{R_r}}, \underline{N_r(\mu_{N_{A_r}})} \cup B_{N_r}(\mu_{N_{A_r}})(x,y) = B_{N_r}(\mu_{N_{A_r}})(x,y) \in \mathcal{T}_{N_{R_r}}$ and so on. Hence, $(X_{N_r}, \mathcal{T}_{N_{R_r}})$ is a nano topology for the diminishing fuzzy neutrosophic set. □

Remark 3.17. The elements of diminishing fuzzy neutrosophic nano topological space (DFNNTS) are called the diminishing fuzzy neutrosophic nano open sets (DFNNOSs) and their complement is the diminishing fuzzy neutrosophic nano closed sets (DFNNCSs).

Theorem 3.18. *Let X_{N_r} be a sub-array of the D_r array and R_{N_r} be an equivalence relation it with the collection of its diminishing fuzzy neutrosophic subsets, $\mathcal{T}_{N_r}$. Then, $\beta_{N_{R_r}}$ forms a basis for diminishing fuzzy neutrosophic nano topological space.*

Proof. The proof of this theorem is obtained by the lemma 3.6. and the above theorem. Let $\mathcal{T}_{N_r}(x,y) = \{0_{N_r}, 1_{N_r}, \underline{N_r(\mu_{N_{A_r}})}(x,y), \overline{N_r(\mu_{N_{A_r}})}(x,y), B_{N_r}(\mu_{N_{A_r}})(x,y)\}$ is a nano topology on X_{N_r} and $\mu_{N_{A_r}}(x,y) \subset X_{N_r}$. We claim that $\beta_{N_{R_r}} = \{1_{N_r}, \underline{N_r(\mu_{N_{A_r}})}(x,y), B_{N_r}(\mu_{N_{A_r}})(x,y)\}$ is an open base for DFNNTS. Since, $1_{N_r} \cup \underline{N_r(\mu_{N_{A_r}})}(x,y) = 1_{N_r}(x,y)$, $1_{N_r} \cup B_{N_r}(\mu_{N_{A_r}})(x,y) = 1_{N_r}(x,y)$, $\underline{N_r(\mu_{N_{A_r}})} \cup B_{N_r}(\mu_{N_{A_r}})(x,y) = B_{N_r}(\mu_{N_{A_r}})(x,y)$ and $1_{N_r} \cup \underline{N_r(\mu_{N_{A_r}})} \cup B_{N_r}(\mu_{N_{A_r}})(x,y) = 1_{N_r}(x,y)$ which belongs to DFNNTS $(X_{N_r}, \mathcal{T}_{N_r})$. If $1_{N_r} \cap \underline{N_r(\mu_{N_{A_r}})}(x,y) = \underline{N_r(\mu_{N_{A_r}})}(x,y)$ and $\underline{N_r(\mu_{N_{A_r}})}(x,y) \subset 1_{N_r} \cap \underline{N_r(\mu_{N_{A_r}})}(x,y)$, then for every element in $1_{N_r} \cap \underline{N_r(\mu_{N_{A_r}})}$ also belongs to $\underline{N_r(\mu_{N_{A_r}})}$ and if $1_{N_r} \cap B_{N_r}(\mu_{N_{A_r}})(x,y) = B_{N_r}(\mu_{N_{A_r}})(x,y)$ and $B_{N_r}(\mu_{N_{A_r}})(x,y) \subset 1_{N_r} \cap B_{N_r}(\mu_{N_{A_r}})(x,y)$, then for every element in $1_{N_r} \cap B_{N_r}(\mu_{N_{A_r}})$ also belongs to $B_{N_r}(\mu_{N_{A_r}})$. As for $B_{N_r}(\mu_{N_{A_r}})$ and $\underline{N_r(\mu_{N_{A_r}})}$, we have $B_{N_r}(\mu_{N_{A_r}}) \cap \underline{N_r(\mu_{N_{A_r}})}(x,y) = \underline{N_r(\mu_{N_{A_r}})}(x,y)$. Thus, the collection $\beta_{N_{R_r}}$ forms a basis for DFNNTS. □

Definition 3.19. Let X_{N_r} and Y_{N_r} be two domains of discourse with the topologies $\mathcal{T}_{A_{N_r}}$ and $\mathcal{T}_{B_{N_r}}$ respectively, where A_{N_r} and B_{N_r} are the two diminishing single valued neutrosophic subsets of X_{N_r} and Y_{N_r} respectively. A function $f_{N_{D_r}}$ is said to be a nano continuous from $(X_{N_r}, \mathcal{T}_{A_{N_r}})$ to $(Y_{N_r}, \mathcal{T}_{B_{N_r}})$ if the inverse of each $\mathcal{T}_{B_{N_r}}$-diminishing nano fuzzy neutrosophic open set is $\mathcal{T}_{A_{N_r}}$-diminishing nano fuzzy neutrosophic open set.

Example 3.20. Without loss of generality, let $X_{N_r} = \{(x_1, y_1), (x_1, y_2), (x_2, y_1), (x_2, y_2)\}$ be a universe of discourse. Let $X_{N_r}/R_{N_r}(x, y) = \{\{(x_1, y_1), (x_2, y_2)\}, (x_1, y_2), (x_2, y_1)\}$ and $\mu_{N_{A_r}}(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\} \subset X_{N_r}$ be a DFNS with $n = 2, k = 1$ for $(x_1, y_1), k = 3$ for (x_2, y_1). Then $\underline{N_r(\mu_{N_{A_r}})}(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}$, $\overline{N_r(\mu_{N_{A_r}})}(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}$ and $B_r(\mu_{N_{A_r}})(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}$. Then, $\mathcal{T}_{N_{A_r}}(x, y) = \{0_{N_r}, 1_{N_r}, \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}$. Let $X_{N_r}/R'_{N_r}(x, y) = \{\{(x_1, y_1), (x_1, y_2)\}, (x_2, y_2), (x_2, y_1)\}$ and $\mu_{N_{B_r}}(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/4,4/5,3/4)}, \frac{(x_2,y_2)}{(1,0,0)}\} \subset X_{N_r}$ be a DFNS with $n = 2, k = 1$ for $(x_1, y_1), k = 2$ for (x_1, y_2) and $(x_2, y_2) = 1_{N_r}$ is free. Then $\underline{N_r(\mu_{N_{B_r}})}(x, y) = \{\frac{(x_1,y_1)}{(1/4,4/5,3/4)}, \frac{(x_1,y_2)}{(1/4,4/5,3/4)}, \frac{(x_2,y_2)}{(1,0,0)}\}, \overline{N_r(\mu_{N_{B_r}})}(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(1,0,0)}\}$ and $B_r(\mu_{N_{B_r}})(x, y) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(0,1,1)}\}$. Then $\mathcal{T}_{N_{B_r}}(x, y) = \{0_{N_r}, 1_{N_r}, \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(1,0,0)}\}, \{\frac{(x_1,y_1)}{(1/4,4/5,3/4)}, \frac{(x_1,y_2)}{(1/4,4/5,3/4)}, \frac{(x_2,y_2)}{(1,0,0)}\}, \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(0,1,1)}\}\}$. Let us define a function $f_{N_{D_r}} : \mathcal{T}_{N_{A_r}} \to \mathcal{T}_{N_{B_r}}$ as $f_{N_{D_r}}(0, 1, 1) = \{\frac{(x_1,y_1)}{(1/4,4/5,3/4)}, \frac{(x_1,y_2)}{(1/4,4/5,3/4)}, \frac{(x_2,y_2)}{(1,0,0)}\}$, $f_{N_{D_r}}(\{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(0,1,1)}\}$, $f_{N_{D_r}}(\{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(1,0,0)}\}$, $f_{N_{D_r}}(1, 0, 0) = (1, 0, 0)$ and $f_{N_{D_r}}(0, 1, 1) = (0, 1, 1)$ Then, $f^(-1)_{N_{D_r}}(\{\frac{(x_1,y_1)}{(1/4,4/5,3/4)}, \frac{(x_1,y_2)}{(1/4,4/5,3/4)}, \frac{(x_2,y_2)}{(1,0,0)}\}) = (0, 1, 1)$, $f^(-1)_{N_{D_r}}(\{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(0,1,1)}\}) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}$, $f^{-1}_{N_{D_r}}(\{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_1,y_2)}{(1/2,2/3,1/2)}, \frac{(x_2,y_2)}{(1,0,0)}\}) = \{\frac{(x_1,y_1)}{(1/2,2/3,1/2)}, \frac{(x_2,y_1)}{(1/8,8/9,7/8)}\}$, $f^{-1}_{N_{D_r}}(1, 0, 0) = (1, 0, 0)$ and $f^{-1}_{N_{D_r}}(0, 1, 1) = (0, 1, 1)$ Clearly, the inverse image of every diminishing nano fuzzy neutrosophic open set in N_{B_r} is the diminishing nano fuzzy neutrosophic open set in N_{A_r}. Hence, $f_{N_{D_r}}$ is a nano continuous on diminishing nano fuzzy neutrosophic topological space.

Remark 3.21. A nano continuous function $f_{N_{D_r}}$ defined on diminishing fuzzy neutrosophic topological space need not be bijective.

Conclusion

The function of diminishing fuzzy sets is extended to the case of fuzzy neutrosophic sets, termed a diminishing fuzzy neutrosophic set. The collections of diminishing fuzzy neutrosophic topology and diminishing fuzzy neutrosophic topology induced by nano topology are determined for the extended fuzzified (neutrosophified) sub-arrays of diminishing cells infinite array. Furthermore, a nano continuous function on the diminishing fuzzy neutrosophic topological spaces is also studied. The approximation space with respect to the conceptualization of pixel neighborhood in the field of image processing for the neighborhoods of diminishing cells and for NFA of the layer word are discussed and the depiction of the neighborhood of arbitrary cells is contrived to correspond to the diminishing fuzzy neutrosophic sets.

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Received: July 06, 2024. Accepted: August 31, 2024