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Modified Explicit Scheme for Solving Neutrosophic Fuzzy Heat Equation

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Abstract: In this paper, an explicit finite difference method is developed and applied for the first time to solve the neutrosophic fuzzy heat equation. For this purpose, the triangular neutrosophic number is used in both the neutrosophic exact and numerical solution. In addition, solutions of neutrosophic fuzzy heat equation were observed at the (α, β, γ)-cut with varied time scales. Also, the error between the analytical and numerical solution obtained on the (α, β, γ)-cut is evaluated and illustrated. A good amount of agreement is seen using closed-form and numerical solutions.

Keywords: Explicit finite difference scheme; Intuitionistic fuzzy number; Intuitionistic fuzzy time fractional diffusion equation

1. Introduction

Zadeh (1965) [1] first introduced the idea of fuzzy sets and fuzzy numbers. After that, different extensions of fuzzy set theory have been developed. One of these important extensions is the fuzzy intuitionistic set introduced by Atanassov (1983) [2]. The intuitionistic set introduced the concepts of the degree of non-membership and falsehood, which led to the creation of intuitionistic fuzzy sets (IFS). This new approach differed from classical fuzzy sets and provided a better way to describe incomplete ideas. Neutrosophy is a new tool to handle problems with unclear, uncertain, and inconsistent information. In 2005, Smarandache (2005) [3] introduced the Neutrosophic sets as a generalization concept of IFS. Neutrosophy is a new tool to handle problems with unclear, uncertain, and inconsistent information. In 2005, Smarandache introduced the Neutrosophic sets as a generalization concept of Intuitionistic fuzzy sets. In the Neutrosophic set, the grade of membership of Truth values (T), Indeterminate values (I) and False values (F) has been defined within the non-standard interval -] 0, 1[+. Non-standard intervals of the neutrosophic set theory work well in the concept of philosophy. But in reality, if we deal with real-life problems in science and engineering it is not possible to put the data into the non-standard interval. To address these issues, Wang et al. (2010) [4] developed single-valued Neutrosophic sets by utilizing the standard form of the unit interval [0, 1]. In addition, some researchers have defined single-valued Neutrosophic numbers [5-7]. Similarly, Ye (2015) [8] introduced Trapezoidal Neutrosophic numbers and discussed their application in decision-making. This approach has since led to a lot of research on many real-world problems.

The Fuzzy Neutrosophic Differential Equations (FNDEs) are one of the important tools to model the uncertainty in particular quantities for certain real-world phenomena. They have fundamental applications in various areas, such as chemistry, engineering, physics, and biology. The exact solution of the FNDEs is usually not obtainable or difficult to obtain. So, many mathematicians have resorted to utilizing the approximation or numerical methods. Among the numerical methods, the finite difference method (FDM) stands out as one of the most widely used methods because of its simplicity and universal applicability. The FDM is used to solve the FNDE numerically. Parikh et al. (2022) [9] solve the first-order non-homogeneous fuzzy differential equation with neutrosophic initial conditions. The triangular neutrosophic numbers used in the solution process. After that, Kamal et al. (2023) [10] used a modified FDM to solve second-order differential equation with neutrosophic fuzzy boundary condition.

Fuzzy heat equation is utilized for modelling certain real-life problems in physics, and biology [11-16]. When dealing with fuzzy heat equations and we are not entirely sure about certain factors, neutrosophic fuzzy numbers (NFNs) can be helpful. They give a clearer way to handle uncertainty, which makes it easier to decide on the best numerical methods and parameters for solving the equations. Based on our literature review, there seems to have been no attempt to solve neutrosophic fuzzy heat equations using FDM. Therefore, in this paper, an explicit FDM is developed and applied to solve the neutrosophic fuzzy heat equations with neutrosophic initial conditions.

2. Heat Equation in Neutrosophic Fuzzy environment

We investigate the Neutrosophic fuzzy heat equation (NFHE) along with its corresponding Neutrosophic initial and Neutrosophic boundary conditions [12]:

$$\frac{\partial \tilde{u}(x,t)}{\partial t} = \tilde{D}(x,t)\frac{\partial^2 \tilde{u}(x,t)}{\partial x^2} + \tilde{q}(x,t), \qquad 0 < x < l, t > 0$$
$$\tilde{u}(x,0) = \tilde{f}(x), \tilde{u}(0,t) = \tilde{g}(t),\ \tilde{u}(l,t) = \tilde{z}(t), \tag{1}$$

where $\tilde{v}(x,t)$ is a fuzzy Neutrosophic fractional function and α is fractional arbitrary order, $\frac{\partial \tilde{u}(x,t)}{\partial t}$ is the fuzzy Neutrosophic time derivative, $\frac{\partial^2 \tilde{u}(x,t)}{\partial x^2}$ is a fuzzy Neutrosophic Hukuhara derivatives. $\tilde{q}(x,t)$, and $\tilde{D}(x,t)$ are fuzzy Neutrosophic functions. $\tilde{u}(x,0)$, is the fuzzy Neutrosophic initial conditions while $\tilde{u}(0,t)$ and also $\tilde{u}(l,t)$ are fuzzy Neutrosophic boundary condition with $\tilde{g}(t)$, $\tilde{z}(t)$ are fuzzy Neutrosophic c convex normalized numbers. Finally, the fuzzy functions $\tilde{D}(x,t)$, $\tilde{q}(x,t)$, and $\tilde{f}(x)$ in Eq. (1) are characterized as follows:

$$\begin{cases} \tilde{D}(x,t) = \tilde{\theta}_1 b_1(x,t), \\ \tilde{q}(x,t) = \tilde{\theta}_2 b_2(x,t), \\ \tilde{f}(x) = \tilde{\theta}_3 b_3(x). \end{cases} \tag{2}$$

where b_1, b_2, and b_3 represent the crisp functions of the independent variable x, while $\tilde{\theta}_1$,$\tilde{\theta}_2$, and $\tilde{\theta}_3$ denote the fuzzy Neutrosophic convex number.

Now, the NFHE in Eq. (1) is defuzzified by using the single parametric form based on the α,β,γ-cut approach described in section 2 for all $0 \le \alpha + \beta + \gamma \le 3$ as the following [21-25]:

$$[\tilde{u}(x,t)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{u_T}(x,t;\alpha), \overline{u_T}(x,t;\alpha)\right], \left[\underline{u_I}(x,t;\beta), \overline{u_I}(x,t;\beta)\right], \left[\underline{u_F}(x,t;\gamma), \overline{u_F}(x,t;\gamma)\right]\right\} \tag{3}$$

$$\left[\frac{\partial \tilde{u}(x,t)}{\partial t}\right]_{\alpha,\beta,\gamma} = \left\{\left[\frac{\partial \underline{u_T}(x,t;\alpha)}{\partial t}, \frac{\partial \overline{u_T}(x,t;\alpha)}{\partial t}\right], \left[\frac{\partial \underline{u_I}(x,t;\beta)}{\partial t}, \frac{\partial \overline{u_I}(x,t;\beta)}{\partial t}\right], \left[\frac{\partial \underline{u_F}(x,t;\gamma)}{\partial t}, \frac{\partial \overline{u_F}(x,t;\gamma)}{\partial t}\right]\right\} \tag{4}$$

$$\left[\frac{\partial^2\tilde{u}(x,t)}{\partial x^2}\right]^{\alpha,\beta,\gamma} = \left\{\left[\frac{\partial^2\underline{u_T}(x,t;\alpha)}{\partial x^2},\frac{\partial^2\overline{u_T}(x,t;\alpha))}{\partial x^2}\right],\left[\frac{\partial^2\underline{u_I}(x,t;\beta)}{\partial x^2},\frac{\partial^2\overline{u_I}(x,t;\beta)}{\partial x^2}\right],\left[\frac{\partial^2\underline{u_F}(x,t;\gamma)}{\partial x^2},\frac{\partial^2\overline{u_F}(x,t;\gamma)}{\partial x^2}\right]\right\} \quad (5)$$

$$\left[\tilde{D}(x,t)\right]_{\alpha,\beta,\gamma} = \left\{\left[\underline{D_T}(x,t;\alpha),\overline{D_T}(x,t;\alpha)\right],\left[\underline{D_I}(x,t;\beta),\overline{D_I}(x,t;\beta)\right],\left[\underline{D_F}(x,t;\gamma),\overline{D_F}(x,t;\gamma)\right]\right\} \quad (6)$$

$$[\tilde{q}(x,t)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{q_T}(x,t;\alpha),\overline{q_T}(x,t;\alpha)\right],\left[\underline{q_I}(x,t;\beta),\overline{q_I}(x,t;\beta)\right],\left[\underline{q_F}(x,t;\gamma),\overline{q_F}(x,t;\gamma)\right]\right\} \quad (7)$$

$$[\tilde{u}(x,0)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{u_T}(x,0;\alpha),\overline{u_T}(x,0;\alpha)\right],\left[\underline{u_I}(x,0;\beta),\overline{u_I}(x,0;\beta)\right],\left[\underline{u_F}(x,0;\gamma),\overline{u_F}(x,0;\gamma)\right]\right\} \quad (8)$$

$$[\tilde{u}(0,t)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{u_T}(0,t;\alpha),\overline{u_T}(0,t;\alpha)\right],\left[\underline{u_I}(0,t;\beta),\overline{u_I}(0,t;\beta)\right],\left[\underline{u_F}(0,t;\gamma),\overline{u_F}(0,t;\gamma)\right]\right\} \quad (9)$$

$$[\tilde{u}(l,t)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{u_T}(l,t;\alpha),\overline{u_T}(l,t;\alpha)\right],\left[\underline{u_I}(l,t;\beta),\overline{u_I}(l,t;\beta)\right],\left[\underline{u_F}(l,t;\gamma),\overline{u_F}(l,t;\gamma)\right]\right\} \quad (10)$$

$$\left[\tilde{f}(x)\right]_{\alpha,\beta,\gamma} = \left\{\left[\underline{f_T}(x;\alpha),\overline{f_T}(x;\alpha)\right],\left[\underline{f_I}(x;\beta),\overline{f_I}(x;\beta)\right],\left[\underline{f_F}(x;\gamma),\overline{f_F}(x;\gamma)\right]\right\} \quad (11)$$

$$\begin{cases}[\tilde{g}(t)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{g_T}(t;\alpha),\overline{g_T}(t;\alpha)\right],\left[\underline{g_I}(t;\beta),\overline{g_I}(t;\beta)\right],\left[\underline{g_F}(x;\gamma),\overline{g_F}(t;\gamma)\right]\right\}\\ [\tilde{z}(t)]_{\alpha,\beta,\gamma} = \left\{\left[\underline{z_T}(t;\alpha),\overline{z_T}(t;\alpha)\right],\left[\underline{z_I}(t;\beta),\overline{z_I}(t;\beta)\right],\left[\underline{z_F}(x;\gamma),\overline{z_F}(t;\gamma)\right]\right\}\end{cases} \quad (12)$$

where

$$\begin{cases}\left[\tilde{D}(x,t)\right]_{\alpha,\beta,\gamma} = \left\{\left[\left[\underline{\theta_T}(\alpha)_1,\overline{\theta_T}(\alpha)_1\right]b_1(x,t)\right],\left[\left[\underline{\theta_I}(\alpha)_1,\overline{\theta_I}(\alpha)_1\right]b_1(x,t)\right],\left[\left[\underline{\theta_F}(\alpha)_1,\overline{\theta_F}(\alpha)_1\right]b_1(x,t)\right]\right\}\\ [\tilde{q}(x,t)]_{\alpha,\beta,\gamma} = \left\{\left[\left[\underline{\theta_T}(\alpha)_2,\overline{\theta_T}(\alpha)_2\right]b_2(x,t)\right],\left[\left[\underline{\theta_I}(\alpha)_2,\overline{\theta_I}(\alpha)_2\right]b_2(x,t)\right],\left[\left[\underline{\theta_F}(\alpha)_2,\overline{\theta_F}(\alpha)_2\right]b_2(x,t)\right]\right\}\\ \left[\tilde{f}(x)\right]_{\alpha,\beta,\gamma} = \left\{\left[\left[\underline{\theta_T}(\alpha)_3,\overline{\theta_T}(\alpha)_3\right]b_3(x,t)\right],\left[\left[\underline{\theta_I}(\alpha)_3,\overline{\theta_I}(\alpha)_3\right]b_3(x,t)\right],\left[\left[\underline{\theta_F}(\alpha)_3,\overline{\theta_F}(\alpha)_3\right]b_3(x,t)\right]\right\}\end{cases} \quad (13)$$

The membership function is established by applying the Zadeh expansion principle described in [1]

$$\begin{cases}\underline{u_T}(x,t;\alpha) = min\{\tilde{u}(\tilde{\mu}(\alpha))|\tilde{\mu}(\alpha) \in \tilde{u}(x,t;\alpha)\}\\ \overline{u_T}(x,t;\alpha) = max\{\tilde{u}(\tilde{\mu}(\alpha))|\tilde{\mu}(\alpha) \in \tilde{u}(t,x;\alpha)\}\end{cases} \quad (14)$$

$$\begin{cases}\underline{u_I}(x,t;\beta) = min\{\tilde{u}(\tilde{\mu}(\beta))|\tilde{\mu}(\beta) \in \tilde{u}(x,t;\beta)\}\\ \overline{u_I}(x,t;\beta) = max\{\tilde{u}(\tilde{\mu}(\beta))|\tilde{\mu}(\beta) \in \tilde{u}(x,t;\beta)\}\end{cases} \quad (15)$$

$$\begin{cases} \underline{u_F}(x,t;\gamma) = min\{\tilde{u}(\tilde{\mu}(\gamma))|\tilde{\mu}(\gamma) \in \tilde{u}(x,t;\gamma)\} \\ \overline{u_F}(x,t;\gamma) = max\{\tilde{u}(\tilde{\mu}(\gamma))|\tilde{\mu}(\gamma) \in \tilde{u}(x,t;\gamma)\} \end{cases} \tag{16}$$

Now Eq. (1) for $0 < x \le l, t > 0$ and $r \in [0,1]$ is rewritten to obtain the general equation of NFHE as follows:

$$\begin{cases} \dfrac{\partial \underline{u_T}(x,t)}{\partial t} = \left[\underline{\theta_T}(\alpha)_1\right] b_1(x,t) \dfrac{\partial^2 \underline{u_T}(x,t;\alpha)}{\partial x^2} + \left[\underline{\theta_T}(\alpha)_2\right] b_2(x,t) \\ \underline{u_T}(x,0;\alpha) = \left[\underline{\theta_T}(\alpha)_3\right] b_3(x,t) \\ \underline{u_T}(0,t;\alpha) = \underline{g}(t,\alpha), \underline{u_T}(l,t;\alpha) = \underline{z}(t,\alpha) \end{cases} \tag{17}$$

$$\begin{cases} \dfrac{\partial \overline{u_T}(x,t)}{\partial t} = \left[\overline{\theta_T}(\alpha)_1\right] b_1(x,t) \dfrac{\partial^2 \overline{u_T}(x,t;\alpha)}{\partial x^2} + \left[\overline{\theta_T}(\alpha)_2\right] b_2(x,t) \\ \overline{u_T}(x,0;\alpha) = \left[\overline{\theta_T}(\alpha)_3\right] b_3(x,t) \\ \overline{u_T}(0,t;\alpha) = \overline{g}(t,\alpha), \overline{u_T}(l,t;\alpha) = \overline{z}(t,\alpha) \end{cases} \tag{18}$$

$$\begin{cases} \dfrac{\partial \underline{u_I}(x,t,\beta)}{\partial t} = \left[\underline{\theta_I}(\beta)_1\right] b_1(x,t) \dfrac{\partial^2 \underline{u_I}(x,t;\beta)}{\partial x^2} + \left[\underline{\theta_I}(\beta)_2\right] b_2(x,t) \\ \underline{u_I}(x,0;\beta) = \left[\underline{\theta_I}(\beta)_3\right] b_3(x,t) \\ \underline{u_I}(0,t;\beta) = \underline{g}(t,\beta), \underline{u_I}(l,t;\beta) = \underline{z}(t,\beta) \end{cases} \tag{19}$$

$$\begin{cases} \dfrac{\partial \overline{u_I}(x,t,\beta)}{\partial t} = \left[\overline{\theta_I}(\beta)_1\right] b_1(x,t) \dfrac{\partial^2 \overline{u_I}(x,t;\beta)}{\partial x^2} + \left[\overline{\theta_I}(\beta)_2\right] b_2(x,t) \\ \overline{u_I}(x,0;\beta) = \left[\overline{\theta_I}(\beta)_3\right] b_3(x,t) \\ \overline{u_I}(0,t;\beta) = \overline{g}(t,\beta), \overline{u_I}(l,t;\beta) = \overline{z}(t,\beta) \end{cases} \tag{20}$$

$$\begin{cases} \dfrac{\partial \underline{u_F}(x,t,\gamma)}{\partial t} = \left[\underline{\theta_F}(\gamma)_1\right] b_1(x,t) \dfrac{\partial^2 \underline{u_F}(x,t;\gamma)}{\partial x^2} + \left[\underline{\theta_F}(\gamma)_2\right] b_2(x,t) \\ \underline{u_F}(x,0;\gamma) = \left[\underline{\theta_F}(\gamma)_3\right] b_3(x,t) \\ \underline{u_F}(0,t;\gamma) = \underline{g}(t,\gamma), \underline{u_F}(l,t;\gamma) = \underline{z}(t,\gamma) \end{cases} \tag{21}$$

$$\begin{cases} \dfrac{\partial \overline{u_F}(x,t,\gamma)}{\partial t} = \left[\overline{\theta_F}(\gamma)_1\right] b_1(x,t) \dfrac{\partial^2 \overline{u_F}(x,t;\gamma)}{\partial x^2} + \left[\overline{\theta_F}(\gamma)_2\right] b_2(x,t) \\ \overline{u_F}(x,0;\gamma) = \left[\overline{\theta_F}(\gamma)_3\right] b_3(x,t) \\ \overline{u_F}(0,t;\gamma) = \overline{g}(t,\gamma), \overline{u_F}(l,t;\gamma) = \overline{z}(t,\gamma) \end{cases} \tag{22}$$

The Eq. (17) and Eq. (18) present the Neutrosophic lower and upper bounds of membership of Truth values (T) for the general form of NFHE. Also, Eq. (19) and Eq. (20) present the Neutrosophic lower and upper bounds of membership of Indeterminate values (I), while Eq. (21) and Eq. (22) present the Neutrosophic lower and upper bounds of membership of False values (F) for the general form of NFHE.

3. Solution of NFHE by FTCS scheme

In this section, the FTCS scheme is reformatted and applied in Neutrosophic single parametric representation with forward difference approximation for time derivatives and central difference approximation for spatial derivatives to solve the NFHE.

The time derivatives $\frac{\partial \widetilde{u_T}(x,t;\alpha)}{\partial t}, \frac{\partial \widetilde{u_I}(x,t;\alpha)}{\partial t}, \frac{\partial \widetilde{u_F}(x,t;\alpha)}{\partial t}$ are discretizes as follows:

$$\frac{\partial \widetilde{u_T}(x,t;\alpha)}{\partial t} = \begin{cases} \dfrac{\underline{u_T}_i^{n+1}(x,t;\alpha) - \underline{u_T}_i^{n}(x,t;\alpha)}{\Delta t} \\ \dfrac{\overline{u_T}_i^{n+1}(x,t;\alpha) - \overline{u_T}_i^{n}(x,t;\alpha)}{\Delta t} \end{cases} \tag{23}$$

$$\frac{\partial \widetilde{u_I}(x,t;\beta)}{\partial t} = \begin{cases} \dfrac{\underline{u_I}_i^{n+1}(x,t;\beta) - \underline{u_I}_i^{n}(x,t;\beta)}{\Delta t} \\ \dfrac{\overline{u_I}_i^{n+1}(x,t;\beta) - \overline{u_I}_i^{n}(x,t;\beta)}{\Delta t} \end{cases} \tag{24}$$

$$\frac{\partial \widetilde{u_F}(x,t;\gamma)}{\partial t} = \begin{cases} \dfrac{\underline{u_F}_i^{n+1}(x,t;\gamma) - \underline{u_F}_i^{n}(x,t;\gamma)}{\Delta t} \\ \dfrac{\overline{u_F}_i^{n+1}(x,t;\gamma) - \overline{u_F}_i^{n}(x,t;\gamma)}{\Delta t} \end{cases} \tag{25}$$

The spatial derivatives $\frac{\partial^2 \widetilde{u_T}(x,t;\alpha)}{\partial x^2}, \frac{\partial^2 \widetilde{u_I}(x,t;\beta)}{\partial x^2}, \frac{\partial^2 \widetilde{u_F}(x,t;\gamma)}{\partial x^2}$ are discretizes as follows:

$$\frac{\partial^2 \widetilde{u_T}(x,t;\alpha)}{\partial x^2} = \begin{cases} \dfrac{\underline{u_T}_{i+1}^{n}(x,t;\alpha) - 2\underline{u_T}_i^{n}(x,t;\alpha) + \underline{u_T}_{i-1}^{n}(x,t;\alpha)}{\Delta x^2} \\ \dfrac{\overline{u_T}_{i+1}^{n}(x,t;\alpha) - 2\overline{u_T}_i^{n}(x,t;\alpha) + \overline{u_T}_{i-1}^{n}(x,t;\alpha)}{\Delta x^2} \end{cases} \tag{26}$$

$$\frac{\partial^2 \widetilde{u_I}(x,t;\beta)}{\partial x^2} = \begin{cases} \dfrac{\underline{u_I}_{i+1}^{n}(x,t;\beta) - 2\underline{u_I}_i^{n}(x,t;\beta) + \underline{u_I}_{i-1}^{n}(x,t;\beta)}{\Delta x^2} \\ \dfrac{\overline{u_I}_{i+1}^{n}(x,t;\beta) - 2\overline{u_I}_i^{n}(x,t;\beta) + \overline{u_I}_{i-1}^{n}(x,t;\beta)}{\Delta x^2} \end{cases} \tag{27}$$

$$\frac{\partial^2 \widetilde{u_F}(x,t;\gamma)}{\partial x^2} = \begin{cases} \dfrac{\underline{u_F}_{i+1}^{n}(x,t;\gamma) - 2\underline{u_F}_i^{n}(x,t;\gamma) + \underline{u_F}_{i-1}^{n}(x,t;\gamma)}{\Delta x^2} \\ \dfrac{\overline{u_F}_{i+1}^{n}(x,t;\gamma) - 2\overline{u_F}_i^{n}(x,t;\gamma) + \overline{u_F}_{i-1}^{n}(x,t;\gamma)}{\Delta x^2} \end{cases} \tag{28}$$

where i represents a point on a spatial grid, while n signifies a point in time.

Now substitute Eq. (23- 28) into Eq.(17 - 22) respectively to get:

$$\frac{\underline{u_T}_i^{n+1}(x,t;\alpha)-\underline{u_T}_i^{n}(x,t;\alpha)}{\Delta t} = \underline{D_T}(x,t,\alpha)\frac{\underline{u_T}_{i+1}^{n}(x,t;\alpha)-2\underline{u_T}_i^{n}(x,t;\alpha)+\underline{u_T}_{i-1}^{n}(x,t;\alpha)}{\Delta x^2}+\underline{q_T}(x,t,\alpha) \quad (29)$$

$$\frac{\overline{u_T}_i^{n+1}(x,t;\alpha)-\overline{u_T}_i^{n}(x,t;\alpha)}{\Delta t} = \overline{D_T}(x,t,\alpha)\frac{\overline{u_T}_{i+1}^{n}(x,t;\alpha)-2\overline{u_T}_i^{n}(x,t;\alpha)+\overline{u_T}_{i-1}^{n}(x,t;\alpha)}{\Delta x^2}+\overline{q_T}(x,t,\alpha) \quad (30)$$

$$\frac{\underline{u_I}_i^{n+1}(x,t;\beta)-\underline{u_I}_i^{n}(x,t;\beta)}{\Delta t} = \underline{D_I}(x,t,\beta)\frac{\underline{u_I}_{i+1}^{n}(x,t;\beta)-2\underline{u_I}_i^{n}(x,t;\beta)+\underline{u_I}_{i-1}^{n}(x,t;\beta)}{\Delta x^2}+\underline{q_I}(x,t,\beta) \quad (31)$$

$$\frac{\overline{u_I}_i^{n+1}(x,t;\beta)-\overline{u_I}_i^{n}(x,t;\beta)}{\Delta t} = \overline{D_I}(x,t,\beta)\frac{\overline{u_I}_{i+1}^{n}(x,t;\beta)-2\overline{u_I}_i^{n}(x,t;\beta)+\overline{u_I}_{i-1}^{n}(x,t;\beta)}{\Delta x^2}+\overline{q_I}(x,t,\beta) \quad (32)$$

$$\frac{\underline{u_F}_i^{n+1}(x,t;\gamma)-\underline{u_F}_i^{n}(x,t;\gamma)}{\Delta t} = \underline{D_F}(x,t,\gamma)\frac{\underline{u_F}_{i+1}^{n}(x,t;\gamma)-2\underline{u_F}_i^{n}(x,t;\gamma)+\underline{u_F}_{i-1}^{n}(x,t;\gamma)}{\Delta x^2}+\underline{q_F}(x,t,\gamma) \quad (33)$$

$$\frac{\overline{u_F}_i^{n+1}(x,t;\gamma)-\overline{u_F}_i^{n}(x,t;\gamma)}{\Delta t} = \overline{D_F}(x,t,\gamma)\frac{\overline{u_F}_{i+1}^{n}(x,t;\gamma)-2\overline{u_F}_i^{n}(x,t;\gamma)+\overline{u_F}_{i-1}^{n}(x,t;\gamma)}{\Delta x^2}+\overline{q_F}(x,t,\gamma) \quad (34)$$

Now let $\tilde{s}=\frac{\tilde{D}(x,t)\Delta t}{\Delta x^2}$, and from Eq. (29 - 34) we obtain for all $\alpha,\beta,\gamma\in[0,1]$

$$\begin{cases}\underline{u_T}_i^{n+1}(x,t;\alpha)=s\,\underline{u_T}_{i+1}^{n}(x,t;\alpha)+(1-2s)\underline{u_T}_i^{n}(x,t;\alpha)+s\,\underline{u_T}_{i-1}^{n}(x,t;\alpha)\ +\Delta t\,\underline{q_T}(x,t,\alpha)\\ \overline{u_T}_i^{n+1}(x,t;\alpha)=s\,\overline{u_T}_{i+1}^{n}(x,t;\alpha)+(1-2s)\overline{u_T}_i^{n}(x,t;\alpha)+s\,\overline{u_T}_{i-1}^{n}(x,t;\alpha)\ +\Delta t\,\overline{q_T}(x,t,\alpha)\end{cases} \quad (35)$$

$$\begin{matrix}\underline{u_I}_i^{n+1}(x,t;\beta)=s\,\underline{u_I}_{i+1}^{n}(x,t;\beta)+(1-2s)\underline{u_I}_i^{n}(x,t;\beta)+s\,\underline{u_I}_{i-1}^{n}(x,t;\beta)\ +\Delta t\,\underline{q_I}(x,t,\beta)\\ \overline{u_I}_i^{n+1}(x,t;\beta)=s\,\overline{u_I}_{i+1}^{n}(x,t;\beta)+(1-2s)\overline{u_I}_i^{n}(x,t;\beta)+s\,\overline{u_I}_{i-1}^{n}(x,t;\beta)\ +\Delta t\,\overline{q_I}(x,t,\beta)\end{matrix} \quad (36)$$

$$\begin{matrix}\underline{u_F}_i^{n+1}(x,t;\beta)=s\,\underline{u_F}_{i+1}^{n}(x,t;\beta)+(1-2s)\underline{u_F}_i^{n}(x,t;\beta)+s\,\underline{u_F}_{i-1}^{n}(x,t;\beta)\ +\Delta t\,\underline{q_F}(x,t,\beta)\\ \overline{u_F}_i^{n+1}(x,t;\beta)=s\,\overline{u_F}_{i+1}^{n}(x,t;\beta)+(1-2s)\overline{u_F}_i^{n}(x,t;\beta)+s\,\overline{u_F}_{i-1}^{n}(x,t;\beta)\ +\Delta t\,\overline{q_F}(x,t,\beta)\end{matrix} \quad (37)$$

4. Numerical Experiment

Consider NFHE along with its corresponding neutrosophic fuzzy initial and boundary conditions [17].

$$\frac{\partial\tilde{u}(x,t)}{\partial t}=\frac{\partial^2\tilde{u}(x,t)}{\partial x^2}\ ,\quad t>0\ ,\ \ 0\le x\le 1 \quad (38)$$

Conditional on the fuzzy intuitionistic boundary condition $\tilde{u}(1,t) = \tilde{u}(0,t) = 0$ and fuzzy neutrosophic initial

$$\tilde{u}(x,0) = \tilde{\mu}\sin(\pi x)\,,\ 0 < x < 1. \tag{39}$$

In α, β, γ-cut of the neutrosophic fuzzy number will be same as follow:

$$\tilde{\mu}(\alpha,\beta,\gamma) = \left\{ \left[\underline{\mu}(\alpha), \overline{\mu}(\alpha)\right], \left[\underline{\mu}(\beta), \overline{\mu}(\beta)\right], \left[\underline{\mu}(\gamma), \overline{\mu}(\gamma)\right] \right\}$$

$$= \{\, [\alpha\,, 2-\alpha], [1-0.5\beta, 1+0.5\beta], [1-0.25\gamma, 1+0.25\gamma] \,\}$$

The neutrosophic analytical solution of Eq. (38) was obtained in [17]:

$$\begin{cases} \underline{V}(x,t;\alpha) = \underline{\mu}(\alpha)e^{-\pi^2 t}\sin(\pi x) \\ \overline{V}(x,t;\alpha) = \overline{\mu}(\alpha)\, e^{-\pi^2 t}\sin(\pi x) \\ \underline{V}(x,t;\beta) = \underline{\mu}(\beta)e^{-\pi^2 t}\sin(\pi x) \\ \overline{V}(x,t;\beta) = \overline{\mu}(\beta)\, e^{-\pi^2 t}\sin(\pi x) \\ \underline{V}(x,t;\gamma) = \underline{\mu}(\gamma)e^{-\pi^2 t}\sin(\pi x) \\ \overline{V}(x,t;\gamma) = \overline{\mu}(\gamma)\, e^{-\pi^2 t}\sin(\pi x) \end{cases} \tag{40}$$

At $\Delta x = 0.1$ and $\Delta t = 0.01$, we obtained the results as the follows:

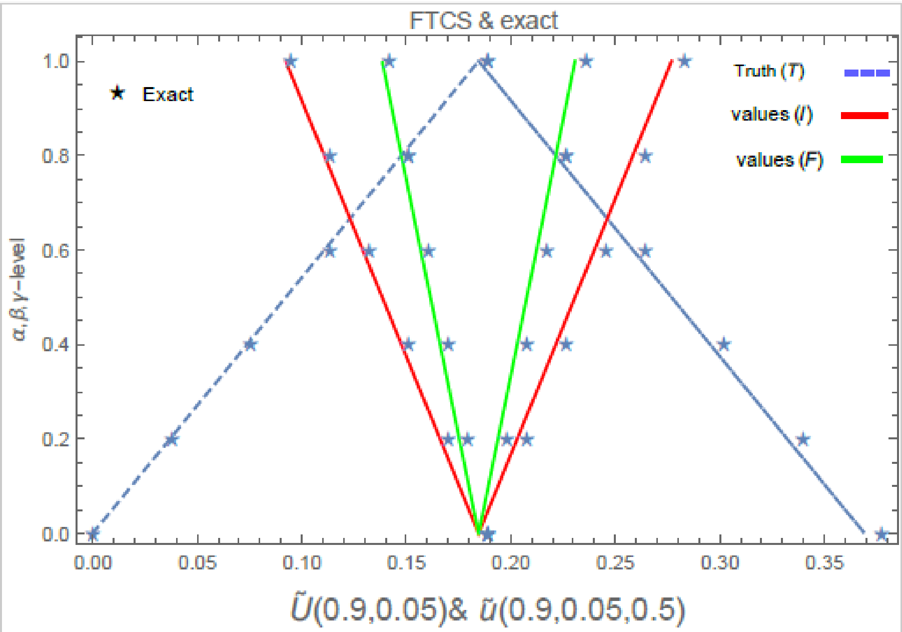

Figure 1: Exact and Neutrosophic fuzzy solution of Eq. (38) by explicit FDM at $x = 0.9, t = 0.05$ and for all $\alpha, \beta, \gamma \in [0,1]$.

The results obtained shows that the proposed explicit FDM methods and exact solution at $x = 0.9, t = 0.05$, and for all $\alpha, \beta, \gamma \in [0,1]$ attaining the triangular fuzzy number shape and thus satisfies the Neutrosophic fuzzy number properties.

Conclusions

In this study, the neutrosophic fuzzy heat equation is examined, incorporating uncertain variables and parameters, where the variables and parameters are considered as neutrosophic fuzzy numbers. An explicit scheme is reformulated and implemented to solve the neutrosophic fuzzy heat equation. The triangular neutrosophic number is used in both the neutrosophic exact and numerical solution on the (α, β, γ)-cut. A numerical example is presented to illustrate the approach. It was found that the results obtained show good agreement with both the exact solution.

Conflicts of Interest: The authors declare no conflict of interest

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