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Various Degrees of Directed Single Valued Neutrosophic Graphs

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Abstract. This paper introduces the concept of Directed Single Valued Neutrosophic Graphs (DSVNG) and explains how to determine the degree and total degree of vertices, considering both indegree and outdegree, along with the maximum and minimum degrees of a DSVNG. The properties of these aspects are analyzed through examples. Additionally, the paper identifies the effective edges of a DSVNG and explores the effective indegree, effective outdegree of vertices, as well as the effective maximum and minimum degrees in relation to indegree and outdegree with examples.

Keywords: Neutrosophic graphs, Directed fuzzy graphs, Indegree, Outdegree, Effective edges.

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1. Introduction

The Neutrosophic Set [6], introduced by Smarandache, is an effective tool for managing incomplete, indeterminate, and inconsistent information in real-world scenarios. A neutrosophic set is defined by three independent membership degrees: truth (T), indeterminacy (I), and falsity (F), each ranging within the real standard or nonstandard unit interval $(0^-, 1^+)$. When these degrees are confined to the real standard unit interval $[0, 1]$, neutrosophic sets become more applicable to engineering problems. Wang et al. [4] introduced the concept of a Single-Valued Neutrosophic Set (SVNS), a subclass of the neutrosophic set, to facilitate its application. They also proposed interval-valued neutrosophic sets [5], where the truth, indeterminacy, and falsity membership degrees are represented as intervals rather than single real numbers. Neutrosophic sets and their extensions, such as SVNS, interval neutrosophic sets, and simplified neutrosophic sets, have found applications in a broad range of fields, including computer science, engineering, mathematics, medicine, and economics.

A directed fuzzy graph is a mathematical structure that extends the concept of a traditional graph by incorporating the idea of fuzziness into the relationships between vertices. In a directed fuzzy graph, each edge has a direction and is associated with a membership value that indicates the strength or degree of connection between two vertices. This membership value, typically ranging between 0 and 1, allows the graph to represent uncertainty, imprecision, or partial relationships in a network. Directed fuzzy graphs are particularly useful when the connections between elements are not strictly binary but instead possess varying degrees of strength, such as in social networks, decision-making processes.

A Single-Valued Neutrosophic Graph (SVNG) is an advanced generalization of fuzzy and intuitionistic fuzzy graphs, designed to model more complex and uncertain relationships. In an SVNG, each edge is characterized by three independent membership degrees: truth (T), indeterminacy (I), and falsity (F). These degrees are real values within the interval [0, 1], allowing the graph to represent not only the certainty and strength of a relationship (as in fuzzy graphs) but also the degree of indeterminacy or ambiguity. This makes SVNGs a powerful tool for handling incomplete, inconsistent, and uncertain information in various applications, such as decision support systems, knowledge representation, and network analysis. The ability to separately account for truth, indeterminacy, and falsity provides a more flexible environment for modeling real-world problems.

In section 2, basic definitions related to neutrosophic sets and graphs are provided. Section 3 introduces the concept of Directed Single Valued Neutrosophic Graphs (DSVNG) and explains how to determine the degree, total degree of vertices in terms of indegree and outdegree, as well as the maximum and minimum degrees of a DSVNG. Their properties are analyzed with examples. In section 4, the effective edges of a DSVNG are identified, and the effective indegree, effective outdegree of vertices, as well as the effective maximum and minimum degrees of a DSVNG in relation to indegree and outdegree, are explored with examples.

2. Preliminaries

In this section basic definitions related to neutrosophic sets and graphs are provided.

Definition 2.1. [2] A neutrosophic set N for an universe X is defined as $\mathsf{N} = \{\langle x, T_{\mathsf{N}}(x), I_{\mathsf{N}}(x), F_{\mathsf{N}}(x)\rangle : x \in \mathsf{X}\}$, where T_{N}, I_{N}, F_{N} denotes the truth, indeterminacy and falsity membership functions respectively from X to $(0^-, 1^+)$ such that $0^- \leq \sup T_{\mathsf{N}}(x) + \sup I_{\mathsf{N}}(x) + \sup F_{\mathsf{N}}(x) \leq 3^+$. The set of all neutrosophic set over X is denoted by $\mathcal{N}(\mathsf{X})$.

Definition 2.2. [4] A single valued neutrosophic set is a neutrosophic set in which the truth, indeterminacy and falsity membership functions are $T_{\mathsf{N}}, I_{\mathsf{N}}, F_{\mathsf{N}}$ respectively, and they are from X to $[0, 1]$.

Definition 2.3. [1] A directed fuzzy graph is a fuzzy graph in which the edges are ordered pair of vertices.

Definition 2.4. [3] A single valued neutrosophic graph (SVN-graph) with underlying set V is defined to be a pair $G = (A, B)$ where

(1) The functions $T_A : V \to [0,1]$,$I_A : V \to [0,1]$, and $F_A : V \to [0,1]$, denote the degree of truth, indeterminacy and falsity membership of the element $v_i \in V$, respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$$

for all $v_i \in V$.

(2) The functions $T_B : E \subseteq V \times V \to [0,1]$,$I_B : E \subseteq V \times V \to [0,1]$, and $F_B : E \subseteq V \times V \to [0,1]$, denote the degree of truth, indeterminacy and falsity membership of the element $(v_i, v_j) \in E$, respectively such that $T_B(v_i, v_j) \leq \min(T_A(v_i), T_A(v_j))$, $I_B(v_i, v_j) \geq \max(I_A(v_i), I_A(v_j))$, $F_B(v_i, v_j) \geq \max(F_A(v_i), F_A(v_j))$ and

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3$$

for all $(v_i, v_j) \in E$.

3. Directed Single Valued Neutrosophic Graphs

In this section the concept of Directed Single Valued Neutrosophic Graph is introduced and degree, total degree of vertices with respect to indegree and outdegree, maximum degree and minimum degree of DSVNG are estabilished and their properties are analysed with example.

Definition 3.1. A directed single valued neutrosophic graph(DSVNG) $\vec{G}(A, B)$ is a single valued neutrosophic graph(SVNG) in which the edges are ordered pair of vertices.

Definition 3.2. Let $\vec{G}(A, B)$ be a DSVNG. Then the indegree of a vertex $v_i \in V$ is denoted by $d^-(v_i) = (d_T^-(v_i), d_I^-(v_i), d_F^-(v_i))$ where $d_T^-(v_i) = \sum_{\overrightarrow{v_j v_i} \in E} T_B(\overrightarrow{v_j v_i})$ denotes the indegree of truth membership of vertex v_i. $d_I^-(v_i) = \sum_{\overrightarrow{v_j v_i} \in E} I_B(\overrightarrow{v_j v_i})$ denotes the indegree of indeterminacy membership of vertex v_i. $d_F^-(v_i) = \sum_{\overrightarrow{v_j v_i} \in E} F_B(\overrightarrow{v_j v_i})$ denotes the indegree of falsity membership of vertex v_i.

Definition 3.3. Let $\vec{G}(A, B)$ be a DSVNG. Then the outdegree of a vertex $v_i \in V$ is denoted by $d^+(v_i) = (d_T^+(v_i), d_I^+(v_i), d_F^+(v_i))$ where $d_T^+(v_i) = \sum_{\overrightarrow{v_i v_j} \in E} T_B(\overrightarrow{v_i v_j})$ denotes the outdegree of truth membership of vertex v_i. $d_I^+(v_i) = \sum_{\overrightarrow{v_i v_j} \in E} I_B(\overrightarrow{v_i v_j})$ denotes the outdegree of indeterminacy membership of vertex v_i. $d_F^+(v_i) = \sum_{\overrightarrow{v_i v_j} \in E} F_B(\overrightarrow{v_i v_j})$ denotes the outdegree of falsity membership of vertex v_i.

Definition 3.4. Let $\vec{G}(A, B)$ be a DSVNG. Then the degree of a vertex $v_i \in V$ is $d(v_i) = d^-(v_i) + d^+(v_i) = (d_T^-(v_i) + d_T^+(v_i), d_I^-(v_i) + d_I^+(v_i), d_F^-(v_i) + d_F^+(v_i))$.

Definition 3.5. Let $\overrightarrow{G}(A,B)$ be a DSVNG. Then the total indegree of a vertex $v_i \in V$ is $td^-(v_i) = (td_T^-(v_i), td_I^-(v_i), td_F^-(v_i))$ where

$$td_T^-(v_i) = \sum_{\overrightarrow{v_jv_i}\in E} T_B(\overrightarrow{v_jv_i}) + T_A(v_i)$$
$$td_I^-(v_i) = \sum_{\overrightarrow{v_jv_i}\in E} I_B(\overrightarrow{v_jv_i}) + I_A(v_i)$$
$$td_F^-(v_i) = \sum_{\overrightarrow{v_jv_i}\in E} F_B(\overrightarrow{v_jv_i}) + F_A(v_i)$$

$td_T^-(v_i)$, $td_I^-(v_i)$, $td_F^-(v_i)$ denotes the total indegree of truth, indeterminacy, falsity membership of vertex v_i respectively.

Definition 3.6. Let $\overrightarrow{G}(A,B)$ be a DSVNG. Then the total outdegree of a vertex $v_i \in V$ is $td^+(v_i)$=($td_T^+(v_i)$, $td_I^+(v_i)$, $td_F^+(v_i)$) where

$$td_T^+(v_i) = \sum_{\overrightarrow{v_iv_j}\in E} T_B(\overrightarrow{v_iv_j}) + T_A(v_i)$$
$$td_I^+(v_i) = \sum_{\overrightarrow{v_iv_j}\in E} I_B(\overrightarrow{v_iv_j}) + I_A(v_i)$$
$$td_F^+(v_i) = \sum_{\overrightarrow{v_iv_j}\in E} F_B(\overrightarrow{v_iv_j}) + F_A(v_i)$$

$td_T^+(v_i), td_I^+(v_i), td_F^+(v_i)$ denotes the total outdegree of truth, indeterminacy, falsity membership of vertex v_i respectively.

Definition 3.7. Let $\overrightarrow{G}(A,B)$ be a DSVNG. Then the total degree of a vertex $v_i \in V$ is $td(v_i) = td^-(v_i) + td^+(v_i) = (td_T^-(v_i) + td_T^+(v_i), td_I^-(v_i) + td_I^+(v_i), td_F^-(v_i) + td_F^+(v_i))$.

Definition 3.8. Let $\overrightarrow{G}(A,B)$ be a DSVNG. Then the minimum indegree of $\overrightarrow{G}$ is $\delta^-(\overrightarrow{G}) = (\delta_T^-(\overrightarrow{G}), \delta_I^-(\overrightarrow{G}), \delta_F^-(\overrightarrow{G}))$ where

$$\delta_T^-(\overrightarrow{G}) = \wedge\{d_T^-(v_i)|v_i \in V\}$$
$$\delta_I^-(\overrightarrow{G}) = \wedge\{d_I^-(v_i)|v_i \in V\}$$
$$\delta_F^-(\overrightarrow{G}) = \wedge\{d_F^-(v_i)|v_i \in V\}$$

$\delta_T^-(\overrightarrow{G})$, $\delta_I^-(\overrightarrow{G})$, $\delta_F^-(\overrightarrow{G})$ denotes the minimum indegree of truth, indeterminacy, falsity membership of $\overrightarrow{G}$ respectively.

Definition 3.9. Let $\vec{G}(A, B)$ be a DSVNG. Then the minimum outdegree of $\vec{G}$ is $\delta^+(\vec{G}) = (\delta_T^+(\vec{G}), \delta_I^+(\vec{G}), \delta_F^+(\vec{G}))$ where

$$\delta_T^+(\vec{G}) = \wedge\{d_T^+(v_i)|v_i \in V\}$$
$$\delta_I^+(\vec{G}) = \wedge\{d_I^+(v_i)|v_i \in V\}$$
$$\delta_F^+(\vec{G}) = \wedge\{d_F^+(v_i)|v_i \in V\}$$

$\delta_T^+(\vec{G})$, $\delta_I^+(\vec{G})$, $\delta_F^+(\vec{G})$ denotes the minimum outdegree of truth, indeterminacy, falsity membership of $\vec{G}$ respectively.

Definition 3.10. Let $\vec{G}(A, B)$ be a DSVNG. Then the minimum degree of a vertex $v_i \in V$ is $\delta(\vec{G}) = \delta^-(\vec{G}) + \delta^+(\vec{G}) = (\delta_T^-(\vec{G}) + \delta_T^+(\vec{G}), \delta_I^-(\vec{G}) + \delta_I^+(\vec{G}), \delta_F^-(\vec{G}) + \delta_F^+(\vec{G}))$.

Definition 3.11. Let $\vec{G}(A, B)$ be a DSVNG. Then the maximum indegree of $\vec{G}$ is $\Delta^-(\vec{G}) = (\Delta_T^-(\vec{G}), \Delta_I^-(\vec{G}), \Delta_F^-(\vec{G}))$ where

$$\Delta_T^-(\vec{G}) = \vee\{d_T^-(v_i)|v_i \in V\}$$
$$\Delta_I^-(\vec{G}) = \vee\{d_I^-(v_i)|v_i \in V\}$$
$$\Delta_F^-(\vec{G}) = \vee\{d_F^-(v_i)|v_i \in V\}$$

$\Delta_T^-(\vec{G})$, $\Delta_I^-(\vec{G})$, $\Delta_F^-(\vec{G})$ denotes the maximum indegree of truth, indeterminacy, falsity membership of $\vec{G}$ respectively.

Definition 3.12. Let $\vec{G}(A, B)$ be a DSVNG. Then the maximum outdegree of $\vec{G}$ is $\Delta^+(\vec{G}) = (\Delta_T^+(\vec{G}), \Delta_I^+(\vec{G}), \Delta_F^+(\vec{G}))$ where

$$\Delta_T^+(\vec{G}) = \vee\{d_T^+(v_i)|v_i \in V\}$$
$$\Delta_I^+(\vec{G}) = \vee\{d_I^+(v_i)|v_i \in V\}$$
$$\Delta_F^+(\vec{G}) = \vee\{d_F^+(v_i)|v_i \in V\}$$

$\Delta_T^+(\vec{G})$, $\Delta_I^+(\vec{G})$, $\Delta_F^+(\vec{G})$ denotes the maximum outdegree of truth, indeterminacy, falsity membership of $\vec{G}$ respectively.

Definition 3.13. Let $\vec{G}(A, B)$ be a DSVNG. Then the maximum degree of $\vec{G}$ is $\Delta(\vec{G}) = \Delta^-(\vec{G}) + \Delta^+(\vec{G}) = (\Delta_T^-(\vec{G}) + \Delta_T^+(\vec{G}), \Delta_I^-(\vec{G}) + \Delta_I^+(\vec{G}), \Delta_F^-(\vec{G}) + \Delta_F^+(\vec{G}))$.

Remark 3.14. For any vertex v_i, $\delta_T(\vec{G}) \leq d_T(v_i) \leq \Delta_T(\vec{G})$, $\delta_I(\vec{G}) \leq d_I(v_i) \leq \Delta_I(\vec{G})$ and $\delta_F(\vec{G}) \leq d_F(v_i) \leq \Delta_F(\vec{G})$.

Proposition 3.15. *For any DSVNG $\vec{G}(A, B)$ with n vertices,*

$$\begin{aligned}\sum_{v_i \in V} d(v_i) &= \big(\sum_{v_i \in V}(d_T^+(v_i) + d_T^-(v_i)), \sum_{v_i \in V}(d_I^+(v_i) + d_I^-(v_i)), \sum_{v_i \in V}(d_F^+(v_i) + d_F^-(v_i))\big) \\ &= \big(\sum_{v_i \neq v_j} T_B^+(\overrightarrow{v_i v_j}) + T_B^-(\overrightarrow{v_j v_i}), \sum_{v_i \neq v_j} I_B^+(\overrightarrow{v_i v_j}) + I_B^-(\overrightarrow{v_j v_i}), \\ &\qquad \sum_{v_i \neq v_j} F_B^+(\overrightarrow{v_i v_j}) + F_B^-(\overrightarrow{v_j v_i})\big).\end{aligned}$$

Proof. Proof follows from the definition of indegree and outdegree of DSVNG $\vec{G}(A, B)$. □

Proposition 3.16. *The maximum indegree or outdegree of any vertex in a DSVNG with n vertices is $n - 1$.*

Proof. Let $\vec{G}(A, B)$ be a DSVNG. The maximum value given to an edge is 1 and the number of edges incident on a vertex can be atmost $n - 1$. Similarly, the maximum indeterminacy and falsity membership value given to an edge is 1 and the number of edges incident on a vertex can be at most $n - 1$. Hence, the maximum truth, indeterminacy and falsity membership degree $d_T(v_i)$, $d_I(v_i)$, $d_F(v_i)$ of any vertex v_i in a DSVNG with n vertices is $n - 1$. Hence the result.
□

Example 3.17. Consider the DSVNG $\vec{G}$ given in Figure 1 and the truth, indeterminacy and falsity membership of vertices and edges of $\vec{G}$ is provided in Table 1, 2.

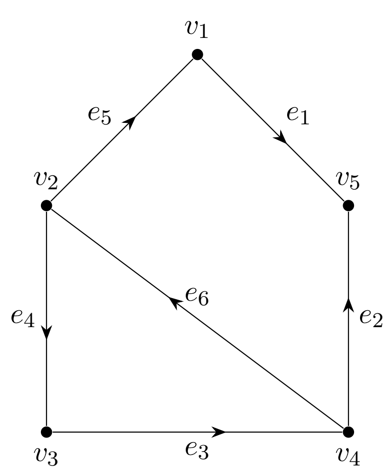

FIGURE 1. DSVNG

TABLE 1. Truth, indeterminacy and falsity membership of vertices

V	v_1	v_2	v_3	v_4	v_5
T_A	0.5	0.5	0.6	0.5	0.3
I_A	0.3	0.7	0.6	0.2	0.4
F_A	0.2	0.3	0.4	0.8	0.7

TABLE 2. Truth, indeterminacy and falsity membership of edges

E	e_1	e_2	e_3	e_4	e_5	e_6
T_B	0.2	0.3	0.4	0.5	0.4	0.4
I_B	0.5	0.5	0.8	0.7	0.7	0.8
F_B	0.7	0.9	0.9	0.6	0.4	0.9

Table 3 provides the degree, indegree, outdegree, total degree, total indegree, total outdegree of vertices of $\vec{G}$.

TABLE 3. Degree and total degree of vertices

(A) Degree

V	v_1	v_2	v_3	v_4	v_5
Indegree of v_i					
d_T^-	0.4	0.4	0.5	0.4	0.5
d_I^-	0.7	0.8	0.7	0.8	1
d_F^-	0.4	0.9	0.6	0.9	1.6
Outdegree of v_i					
d_T^+	0.2	0.9	0.4	0.7	0
d_I^+	0.5	1.4	0.8	1.3	0
d_F^+	0.7	1	0.9	1.8	0
Degree of v_i					
d_T	0.6	1.3	0.9	1.1	0.5
d_I	1.2	2.1	1.3	2.1	1
d_F	1.1	2.4	1.5	2.7	1.6

(B) Total degree

V	v_1	v_2	v_3	v_4	v_5
Total indegree of v_i					
td_T^-	0.9	0.9	1.1	0.9	0.8
td_I^-	1	1.5	1.3	1	0.5
td_F^-	0.6	1.2	1	1.7	2.3
Total outdegree of v_i					
td_T^+	0.7	1.4	1	1.2	0.3
td_I^+	0.8	2.1	1.4	1.5	0.4
td_F^+	0.9	1.3	1.3	2.6	0.7
Total degree of v_i					
td_T	1.6	2.3	2.1	2.1	1.1
td_I	1.8	3.6	2.7	2.5	0.9
td_F	1.5	2.5	2.3	4.3	3

Table 4, 5 provides the minimum degree, indegree, outdegree and maximum degree, indegree, outdegree of $\vec{G}$.

Table 4. Minimum degree of DSVNG.

$\vec{G}$	$\delta^-(\vec{G})$	$\vec{G}$	$\delta^+(\vec{G})$	$\vec{G}$	$\delta(\vec{G})$
$\delta_T^-(\vec{G})$	0.4	$\delta_T^+(\vec{G})$	0	$\delta_T(\vec{G})$	0.4
$\delta_I^-(\vec{G})$	0.7	$\delta_I^+(\vec{G})$	0	$\delta_I(\vec{G})$	0.7
$\delta_F^-(\vec{G})$	0.4	$\delta_F^+(\vec{G})$	0	$\delta_F(\vec{G})$	0.4

Table 5. Maximum degree of DSVNG.

$\vec{G}$	$\Delta^-(\vec{G})$	$\vec{G}$	$\Delta^+(\vec{G})$	$\vec{G}$	$\Delta(\vec{G})$
$\Delta_T^-(\vec{G})$	0.5	$\Delta_T^+(\vec{G})$	0.9	$\Delta_T(\vec{G})$	1.4
$\Delta_I^-(\vec{G})$	1	$\Delta_I^+(\vec{G})$	1.4	$\Delta_I(\vec{G})$	2.4
$\Delta_F^-(\vec{G})$	1.6	$\Delta_F^+(\vec{G})$	1.8	$\Delta_F(\vec{G})$	3.4

4. Effective edges

In this section the effective edges of a Directed Single Valued Neutrosophic Graph is identified and efffective indegree, efffective outdegree of vertex, efffective maximum degree, efffective minimum degree of DSVNG with respect to indegree and outdegree are investigated with example.

Definition 4.1. An edge $\overrightarrow{v_iv_j}$ of a DSVNG $\vec{G}(A,B)$ is called an effective edge if $T_B(\overrightarrow{v_i,v_j}) = T_A(v_i) \wedge T_A(v_j)$, $I_B(\overrightarrow{v_i,v_j}) = I_A(v_i) \vee I_A(v_j)$, and $F_B(\overrightarrow{v_i,v_j}) = F_A(v_i) \vee F_A(v_j)$.

Definition 4.2. The effective indegree of a vertex v_i in $\vec{G}(A,B)$ is defined by $d_E^-(v_i) = (d_{ET}^-(v_i), d_{EI}^-(v_i), d_{EF}^-(v_i))$, where $d_{ET}^-(v_i)$ is the sum of the truth membership values of the effective edges with v_i as the terminal vertex, $d_{EI}^-(v_i)$ is the sum of the indeterminacy membership values of the effective edges with v_i as the terminal vertex, $d_{EF}^-(v_i)$ is the sum of the falsity membership values of the effective edges with v_i as the terminal vertex.

Definition 4.3. The effective outdegree of a vertex v_i in $\vec{G}(A,B)$ is defined by $d_E^+(v_i) = (d_{ET}^+(v_i), d_{EI}^+(v_i), d_{EF}^+(v_i))$, where $d_{ET}^+(v_i)$ is the sum of the truth membership values of the effective edges with v_i as the terminal vertex, $d^{+}_{EI}(v_i)$ is the sum of the indeterminacy membership values of the effective edges with v_i as the terminal vertex, $d^{+}_{EF}(v_i)$ is the sum of the falsity membership values of the effective edges with v_i as the terminal vertex.

Definition 4.4. Let $\vec{G}(A, B)$ be a DSVNG. Then the effective degree of a vertex $v_i \in V$ is $d_E(v_i) = d^{-}_{E}(v_i) + d^{+}_{E}(v_i) = (d^{-}_{ET}(v_i) + d^{+}_{ET}(v_i), d^{-}_{EI}(v_i) + d^{+}_{EI}(v_i), d^{-}_{EF}(v_i) + d^{+}_{EF}(v_i))$.

Definition 4.5. The minimum effective indegree of $\vec{G}$ is $\delta^{-}_{E}[\vec{G}] = (\delta^{-}_{ET}[\vec{G}],\ \delta^{-}_{EI}[\vec{G}],\ \delta^{-}_{EF}[\vec{G}])$ where

$$\delta^{-}_{ET}[\vec{G}] = \wedge\{d^{-}_{ET}[v_i] \mid v_i \in V\}$$
$$\delta^{-}_{EI}[\vec{G}] = \wedge\{d^{-}_{EI}[v_i] \mid v_i \in V\}$$
$$\delta^{-}_{EF}[\vec{G}] = \wedge\{d^{-}_{EF}[v_i] \mid v_i \in V\}$$

$\delta^{-}_{ET}[\vec{G}]$, $\delta^{-}_{EI}[\vec{G}]$, $\delta^{-}_{EF}[\vec{G}]$ denotes the minimum effective T-indegree, I-indegree, F-indegree respectively.

Definition 4.6. The minimum effective outdegree of $\vec{G}$ is $\delta^{+}_{E}[\vec{G}] = (\delta^{+}_{ET}[\vec{G}],\ \delta^{+}_{EI}[\vec{G}],\ \delta^{+}_{EF}[\vec{G}])$ where

$$\delta^{+}_{ET}[\vec{G}] = \wedge\{d^{+}_{ET}[v_i] \mid v_i \in V\}$$
$$\delta^{+}_{EI}[\vec{G}] = \wedge\{d^{+}_{EI}[v_i] \mid v_i \in V\}$$
$$\delta^{+}_{EF}[\vec{G}] = \wedge\{d^{+}_{EF}[v_i] \mid v_i \in V\}$$

$\delta^{+}_{ET}[\vec{G}]$, $\delta^{+}_{EI}[\vec{G}]$, $\delta^{+}_{EF}[\vec{G}]$ denotes the maximum effective T-indegree, I-indegree, F-indegree respectively.

Definition 4.7. Let $\vec{G}(A, B)$ be a DSVNG. Then the minimum effective degree of $\vec{G}$ is $\delta_E(\vec{G}) = \delta^{-}_{E}(\vec{G}) + \delta^{+}_{E}(\vec{G}) = (\delta^{-}_{ET}(\vec{G}) + \delta^{+}_{ET}(\vec{G}), \delta^{-}_{EI}(\vec{G}) + \delta^{+}_{EI}(\vec{G}), \delta^{-}_{EF}(\vec{G}) + \delta^{+}_{EF}(\vec{G}))$.

Definition 4.8. The maximum effective indegree of $\vec{G}$ is $\Delta^{-}_{E}[\vec{G}] = (\Delta^{-}_{ET}[\vec{G}], \Delta^{-}_{EI}[\vec{G}], \Delta^{-}_{EF}[\vec{G}])$ where

$$\Delta^{-}_{ET}[\vec{G}] = \vee\{d^{-}_{ET}[v_i] \mid v_i \in V\}$$
$$\Delta^{-}_{EI}[\vec{G}] = \vee\{d^{-}_{EI}[v_i] \mid v_i \in V\}$$
$$\Delta^{-}_{EF}[\vec{G}] = \vee\{d^{-}_{EF}[v_i] \mid v_i \in V\}$$

$\Delta^{-}_{ET}[\vec{G}]$, $\Delta^{-}_{EI}[\vec{G}]$, $\Delta^{-}_{EF}[\vec{G}]$ denotes the minimum effective T-indegree, I-indegree, F-indegree respectively.

Definition 4.9. The maximum effective outdegree of $\vec{G}$ is $\Delta^{+}_{E}[\vec{G}] = (\Delta^{+}_{ET}[\vec{G}],\ \Delta^{+}_{EI}[\vec{G}],\ \Delta^{+}_{EF}[\vec{G}])$ where

$$\Delta^+_{ET}[\vec{G}] = \vee\{d^+_{ET}[v_i] \mid v_i \in V\}$$
$$\Delta^+_{EI}[\vec{G}] = \vee\{d^+_{EI}[v_i] \mid v_i \in V\}$$
$$\Delta^+_{EF}[\vec{G}] = \vee\{d^+_{EF}[v_i] \mid v_i \in V\}$$

$\Delta^+_{ET}[\vec{G}]$, $\Delta^+_{EI}[\vec{G}]$, $\Delta^+_{EF}[\vec{G}]$ denotes the maximum effective T-indegree, I-indegree, F-indegree respectively.

Definition 4.10. Let $\vec{G}(A, B)$ be a DSVNG. Then the maximum effective degree of $\vec{G}$ is $\Delta_E(\vec{G}) = \Delta^-_E(\vec{G}) + \Delta^+_E(\vec{G}) = (\Delta^-_{ET}(\vec{G}) + \Delta^+_{ET}(\vec{G}), \Delta^-_{EI}(\vec{G}) + \Delta^+_{EI}(\vec{G}), \Delta^-_{EF}(\vec{G}) + \Delta^+_{EF}(\vec{G}))$.

Example 4.11. Consider the DSVNG $\vec{G}$ given in Figure 2 and the truth, indeterminacy and falsity membership of vertices and edges of $\vec{G}$ is provided in Table 6.

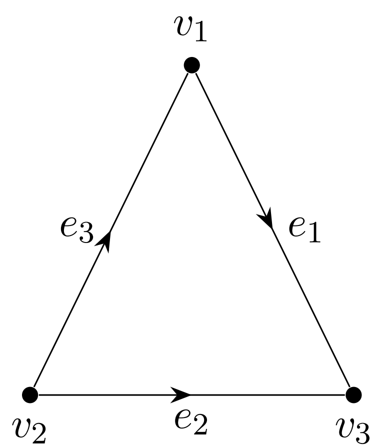

FIGURE 2. DSVNG

TABLE 6. Truth, indeterminacy and falsity membership of vertices and edges

(A) Membership of vertices

$\vec{G}$	v_1	v_2	v_3
T_A	0.5	0.2	0.3
I_A	0.3	0.6	0.4
F_A	0.2	0.7	0.2

(B) Membership of edges

$\vec{G}$	e_1	e_2	e_3
T_B	0.3	0.2	0.2
I_B	0.4	0.6	0.6
F_B	0.2	0.7	0.7

Table 7 provides the effective degree, effective indegree, effective outdegree of vertices of $\vec{G}$. Table 8, 9 provides the minimum effective degree, indegree, outdegree and maximum effective degree, indegree, outdegree of $\vec{G}$.

TABLE 7. Effective degree of DSVNG

(A) Indegree of DSVNG

$\vec{G}$	v_1	v_2	v_3
Effective indegree			
d^-_{ET}	0.2	0	0.5
d^-_{EI}	0.6	0	1.0
d^-_{EF}	0.7	0	0.9

(B) Outdegree of DSVNG

$\vec{G}$	v_1	v_2	v_3
Effective outdegree			
d^+_{ET}	0.3	0.4	0
d^+_{EI}	0.4	1.2	0
d^+_{EF}	0.2	1.4	0

(C) Degree of DSVNG

$\vec{G}$	v_1	v_2	v_3
Effective degree			
d_{ET}	0.5	0.4	0.5
d_{EI}	1.0	1.2	1.0
d_{EF}	0.9	1.4	0.9

TABLE 8. Minimum effective degree of DSVNG.

$\vec{G}$	$\delta^-(\vec{G})$	$\vec{G}$	$\delta^+(\vec{G})$	$\vec{G}$	$\delta(\vec{G})$
$\delta^-_{ET}(\vec{G})$	0	$\delta^+_{ET}(\vec{G})$	0	$\delta_{ET}(\vec{G})$	0
$\delta^-_{EI}(\vec{G})$	0	$\delta^+_{EI}(\vec{G})$	0	$\delta_{EI}(\vec{G})$	0
$\delta^-_{EF}(\vec{G})$	0	$\delta^+_{EF}(\vec{G})$	0	$\delta_{EF}(\vec{G})$	0

TABLE 9. Maximum effective degree of DSVNG.

$\vec{G}$	$\Delta^-(\vec{G})$	$\vec{G}$	$\Delta^+(\vec{G})$	$\vec{G}$	$\Delta(\vec{G})$
$\Delta^-_{ET}(\vec{G})$	0.5	$\Delta^+_{ET}(\vec{G})$	0.4	$\Delta_{ET}(\vec{G})$	0.9
$\Delta^-_{EI}(\vec{G})$	1	$\Delta^+_{EI}(\vec{G})$	1.2	$\Delta_{EI}(\vec{G})$	2.2
$\Delta^-_{EF}(\vec{G})$	0.9	$\Delta^+_{EF}(\vec{G})$	1.4	$\Delta_{EF}(\vec{G})$	2.3

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