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Identification of influential factors affecting student performance in semester examinations in the educational institution using score topological indices in Single Valued Neutrosophic Graphs

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Abstract: Education not only measures the progress of an individual, but it also contributes to the development of a community and a nation. To understand the factors that affect the academic performance of students in institutions of higher education, there are many factors within the institution and outside the institution that affect the academic performance of students. Includes affecting student academic performance are class size, parent-related factors, academic performance, the contribution of institutional factors to student academic performance, and the impact of poverty on students. The interplay of personal attributes, learning habits, prior academic preparation, and the school environment is essential to understanding and supporting student academic achievement. Each of these factors contributes uniquely to academic performance, and their relative importance may vary according to each student's circumstances. All these factors include uncertainty in nature. Neutrosophic graphs provide a powerful mathematical framework for dealing with uncertainty and complexity in various mathematical and real-world problems. This manuscript comprises the study of some innovative results based on the score topological indices in SVN_{graphs}. Finally, we identify the influential factors in the successful conduct of semester examinations using these new parameters.

Keywords: *Score Function*, SVN_{graph}, *Score Topological Indices*

Abbreviation	Description
F_{set}	Fuzzy set
$F_{relation}$	Fuzzy relation
F_{graph}	Fuzzy graph
IF_{set}	Intuitionistic Fuzzy set
CF_{set}	Classical fuzzy set
N_{set}	Neutrosophic set

SVN_{set}	Single Valued Neutrosophic set
N_{graph}	Neutrosophic graph
$F_{numbers}$	Fuzzy number
IF_{number}	Intuitionistic Fuzzy number
$TriIF_{numbers}$	Triangular Intuitionistic Fuzzy number
SVN_{values}	Single Valued Neutrosophic values
U_{set}	Universe set
FV_{set}	Fuzzy vertex set
FI_{set}	Fuzzy index set
NV_{set}	Neutrosophic vertex set
NI_{set}	Neutrosophic index set
$T_{mem}, I_{mem}, F_{mem}$	Truth membership, Indeterminacy membership, Falsity membership
$SVNV_{set}$	Single Value Neutrosophic vertex set
$SVNI_{set}$	Single Value Neutrosophic index set
N_{cycle}	Neutrosophic cycle
SD_{vertex}	Score Degree of vertex
$SVN_{subgraph}$	Single-valued neutrosophic sub-graph

1. Introduction

In 1965, Zadeh [1] introduced the concept of the degree of membership/truth (T) and defined the F_{set}. Kaufmann [2] later extended this by applying the concept of fuzziness to graph theory using $F_{relation}$. Rosenfeld [3] introduced several concepts related to F_{graph}, including bridges, cycles, paths, trees, and connectedness, and explored some properties ofF_{graph}. An IF_{set}Considers both the membership grade and the non-membership grade for any entity, with the requirement that their sum does not exceed one. Atanassov [4–5] introduced theIF_{set} as a modified version of theCF_{set}.

In 1995, Smarandache [7] introduced the concept of indeterminacy/neutrality (I) as an independent component and defined theN_{set}, which consists of three components: Truth (T), Indeterminacy (I), and Falsity (F). N_{set} are characterized by three functions: $T_{mem}, I_{mem}, and\ F_{mem}$ where their values are real standard or non-standard subsets of the unit interval]–0, 1+[. The SVN_{set} [8], which takes values from the subset of [0, 1], is a specific instance of aN_{set} and is particularly useful for addressing real-world problems, especially in decision support systems.

Topological indices are molecular descriptors to study the properties of a chemical compound in chemical graph theory. These numerical quantities of a chemical graph describe its topology. A molecular graph can be represented using an atom as a vertex and a bond between two atoms represents an edge. There are many indices available in the literature such as the Wiener index [13], Zagreb index [22-23], Randic index [16], Atom-Bond-Connectivity index [17], Harmonic index[18], sum-connectivity and general sum-connectivity indices [19-20], Schultz index or degree distance index [21] and Gutman index [22].

InF_{graphs}, the study of these indices has been much studied in recent years by many researchers. Shriram Kalathian et. al [23] introduced some topological indices in F_{graphs}. Masoud Ghods and Zahra Rostami studied the first and second Zagreb indices, the Harmonic index, the Randic index, and the Connectivity index in N_{graphs} [24]. The score and accuracy of uncertainty graphs made it easy to rank them. In this work, we introduce a new kind of topological indices based on score function and prove a few theorems related to these indices. Ranking of IF_{number} Was done by Mitchell, et. al and Nayagam et. al. [26 -29]. Also, Sahin, R. [30] analyzed a multi-criteria decision-making method in a neutrosophic environment, utilizing score and accuracy functions. This approach aims to enhance decision-making processes by effectively addressing the uncertainty and indeterminacy inherent inN_{set}.

Ranking$F_{numbers}$Was first introduced by Jain [27]. Some of the researchers proposed ranking methods based on triangular and trapezoidal membership functions. Later, a few of them also introduced ranking $F_{numbers}$ Based on the centroid point. Yager discussed centroid-based ranking. $F_{numbers}$Initially. Mitchell discussed a new ranking method for $IF_{numbers}$ [28]. Gomthi Nayagam et.al introduced the concept of ranking the $IF_{numbers}$ and Modified ranking of $IF_{numbers}$ [29,30]. A ranking Method of $TriIF_{numbers}$ and its applications to decision-making were studied by Li [31]. In 2016, Sahin [32] introduced score and accuracy functions within a neutrosophic environment to solve multi-criteria decision-making problems.

Grzegorzewski [33] treated $IF_{numbers}$As two families of metrics and developed a ranking method for these numbers. In 2016, Nancy and Harish Garg [34] introduced an improved score function for ranking N_{set}, providing a more refined approach for evaluating and comparing alternatives in a neutrosophic decision-making environment. In this work, we introduce some of the topological indices based on the following score function which is useful for ranking. SVN_{set}.

$$\tilde{S} = \frac{\tilde{T} + \tilde{I} + 1 - \tilde{F}}{3}, \tilde{S} \in [0,1] (1).$$

Education is a process through which a person can transmit his findings and experiences for survival and development over the generations. Education aims to strengthen the individual's as well as society’s culture. Examination is a very interesting and important tool to examine a student's achievement. Evaluating a student's performance through examination leads to both government and non-government organizations' support and treatment for improving the quality of education for the benefit of social changes and carrying out education sector reforms. The performance of a student in examinations is influenced by a variety of factors, and understanding these factors is crucial for improving educational outcomes. Saima Rasul and Qadir Bukhsh designed to measure the factors affecting students’ performance in examinations at the university level [35]. Kassu Mehari Beyene Jemal Ayalew Yimam discussed the Multilevel Analysis for Identifying Factors Influencing Academic Achievement of Students in Higher Education Institutions in Wollo University [36]. Yousuf Nasser Said Al Husaini andNur Syufiza Ahmad Shukor [37] analyzed a comprehensive review of the factors affecting student academic performance based on low entry grades, family background, food and accommodation facility, gender, past assessment grade, and students' learning activity. Kawtar Tani et.al. [38] evaluated the factors contributing to the poor academic performance of students in higher education and the effect of factors relating to family obligations, work, social commitments, and financial concerns were analyzed. They showed a relationship between the academic performance of students and their attendance. They also concluded that the effect of absenteeism is a major one for determining student performance in most educational institutions and encouraged students to attend classes to achieve better grades in their future courses. Wei Liu and Lei Zhang [39] showed the evaluation of student performance and identified the key influencing factors based on the Entropy-Weighted TOPSIS Model.

In all the above studies, the amount of uncertainty was not an important factor in finding the key factors of a student's performance in the end-semester examination. This motivates us to study the performance of a student using a new mathematical tool namely score-based topological indices using SVN_{values}. In this study, we aim to analyze the impact of different factors that affect student performance at the higher education level based on the factors psychological, socioeconomic, and physical influences, changes in question paper patterns, guidance adequacy, exam-related anxieties, internal environment, question paper difficulty, and evaluator impact. A survey of over 500 students was conducted to gather data. The neutrosophic theory was employed to handle uncertainties in student satisfaction levels.

In sectionδ.2, all the basic definitions are presented. Section δ.3 introduces the concept of score-based topological indices, along with illustrations to demonstrate their application.

2. Preliminaries

Definition 1: [1] Let X be theU_{set}. A $F_{set}\tilde{A}$ on X is defined as $\tilde{A} = \{\langle x, T_{\tilde{A}}(x)\rangle : x \in X\}$, where $T_{\tilde{A}}(x): X \to ^{-}]0,1[^{+}$ is said to be the T_{mem}function, which represents the degree of confidence.

Definition 2:[3]Let $G = (V_G, E_G)$ be a simple graph, where V_G, E_G be the set of vertices and edges, respectively. Then, a $F_{graph}G$ is denoted by $\tilde{G} = (V_G,\ \tilde{\mu}, \tilde{\rho})$, where $\tilde{\mu} = T_{\tilde{\mu}}$is the FV_{set} on $V_G$and $\tilde{\rho} = T_{\tilde{\rho}}$ is the FI_{set} on $E_G \subseteq V_G \times V_G\ where$ $(T_{\tilde{\mu}}: V_G \to [0,1]$:$T_{\tilde{\rho}}: V_G \times V_G \to [0,1])$, and is defined as$T_{\tilde{\rho}}(u_i, v_j) \leq T_{\tilde{\mu}}(u_i) \wedge T_{\tilde{\mu}}(v_j)$, $(u_i, v_j) \in E_G (i, j = 1,2,3 \dots n)$.

Definition 3: [4,5] A $F_{set}\tilde{A}$ in the universal discourse X. An $IF_{set}\tilde{A}$ on X is defined as $\tilde{A} = \{\langle x, T_{\tilde{A}}(x),\ F_{\tilde{A}}(x)\rangle : x \in X\}$, where $T_{\tilde{A}}(x): X \to ^{-}]0,1[^{+}$ is said to be the certainty membership function, which specifies the degree of confidence, $I_{\tilde{A}}(x): X \to ^{-}]0,1[^{+}$is said to be the uncertainty membership, which represents the degree of indistinctness, respectively of the element $x \in X\ in\ \tilde{A}$, such that $0 \leq T_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 1$.

Definition 4:[5] An IF_{graph} of G is of the form $\tilde{G} = (V_G,\ \tilde{\mu}, \tilde{\rho})$, where $\tilde{\mu} = (T_{\tilde{\mu}}, F_{\tilde{\mu}})$ is aNV_{set} on V_G and $\tilde{\rho} = (T_{\tilde{\rho}}, F_{\tilde{\rho}})$ is the NI_{set} on $E_G \subseteq V_G \times V_G$.$V_G = \{v_1, v_2, v_3, \dots v_n\}$ such that $(T_{\tilde{\mu}}: V_G \to [0,1]$, $F_{\tilde{\mu}}: V_G \to [0,1]$, denote the degree of certainty membership, and degree of indistinctness membership, respectively of the element $v_i \in V_G$and ,$0 \leq T_{\tilde{\mu}}(v_i) + F_{\tilde{\mu}}(v_i) \leq 1\ for\ every\ v_i \in V_G$.$E_G \subseteq V_G \times V_G where\ T_{\tilde{\rho}}: V_G \times V_G \to [0,1], F_{\tilde{\rho}}: V_G \times V_G \to [0,1]$are such that $T_{\tilde{\rho}}(v_i, v_j) \leq [\ T_{\tilde{\mu}}(v_i) \wedge\ T_{\tilde{\mu}}(v_j)]$, $F_{\tilde{\rho}}(v_i, v_j) \leq [\ F_{\tilde{\mu}}(v_i) \vee\ F_{\tilde{\mu}}(v_j)]$ and $0 \leq T_{\tilde{\rho}}(v_i, v_j) + F_{\tilde{\rho}}(v_i, v_j) \leq 1\ for\ every\ (v_i, v_j) \in E_G (i, j = 1,2,3 \dots n)$.

Definition 5: [7] Let X be a U_{set}. A $N_{set}\tilde{A}$ on X is defined as $\tilde{A} = \{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle : x \in X\}$, where $T_{\tilde{A}}(x): X \to ^{-}]0,1[^{+}$ is said to be the T_{mem}function, which represents the degree of confidence, $I_{\tilde{A}}(x): X \to ^{-}]0,1[^{+}$is said to be the I_{mem}, which represents the degree of uncertainty, and $F_{\tilde{A}}(x): X \to ^{-}]0,1[^{+}$ is said to be the F_{mem}, which represents the degree of skepticism, respectively of the element $x \in X\ in\ \tilde{A}$, such that $^{-}0 \leq T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 3^{+}, \forall x \in X$.

Definition 6: N_{graph} of G is of the form $\tilde{G} = (V_G,\ \tilde{\mu}, \tilde{\rho})$, where $\tilde{\mu} = (T_{\tilde{\mu}}, I_{\tilde{\mu}}, F_{\tilde{\mu}})$ is a NV_{set}on V_G and $\tilde{\rho} = (T_{\tilde{\rho}}, I_{\tilde{\rho}}, F_{\tilde{\rho}})$ is the NI_{set} on $E_G \subseteq\ V_G \times V_G$.

i. $V_G = \{v_1, v_2, v_3, \dots v_n\}$ such that $(T_{\tilde{\mu}}: V_G \to [0,1], I_{\tilde{\mu}}: V_G \to [0,1], F_{\tilde{\mu}}: V_G \to [0,1]$, denote the degree of $T_{mem}, I_{mem}, F_{mem}$, respectively of the element $v_i \in V_G$and , $^{-}0 \leq T_{\tilde{\mu}}(v_i) + I_{\tilde{\mu}}(v_i) + F_{\tilde{\mu}}(v_i) \leq 3^{+}\ for\ every\ v_i \in V_G$.

ii. $E_G \subseteq V_G \times V_G where\ T_{\tilde{\rho}}: V_G \times V_G \to [0,1], I_{\tilde{\rho}}: V_G \times V_G \to [0,1], F_{\tilde{\rho}}: V_G \times V_G \to [0,1]$ are such that $T_{\tilde{\rho}}(v_i, v_j) \leq [\ T_{\tilde{\mu}}(v_i) \wedge\ T_{\tilde{\mu}}(v_j)]$, $I_{\tilde{\rho}}(v_i, v_j) \leq [\ I_{\tilde{\mu}}(v_i) \wedge\ I_{\tilde{\mu}}(v_j)]$, $F_{\tilde{\rho}}(v_i, v_j) \leq [\ F_{\tilde{\mu}}(v_i) \wedge\ F_{\tilde{\mu}}(v_j)]$ and $^{-}0 \leq T_{\tilde{\rho}}(v_i, v_j) + I_{\tilde{\rho}}(v_i, v_j) + F_{\tilde{\rho}}(v_i, v_j) \leq 3^{+}\ for\ every\ (v_i, v_j) \in E_G (i, j = 1,2,3 \dots n)$.

Definition 7: Let X be a universe set. A$SVN_{set}\tilde{S}$ on X is defined as $\tilde{S} = \{\langle x, T_{\tilde{S}}(x), I_{\tilde{S}}(x), F_{\tilde{S}}(x)\rangle : x \in X\}$, where $T_{\tilde{S}}(x): X \to ^{-}]0,1[^{+}$ is said to be the T_{mem} function, $I_{\tilde{S}}(x): X \to ^{-}]0,1[^{+}$is said to be the I_{mem}, and $F_{\tilde{S}}(x): X \to ^{-}]0,1[^{+}$ is said to be the F_{mem}, respectively of the element $x \in X\ on\ \tilde{S}$, such that $^{-}0 \leq T_{\tilde{S}}(x) + I_{\tilde{S}}(x) + F_{\tilde{S}}(x) \leq 3^{+}, \forall x \in X$.

Definition 8: A $SVN_{graph}\tilde{S}$ of G is of the form $\tilde{G} = (V_G,\ \tilde{\mu}, \tilde{\rho})$, where $\tilde{\mu} = (T_{\tilde{\mu}}, I_{\tilde{\mu}}, F_{\tilde{\mu}})$ is a$SVNV_{set}$on V_G

and $\tilde{\rho} = (T_{\tilde{\rho}}, I_{\tilde{\rho}}, F_{\tilde{\rho}})$ is the $SVNI_{set}$ on $E_G \subseteq V_G \times V_G$.

i. $V_G = \{v_1, v_2, v_3, \dots v_n\}$ such that $T_{\tilde{\mu}}: V_G \to [0,1], I_{\tilde{\mu}}: V_G \to [0,1], F_{\tilde{\mu}}: V_G \to [0,1]$, denote the degree of $T_{mem}, I_{mem}, F_{mem}$, respectively of the element $v_i \in V_G$ and , $^{-}0 \leq T_{\tilde{\mu}}(v_i) + I_{\tilde{\mu}}(v_i) + F_{\tilde{\mu}}(v_i) \leq 3^{+}$ $for\ every\ v_i \in V_G$.

ii. $E_G \subseteq V_G \times V_G where\ T_{\tilde{\rho}}: V_G \times V_G \to [0,1], I_{\tilde{\rho}}: V_G \times V_G \to [0,1], F_{\tilde{\rho}}: V_G \times V_G \to [0,1]$ are such that $T_{\tilde{\rho}}(v_i, v_j) \leq [\, T_{\tilde{\mu}}(v_i) \wedge T_{\tilde{\mu}}(v_j)], I_{\tilde{\rho}}(v_i, v_j) \leq [\, I_{\tilde{\mu}}(v_i) \vee I_{\tilde{\mu}}(v_j)], F_{\tilde{\rho}}(v_i, v_j) \geq [\, F_{\tilde{\mu}}(v_i) \vee F_{\tilde{\mu}}(v_j)]$ and $^{-}0 \leq T_{\tilde{\rho}}(v_i, v_j) + I_{\tilde{\rho}}(v_i, v_j) + F_{\tilde{\rho}}(v_i, v_j) \leq 3^{+}$ $for\ every\ (v_i, v_j) \in E_G (i, j = 1,2,3 \dots n)$. $T_{\tilde{\rho}}(v_i, v_j) \leq min[\, T_{\tilde{\mu}}(v_i), T_{\tilde{\mu}}(v_j)]$, $I_{\tilde{\rho}}(v_i, v_j) \geq max[\, I_{\tilde{\mu}}(v_i), I_{\tilde{\mu}}(v_j)$, $F_{\tilde{\rho}}(v_i, v_j) \geq max[\, F_{\tilde{\mu}}(v_i) F_{\tilde{\mu}}(v_j)]$,

Definition 9: Let $\tilde{G} = (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph} and $\check{P}$ is a path in $\tilde{G}$. $\check{P}$ is a collection of vertices, $v_0, v_1, v_2, \cdots, v_n$ such that $[T_{\tilde{\mu}}(v_{i-1}, v_i), I_{\tilde{\mu}}(v_{i-1}, v_i), F_{\tilde{\mu}}(v_{i-1}, v_i)] > 0$ for $0 \leq i \leq n$. $\check{C}$ is a N_{cycle} if $v_0 = v_n$ and $n \geq 3$.

3. Topological indices of SVN_{graph}

Definition 10: Let $\tilde{G} = (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph} and $v \in V_G$. Then the $SD_{vertex} v$ is defined as $d_s(v) = \sum_{u_i \epsilon N(v)} \tilde{S}(u_i, v)$, where $N(v)$ represents the number of neighbors of v.

Definition 11: Let $\tilde{G} = (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph} and $v \in V_G$. Then the k- $SD_{vertex} v$ is defined as $d_s^k(v) = \sum_{u_i \epsilon N(v)} \tilde{S}(u_i, v)^k$, where $N(v)$ represents the number of vertices which are adjacent to v.

3.1 Zagreb Index

Definition 12: Let $\tilde{G}: (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph} with a nonempty vertex set. The first kind of Zagreb index $ZI(\tilde{G})$ of a SVN_{graph} is defined as

$$ZI(\tilde{G}) = \sum_{u_i \epsilon \sigma^*} \tilde{S}(u_i) d_s^2(u_i), u_i \in V_G.$$

Definition 13: The second kind of Zagreb index $ZI^*(\tilde{G})$ of a SVN_{graph} is defined as

$$ZI^*(\tilde{G}) = \sum_{(u_i, u_j) \epsilon \mu} \tilde{S}(u_i) d_s(u_i) \tilde{S}(u_j) d_s(u_j), \quad \forall i \neq j.$$

Example 14: Let $\tilde{G}$ be the SVN_{graph} as shown in Fig. 1 such that $V_{\tilde{G}} = \{v_1, v_2, v_3, v_4\}$.

The score function for all vertices is

$\tilde{S}(v_1) = \frac{0.5+0.1+1-0.4}{3} = \frac{1.2}{3} = 0.4$, $\tilde{S}(v_2) = \frac{0.6+0.3+1-0.2}{3} = \frac{1.7}{3} = 0.567$, $\tilde{S}(v_3) = \frac{0.2+0.3+1-0.4}{3} = \frac{1.1}{3} = 0.367$, $\tilde{S}(v_4) = \frac{0.4+0.2+1-0.5}{3} = \frac{1.1}{3} = 0.367$, $\tilde{S}(v_1, v_2) = \frac{0.5+0.4+1-0.5}{3} = \frac{1.4}{3} = 0.467$; $\tilde{S}(v_2, v_3) = \frac{0.2+0.3+1-0.4}{3} = \frac{1.1}{3} = 0.367$, $\tilde{S}(v_3, v_4) = \frac{0.2+0.4+1-0.5}{3} = \frac{1.1}{3} = 0.367$, $\tilde{S}(v_4, v_1) = \frac{0.4+0.3+1-0.6}{3} = \frac{1.1}{3} = 0.367$.

Now, we have

$$d_s(v_1) = \tilde{S}(v_1, v_2) + \tilde{S}(v_4, v_1) = 0.467 + 0.367 = 0.834$$
$$d_s(v_2) = \tilde{S}(v_1, v_2) + \tilde{S}(v_2, v_3) = 0.467 + 0.367 = 0.834$$
$$d_s(v_3) = \tilde{S}(v_2, v_3) + \tilde{S}(v_3, v_4) = 0.367 + 0.367 = 0.734$$
$$d_s(v_4) = \tilde{S}(v_3, v_4) + \tilde{S}(v_4, v_1) = 0.367 + 0.367 = 0.734$$

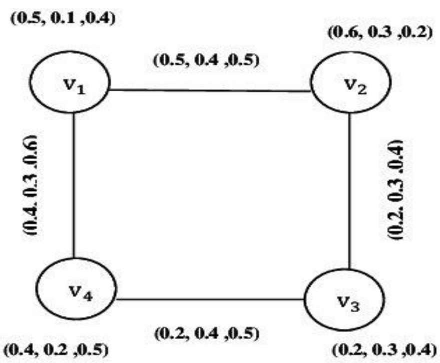

Fig.1.SVN_{graph} *with* $PI(\tilde{G}) = 0.956$, $ZI(\tilde{G}) = .0606$, $ZI^*(\tilde{G}) = 0.224$

From definitions [12, 13], we have

$$ZI(\tilde{G}) = 0.4(0.467^2 + 0.367^2) + 0.567(0.467^2 + 0.367^2) + 0.367(0.367^2 + 0.367^2) + 0.367(0.367^2 + 0.367^2)$$

$$= 0.4(0.3523) + 0.567(0.3523) + 0.367(0.1347) + 0.367(0.1347) = 0.0606.$$

$$ZI^*(\tilde{G}) = \sum_{(v_i,v_j)\epsilon\mu} \tilde{S}(v_i)d_s(v_i)\tilde{S}(v_j)d_s(v_j), \forall\, i \neq j \text{ and } v_i, v_j \in V_{\tilde{G}}, i,j = \{1,2,3,4\}$$

$$ZI^*(\tilde{G}) = \{0.1578 + 0.1274 + 0.0726 + 0.0899\} = 0.4477.$$

Observation 15: Let $\tilde{G}$ is the SVN_{graph} and $\widetilde{H}$ is the $SVN_{subgraph}\tilde{G}$ such that $\widetilde{H} = \tilde{G} - u$ then $ZI(\widetilde{H})$< ZI $(\tilde{G})$ and $ZI^*(\widetilde{H})$<$ZI^*(\tilde{G})$.

Definition 16: A $SVN_{graph}\tilde{G} = (\tilde{\mu}, \tilde{\rho})$ is said to be a score regular SVN_{graph} if $d_s(v) = k$, for all vertices v in V_G.

Observation 17: Let $\tilde{G}$: $(\tilde{\mu}, \tilde{\rho})$be the score regular N_{graph}. Then, $ZI^*(\tilde{G}) = k^2 d_s(v)$.

Theorem 18: Suppose $\tilde{G}$: $(\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph} with $\tilde{n}$ vertices and $\tilde{m}$ Edges. Then $ZI^*(\tilde{G}) \leq \left(\frac{16}{81}\right)\tilde{\Delta}^2$.

Proof. As $d_s(v) \leq \frac{2}{3}$ and $\tilde{S}(v) \leq \frac{2}{3}$,. $\tilde{S}(u_i)d_s(u_i)\tilde{S}(u_j)d_s(u_j) \leq \left(\frac{16}{81}\right)\tilde{\Delta}^2$where $\tilde{\Delta}$ represents the maximum degree of $\tilde{G}$.

Theorem 19: Let $\check{P}$ be a path with n vertices. Then $ZI^*(\check{P}) \leq \frac{64}{81}(n-2)$.

Proof. Let $(v_1, \cdots, v_n)$ be a vertex set of path $\check{P}$. Then $d_s(v_1) = \tilde{S}(u_i, v_1) = \tilde{S}(v_1, v_2) \leq \frac{2}{3} d_s(v_n) = \sum_{u_i \epsilon N(v_n)} \tilde{S}(u_i, v_n) = \tilde{S}(v_n, v_{n-1}) \leq \frac{2}{3}$, and $d_s(v_i) = \sum_{u_i \epsilon N(v_i)} \tilde{S}(u_i, v_i) \leq \frac{4}{3}$, for $i = 2, 3, \cdots, n-1$ and $\tilde{S}(v_i) \leq \frac{2}{3}, \forall i$.

Therefore, $ZI^*(\check{P}) = \sum_{(u_i,u_j)\epsilon\mu} \tilde{S}(u_i)d_s(u_i)\tilde{S}(u_j)d_s(u_j)$

$$= \left[\tilde{S}(v_1)d_s(v_1)\tilde{S}(v_2)d_s(v_2) + \tilde{S}(v_n)d_s(v_n)\tilde{S}(v_{n-1})d_s(v_n) + \sum_{i=2}^{n-2} \tilde{S}(v_i)d_s(v_i)\tilde{S}(v_{i-1})d_s(v_{i-1})\right]$$

$$\leq \left[\frac{32}{81} + \frac{32}{81} + \frac{64}{81}(n-3)\right] = \frac{64}{81}(n-2).$$

Example 19:

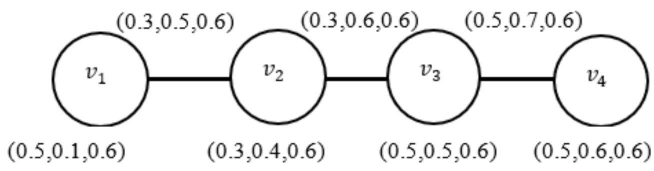

Fig.2. *SVN_{graph} $\check{P}$ on 4 vertices with $ZI^*(\tilde{G}) = 0$.*

The score function for all vertices is

$\tilde{S}(v_1) = \frac{0.5+0.1+1-0.6}{3} = 0.33, \tilde{S}(v_2) = \frac{0.3+0.4+1-0.6}{3} = 0.37, \tilde{S}(v_3) = \frac{0.5+0.5+1-0.6}{3} = 0.47;$ $\tilde{S}(v_4) = \frac{0.5+0.6+1-0.6}{3} = \frac{1.5}{3} = 0.5, \tilde{S}(v_1, v_2) = \frac{0.3+0.5+1-0.6}{3} = \frac{1.2}{3} = 0.4, \tilde{S}(v_2, v_3) = \frac{0.3+0.6+1-0.6}{3} = \frac{1.3}{3} = 0.43, \tilde{S}(v_3, v_4) = \frac{0.5+0.7+1-0.6}{3} = \frac{1.6}{3} = 0.53.$

Now, we have $d_s(v_1) = \tilde{S}(v_1, v_2) = 0.4$; $d_s(v_2) = \tilde{S}(v_1, v_2) + \tilde{S}(v_2, v_3) = 0.4 + 0.43 = 0.83$; $d_s(v_3) = \tilde{S}(v_2, v_3) + \tilde{S}(v_3, v_4) = 0.43 + 0.53 = 0.96$; $d_s(v_4) = \tilde{S}(v_3, v_4) = 0.53$

From definitions [13], we have

$$ZI^*(\tilde{G}) = \tilde{S}(v_1)d_s(v_1)\tilde{S}(v_2)d_s(v_2) + \tilde{S}(v_2)d_s(v_2)\tilde{S}(v_3)d_s(v_3) + \tilde{S}(v_3)d_s(v_3)\tilde{S}(v_4)d_s(v_4)$$
$$= .33 \times .4 \times .37 \times .83 + .37 \times .83 \times .96 \times .43 + .47 \times .43 \times .5 \times .53 = 0.279\,.$$

Theorem 20: Let $\check{C}$ is a cycle with n vertices. Then $ZI^*(\check{C}) \leq \frac{64}{81}(n-1)$.

Proof. Let $(v_1, \cdots, v_n)$ be a vertex set of cycle $\check{C}$. Then $d_s(v_1) = \sum_{u_i \epsilon N(v_1)} \tilde{S}(u_i, v_n) = \tilde{S}(v_1, v_n) \leq \frac{2}{3}$ and $d_s(v_i) = \sum_{u_i \epsilon N(v_i)} \tilde{S}(u_i, v_i) = \tilde{S}(v_{i-1}, v_i) \leq \frac{4}{9}$, for $i = 2, 3, \cdots, n-1$ and $\tilde{S}(v_i) \leq \frac{2}{3}, \forall i$.

Therefore,

$$ZI^*(\check{C}) = \sum_{(u_i, u_j)\epsilon \mu} \tilde{S}(u_i)d_s(u_i)\tilde{S}(u_j)d_s(u_j)$$
$$= \left[\tilde{S}(v_1)d_s(v_1)\tilde{S}(v_2)d_s(v_2) + \tilde{S}(v_1)d_s(v_n)\tilde{S}(v_n)d_s(v_n) + \sum_{i=2}^{n-2} \tilde{S}(v_i)d_s(v_{i+1})\tilde{S}(u_{i+1})d_s(v_{i+1})\right]$$
$$\leq \left[\frac{64}{81} + \frac{64}{81} + \frac{64}{81}(n-3)\right] = \frac{64}{81}(n-1).$$

Example 20:

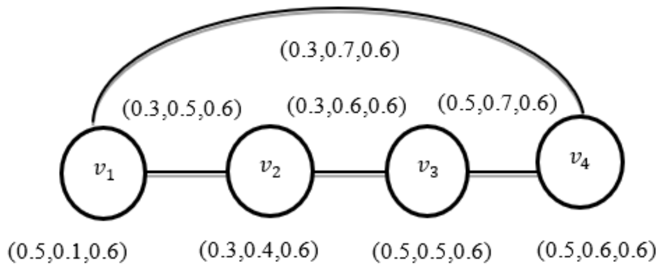

Fig.3. *$SVN_{graph}\check{C}$ on 4 vertices with $ZI^*(\check{C}) = 0$.*

The score function for all vertices is

$\tilde{S}(v_1) = \frac{0.5+0.1+1-0.6}{3} = 0.33, \tilde{S}(v_2) = \frac{0.3+0.4+1-0.6}{3} = 0.37, \tilde{S}(v_3) = \frac{0.5+0.5+1-0.6}{3} = 0.47;$ $\tilde{S}(v_4) = \frac{0.5+0.6+1-0.6}{3} = \frac{1.5}{3} = 0.5, \tilde{S}(v_1, v_2) = \frac{0.3+0.5+1-0.6}{3} = \frac{1.2}{3} = 0.4, \tilde{S}(v_2, v_3) = \frac{0.3+0.6+1-0.6}{3} = \frac{1.3}{3} = 0.43,$ $\tilde{S}(v_3, v_4) = \frac{0.5+0.7+1-0.6}{3} = \frac{1.6}{3} = 0.53, \tilde{S}(v_1, v_4) = \frac{0.3+0.7+1-0.6}{3} = \frac{1.4}{3} = 0.47.$

Now, we have $d_s(v_1) = \tilde{S}(v_1, v_2) + \tilde{S}(v_1, v_4) = 0.4 + .47 = .87$; $d_s(v_2) = \tilde{S}(v_1, v_2) + \tilde{S}(v_2, v_3) = 0.4 + 0.43 = 0.83$; $d_s(v_3) = \tilde{S}(v_2, v_3) + \tilde{S}(v_3, v_4) = 0.43 + 0.53 = 0.96$; $d_s(v_4) = \tilde{S}(v_3, v_4) + \tilde{S}(v_1, v_4) = 0.53 + .47 = 1$.

From definitions [13], we have

$$ZI^*(\check{C}) = \tilde{S}(v_1)d_s(v_1)\tilde{S}(v_2)d_s(v_2) + \tilde{S}(v_2)d_s(v_2)\tilde{S}(v_3)d_s(v_3) + \tilde{S}(v_3)d_s(v_3)\tilde{S}(v_4)d_s(v_4) + \tilde{S}(v_4)d_s(v_4)\tilde{S}(v_1)d_s(v_1)$$
$$= .33 \times .4 \times .37 \times .. 83 + .37 \times .83 \times .96 \times .13 + .13 \times .5 \times .53 \times .96 + .5 \times .33 \times 1 \times .87$$
$$= .255.$$

Theorem 20: Let $\check{S}$ be a star with n vertices. Then $ZI^*(\check{S}) \leq \frac{8}{9}(n-1)$.

Proof. Let $(u, v_1, \cdots, v_{n-1})$ be a vertex set of $\check{S}$. $d_s(u) = \sum_{u_i \epsilon N(u)} \tilde{S}(u_i, u) \leq \frac{2}{3}(n-1)$ and $d_s(v_i) = \sum_{u_i \epsilon N(v_i)} \tilde{S}(u_i, v_i) \leq \frac{2}{3}$, for $i = 2, 3, \cdots, n-1$ and $\tilde{S}(v_i) \leq \frac{2}{3}, \forall i$.

Therefore,

$$ZI^*(\check{C}) = \sum_{(u_i,u_j)\epsilon\mu} \tilde{S}(u_i) d_s(u_i) \tilde{S}(u_j) d_s(u_j) = \left[\tilde{S}(u) \sum_{i=2}^{n-2} d_s(u) \tilde{S}(v_i) d_s(v_i)\right]$$

$$\leq \frac{2}{3}\left[\frac{4}{3}(n-1)\right] = \frac{8}{9}(n-1).$$

3.2 *Connectivity Index*

Definition 18: Let $\tilde{G}: (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph}. Then the strength of score connectedness $SCONN_{\tilde{G}}(u_i, v_j)$ is between two vertices u_i and v_j is defined as $SCONN_{\tilde{G}}(u_i, v_j) = \tilde{S}(u_i, v_j)$.

Definition 19: Let $\tilde{G}: (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph}. The Connectivity index $CI(\tilde{G})$ of a SVN_{graph} is defined as $CI(\tilde{G}) = \sum_{(u_i,v_j)\epsilon\sigma^*} \tilde{S}(u_i)\tilde{S}(v_j) SCONN_{\tilde{G}}(u_i, v_j), \forall i \neq j \text{ and } (v_i, v_j) \in E_G$.

Example 20: Let $\tilde{G}: (V_G, \tilde{\mu}, \tilde{\rho})$ be a SVN_{graph} in Fig.4 with $\sigma^* = [a, b, c, d]$.

The score function for all vertices is given by,

$$\tilde{S}(a) = \frac{\tilde{T}_a + \tilde{I}_a + 1 - \tilde{F}_a}{3} = 0.33;\ \tilde{S}(b) = \frac{\tilde{T}_b + \tilde{I}_b + 1 - \tilde{F}_b}{3} = 0.367;$$

$$\tilde{S}(c) = \frac{\tilde{T}_c + \tilde{I}_c + 1 - \tilde{F}_c}{3} = 0.43;\ \tilde{S}(d) = \frac{\tilde{T}_d + \tilde{I}_d + 1 - \tilde{F}_d}{3} = 0.5.$$

and the edges the score function is given by,

$$\tilde{S}(a,b) = \frac{\tilde{T}_{ab} + \tilde{I}_{ab} + 1 - \tilde{F}_{ab}}{3} = 0.3;\ \tilde{S}(b,d) = \frac{\tilde{T}_{bd} + \tilde{I}_{bd} + 1 - \tilde{F}_{bd}}{3} = 0.33;$$

$$\tilde{S}(d,c) = \frac{\tilde{T}_{dc} + \tilde{I}_{dc} + 1 - \tilde{F}_{dc}}{3} = 0.3;\ \tilde{S}(c,a) = \frac{\tilde{T}_{ca} + \tilde{I}_{ca} + 1 - \tilde{F}_{ca}}{3} = 0.3.$$

Using Eq.(1), the strength of connectedness $(CONN_{\tilde{G}}(u_i, v_j)$ can be determined to be
$SCONN_{\tilde{G}}(a,b) = 0.3;\ SCONN_{\tilde{G}}(a,c) = 0.3;\ SCONN_{\tilde{G}}(a,d) = 0.3;$
$SCONN_{\tilde{G}}(b,c) = 0.3;\ SCONN_{\tilde{G}}(b,d) = 0.33; SCONN_{\tilde{G}}(c,d) = 0.3.$

After computing the connectedness between all pairs of vertices, we found that,
$CI(\tilde{G}) = 0.590$.

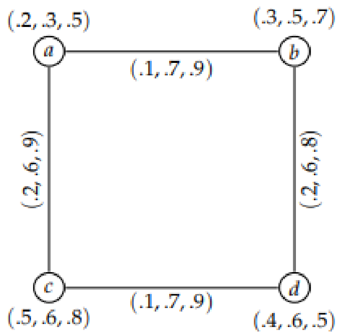

Fig.4. A SVN_{graph} with $CI(\tilde{G}) = 0.590, WI(\tilde{G}) = 0.795, HWI(\tilde{G}) = 0.427$.

3.3 *Wiener Index*

Definition 21: The Wiener index $WI(\tilde{G})$ of a SVN_{graph} is defined as

$$WI(\tilde{G}) = \sum_{(u_i,v_j)\epsilon\sigma^*} \tilde{S}(u_i)\tilde{S}(v_j) d_s(u_i, v_j), \forall i \neq j \text{ and } (v_i, v_j) \in E_G$$

where $d_s(u_i, v_j)$ is the sum of minimum average score functions from u_i to v_j.

Example 22: Consider the $SVN_{graph}\tilde{G}: (V_G, \tilde{\mu}, \tilde{\rho})$ $with$ $V_G = \{a, b, c, d\}, \tilde{S}(a, b) = 0.3, \tilde{S}(b, d) = 0.33, \tilde{S}(d, c) = 0.3, \tilde{S}(c, a)0.3$ (Fig.4).

To find the $d_s(u_i, v_j)$for two vertices $u_i$and v_j Such that the distance between the vertices has the minimum value. Therefore, $d_s(a, b) = 0.3;$ $d_s(a, c) = 0.3;$

$d_s(b, c) = \tilde{S}(b, a) + \tilde{S}(a, c) = 0.3 + 0.3 = 0.6;$ $d_s(b, d) = 0.33;$ $d_s(c, d) = 0.3$

Hence,

$$WI(\tilde{G}) = \sum_{(u_i, v_j)\epsilon\sigma^*} \tilde{S}(u)\tilde{S}(v)d_s(u, v) =$$
$$= \tilde{S}(a)\tilde{S}(b)d_s(a, b) + \tilde{S}(a)\tilde{S}(c)d_s(a, c) + \tilde{S}(a)\tilde{S}(d)d_s(a, d) + \tilde{S}(b)\tilde{S}(a)d_s(b, a) + \tilde{S}(b)\tilde{S}(c)d_s(b, c) + \tilde{S}(b)\tilde{S}(d)d_s(b, d) + \tilde{S}(c)\tilde{S}(a)d_s(c, a) + \tilde{S}(c)\tilde{S}(b)d_s(c, b) + \tilde{S}(c)\tilde{S}(d)d_s(c, d) + \tilde{S}(d)\tilde{S}(a)d_s(d, a) + \tilde{S}(d)\tilde{S}(b)d_s(d, b) + \tilde{S}(d)\tilde{S}(c)d_s(d, c) = 0.795.$$

3.4 Modified Wiener Index

Definition 23: The Modified Wiener index $MWI(\tilde{G})$of a SVN_{graph} is defined as

$$MWI(\tilde{G}) = \frac{1}{2} \sum_{(u_i, v_j)\epsilon\sigma^*} \tilde{S}(u_i)\tilde{S}(v_j)d_s(u_i, v_j), \forall i \neq j \text{ and } (v_i, v_j) \in E_G.$$

Example 24: From Fig.4

$$MWI(\tilde{G}) = \frac{1}{2} \sum_{(u_i, v_j)\epsilon\sigma^*} \tilde{S}(u_i)\tilde{S}(v_j)d_s(u_i, v_j) = \frac{1}{2} \times WI(\tilde{G}) = 0.398.$$

3.5 Hyper Wiener Index

Definition 25: The Hyper Wiener index $HWI(\tilde{G})$of a SVN_{graph} is defined as

$$HWI(\tilde{G}) = \frac{1}{2} \sum_{(u_i, v_j)\epsilon\sigma^*} \left(\tilde{S}(u_i)\tilde{S}(v_j)d_s(u_i, v_j) + \left(\tilde{S}(u_i)\tilde{S}(v_j)d_s(u_i, v_j)\right)^2\right), \forall i \neq j \text{ and } (v_i, v_j) \in E_G.$$

Example 26: From Fig.4

$$HWI(\tilde{G}) = \frac{1}{2}.2[((0.33 \times 0.367 \times 0.3) + (0.33 \times 0.367 \times 0.3)^2) + ((0.33 \times 0.43 \times 0.3) + (0.33 \times 0.43 \times 0.3)^2) + ((0.33 \times 0.5 \times 0.6) + (0.33 \times 0.5 \times 0.6)^2) + ((0.367 \times 0.43 \times 0.6) + (0.367 \times 0.43 \times 0.6)^2) + ((0.367 \times 0.5 \times 0.33) + (0.367 \times 0.5 \times 0.33)^2) + ((0.43 \times 0.5 \times 0.3) + (0.43 \times 0.5 \times 0.3)^2)] = 0.427.$$

3.6 Schultz Index

Definition 27: The Schultz index $SCI(\tilde{G})$of a SVN_{graph} is defined as Schultz index given by

$$SCI(\tilde{G}) = \sum_{u(u_i, v_j)\epsilon\sigma^*} \tilde{S}(u_i)\tilde{S}(v_j)d_s(u_i, v_j)[d_s(u_i) + d_s(v_j)], \forall i \neq j \text{ and } (v_i, v_j) \in E_G.$$

Example 28: Consider the $SVN_{graph}\tilde{G}: (V_{\tilde{G}}, \tilde{\mu}, \tilde{\rho})$ as shown in Figure 5, with the vertex set $V_{\tilde{G}} = \{v_1, v_2, v_3, v_4\}$ such that $(\tilde{T}, \tilde{I}, \tilde{F})(v_1) = (0.5, 0.1, 0.4)$, $(\tilde{T}, \tilde{I}, \tilde{F})(v_2) = (0.6, 0.3, 0.2), (\tilde{T}, \tilde{I}, \tilde{F})(v_3) = (0.2, 0.3, 0.4), and\ (\tilde{T}, \tilde{I}, \tilde{F})(v_4) = (0.4, 0.2, 0.5)$ The edge set contains $(\tilde{T}, \tilde{I}, \tilde{F})(v_1, v_2) = (0.5, 0.4, 0.5)$, $(\tilde{T}, \tilde{I}, \tilde{F})(v_2, v_3) = (0.2, 0.3, 0.4), (\tilde{T}, \tilde{I}, \tilde{F})(v_3, v_4) = (0.2, 0.4, 0.4), and\ (\tilde{T}, \tilde{I}, \tilde{F})(v_4, v_1) = (0.4, 0.3, 0.6)$.

We have,

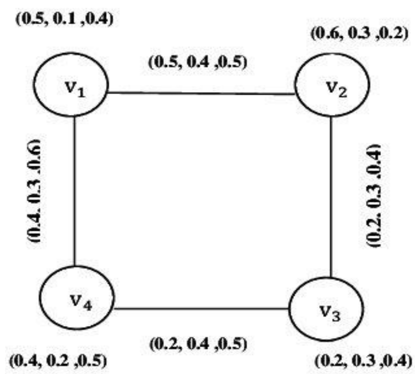

Fig.5.$SVN_{graph}$with $SCI(\tilde{G}) = 4.491$, $GI(\tilde{G}) = 1.764$, $HI(\tilde{G}) = 3.051$.

The score function for all vertices is

$\tilde{S}(v_1) = \frac{0.5+0.1+1-0.4}{3} = \frac{1.2}{3} = 0.4$; $\tilde{S}(v_2) = \frac{0.6+0.3+1-0.2}{3} = \frac{1.7}{3} = 0.567$,$\tilde{S}(v_3) = \frac{0.2+0.3+1-0.4}{3} = \frac{1.1}{3} = 0.367$; $\tilde{S}(v_4) = \frac{0.4+0.2+1-0.5}{3} = \frac{1.1}{3} = 0.367$,$\tilde{S}(v_1, v_2) = \frac{0.5+0.4+1-0.5}{3} = \frac{1.4}{3} = 0.467$; $\tilde{S}(v_2, v_3) = \frac{0.2+0.3+1-0.4}{3} = \frac{1.1}{3} = 0.367$,$\tilde{S}(v_3, v_4) = \frac{0.2+0.4+1-0.5}{3} = \frac{1.1}{3} = 0.367$; $\tilde{S}(v_4, v_1) = \frac{0.4+0.3+1-0.6}{3} = \frac{1.1}{3} = 0.367$.

now, we have,

$d_s(v_1) = \tilde{S}(v_1, v_2) + \tilde{S}(v_4, v_1) = 0.467 + 0.367 = 0.834$;
$d_s(v_2) = \tilde{S}(v_1, v_2) + \tilde{S}(v_2, v_3) = 0.467 + 0.367 = 0.834$;
$d_s(v_3) = \tilde{S}(v_2, v_3) + \tilde{S}(v_3, v_4) = 0.367 + 0.367 = 0.734$;
$d_s(v_4) = \tilde{S}(v_3, v_4) + \tilde{S}(v_4, v_1) = 0.367 + 0.367 = 0.734$;
and $d_s(v_1, v_2) = 1, d_s(v_1, v_3) = 2, d_s(v_1, v_4) = 1, d_s(v_2, v_3) = 1, d_s(v_2, v_4) = 2, d_s(v_3, v_4) = 1$.

$$SCI(\tilde{G}) = 0.4 \times 0.567 \times 1 \times (0.834 + 0.834) + 0.4 \times 0.367 \times 2 \times (0.834 + 0.734) + 0.4 \times 0.367 \times 1(0.834 + 0.734) + 0.4 \times 0.567 \times 1 \times (0.834 + 0.834) + 0.567 \times 0.367 \times 1 \times (0.834 + 0.734) + 0.567 \times 0.367 \times 2 \times (0.834 + 0.734) + 0.4 \times 0.367 \times 2 \times (0.834 + 0.734) + 0.567 \times 0.367 \times 1 \times (0.834 + 0.734) + 0.367 * 0.367 \times 1 \times (0.734 + 0.734) + 0.4 \times 0.367 \times 1 \times (0.834 + 0.734) + 0.567 \times 0.367 \times 2 \times (0.834 + 0.734) + 0.367 \times 0.367 \times 1 \times (0.734 + 0.734)$$

$$SCI(\tilde{G}) = 2 \times [0.3783024 + 0.4603648 + 0.2301824 + 0.32628355 + 0.6525671 + 0.19772345]$$

$$SCI(\tilde{G}) = 2 \times 2.2454237 = 4.491.$$

3.7 *Gutman Index*

Definition 29: The Gutman index $GI(\tilde{G})$of a SVN_{graph} is defined as

$GI(\tilde{G}) = \sum_{(u_i, v_j) \epsilon \sigma^*} \tilde{S}(u_i)\tilde{S}(v_j) d_s(u_i, v_j) d_s(u_i) d_s(v_j), \forall i \neq j \text{ and } (v_i, v_j) \in E_G$.

Example 30: Consider the graph in Figure 5,

$$GI(\tilde{G}) = \sum_{u,v \epsilon \sigma^*} \tilde{S}(u_i)\tilde{S}(v_j) d_s(u_i, v_j) d_s(u_i) d_s(v_j),$$

$$= 0.4 \times 0.567 \times 1 \times 0.834 \times 0.834 + 0.4 \times 0.367 \times 2 \times 0.834 \times 0.734 + 0.4 \times 0.367 \times 1 \times 0.834 \times 0.734 + 0.4 \times 0.567 \times 1 \times 0.834 \times 0.834 + 0.567 \times 0.367 \times 1 \times 0.834 \times 0.734 + 0.567 \times 0.367 \times 2 \times 0.834 \times 0.734 + 0.4 \times 0.367 \times 2 \times 0.834 \times 0.734 + 0.567 \times 0.367 \times 1 \times 0.834 \times 0.734 + 0.367 \times 0.367 \times 1 \times 0.734 \times 0.734 + 0.4 \times 0.367 \times 1 \times 0.834 \times 0.734 + 0.567 \times 0.367 \times 2 \times 0.834 \times 0.734 + 0.367 \times 0.367 \times 1 \times 0.734 \times 0.734$$

$$= 2 \times [0.1577521 + 0.179729 + 0.0898645 + 0.12738293 + 0.25476586 + 0.07256451 = 1.764.$$

3.8 *Planarity Index*

Definition 31: The Planarity index $PI(\tilde{G})$of a SVN_{graph} is defined as

$$PI(\tilde{G}) = min\left\{\frac{1}{2}\sum_{\tilde{G}-xy} \tilde{S}(u_i)\tilde{S}(v_j)d_s(u_i,v_j)\right\}.$$

Example 32: For the SVN_{graph} in Fig. 6, $\tilde{S}(v_1) = 0.6$, $\tilde{S}(v_2) = 0.5$, $\tilde{S}(v_3) = 0.433, \tilde{S}(v_4) = 0.567$; $\tilde{S}(v_1,v_2) = 0.533, \tilde{S}(v_2,v_3) = 0.467, \tilde{S}(v_3,v_4) = 0.433, \tilde{S}(v_4,v_1) = 0.6, \tilde{S}(v_1,v_3) = .5, \tilde{S}(v_2,v_4) = 0.5$; $d_s(v_1,v_2) = 0.533, d_s(v_1,v_3) = 1, d_s(v_1,v_4) = 0.6, d_s(v_2,v_3) = 0.467, d_s(v_2,v_4) = 0.5, d_s(v_3,v_4) = 0.433$ ($when\ v_1v_3 is\ deleted$); $d_s(v_1,v_2) = 0.533, d_s(v_1,v_3) = 0.5$, $d_s(v_1,v_4) = 0.6, d_s(v_2,v_3) = 0.467, d_s(v_2,v_4) = 0.9$, $d_s(v_3,v_4) = 0.433$ ($when\ v_2v_4\ is\ deleted$)and$PI(\tilde{G}) = min\{0.9729, 0.95648\}$ = 0.956.

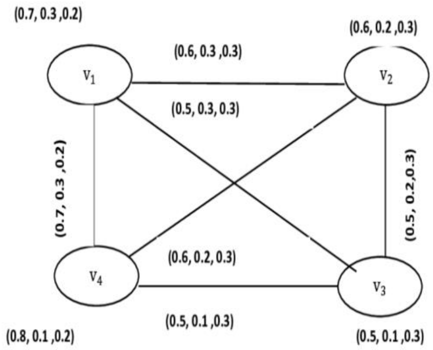

Fig.6.SVN_{graph} with $PI(\tilde{G}) = 0.956$, $RI(\tilde{G}) = 6.184$.

3.9 Randic Index

Definition 33: Let $\tilde{G}: (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph}.The Randic index $RI(\tilde{G})$of a SVN_{graph} is defined as

$$RI(\tilde{G}) = \frac{1}{2}\sum_{(u_i,u_j)\epsilon\mu} \left[\tilde{S}(u_i)\tilde{S}(u_j)d_s(u_i)d_s(u_j)\right]^{\frac{-1}{2}} ,\forall i \neq j.$$

Example 34: Consider the SVNG as shown in Figure 6, by simple calculations, we get

$$RI(\tilde{G}) = \frac{1}{2}\Big\{[\tilde{S}(v_1)\tilde{S}(v_2)d_s(v_1)d_s(v_2)]^{\frac{-1}{2}} + [\tilde{S}(v_2)\tilde{S}(v_3)d_s(v_2)d_s(v_3)]^{\frac{-1}{2}} + [\tilde{S}(v_3)\tilde{S}(v_4)d_s(v_3)d_s(v_4)]^{\frac{-1}{2}} + [\tilde{S}(v_4)\tilde{S}(v_1)d_s(v_4)d_s(v_1)]^{\frac{-1}{2}}\Big\}$$

$$= \frac{1}{2}\{2.51774893 + 2.80184681 + 3.7122555 + 3.33584545\} = 6.184.$$

3.10Harmonic Index

Definition 35: Let $\tilde{G}: (\tilde{\mu}, \tilde{\rho})$ be a SVN_{graph}.The Harmonic index $HI(\tilde{G})$of a SVN_{graph} is defined as

$$HI(\tilde{G}) = \frac{1}{2}\sum_{(u_i,u_j)\epsilon\mu} \left[\tilde{S}(u_i)d_s(u_i) + \tilde{S}(u_j)d_s(u_j)\right]^{-1} ,\forall i \neq j.$$

Example 36: Consider the SVN_{graph} as shown in Figure 6.

$$HI(\tilde{G}) = \frac{1}{2}\Big\{\left[\tilde{S}(v_1)d_s(v_1) + \tilde{S}(v_2)d_s(v_2)\right]^{-1} + \left[\tilde{S}(v_2)d_s(v_2) + \tilde{S}(v_3)d_s(v_3)\right]^{-1} + \left[\tilde{S}(v_3)d_s(v_3) + \tilde{S}(v_4)d_s(v_4)\right]^{-1} + \left[\tilde{S}(v_4)d_s(v_4) + \tilde{S}(v_1)d_s(v_1)\right]^{-1}\Big\}$$

$$= \frac{1}{2}(1.23995943 + 1.34724408 + 1.85612782 + 1.6584353) = 3.051.$$

4 Identification of influential factors affecting students' performance in semester examinations in the educational institution using score topological indices

The conduct of a semester examination in an education institute depends on many factors, out of which few factors affect the students' performance either directly or indirectly in the examination. These factors are classified into the following four categories.

Category 1: Examination hall related issues: (Temperature, Light disturbing sounds both inside and outside the examination hall) Environment of exam Hall (H_1), Attitude of the invigilator (H_2), Seating Plan (H_3).

Category 2: Question paper-related issues: Change in question pattern (Q_1), Similarities in question paper (Q_2), Ambiguity in questions of question paper (Q_3), Mood of paper corrector (Q_4), Difficulty levels in question paper (Q_5), Extra and detailed study for examination (Q_6), Selection of questions in the choice-based questions (Q_7).

Category 3: Personal issues of a student: Shortage of attendance (P_1), Domestic problems (P_2), financial condition (P_3), Attention and interest of parents (P_4).

Category 4: Exam Preparation related issues: Preparation without determination of objectives (E_1), Overconfidence (E_2), Methods for approaching the questions (E_3), Handwriting (E_4), Examination fever (E_5).

The questionnaire was prepared based on the above main four factors and used as a tool for this research. The questionnaire was circulated among more than 50 engineering colleges in the city, around 250 students, and 50 controllers of examination in various institutions and universities in and around Chennai city. The main objective of the study is to determine influential factors in each of the above-mentioned four classes that affect the student's performance in the examination. Based on the collected questionnaire data and discussion with the CoEs from different institutions in the city, we draw the relation SVN_{graph} of the most influential factors affecting students' performance in semester examinations with neutrosophic values, represented in Figures 8-11. Also, vertex and edge-score values, and the score degree of the vertex are calculated for different neutrosophic values, illustrated in Tables 1-4.

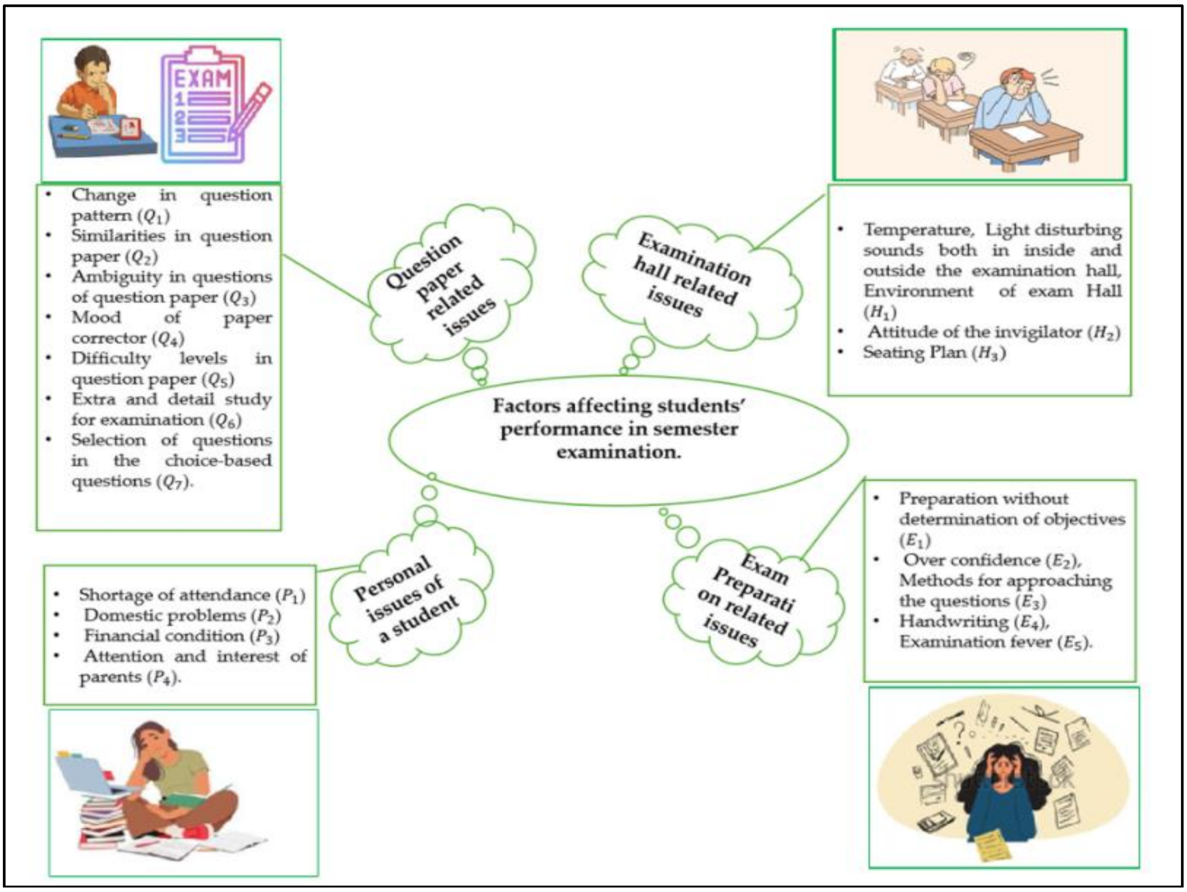

Fig.7. Influential factors affecting student performance

4.1 Analysis of Category 1

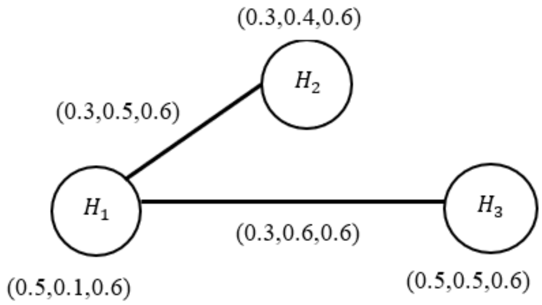

Fig.8.SVN_{graph} −Examination Hall related issues

Table 1. Calculated values of vertex and edge --score, and score degree of vertex for category 1

Vertex-score values	Edge-score values	Score degree of the vertex
$\tilde{S}(H_1) = \frac{0.5+0.1+1-0.6}{3} = 0.33$	$\tilde{S}(H_1,H_2) = \frac{0.3+0.5+1-0.6}{3} = 0.27$	$d_s(H_1) = \tilde{S}(H_1,H_2) + \tilde{S}(H_1,H_3) = 0.27 + 0.43 = 0.70$
$\tilde{S}(H_2) = \frac{0.3+0.4+1-0.6}{3} = 0.37$	$\tilde{S}(H_1,H_3) = \frac{0.3+0.6+1-0.6}{3} = 0.43$	$d_s(H_2) = \tilde{S}(H_1,H_2) = 0.27$
$\tilde{S}(H_3) = \frac{0.5+0.5+1-0.6}{3} = 0.47$		$d_s(H_3) = \tilde{S}(H_1,H_3) = 0.43$

The Zagreb's indices of $\tilde{G}$ On first and second kinds are $ZI(\tilde{G}) = \tilde{S}(H_i)d_s^2(H_i), H_i \in V_G$ *where* $i = \{1, 2, 3\}$

$ZI(\tilde{G}) = 0.33(0.27^2 + 0.43^2) + 0.37(0.27^2) + 0.47(0.43^2)$

$= 0.33(0.2578) + 0.37(0.0729) + 0.47(0.0729)$

$= 0.0367.$

4.2 *Analysis of Category 2*

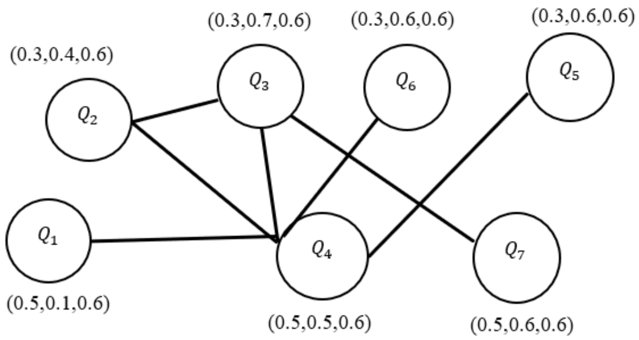

Fig.9.SVN_{graph} −Question paper-related issues

Table 2. Calculated values of vertex and edge-score, and score degree of the vertex for category 2

Vertex-score values	Edge-score values	Score degree of the vertex
$\tilde{S}(Q_1) = \frac{0.5+0.1+1-0.4}{3} = 0.4$	$\tilde{S}(Q_1,Q_4) = \frac{0.5+0.5+1-0.6}{3} = 0.4$	$d_s(Q_1) = \tilde{S}(Q_1,Q_4) = 0.4$
$\tilde{S}(Q_2) = \frac{0.3+0.4+1-0.6}{3} = 0.37$	$\tilde{S}(Q_2,Q_3) = \frac{0.3+0.7+1-0.6}{3} = 0.47$	$d_s(Q_2) = \tilde{S}(Q_2,Q_3) + \tilde{S}(Q_2,Q_4) = 0.47 + 0.47 = 0.94$
$\tilde{S}(Q_3) = \frac{0.5+0.7+1-0.6}{3} = 0.533$	$\tilde{S}(Q_2,Q_4) = \frac{0.3+0.7+1-0.6}{3} = 0.47$	$d_s(Q_3) = \tilde{S}(Q_3,Q_4) + \tilde{S}(Q_3,Q_7) = 0.47 + 0.47 = 1.03$
$\tilde{S}(Q_4) = \frac{0.5+0.6+1-0.6}{3} = 0.5$	$\tilde{S}(Q_3,Q_4) = \frac{0.3+0.7+1-0.6}{3} = 0.47$ $\tilde{S}(Q_3,Q_7) = \frac{0.3+0.7+1-0.6}{3} = 0.47$	$d_s(Q_4) = \tilde{S}(Q_1,Q_4) + \tilde{S}(Q_2,Q_4) + \tilde{S}(Q_3,Q_4) + \tilde{S}(Q_4,Q_5) + \tilde{S}(Q_4,Q_6)$

	$\tilde{S}(Q_4,Q_5)=\frac{0.3+0.7+1-0.6}{3}=0.47$	$=0.4+0.47+0.47+0.47+0.47$
$\tilde{S}(Q_5)=\frac{0.5+0.1+1-0.4}{3}=0.4$		$d_s(Q_5)=\tilde{S}(Q_4,Q_5)=0.47$
$\tilde{S}(Q_6)=\frac{0.5+0.1+1-0.4}{3}=0.4$	$\tilde{S}(Q_4,Q_6)=\frac{0.3+0.7+1-0.6}{3}$	$d_s(Q_6)=\tilde{S}(Q_4,Q_6)=0.47$
$\tilde{S}(Q_7)=\frac{0.5+0.1+1-0.4}{3}=0.4$	$=0.47$	$d_s(Q_7)=\tilde{S}(Q_3,Q_7)=0.47$

The Zagreb's indices of $\tilde{G}$ On first and second kinds are $ZI(\tilde{G})=\sum_{Q_i\epsilon\sigma^*}\tilde{S}(Q_i)d_s^2(Q_i), Q_i\in V_G$ *where* $i=\{1,2,3,4,5,6,7\}$.

$$ZI(\tilde{G})=0.4(0.4^2)+0.37(0.47^2+0.47^2)+0.533(0.47^2+0.47^2)$$
$$+0.5(0.4^2+0.47^2+0.47^2+0.47^2+0.47^2)+0.4(0.47^2)+0.4(0.47^2)+0.4(0.47^2)$$
$$=0.4(0.16)+0.37(0.2697)+0.533(0.2697)+0.5(1.0436)+3\ X\ 0.4(0.2209)$$
$$=0.3274.$$

4.3 Analysis of Category 3

The first and second kinds of Zagreb's indices for Fig.10 is

$$ZI(\tilde{G})=0.4(0.4^2+0.47^2)+0.37(0.4^2+0.47^2+0.5^2)+0.533(0.53^2+0.47^2+0.47^2)$$
$$+0.5(0.5^2+0.53^2)$$
$$=0.4(0.3809)+0.567(0.6309)+0.367(0.7227)+0.367(0.5309)$$
$$=0.5098.$$

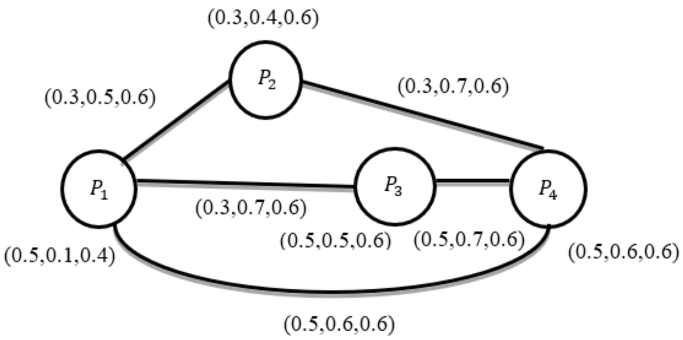

Fig.10.SVN_{graph} –Personal issues of a student

Table 3. Calculated values of vertex and edge-score, and score degree of vertex for category 3

Vertex-score values	Edge-score values	Score degree of the vertex
$\tilde{S}(P_1)=\frac{0.5+0.1+1-0.4}{3}=0.4$	$\tilde{S}(P_1,P_2)=\frac{0.3+0.5+1-0.6}{3}=0.4$	$d_s(P_1)=\tilde{S}(P_1,P_2)+\tilde{S}(P_1,P_3)=0.4+0.47=0.87$
$\tilde{S}(P_2)=\frac{0.3+0.4+1-0.6}{3}=0.37$	$\tilde{S}(P_1,P_3)=\frac{0.3+0.7+1-0.6}{3}=0.47$	$d_s(P_2)=\tilde{S}(P_1,P_2)+\tilde{S}(P_2,P_3)+\tilde{S}(P_2,P_4)=0.4+0.47+.5=1.37$
$\tilde{S}(P_3)=\frac{0.5+0.7+1-0.6}{3}=0.533$	$\tilde{S}(P_2,P_3)=\frac{0.3+0.7+1-0.6}{3}=0.47$	$d_s(P_3)=\tilde{S}(P_3,P_4)+\tilde{S}(P_2,P_3)+\tilde{S}(P_1,P_3)=0.53+0.47+0.47=1.47$
$\tilde{S}(P_4)=\frac{0.5+0.6+1-0.6}{3}=0.5$	$\tilde{S}(P_2,P_4)=\frac{0.5+0.6+1-0.6}{3}=0.5$ $\tilde{S}(P_3,P_4)=\frac{0.5+0.7+1-0.6}{3}=0.53$	$d_s(P_4)=\tilde{S}(P_3,P_4)+\tilde{S}(P_2,P_4)=0.5+0.53=1.03$

4.4 Analysis of Category 4

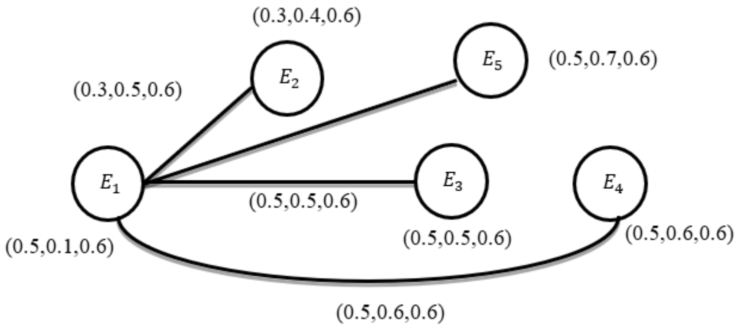

Fig.11.SVN_{graph} −Examination Hall related issues

Table 4. Calculated values of vertex and edge-score, and score degree of vertex for category 4

Vertex-score values	Edge-score values	Score degree of the vertex
$\tilde{S}(E_1) = \frac{0.5+0.1+1-0.6}{3} = 0.33$	$\tilde{S}(E_1,E_2) = \frac{0.3+0.5+1-0.6}{3} = 0.4$	$d_s(E_1) = \tilde{S}(E_1,E_2) + \tilde{S}(E_1,E_3) + \tilde{S}(E_1,E_4) + \tilde{S}(E_1,E_5) = 0.4 + 0.47 + 0.47 + .5 = 1.84$
$\tilde{S}(E_2) = \frac{0.3+0.4+1-0.6}{3} = 0.37$	$\tilde{S}(E_1,E_5) = \frac{0.3+0.7+1-0.6}{3} = 0.47$	$d_s(E_2) = \tilde{S}(E_1,E_2) = 0.4$
$\tilde{S}(E_3) = \frac{0.5+0.5+1-0.6}{3} = 0.47$	$\tilde{S}(E_1,E_3) = \frac{0.5+0.5+1-0.6}{3} = 0.47$	$d_s(E_3) = \tilde{S}(E_1,E_3) = 0.47$
$\tilde{S}(E_4) = \frac{0.5+0.6+1-0.6}{3} = 0.5$	$\tilde{S}(E_1,E_4) = \frac{0.5+0.6+1-0.6}{3} = 0.5$	$d_s(E_4) = \tilde{S}(E_1,E_4) = 0.5$
$\tilde{S}(E_5) = \frac{0.5+0.7+1-0.6}{3} = 0.53$		$d_s(E_5) = \tilde{S}(E_1,E_5) = .47$

The Zagreb's indices of Figure 11 are calculated as follows:

$ZI(\tilde{G}) = 0.4(0.4^2 + 0.47^2 + 0.47^2 + 0.5^2) + 0.37(0.4^2) + 0.47(0.47^2) + 0.5(0.47^2) + .53(.47^2) +$
$= 0.4(0.8518) + 0.37(0.16) + 0.47(0.2209) + 0.5(0.2209) + 0.53(0.2209)$
$= 0.1478.$

5 Conclusion

We concluded that the personal issues of a student play an important role in their performance in the examination even though the change in question paper pattern or other hall environment issues or evaluation process is based on the values of the first Zagreb index of all the above four categories. The other factors affect the student's performance to a negligible level compared with individual student's issues.

References:

1. Zadeh, L.A. Fuzzy sets. Inf. Control, 8, 338–353, 1965.
2. Kaufmann, A, Introduction to the Theory of Fuzzy Subsets; Academic Press: Cambridge, MA, USA, 2.
3. Rosenfeld, A, Fuzzy graphs. In Fuzzy Sets and Their Applications to Cognitive and Decision Processes; Elsevier: Amsterdam, The Netherlands, 77–95,1975.
4. Atanassov, K.T. Intuitionistic fuzzy sets. Fuzzy Sets Syst., 20, 87–96,1986.
5. Atanassov, K.T. Intuitionistic fuzzy sets. In Intuitionistic Fuzzy Sets; Springer: Berlin, Germany, 1–137,1999.
6. K. Atanassov and A. Shannon,On a generalisation of Intuitionistic fuzzy graph,Notes on Intuitionistic fuzzy Sets, 8, 73-78, 2002.

7. Smarandache, F. A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics; Infinite Study; Modern Science Publisher: New York, NY, USA, 2005.
8. F. Smarandache," Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies," Neutrosophic Sets and Systems,9, 58 - 63, 2015.
9. F. Smarandache, "Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology," seminar, Universitatea Transilvania din Brasov, Facultatea de Design de Produs si Mediu, Brasov, Romania. 06 June 2015.
10. S. Broumi, M. Talea, A. Bakali, F. Smarandache, "Single Valued Neutrosophic Graphs," Journal of New Theory, N 10, 86-101, 2016.
11. Broumi, S., Talea, M., Smarandache, F. and Bakali, A. Single Valued Neutrosophic Graphs: Degree, Order and Size, IEEE International Conference on Fuzzy Systems, 11, 2016,84-102.
12. Mitchell, H. B., Ranking intuitionistic fuzzy numbers, Int. J. Uncertainty, Fuzziness Knowledge Based Syst., 12(3), 377–386, 2007.
13. Eliasi M, Raeisi G and Taeri B, Wiener index of some graph operations Discrete. Appl. Math. 160 1333–44,2012.
14. Ghorbani M and Hosseinzadeh M A, A new version of Zagreb indices Filomat, 26, 93–100, 2012.
15. Idrees N, Saif M J, Rauf A and Mustafa S, First and second Zagreb eccentricity indices of thorny graphs Symmetry (Basel), 9, 1–9,2017.
16. Li X and Shi Y, A survey on the Randic index MATCH Commun. Math. Comput. Chem, 59, 127– 56,2008.
17. Xing R, Zhou B and Dong F, On atom-bond connectivity index of connected graphs, Discrete Appl. Math. ,2011.
18. Zhong L, The harmonic index for graphs Appl. Math. Lett., 25 561–6,2012.
19. Lučić B, Trinajstić N and Zhou B 2009 Comparison between the sum-connectivity index and productconnectivity index for benzenoid hydrocarbons Chem. Phys. Lett. 475 146–8
20. Zhou B and Trinajstić N 2010 On general sum-connectivity index J. Math. Chem. 47 210–8
21. HUA H, Wiener and Schultz molecular topological indices of graphs with specified cut edges MATCH Commun. Math. Comput. Chem. 61,643, 2009.
22. Andova V, Dimitrov D, Fink J and Škrekovski R, Bounds on Gutman index MatchCommunications Math. Comput. Chem. 67, 515, 2012.
23. Shriram Kalathian, Sujatha Ramalingam, Sundareswaran Raman and Narasimman Srinivasan, Some topological indices in fuzzy graphs, Journal of Intelligent & Fuzzy Systems, 39(5), 6033-6046, 2020.
24. Masoud Ghods, Zahra Rostami, Introduction to Topological Indices in Neutrosophic Graphs, Neutrosophic Sets and Systems, 35, 2020.
25. P.T. Chang, E.S. Lee, Fuzzy decision networks and deconvolution Comput. Math. Appl., 37, 53-63, 1999.
26. H. Lee-Kwang, J.-H. Lee, A method for ranking fuzzy numbers and its application to decision-making
27. IEEE Trans. Fuzzy Sets Syst., 7, 677-685, 1999.
28. R. Jain, Decision-making in the presence of fuzzy variables IEEE Trans. Syst. Man Cybern., 6 , 698-703, 1976,
29. Mitchell, H.B., Ranking - Intuitionistic Fuzzy numbers. Int. J. Uncert. Fuzz. Knowl. Based Syst. 12, 377–386, 2004.
30. Nayagam, V. L. G., Venkateshwari, G., and Sivaraman, G., Ranking of intuitionistic fuzzy numbers, In Proc. of the IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE'08), 1971–1974, 2008.
31. Nayagam, V.L.G., Venkateshwari, G., Sivaraman, G., Modified ranking of intuitionistic fuzzy numbers. Notes on Intuitionistic Fuzzy Sets 17, 5–22, 2011.

32. Li, D.F., Nan J.X., Zhang M.J., A ranking Method of triangular intuitionistic fuzzy numbers and applications to decision making, international Journal of Computational Intelligence Systems 3, 522–530, 2010.
33. Sahin, R., Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment, International Journal for Uncertainty Quantification, 6 (5), 377–385, 2016.
34. Przemysław Grzegorzewski, Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric, Fuzzy Sets and Systems, 148(2), 319-328, 2004.
35. Nancy and Harish Garg, an improved score function for ranking neutrosophic sets and its application to decision-making process,International Journal for Uncertainty Quantification, 6 (5), 377–385, 2016.
36. Saima Rasul, Qadir Bukhsh, A study of factors affecting students' performance in examination at university level, Procedia Social and Behavioral Sciences, 15, 2042–2047, 2011.
37. Kassu Mehari Beyene Jemal Ayalew Yimam,Multilevel Analysis for Identifying Factors Influencing Academic Achievement of Students in Higher Education Institution: The Case of Wollo University, Journal of Education and Practice, 7(13), 2016.
38. Yousuf Nasser Said Al Husaini,Nur Syufiza Ahmad Shukor, Factors Affecting Students' Academic Performance: A review,Published/ publié in Res Militaris (resmilitaris.net), 12(6), Winter 2022.
39. Kawtar Tani, Elizabeth Dalzell, Nathan Ehambaranathan, Sheela Murugasu and Anne Steele,Evaluation of factors affecting students' performance in tertiary education,Journal of Pedagogical Research ,3 (2), 2019.
40. Wei Liu and Lei Zhang, Performance Evaluation and Identification of Key Influencing Factors for Student Achievement Based on the Entropy-Weighted TOPSIS Model, Computational Intelligence and Neuroscience Volume 2023, Article ID 1253824, 12 pages.

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