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A Neutrosophic Solution of Heat Equation by Neutrosophic Laplace Transform

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Abstract. This article aims to study the one-dimensional neutrosophic heat equation. In this study, we discuss the one-dimensional heat equation using neutrosophic numbers and provide numerical example to demonstrate the effectiveness of the neutrosophic Laplace transform method.

Keywords: neutrosophic real number; neutrosophic differential equation; neutrosophic laplace transformation; one-dimensional geometric AH-Isometry.

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1. Introduction

Partial differential equations (PDEs) are instrumental in modeling a wide array of phenomena across physical, biological, and social sciences. They are particularly useful for describing dynamic systems, such as heat conduction, where classical models often encounter uncertainties in variables and parameters, such as initial and boundary conditions or material properties. Neutrosophic differential equations (NDEs), which extend classical differential equations by incorporating neutrosophic numbers to address these uncertainties, offer a more robust framework for handling such imprecise or vague conditions. Neutrosophic set theory, introduced by Smarandache [1] as an extension of fuzzy sets invented by Zadeh (1965), provides a valuable tool for managing uncertainty in mathematical models, leading to more accurate and stable solutions. For example, in modeling heat diffusion, the neutrosophic heat equation accounts for the vagueness inherent in real-world conditions, such as varying ambient temperatures and material impurities. While exact analytical solutions for neutrosophic heat equations can be challenging to derive, numerical methods are employed to obtain practical solutions. The application of neutrosophic differential equations spans various fields, including engineering and medicine, demonstrating their versatility in modeling dynamic systems with inherent uncertainties. As such, neutrosophic calculus enriches the theory of differential equations, offering a more comprehensive approach than interval computations for addressing real-world complexities.

Smarandache proposed neutrosophic logic to represent a mathematical model of uncertainty, inaccuracy, ambiguity, imprecision, vagueness, unknown, incompleteness, inconsistency, redundancy, and contradiction, the concept of neutrosophy being a new branch of philosophy introduced by Smarandache (1998) [2–14]. Also, he introduced the meaning of the standard form of a neutrosophic real number and the circumstances for the division of two neutrosophic real numbers, characterized the standard form of a complex neutrosophic number, and found the root index $n \geq 2$ of a neutrosophic real and complex number (Smarandache, 1998 - 2014) [2–5]. While studying the concept of neutrosophic probability and neutrosophic statistics, professor Smarandache (1998 - 2015) [2–9] entered the concept of provisional calculus, where he first introduced the concepts of the neutrosophic mereo limit, neutrosophic mereo continuity, neutrosophic mereo derivative and neutrosophic mereo integral. Edalatpanah proposed a new simple algorithm for solving linear neutrosophic programming, in which the variables and the Right hand si de represent the Triangular neutrosophic numbers [12]. Mondal et al. (2021b) described the application of the neutrosophic differential equation on mine safety via a single-valued neutrosophic number. Sumathi et al. (2019) discussed the differential equation in a neutrosophic environment and the solution of a second-order linear differential equation with trapezoidal neutrosophic numbers as boundary conditions. Acharya et al. (2023a) used the differential equations in a neutrosophic environment under Hukuhara differentiability to model the amount of glucose distribution and absorption rates in blood. Lathamaheswari et al. (2022c) solved the neutrosophic differential equation by using bipolar trapezoidal neutrosophic number and applied this concept in predicting bacterial reproduction over separate bodies. Parikh et al. (2022d) describe the solution of a first-order linear non-homogeneous fuzzy differential equation with initial conditions in a neutrosophic environment. He also introduced the neutrosophic analytical method and the fourth-order Runge-Kutta numerical method by using triangular neutrosophic numbers.

Recently Alhasan (2021a - 2022b) [15–17] discussed some basics of differential and integral methods based on neutrosophic real numbers. Salamah et al. (2023b) used the concepts of continuity, differentiability, and integrability from real analysis to study the derivative and integration of a neutrosophic real function with one variable depending on the geometry isometry (AH-Isometry), also they studied the neutrosophic differential equation by using one-dimensional geometry AH-Isometry (Salamah et al., 2023c) [18], where they discussed the methods of finding the solution of neutrosophic identical linear differential equation and neutrosophic non-homogeneous linear differential equation. Moreover several researches have made multiple contribution to neutrosophic topology [19–25] and neutrosophic analysis.

The structure of this paper is organized as follow: In the first section, we provides a scientific overview of neutrosophists. The second section covers some basic concepts which will be used in this work. The third section, defined the fuzzy heat equation, its solution by a neutrosophic laplace transform method and an example is illustrated. Lastly, in the fourth section, the conclusion of this article is drawn.

2. Preliminaries

2.1. Definition: Neutrosophic Real Number. [3]

Let η be a neutrosophic real number, then it takes the standard form $\eta = \eta_1 + \eta_2 I$, where $\eta_1, \eta_2 \in R$, and I represent indeterminacy, such that $I.0 = 0$ and $I^n = I \quad \forall n \in Z^+$.

2.2. Definition: Division of two neutrosophic real numbers. [3]

If η and γ are two neutrosophic real numbers where, $\eta = \eta_1 + \eta_2 I, \quad \gamma = \gamma_1 + \gamma_2 I$.

Then, $\dfrac{\eta_1 + \eta_2 I}{\gamma_1 + \gamma_2 I} = \dfrac{\eta_1}{\gamma_1} + \dfrac{\gamma_1\eta_2 - \eta_1\gamma_2}{\gamma_1(\gamma_1 + \gamma_2)}.I$, provided, $\gamma_1 \neq 0$ and $\gamma_1 \neq -\gamma_2$.

2.3. Definition: [18]

Let $R(I) = \{\eta_1 + \eta_2 I; \eta_1, \eta_2 \in R\}$ be the neutrosophic field of reals. Then the one-dimensional isometry (AH-Isometry) is defined as follows

$$\mathcal{T} : R(I) \to R \times R$$
$$\mathcal{T}(\eta_1 + \eta_2 I) = (\eta_1, \eta_1 + \eta_2)$$

Remark: $\mathcal{T}$ is an algebraic isomorphism between two rings, it has the following properties:

(1) $\mathcal{T}$ is bijective.

(2) $\mathcal{T}$ preserves addition and multiplication, i.e.

$$\mathcal{T}[(\eta_1 + \eta_2 I) + (\gamma_1 + \gamma_2 I)] = \mathcal{T}(\eta_1 + \eta_2 I) + \mathcal{T}(\gamma_1 + \gamma_2 I)$$
$$\mathcal{T}[(\eta_1 + \eta_2 I).(\gamma_1 + \gamma_2 I)] = \mathcal{T}(\eta_1 + \eta_2 I).T(\gamma_1 + \gamma_2 I)$$

(3) $\mathcal{T}$ is invertible, i.e.

$$\mathcal{T}^{-1} : R \times R \to R(I)$$
$$\mathcal{T}^{-1}(\eta_1, \eta_2) = \eta_1 + (\eta_2 - \eta_1)I$$

(4) $\mathcal{T}$ preserves distances, i.e.

If $A = \eta_1 + \eta_2 I, \quad \gamma_1 + \gamma_2 I$ are two neutrosophic real numbers, then

$$\mathcal{T}(||\vec{AB}||) = ||\mathcal{T}(\vec{AB})||$$

2.4. **Definition: Neutrosophic Real Function [18]**

Let $f : R(I) \rightarrow R(I)$; where $f = f(x)$ and $x = x_1 + x_2 I \in R(I)$, then f is said to be a neutrosophic real function having one neutrosophic variable. A nuetrosophic real function $f(x)$ is written as

$$f(x) = f(x_1 + x_2 I) = f(x_1) + I[f(x_1 + x_2) - f(x_1)]$$

2.5. **Definition: Neutrosophic Laplace Transformation. [18]**

Let $f(x) = f(x_1 + x_2 I) = f(x_1) + [f(x_1 + x_2) - f(x_1)]I$ be a neutrosophic function on $R(I)$, where $x = x_1 + x_2 I$, then Neutrosophic Laplace transformation of $f(X)$ is defined as

$$L\{f(x), s\} = L\{f(x)\} = F(s) = \int_0^{-\infty} e^{-sx} f(x) dx \tag{1}$$

Where $s = s_1 + s_2 I \in R(I)$ and L is the Laplace transformation operator.
Equation (1) can be written as-

$$\Rightarrow F(s_1 + s_2 I) = \int_0^{-\infty} e^{-s_1 x_1} f(x_1) d(x_1) + \Big[\int_0^{-\infty} e^{-(s_1+s_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2) - \int_0^{-\infty} e^{-s_1 x_1} f(x_1) d(x_1)\Big] I \tag{2}$$

This is the Neutrosophic Laplace transformation of $f(x)$ or $f(x_1 + x_2 I)$

Method of finding solution:

(1) Taking AH-Isometry on both sides of the equation (2), we get

$F(s_1) = \int_0^{-\infty} e^{-s_1 x_1} f(x_1) d(x_1)$

$\Rightarrow F(s_1 + s_2) = \int_0^{-\infty} e^{-(s_1+s_2)(x_1+x_2)} f(x_1 + x_2) d(x_1 + x_2)$

This $F(s_1)$ and $F(s_1 + s_2)$ are two classical Laplace transformation.

(2) We find $F(s_1)$ and $F(s_1 + s_2)$.

(3) Taking invertible AH-Isometry, then we get the required Neutrosophic Laplace transformation.

$T^{-1}\big(F(s_1), F(s_1 + s_2)\big) = F(s_1) + \big(F(s_1 + s_2) - F(s_1)\big)I = F(s_1 + s_2 I).$

2.6. **Definition: Neutrosophic Inverse Laplace Transformation.**

If $F(s)$ be the Laplace transform of a function $f(x)$, then $f(x)$ is called the Neutrosophic Laplace Transform of the function $F(s)$ and is written as $f(x) = L^{-1}[F(s)]$, where L^{-1} is the Inverse Laplace Transformation operator and $x = x_1 + x_2 I \in R(I), \quad s = s_1 + s_2 I \in R(I)$.

3. Neutrosophic Heat Equation

The field of partial differential equations holds a unique and fascinating place in mathematics. Problems that involve time 'T' as an independent variable generally lead to parabolic or hyperbolic equations. The simplest parabolic equation, $\frac{\partial U}{\partial T} = k\frac{\partial^2 U}{\partial X^2}$, arises from the theory of heat conduction. Its solution provides, for instance, the temperature 'U' at a distance 'X' units from one end of a thermally insulated bar after 'T' seconds of heat conduction. In reality, information about dynamical systems modeled by partial differential equations, especially in heat equations, is often incomplete or unclear. This uncertainty may manifest in different parts of heat equations, such as initial conditions, boundary conditions, etc. Therefore, we solve the heat equations based on neutrosophic conditions to obtain more accurate results than those based on real conditions.

We now assume that the temperature $U(X,T)$ of bar of constant cross-section and homogeneous material, lying along the axis and completely insulated laterally may be modeled by the one-dimensional heat equation

$$\frac{\partial U}{\partial T} = k\frac{\partial^2 U}{\partial X^2}$$

where k is the diffusivity of the material of the bar.
with the neutrosophic initial condition

$$U(X,T) = f(X,I) \in R(I), \quad 0 \leq X \leq a$$

and the neutrosophic boundary condition

$$[U(0,T)]_{X=0} \in R(I), \quad and \quad T > 0$$

$$[U(a,T)]_{X=a} \in R(I), \quad and \quad T > 0$$

In this work, without loss of generality, we will consider the diffusivity of the material of the bar k=1.

Example:

We consider the following heat equation

$$\frac{\partial U}{\partial T} = \frac{\partial^2 U}{\partial X^2} \tag{3}$$

with initial condition

$$U(X,T) = \sin\frac{\pi X}{a} \in R(I) \tag{4}$$

and boundary condition

$$U(0,T) = 0 \quad \& \quad U(a,T) = 0 \tag{5}$$

where $U = U_1 + U_2 I, T = T_1 + T_2 I, X = X_1 + X_2 I \in R(I)$ and $0 = 0 + 0.I$ is also a neutrosophic number.

We now find the solution by applying Neutrosophic laplace transform

Taking AH-Isometry on (3), (4) and (5) respectively, we get

$$\frac{\partial U_1}{\partial T} = \frac{\partial^2 U_1}{\partial X^2} \quad \& \quad \frac{\partial U_1 + U_2}{\partial T} = \frac{\partial^2 U_1 + U_2}{\partial X^2} \tag{6}$$

$$U_1(X_1,T_1) = \sin\frac{\pi X_1}{a} \quad \& \quad (U_1+U_2)(X,T) = \sin\frac{\pi X_1}{a} + \sin\frac{\pi X_2}{a} \tag{7}$$

$$U_1(0,T_1) = (U_1+U_2)(0,T) = 0 \quad \& \quad U_1(a,T_1) = (U_1+U_2)(a,T) = 0 \tag{8}$$

Taking Laplace Transform on (6), we get

$$S_1\overline{U_1} - \sin\frac{\pi X_1}{a} = \frac{d^2\overline{U_1}}{dX^2} \tag{9}$$

$$(S_1+S_2)(\overline{U_1+U_2}) - \big(\sin\frac{\pi X_1}{a} + \sin\frac{\pi X_2}{a}\big) = \frac{d^2\overline{U_2}}{dX^2} \tag{10}$$

The A.E. of equation (9) is given by

$$D^2 - S_1 = 0 \Rightarrow D = \pm\sqrt{S_1}$$

Its C.F. is

$$Ae^{\sqrt{s_1}X_1} + Be^{-\sqrt{s_1}X_1}$$

Therefore P.I. of equation (9) is given by

$$\frac{1}{D^2 - S_1}\left[sin\frac{\pi X_1}{a}\right] = \frac{1}{\dfrac{\pi^2}{a^2} + S_1} sin\frac{\pi X_1}{a}$$

Thus the General solution of equation (9) is

$$\overline{U_1}(X,S_1) = Ae^{\sqrt{s_1}X_1} + Be^{-\sqrt{s_1}X_1} + \frac{1}{\dfrac{\pi^2}{a^2} + S_1} sin\frac{\pi X_1}{a} \tag{11}$$

Now taking Laplace Transform on (8), we get

$$\overline{U_1}(0,S_1) = \overline{(U_1+U_2)}(0,S_1+S_2) = 0 \quad \& \quad \overline{U_1}(a,S_1) = \overline{(U_1+U_2)}(a,S_1+S_2) = 0 \tag{12}$$

When $X = 0$ Then

Equation (11) gives,

$$\overline{U_1}(0,S_1) = A + B \Rightarrow A + B = 0 \tag{13}$$

When $X = a$ Then

Equation (11) gives,

$$\overline{U_1}(a, S_1) = Ae^{\sqrt{s_1}a} + Be^{-\sqrt{s_1}a} + 0 \Rightarrow Ae^{\sqrt{s_1}a} + Be^{-\sqrt{s_1}a} = 0 \tag{14}$$

Solving equation (13) and (14), we get $A = B = 0$.

Now, using these values in (11), the solution of equation (9) is given

$$\overline{U_1(X, S_1)} = \frac{sin\frac{\pi X_1}{a}}{\frac{\pi^2}{a^2} + S_1}$$

$$U_1(X, T_1) = sin\frac{\pi X_1}{a} L^{-1}\left[\frac{1}{\frac{\pi^2}{a^2} + S_1}\right]$$

$$U_1(X, T_1) = sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}} \tag{15}$$

Proceeding the same way, the solution of equation (10) gives,

$$(U_1 + U_2)(X, T_1 + T_2) = sin\frac{\pi(X_1 + X_2)}{a} e^{\frac{-\pi^2(T_1 + T_2)}{a^2}} \tag{16}$$

Now, taking inverse AH-Isometry on (15) and (16), we get

$$U(X, T) = \mathcal{T}^{-1}\left[\sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}}, \sin\frac{\pi(X_1 + X_2)}{a} e^{\frac{-\pi^2(T_1 + T_2)}{a^2}}\right]$$

$$U(X, T) =$$

$$\sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}} + \left[\left[\sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}} + \sin\frac{\pi(X_1 + X_2)}{a} e^{\frac{-\pi^2(T_1 + T_2)}{a^2}}\right] - \sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}}\right].I$$

This is the required temperatrure, where $\sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}}$ and $\left[\left[\sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}} + \sin\frac{\pi(X_1+X_2)}{a} e^{\frac{-\pi^2(T_1 + T_2)}{a^2}}\right] - \sin\frac{\pi X_1}{a} e^{\frac{-\pi^2 T_1}{a^2}}\right]$ are the determinant and indeterminant temperature.

4. **Conclusions**

In this article, we discussed the neutrosophic heat equation and its solution by taking numerical example. We have solved the practical problem by using one-dimensional geometric AH-Isometry of neutrosophic Laplace transformation. In the future, we will try to find general theoretical solutions so that practical problems can be solved quickly. In addition, we will try to solve the sensible solution to the neutrosophic heat equation by using different methods.

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