Neutrosophic Sets and Systems, Vol. 73, 2024
University of New Mexico

Key performance indicators in technology firms using generalized fermatean neutrosophic competition graph

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Abstract. This study investigates fermatean neutrosophic digraphs, generalized fermatean neutrosophic digraphs, and the out-neighborhood of vertices inside generalized fermatean neutrosophic digraphs. It looks at the qualities and characteristics of generalized fermatean neutrosophic competition graphs and their matrix representations. It also establishes the minimal graph, competition number for generalized fermatean neutrosophic competition graphs, and relevant features. Finally, the paper addresses a practical implementation of these ideas.

Keywords: Neutrosophicgraph, fermatean neutrosophic graph, fermatean neutrosophic digraphs, generalized fermatean neutrosophic digraphs.

1. **Introduction**

An important area of applied mathematics is graph theory, which is utilized to address a wide range of issues in computer science, geometry, algebra, social networks, optimization, and other fields [1]. Cohen [2] introduced the competition graph and its use in ecosystems, focusing on species competition within food webs. When two species have at least one common prey, they are considered to be in competition in this context. Roberts et al. [3, 4] investigated the possibility of representing all networks with isolated vertices as competition graphs. The competition number represents the least number of such vertices. Opsut [5] studied how to calculate a graph's competition number. Kim and colleagues [6, 7] provided the p-competition number and graph. Brigham et al. [8] expanded the p-competition graph to incorporate the ∅-tolerance graph, enhancing its generality.

Cho and Kim [9] investigated the competition number of a graph with one hole. Li and Chang [10] investigated competition graphs with h holes. Factor and Merz initially proposed the (1,2)-step competition graph for tournaments [11], and then expanded the concept to the (1,2)-step competition graph. Kaufman [12] created the fuzzy graph, where each vertex and edge has various degrees of membership, to account for erroneous data in real life. Numerous scientific investigations have been conducted on fuzzy graphs [13]. Parvathi and Karunambigal introduced intuitionistic fuzzy graphs in [14]. It is a graph composed of vertices and edges with variable degrees of membership and non-membership. Akram and Dubek [15] introduced interval-valued fuzzy graphs, where vertices and edges' membership values are represented as intervals. However, even when competition is portrayed using competition graphs, these features are not fully realized.

Samanta and Pal [16] represented competition in a fuzzy environment more realistically, taking into account the ambiguity of prey and species in a food chain. Samanta and Sarkar [17, 18] proposed the generalized fuzzy competition graph and generalized fuzzy graph, where the vertices' membership values dictate edge membership values. Pramanik et al. [19] merged fuzzy tolerance graphs with fuzzy φ tolerance competition graphs.

Smarandache [20] developed neutrosophic logic, a cohesive framework for dealing with indeterminate and inconsistent information that extends classical and fuzzy logics. This fundamental discovery laid the groundwork for later discoveries in a wide range of fields, including decision-making and graph theory. Ye [21] expanded neutrosophic logic by developing a multi-criteria decision-making technique that leverages the correlation coefficient in a single-valued neutrosophic environment, proving its practical applicability in challenging decision circumstances.Akram, Siddique, and Davvaz [22] presented new concepts in neutrosophic graphs and studied their applications, proving the flexibility of neutrosophic logic in mathematical modeling. Quek et al. [23] made additional contributions to the topic by investigating graph theory in the context of complicated neutrosophic sets, revealing neutrosophic sets' ability to handle increasingly complex relational data. ahin [24] presented a practical approach to neutrosophic graph theory, highlighting its usefulness in tackling real-world situations. Huang et al. [25] investigated regular and irregular neutrosophic graphs, applying these principles to real-world circumstances and emphasizing their practical relevance. Mohanta et al. [26] investigated m-polar neutrosophic graphs, expanding the use of neutrosophic graph theory in intelligent and fuzzy systems and enlarging the scope and usefulness of neutrosophic logic in contemporary computing issues. Recent advances in neutrosophic logic and graph theory have greatly broadened the scope of decision-making and problem-solving in complicated and ambiguous situations. Mohanta, Dey, and Pal [27] investigated several neutrosophic graph products, stressing their potential for managing complex interactions in intelligent systems. Broumi et al. [28, 29] proposed interval-valued Fermatean neutrosophic graphs as a complete framework for dealing with data with a wider range of uncertainty and indeterminacy. Their contributions to ”Collected Papers” and ”Neutrosophic Sets and Systems” highlighted the theoretical underpinnings and practical uses of these graphs, particularly in scenarios needing improved decision-making capacities. Broumi et al. [30] extended the use of neutrosophic graph theory by developing complicated Fermatean neutrosophic graphs that were used in decision-making processes in management and engineering. This novel technique revealed the usefulness of neutrosophic graphs in solving diverse choice issues. Dhouib et al. [31] solved the Minimum Spanning Tree Problem with interval-valued Fermatean neutrosophic domains, demonstrating the usefulness of these graphs in optimizing network-related tasks. Additionally, Saeed and Shafique [32] examined the relationship of Fermatean neutrosophic soft sets with applications to sustainable agriculture, their findings showed that neutrosophic sets can help improve decision-making processes in agricultural sustainability. AL-Omeri et al. [[33]- [38]] discussed identify internet streaming services using max product of complement in neutrosophic graphs and give some real time applications.

1.1. *Motivation*

(1) To broaden graph theory by including fermatean neutrosophic graph, which covers membership, indeterminacy, and non membership.
(2) To develop powerful tools for modeling complicated real-world issues that go beyond the capability of classical binary logic.
(3) The inspiration for this study derives from the desire to better capture and evaluate dynamic interactions in such systems, where classical graph theory falls short.

1.2. *Novelty*

(1) The paper extends the notion of fermatean neutrosophic graphs to extended fermatean neutrosophic competition graphs, increasing the scope of fermatean neutrosophic graph theory.
(2) It defines and examines the minimal graph and competition number for generalized fermatean neutrosophic competition graphs, yielding novel theoretical insights and characteristics.
(3) The research presents a matrix form of generalized fermatean neutrosophic competition graphs that makes them easier to compute and see.
(4) The paper describes a practical application of generalized fermatean neutrosophic competition graphs in the context of technology firms, illustrating the relevance of the suggested ideas in capturing real-world contests and interactions.

1.3. *Structure of the article*

The research begins with an overview of fermatean neutrosophic digraphs(FNDG) and the reason behind expanding graph theory to include fermatean neutrosophic graph(FNG).
Section 2 covers the fundamental terminology and preliminary information required. Section 3 present generalized fermatean neutrosophic graph(GFNG), fermatean neutrosophic digraphs, generalized fermatean neutrosophic digraphs, fermatean neutrosophic competition graph(FNCG), generalized fermatean neutrosophic competition graph(GFNCG), minimal graph and competition number for generalized fermatean neutrosophic competition graphs, along with their features.
Section 4 presents the matrix form of GNCGs, followed by an appropriate example to demonstrate its use. section 5 discusses a practical application of the explored theoretical principles, demonstrating the importance and value of generalized fermatean neutrosophic digraphs(GFNDG) in solving real-world problems.
Summarizes the findings and makes recommendations for future study in fermatean neutrosophic graph(FNG) theory.

2. **Basic Definitions**

This part provides the essential components required for understanding the article.

Definition 2.1. Let X represent a universal set. A Fermatean Neutrosophic relation on X is a mapping $g = (\chi_\upsilon, \chi_\xi, \chi_\varsigma) : X \times X \to [0, 1]$ where $\chi_\upsilon(\kappa_r, \kappa_s), \chi_\xi(\kappa_r, \kappa_s), \chi_\varsigma(\kappa_r, \kappa_s) \in [0, 1]$.

Definition 2.2. Let X be a universal set. Let $G = (g, \vartheta)$ be FNG, where g is a fermatean neutrosophic set on X and ϑ is a fermatean neutrosophic relation on X. The pair fulfills the following requirements

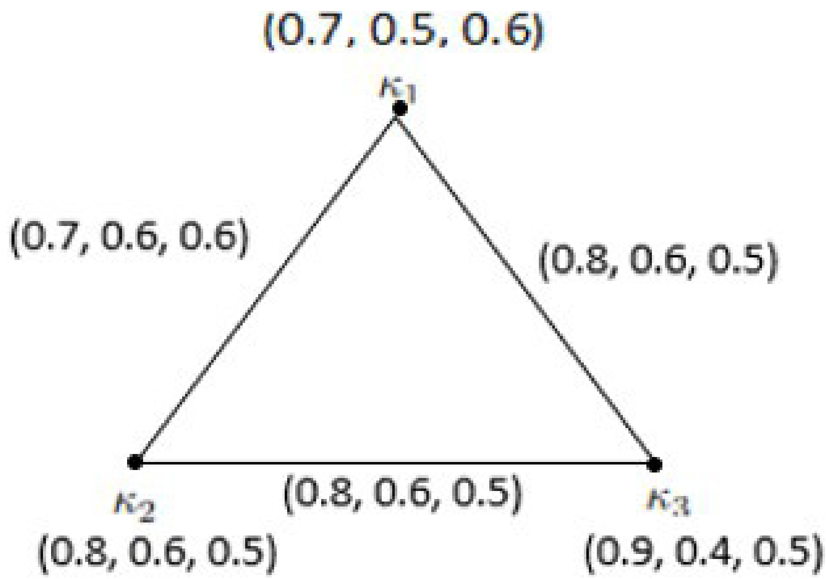
$\delta_\upsilon(\kappa_r, \kappa_s) \leq min\{\chi_\upsilon(\kappa_r), \chi_\upsilon(\kappa_s)\}$

$\delta_\xi(\kappa_r, \kappa_s) \geq max\{\chi_\xi(\kappa_r), \chi_\xi(\kappa_s)\}$

$\delta_\varsigma(\kappa_r, \kappa_s) \geq max\{\chi_\varsigma(\kappa_r), \chi_\varsigma(\kappa_s)\}$

$0 \leq \delta^3_\upsilon(\kappa_r, \kappa_s) + \delta^3_\xi(\kappa_r, \kappa_s) + \delta^3_\varsigma(\kappa_r, \kappa_s) \leq 2$ for all $\kappa_r, \kappa_s \in X$ where $\delta_\upsilon : X \times X \to [0, 1], \delta_\xi : X \times X \to [0, 1]$ $and \delta_\varsigma : X \times X \to [0, 1]$ represents the degree of membership, indeterminacy-membership, and non-membership of ϑ, respectively. Here, the Fermatean Neutrosophic edge set of G is represented by ϑ, while the Fermatean Neutrosophic vertex set of G is represented by g.

Example 2.3. Consider the FNG $G = (g, \vartheta)$ where edge set of G is represented by ϑ, and vertex set of G is represented by g defined by $g = \{\kappa_1, \kappa_2, \kappa_3\}$ and edges $\vartheta = \{(\kappa_1, \kappa_2), (\kappa_1, \kappa_3), (\kappa_2, \kappa_3)\}$ as in figure 1.



FNG.PNG

Figure 1. FNG .

Definition 2.4. The cardinality of FNS χ is denoted as $|\chi| = (|\chi|_{\upsilon}, |\chi|_{\xi}\ |\chi|_{\varsigma})$. The total membership values are represented by $|\chi|_{\upsilon}$, indeterminacy values are represented by $|\chi|_{\xi}$ and non membership values are represented by $|\chi|_{\varsigma}$.

Definition 2.5. The height of an FNS $\chi = (X, \chi_{\upsilon}, \chi_{\xi}\ \chi_{\varsigma})$ is defined as $h(\chi) = (sup_{x \in X}\chi_{\upsilon}(x), inf_{x \in X}\chi_{\xi}(x), inf_{x \in X}\chi_{\varsigma}(x)) = (h_1(\chi), h_2(\chi))$

Definition 2.6. If $\overrightarrow{G} = (g, \overrightarrow{\vartheta})$ is defined as FNDG if

(i) $\chi_{\upsilon} : g \to [0, 1]$, $\chi_{\xi} : g \to [0, 1]$ and $\chi_{\varsigma} : g \to [0, 1]$ it denotes the degree of membership, indeterminacy and non membership respectively, such that $0 \leq \chi_{\upsilon}^3 + \chi_{\xi}^3 + \chi_{\varsigma}^3 \leq 2\ \forall \kappa_r \in g$.

(ii) $\delta_{\upsilon} : \vartheta \to [0, 1]$, $\delta_{\xi} : \vartheta \to [0, 1]$ and $\delta_{\varsigma} : \vartheta \to [0, 1]$ it denotes the degree of membership, indeterminacy and non membership of edge respectively.

$$\delta_{\upsilon}(\overrightarrow{\kappa_r, \kappa_s}) \leq min\{\chi_{\upsilon}(\kappa_r), \chi_{\upsilon}(\kappa_s)\}$$
$$\delta_{\xi}(\overrightarrow{\kappa_r, \kappa_s}) \geq max\{\chi_{\xi}(\kappa_r), \chi_{\xi}(\kappa_s)\}$$
$$\delta_{\varsigma}(\overrightarrow{\kappa_r, \kappa_s}) \geq max\{\chi_{\varsigma}(\kappa_r), \chi_{\varsigma}(\kappa_s)\}$$
$$0 \leq \delta_{\upsilon}^3(\overrightarrow{\kappa_r, \kappa_s}) + \delta_{\xi}^3(\overrightarrow{\kappa_r, \kappa_s}) + \delta_{\varsigma}^3(\overrightarrow{\kappa_r, \kappa_s}) \leq 2$$

Example 2.7. The graph in Figure 2 is represented by the notation $\overrightarrow{G} = (g, \overrightarrow{\vartheta})$, with vertices $g = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ and edges $\vartheta = \{(\overrightarrow{\kappa_1, \kappa_2}), (\overrightarrow{\kappa_1, \kappa_3}), (\overrightarrow{\kappa_2, \kappa_3}), (\overrightarrow{\kappa_3, \kappa_4})\}$

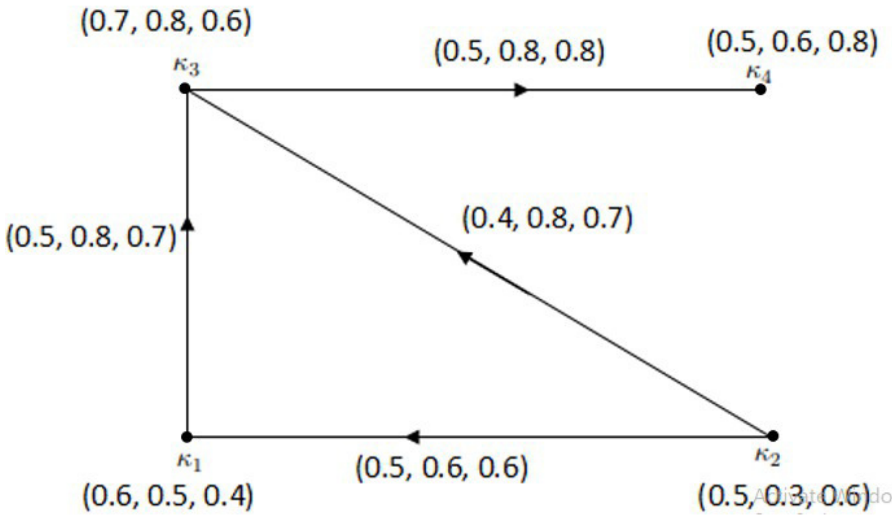


FIG1.PNG

Figure 2. FNDG .

Definition 2.8. The Fermatean Neutrosophic out-neighborhood of a vertex κ_r in a directed Fermatean Neutrosophic graph $\vec{G} = (g, \chi, \delta)$ is a FNS $F^+(\kappa_r) = \left(X^+_{\kappa_r}, \chi^+_{\upsilon_{\kappa_r}}, \chi^+_{\xi_{\kappa_r}}, \chi^+_{\varsigma_{\kappa_r}}\right)$, where $X^+_{\kappa_r} = \{\kappa_s | \delta_\upsilon(\overrightarrow{\kappa_r, \kappa_s}) > 0, \delta_\xi(\overrightarrow{\kappa_r, \kappa_s}) > 0, \delta_\varsigma(\overrightarrow{\kappa_r, \kappa_s}) > 0\}$ and $\chi^+_{\upsilon_{\kappa_r}} : X^+_{\kappa_r} \to [0, 1]$ defined by $\chi^+_{\upsilon_{\kappa_r}}(\kappa_s) = \delta_\upsilon(\overrightarrow{\kappa_r, \kappa_s})$,
$\chi^+_{\xi_{\kappa_r}} : X^+_{\kappa_r} \to [0, 1]$ defined by $\chi^+_{\xi_{\kappa_r}}(\kappa_s) = \delta_\xi(\overrightarrow{\kappa_r, \kappa_s})$ and $\chi^+_{\varsigma_{\kappa_r}} : X^+_{\kappa_r} \to [0, 1]$ defined by $\chi^+_{\varsigma_{\kappa_r}}(\kappa_s) = \delta_\varsigma(\overrightarrow{\kappa_r, \kappa_s})$.

Definition 2.9. The Fermatean Neutrosophic in-neighborhood of a vertex κ_r in a directed Fermatean Neutrosophic graph $\vec{G} = (g, \chi, \delta)$ is a FNS $F^-(\kappa_r) = \left(X^-_{\kappa_r}, \chi^-_{\upsilon_{\kappa_r}}, \chi^-_{\xi_{\kappa_r}}, \chi^-_{\varsigma_{\kappa_r}}\right)$, where $X^-_{\kappa_r} = \{\kappa_s | \delta_\upsilon(\overrightarrow{\kappa_s, \kappa_r}) > 0, \delta_\xi(\overrightarrow{\kappa_s, \kappa_r}) > 0, \delta_\varsigma(\overrightarrow{\kappa_s, \kappa_r}) > 0\}$ and $\chi^-_{\upsilon_{\kappa_r}} : X^+_{\kappa_r} \to [0, 1]$ defined by $\chi^-_{\upsilon_{\kappa_r}}(\kappa_s) = \delta_\upsilon(\overrightarrow{\kappa_s, \kappa_r})$, $\chi^-_{\xi_{\kappa_r}} : X^+ \to [0, 1]$ defined by $\chi^-_{\xi_{\kappa_r}}(\kappa_s) = \delta_\xi(\overrightarrow{\kappa_s, \kappa_r})$ and $\chi^-_{\varsigma_{\kappa_r}} : X^+_{\kappa_r} \to [0, 1]$ defined by $\chi^-_{\varsigma_{\kappa_r}}(\kappa_s) = \delta_\varsigma(\overrightarrow{\kappa_s, \kappa_r})$.

3. **Generalized fermatean neutrosophic competition graph**

Definition 3.1. A GFNG $G = (g, \vartheta)$ where $\vartheta \subseteq g \times g$ is examined if certain functions exist

$\chi_\upsilon : g \to [0, 1]$, $\chi_\xi : g \to [0, 1]$ and $\chi_\varsigma : g \to [0, 1]$.

$\delta_\upsilon : \vartheta \to [0, 1]$, $\delta_\xi : \vartheta \to [0, 1]$ and $\delta_\varsigma : \vartheta \to [0, 1]$.

$E_\upsilon : \vartheta_\upsilon \to [0, 1]$, $E_\xi : \vartheta_\xi \to [0, 1]$ and $E_\varsigma : \vartheta_\varsigma \to [0, 1]$,
such that $0 \leq \chi^3_\upsilon(\kappa_r) + \chi^3_\xi(\kappa_r) + \chi^3_\varsigma(\kappa_r) \leq 2 \ \forall \kappa_r \in g \ (r = 1, 2, \ldots, n)$ and

$\delta_\upsilon(\kappa_r, \kappa_s) = E_\upsilon(\chi_\upsilon(\kappa_r), \chi_\upsilon(\kappa_s))$

$\delta_\xi(\kappa_r, \kappa_s) = E_\xi(\chi_\xi(\kappa_r), \chi_\xi(\kappa_s))$

$\delta_\varsigma(\kappa_r, \kappa_s) = E_\varsigma(\chi_\varsigma(\kappa_r), \chi_\varsigma(\kappa_s))$

where

$\vartheta_\upsilon = \{(\chi_\upsilon(\kappa_r), \chi_\upsilon(\kappa_s)) : \delta_\upsilon(\kappa_r, \kappa_s) \geq 0\}$

$\vartheta_{\xi} = \{(\chi_{\xi}(\kappa_r), \chi_{\xi}(\kappa_s)) : \delta_{\xi}(\kappa_r, \kappa_s) \geq 0\}$

$\vartheta_{\varsigma} = \{(\chi_{\varsigma}(\kappa_r), \chi_{\varsigma}(\kappa_s)) : \delta_{\varsigma}(\kappa_r, \kappa_s) \geq 0\}$

and $\chi_{\upsilon}(\kappa_r)$, $\chi_{\xi}(\kappa_r)$ and $\chi_{\varsigma}(\kappa_r)$ denotes the degree of membership, indeterminacy and non membership of vertex respectively and $\delta_{\upsilon}(\kappa_r, \kappa_s)$, $\delta_{\xi}(\kappa_r, \kappa_s)$ and $\delta_{\varsigma}(\kappa_r, \kappa_s)$ denotes the degree of membership, indeterminacy and non membership of edges respectively.

Definition 3.2. A GFNG $\vec{Ģ} = (g, \vec{\vartheta})$ where $\vec{\vartheta} \subseteq g \times g$ is examined if certain functions exist

$\chi_{\upsilon} : g \to [0, 1]$, $\chi_{\xi} : g \to [0, 1]$ and $\chi_{\varsigma} : g \to [0, 1]$.

$\delta_{\upsilon} : \vec{\vartheta} \to [0, 1]$, $\delta_{\xi} : \vec{\vartheta} \to [0, 1]$ and $\delta_{\varsigma} : \vec{\vartheta} \to [0, 1]$.

$E_{\upsilon} : \vartheta_{\upsilon} \to [0, 1]$, $E_{\xi} : \vartheta_{\xi} \to [0, 1]$ and $E_{\varsigma} : \vartheta_{\varsigma} \to [0, 1]$,

such that $0 \leq \chi^3_{\upsilon}(\kappa_r) + \chi^3_{\xi}(\kappa_r) + \chi^3_{\varsigma}(\kappa_r) \leq 2 \ \forall \kappa_r \in g \ (r = 1, 2, ..., n)$ and

$\delta_{\upsilon}(\overrightarrow{\kappa_r, \kappa_s}) = E_{\upsilon}(\chi_{\upsilon}(\kappa_r), \chi_{\upsilon}(\kappa_s))$

$\delta_{\xi}(\overrightarrow{\kappa_r, \kappa_s}) = E_{\xi}(\chi_{\xi}(\kappa_r), \chi_{\xi}(\kappa_s))$

$\delta_{\varsigma}(\overrightarrow{\kappa_r, \kappa_s}) = E_{\varsigma}(\chi_{\varsigma}(\kappa_r), \chi_{\varsigma}(\kappa_s))$

where

$\vartheta_{\upsilon} = \{(\chi_{\upsilon}(\kappa_r), \chi_{\upsilon}(\kappa_s)) : \delta_{\upsilon}(\kappa_r, \kappa_s) \geq 0\}$

$\vartheta_{\xi} = \{(\chi_{\xi}(\kappa_r), \chi_{\xi}(\kappa_s)) : \delta_{\xi}(\kappa_r, \kappa_s) \geq 0\}$

$\vartheta_{\varsigma} = \{(\chi_{\varsigma}(\kappa_r), \chi_{\varsigma}(\kappa_s)) : \delta_{\varsigma}(\kappa_r, \kappa_s) \geq 0\}$

and $\chi_{\upsilon}(\kappa_r)$, $\chi_{\xi}(\kappa_r)$ and $\chi_{\varsigma}(\kappa_r)$ denotes the degree of membership, indeterminacy and non membership of vertex respectively and $\delta_{\upsilon}(\overrightarrow{\kappa_r, \kappa_s})$, $\delta_{\xi}(\overrightarrow{\kappa_r, \kappa_s})$ and $\delta_{\varsigma}(\overrightarrow{\kappa_r, \kappa_s})$ denotes the degree of membership, indeterminacy and non membership of edges respectively.

Example 3.3. The graph in Figure 3 is represented by the notation $\vec{Ģ} = (g, \vec{\vartheta})$, with vertices $g = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ and edges $\vartheta = \{(\overrightarrow{\kappa_1, \kappa_2}), (\overrightarrow{\kappa_1, \kappa_3}), (\overrightarrow{\kappa_4, \kappa_1}), (\overrightarrow{\kappa_3, \kappa_2})\}$

Definition 3.4. A GFNG $\vec{Ģ} = (g, \vec{\vartheta})$ is defined as GFNDG.The out-neighbourhood $\mathsf{F}(\kappa_r)$ of a vertex $\kappa_r \in g$ is denoted as $\mathsf{F}(\kappa_r) = \{\kappa_s, (\delta_{\upsilon}(\overrightarrow{\kappa_r, \kappa_s}), \delta_{\xi}(\overrightarrow{\kappa_r, \kappa_s}), \delta_{\varsigma}(\overrightarrow{\kappa_r, \kappa_s})) / (\overrightarrow{\kappa_r, \kappa_s}) \in \vartheta\}$.

Example 3.5. The graph in Figure 4 is represented by the notation $\vec{Ģ} = (g, \vec{\vartheta})$, with vertices $g = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ and edges $\vartheta = \{(\overrightarrow{\kappa_1, \kappa_2}), (\overrightarrow{\kappa_1, \kappa_3}), (\overrightarrow{\kappa_1, \kappa_4}), (\overrightarrow{\kappa_3, \kappa_4}), (\overrightarrow{\kappa_2, \kappa_3})\}$

$\mathsf{F}(\kappa_1) = \{(\kappa_2, (0.7, 0.8, 0.6)), (\kappa_3, (0.9, 0.5, 0.6)), (\kappa_4, (0.6, 0.6, 0.7))\}$

$\mathsf{F}(\kappa_2) = \{(\kappa_3, (0.9, 0.8, 0.7))\}$ $\mathsf{F}(\kappa_3) = \{(\kappa_4, (0.9, 0.6, 0.7))\}$ $\mathsf{F}(\kappa_4) = \emptyset$

Definition 3.6. If $\vec{Ģ} = (g, \vec{\vartheta})$ is defined as GFNDG. The GFNCG $C(\vec{Ģ})$ of $\vec{Ģ} = (g, \vec{\vartheta})$ is GFNG that has the same vertex set g and contains a fermatean neutrosophic edge between κ_1 and κ_2 iff $\mathsf{F}(\kappa_1) \cap \mathsf{F}(\kappa_2) \neq \emptyset$. Furthermore, there exist sets

$\lambda_1 = \{\aleph^{\upsilon}_{\kappa_1}\kappa_1 \in g\}$, $\lambda_2 = \{\aleph^{\xi}_{\kappa_1}\kappa_1 \in g\}$, $\lambda_3 = \{\aleph^{\varsigma}_{\kappa_1}\kappa_1 \in g\}$ and functions $E_1 : \lambda_1 \times \lambda_1 \to [0, 1]$, $E_2 : \lambda_2 \times \lambda_2 \to [0, 1]$, $E_3 : \lambda_3 \times \lambda_3 \to [0, 1]$ for each $(\kappa_1, \kappa_2) \in \vartheta$ where

FIG2.PNG

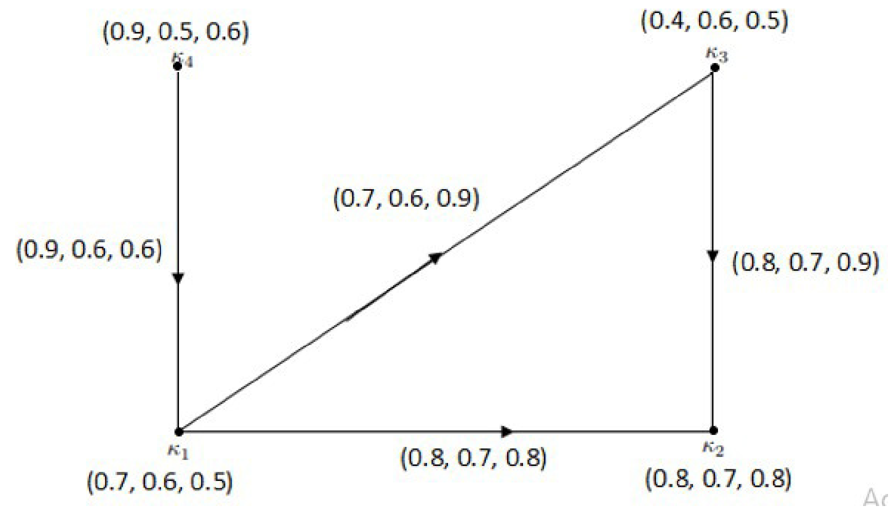


Figure 3. GFNDG .

FIG3.PNG

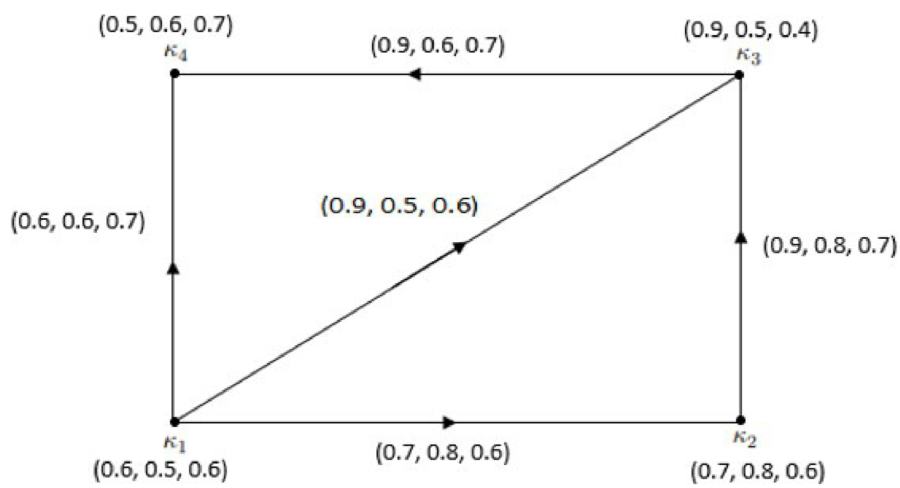


Figure 4. FNDG.

$$\delta_\upsilon(\kappa_1,\kappa_2) = E\left(\aleph^{\upsilon}_{\kappa_1}, \aleph^{\upsilon}_{\kappa_2}\right)$$
$$\delta_\xi(\kappa_1,\kappa_2) = E\left(\aleph^{\xi}_{\kappa_1}, \aleph^{\xi}_{\kappa_2}\right)$$
$$\delta_\varsigma(\kappa_1,\kappa_2) = E\left(\aleph^{\varsigma}_{\kappa_1}, \aleph^{\varsigma}_{\kappa_2}\right)$$
$$\aleph^{\upsilon}_{\kappa_1} = min\,\{\delta_\upsilon(\overrightarrow{\kappa_1,\kappa_3}), \forall\kappa_3 \in F(\kappa_1) \cap F(\kappa_2)\}$$
$$\aleph^{\upsilon}_{\kappa_2} = min\,\{\delta_\upsilon(\overrightarrow{\kappa_2,\kappa_3}), \forall\kappa_3 \in F(\kappa_1) \cap F(\kappa_2)\}$$
$$\aleph^{\xi}_{\kappa_1} = max\,\{\delta_\xi(\overrightarrow{\kappa_1,\kappa_3}), \forall\kappa_3 \in F(\kappa_1) \cap F(\kappa_2)\}$$
$$\aleph^{\xi}_{\kappa_2} = max\,\{\delta_\xi(\overrightarrow{\kappa_2,\kappa_3}), \forall\kappa_3 \in F(\kappa_1) \cap F(\kappa_2)\}$$
$$\aleph^{\varsigma}_{\kappa_1} = max\,\{\delta_\varsigma(\overrightarrow{\kappa_1,\kappa_3}), \forall\kappa_3 \in F(\kappa_1) \cap F(\kappa_2)\}$$
$$\aleph^{\varsigma}_{\kappa_2} = max\,\{\delta_\varsigma(\overrightarrow{\kappa_2,\kappa_3}), \forall\kappa_3 \in F(\kappa_1) \cap F(\kappa_2)\}$$

Example 3.7. The graph in Figure 4 is represented GFNDG $\vec{\c{G}} = (g, \vec{\vartheta})$, with vertices $g = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ and edges $\vartheta = \{(\overrightarrow{\kappa_1, \kappa_2}), (\overrightarrow{\kappa_1, \kappa_3}), (\overrightarrow{\kappa_1, \kappa_4}), (\overrightarrow{\kappa_3, \kappa_4}), (\overrightarrow{\kappa_2, \kappa_3})\}$. The consequent competition graph (Figure 5)

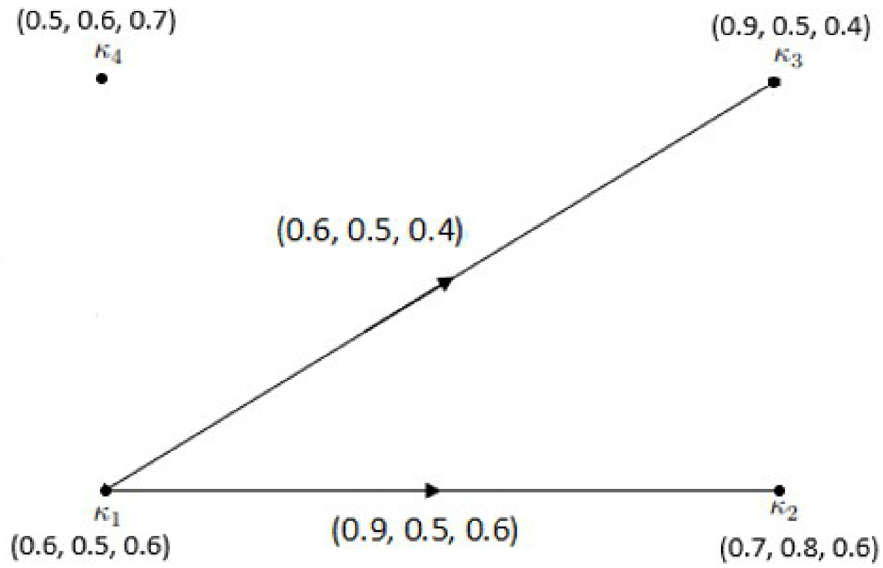


FIG4.PNG

Figure 5. GFNCG of graph (Figure 4).

Theorem 3.8. *If Ģ represent a GFNG, then there exist a GFNDG $\vec{\c{G}}$ such that $C(\vec{\c{G}}) = \c{G}$*
proof Given a GFNG $\c{G} = (g, \vartheta)$. (u_1, u_2) indicates an edge in Ģ, the goal is to create a GFNDG $\vec{\c{G}}$, the competition graph $C(\vec{\c{G}})$ is equal to Ģ. Let $\check{u}_1$ and $\check{u}_2$ be the corresponding vertices of u_1 and u_2 in Ģ. Then, from vertices $\check{u}_1, \check{u}_2$ we may construct two directed edges to a vertex $\check{u}_3 \in \vec{\c{G}}$ such that $\check{u}_3 \in F(\check{u}_1) \cap F(\check{u}_2)$. In a similar manner, we may do this for each vertex and edge of Ģ, and as a result, $C(\vec{\c{G}}) = \c{G}$

Definition 3.9. Let Ģ represent a GFNG. The minimal graph $\vec{\c{G}}$ of Ģ is a GFNDG with $C(\vec{\c{G}}) = \c{G}$ and $\vec{\c{G}}$ has the minimum number of edges, i.e, if another graph, $\vec{\c{G}}'$ exists and $C(\vec{\c{G}}') = \c{G}$, then number of edges of $\vec{\c{G}} \leq$ number of edges of $\vec{\c{G}}'$.

Given a GFNCG, we may create a directed variant(a GFNDG) that emphasizes these competitive interactions. However, for a single GFNCG there may be many comparable digraphs with various amount of edges. Our objective is to identify the most compact digraph-one with the minimum number of edges- that appropriately represents the competition.

Theorem 3.10. *In a generalized fermatean neutrosophic connected graph $\vec{\c{G}}$ which has an underlying complete graph with vertex n. The minimal graph of $\vec{\c{G}}$ has $2n$ edges where $n \geq 2$.*

proof

Let $\vec{\check{G}}$ be the connected GFNG which has an underlying complete graph with vertex n this means that each vertex is linked to every other vertex. Let k and r be two neighboring vertices in $\vec{\check{G}}$ and k_1, r_1 be the corresponding vertices in the minimal graph $\vec{\check{G}}$. Let $\vec{\check{G}}$ is a GFNDG every vertex except k_1 has only out neighbourhood as k_1. Hence $\vec{\check{G}}_1$ has $n-1$ edges. In a similar way, a GFNDG $\vec{\check{G}}_2$ is taken into consideration r_1 and consequently $\vec{\check{G}}_2$. There are $n-1$ edges in $\vec{\check{G}}_2$. Let us now examine GFNDG $\vec{\check{G}}_3$ including only the edges $\overrightarrow{(k_1, u_1)}\overrightarrow{(r_1, u_1)}$. Consequently, the combined graph $\vec{\check{G}} = \vec{\check{G}}_1 \cup \vec{\check{G}}_2 \cup \vec{\check{G}}_3$ has a total $(n-1)+(n-1)+2 = 2n$ edges.

Definition 3.11. In a GFNG, the score of an edge (κ_1, κ_2) connecting the vertices is denoted by $S(\kappa_1, \kappa_2) = \frac{2\delta_\upsilon + \delta_\xi - 2\delta_\upsilon\delta_\varsigma}{3}$

Example 3.12. The GFNG in Figure 6 is represented by the notation $\text{Ģ} = (g, \vartheta)$, with vertices $g = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ and edges $\vartheta = \{\overrightarrow{(\kappa_1, \kappa_2)}, \overrightarrow{(\kappa_1, \kappa_4)}, \overrightarrow{(\kappa_2, \kappa_3)}, \overrightarrow{(\kappa_3, \kappa_4)}, \overrightarrow{(\kappa_2, \kappa_4)}\}$

.

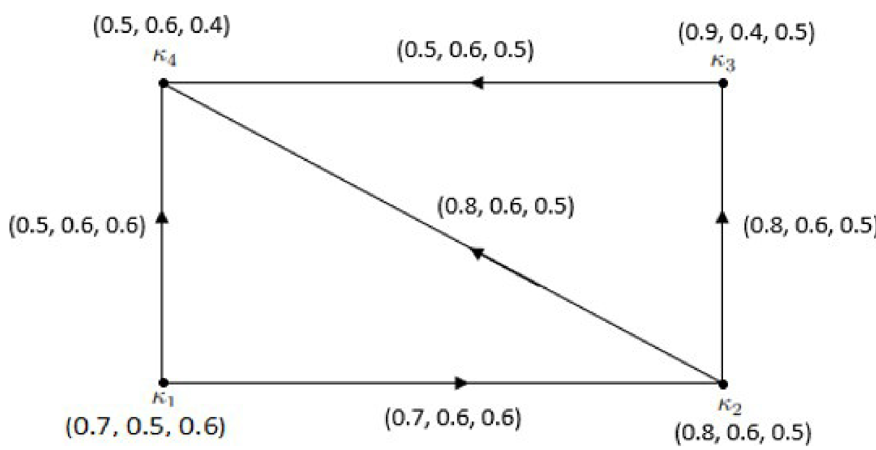


FIG5.PNG

Figure 6. GFNG .

Definition 3.13. The vertexκ_1 with neighbouring vertices $r_1, r_2, \ldots, r_h$is considered isolated in GFNG if $S(\kappa_1, r_l)$ if $l = 1, 2, \ldots, h$.

Example 3.14. The GFNG in Figure 7 is represented by the notation $\text{Ģ} = (g, \vartheta)$, with vertices $g = \{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ and edges $\vartheta = \{\overrightarrow{(\kappa_1, \kappa_2)}, \overrightarrow{(\kappa_1, \kappa_3)}, \overrightarrow{(\kappa_2, \kappa_3)}, \overrightarrow{(\kappa_2, \kappa_4)}\}$

. The neighboring vertex of κ_4 is κ_2, with the edge score (κ_2, κ_4) is 0, indicating the κ_4 is an isolated vertex.

Table 1. Edge score values.

Edges	Score Value
$\kappa_1\kappa_2$	0.386
$\kappa_2\kappa_4$	0.466
$\kappa_2\kappa_3$	0.466
$\kappa_3\kappa_4$	0.366
$\kappa_4\kappa_1$	0.33

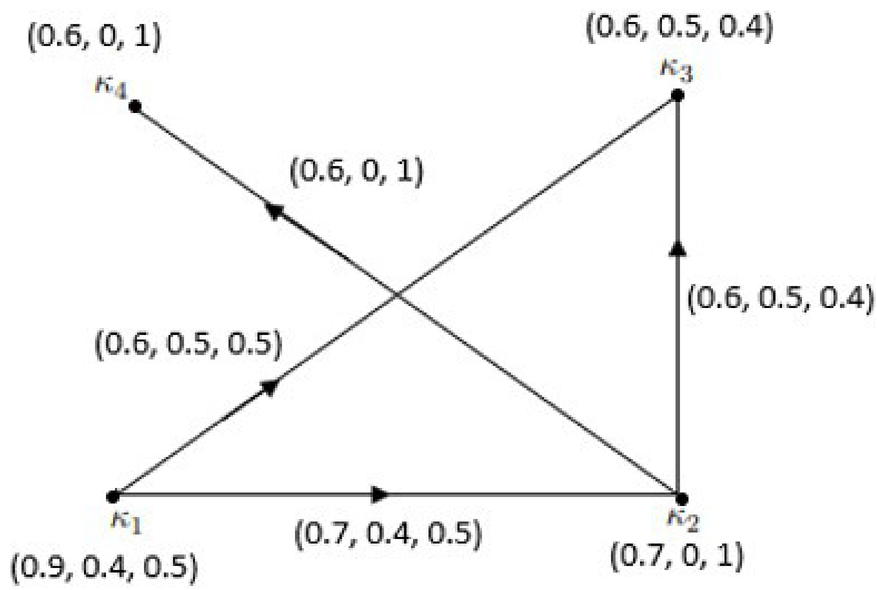


FIG6.PNG

Figure 7. GFNG with isolated vertex .

Definition 3.15. In a GFNG, a cycle with a length of more than 4 is referred to as a hole if each edge in the cycle has a score that is not zero.

Example 3.16. In Example 5, the graph $\kappa_1 - \kappa_2 - \kappa_3 - \kappa_4 - \kappa_1$ shows a 4-cycle with non-zero scores, indicating a hole.

Definition 3.17. In a generalized neighborhood graph, the competition number refers to the smallest isolated vertex, denoted by $C_F(\text{Ģ})$.

Lemma 3.18. *A crisp graph with a single hole has a maximum completion number of two. A GFNG with a single hole may have a competition number larger than two.*

Consider a graph (Figure 8) with a single hole with competition number two. Edge Scores $(\overrightarrow{\kappa_1,\kappa_2})$, $(\overrightarrow{\kappa_2,\kappa_3})$, $(\overrightarrow{\kappa_3,\kappa_4})$ *and* $(\overrightarrow{\kappa_4,\kappa_1})$ *are non- zero by definition. However, the score of*

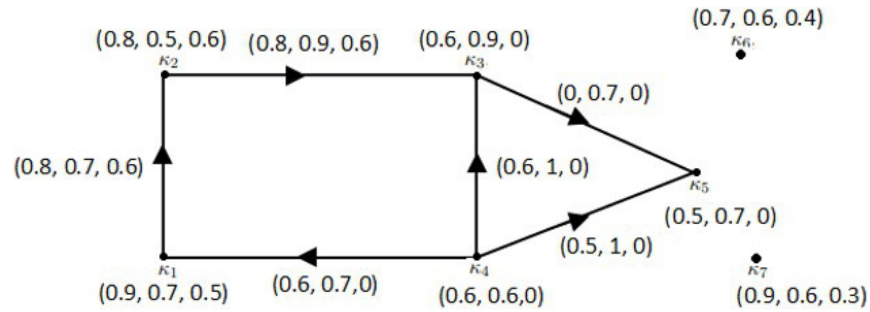


FIG7.PNG

Figure 8. GFNG with competition number 2 .

(κ_4, κ_5) and (κ_3, κ_5) may be zero. Hence, κ_5 is an isolated vertex. The competition number is three.

Definition 3.19. A fermatean neutrosophic chordal graph(FNCG) is one in which every hole has a chord with a score than zero.

Example 3.20. In Example 5, the graph if FNCG if the edges (κ_2, κ_4) are chords with non-zero scores and κ_1 is a hole.

Lemma 3.21. *A FNCG with a pendent vertex must have a competition number larger than one. The isolation of vertex κ_5 in the FNCG (Figure 9) results in a competition number larger than two.*

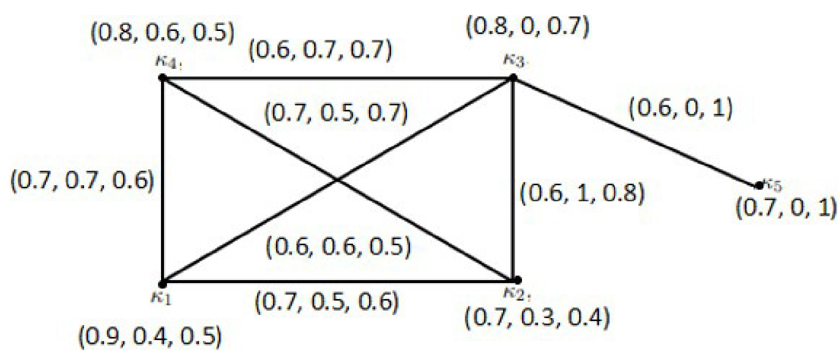


FIG8.PNG

Figure 9. FNCG .

4. GFNCG represented as a matrix

The following procedure calculates the elements of the adjacency matrix of a GFNCG.

(1) Consider GFNDG.

(2) Identify the vertices k_l and r_l for $l = 1, 2, ..., n$ such that their exist an edge $(k_l, u_p)\,(r_l, u_p)$ for $p = 1, 2, ..., m$ with $\digamma(k_l)$ and $\digamma(r_l)$

(3) Determine the set $\digamma(k_l) \cap \digamma(r_l) = \{u_q, q = 1, 2, ..., n\}$

(4) Compute

$$\aleph_k^{\upsilon} = \min\left\{\delta_\upsilon\left(\overrightarrow{k_l, u_1}\right), \delta_\upsilon\left(\overrightarrow{k_l, u_2}\right), ..., \delta_\upsilon\left(\overrightarrow{k_l, u_n}\right)\right\}$$
$$\aleph_r^{\upsilon} = \min\left\{\delta_\upsilon\left(\overrightarrow{r_l, u_1}\right), \delta_\upsilon\left(\overrightarrow{r_l, u_2}\right), ..., \delta_\upsilon\left(\overrightarrow{r_l, u_n}\right)\right\}$$
$$\aleph_k^{\xi} = \max\left\{\delta_\xi\left(\overrightarrow{k_l, u_1}\right), \delta_\xi\left(\overrightarrow{k_l, u_2}\right), ..., \delta_\xi\left(\overrightarrow{k_l, u_n}\right)\right\}$$
$$\aleph_r^{\xi} = \max\left\{\delta_\xi\left(\overrightarrow{r_l, u_1}\right), \delta_\xi\left(\overrightarrow{r_l, u_2}\right), ..., \delta_\xi\left(\overrightarrow{r_l, u_n}\right)\right\}$$
$$\aleph_k^{\varsigma} = \max\left\{\delta_\varsigma\left(\overrightarrow{k_l, u_1}\right), \delta_\varsigma\left(\overrightarrow{k_l, u_2}\right), ..., \delta_\varsigma\left(\overrightarrow{k_l, u_n}\right)\right\}$$
$$\aleph_r^{\varsigma} = \max\left\{\delta_\varsigma\left(\overrightarrow{r_l, u_1}\right), \delta_\varsigma\left(\overrightarrow{r_l, u_2}\right), ..., \delta_\varsigma\left(\overrightarrow{r_l, u_n}\right)\right\}$$

(5) For the pair of vertices k, r use functions E_1, E_2 and E_3 to get the combined membership degrees.

$$\delta_\upsilon(k, r) = E_1\left(\aleph_k^{\upsilon}, \aleph_r^{\upsilon}\right)$$
$$\delta_\xi(k, r) = E_1\left(\aleph_k^{\xi}, \aleph_r^{\xi}\right)$$
$$\delta_\varsigma(k, r) = E_1\left(\aleph_k^{\varsigma}, \aleph_r^{\varsigma}\right)$$

For the sake of simplicity, the functions E_1, E_2 and E_3 may be replaced by a single function E.

(6) A competition matrix is a square matrix. The number of vertices is equal to its order. The entries are as follows

$$\alpha_l = \begin{cases} \left(E_1\left(\aleph_l^{\upsilon}, \aleph_p^{\upsilon}\right), E_2\left(\aleph_l^{\xi}, \aleph_p^{\xi}\right), E_3\left(\aleph_l^{\varsigma}, \aleph_p^{\varsigma}\right)\right) & \text{if there exists an edge between l and p.} \\ (0, 0, 0) & \text{if there is no edge between l and p.} \end{cases}$$

Example 4.1. A matrix representation example is provided, complete with all phases.

Step 1: Consider GFNDG.

Step 2: $\digamma(k_1) = \{k_2\}$, $\digamma(k_2) = \{k_5\}$, $\digamma(k_3) = \{k_1, k_2\}$, $\digamma(k_4) = \{k_1, k_2, k_3\}$, $\digamma(k_5) = \{k_3\}$, $\digamma(k_6) = \{k_5\}$, $\digamma(k_7) = \{k_5\}$

Step 3: $\digamma(k_1) \cap \digamma(k_2) = \digamma(k_1) \cap \digamma(k_5) = \digamma(k_1) \cap \digamma(k_6) = \digamma(k_1) \cap \digamma(k_7) = \emptyset$, $\digamma(k_1) \cap \digamma(k_3) = \{k_2\}$, $\digamma(k_1) \cap \digamma(k_4) = \{k_2\}$, $\digamma(k_2) \cap \digamma(k_3) = \digamma(k_2) \cap \digamma(k_4) = \digamma(k_2) \cap \digamma(k_5) = \emptyset$, $\digamma(k_2) \cap \digamma(k_6) = \{k_5\}$, $\digamma(k_2) \cap \digamma(k_7) = \{k_5\}$, $\digamma(k_3) \cap \digamma(k_4) = \{k_1\}$, $\digamma(k_3) \cap \digamma(k_5) = \digamma(k_3) \cap \digamma(k_6) = \digamma(k_3) \cap \digamma(k_7) = \emptyset$, $\digamma(k_4) \cap \digamma(k_5) = \{k_3\}$, $\digamma(k_4) \cap \digamma(k_6) = \digamma(k_4) \cap \digamma(k_7) = \digamma(k_5) \cap \digamma(k_6) = \digamma(k_5) \cap \digamma(k_7) = \digamma(k_6) \cap \digamma(k_7) = \emptyset$

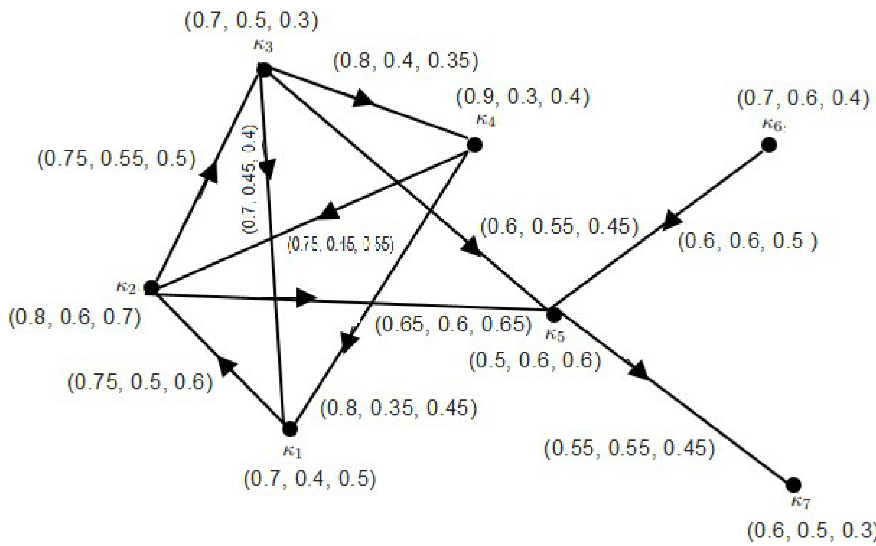


FIG9.PNG

Figure 10. GFNG with 7 vertices .

Step 4:

$\aleph^{\upsilon}_{1} = 0.75$	$\aleph^{\xi}_{1} = 0.5$	$\aleph^{\varsigma}_{1} = 0.6$
$\aleph^{\upsilon}_{3} = 0.75$	$\aleph^{\xi}_{3} = 0.55$	$\aleph^{\varsigma}_{3} = 0.5$
$\aleph^{\upsilon}_{4} = 0.8$	$\aleph^{\xi}_{4} = 0.4$	$\aleph^{\varsigma}_{4} = 0.35$
$\aleph^{\upsilon}_{4} = 0.8$	$\aleph^{\xi}_{4} = 0.35$	$\aleph^{\varsigma}_{4} = 0.45$
$\aleph^{\upsilon}_{31} = 0.7$	$\aleph^{\xi}_{31} = 0.45$	$\aleph^{\varsigma}_{31} = 0.4$
$\aleph^{\upsilon}_{4} = 0.75$	$\aleph^{\xi}_{4} = 0.45$	$\aleph^{\varsigma}_{4} = 0.55$
$\aleph^{\upsilon}_{53} = 0.6$	$\aleph^{\xi}_{53} = 0.55$	$\aleph^{\varsigma}_{53} = 0.45$
$\aleph^{\upsilon}_{7} = 0.55$	$\aleph^{\xi}_{7} = 0.55$	$\aleph^{\varsigma}_{7} = 0.45$
$\aleph^{\upsilon}_{25} = 0.65$	$\aleph^{\xi}_{25} = 0.6$	$\aleph^{\varsigma}_{25} = 0.65$
$\aleph^{\upsilon}_{6} = 0.6$	$\aleph^{\xi}_{6} = 0.6$	$\aleph^{\varsigma}_{6} = 0.5$

Step 5:

$$\delta_1^{\upsilon} = 0 \qquad \delta_1^{\xi} = 0.1 \qquad \delta_1^{\varsigma} = 0.25$$

$$\delta_1^{\upsilon} = 0.3 \qquad \delta_1^{\xi} = 0.25 \qquad \delta_1^{\varsigma} = 0.25$$

$$\delta_{34}^{\upsilon} = 0.3 \qquad \delta_{34}^{\xi} = 0.25 \qquad \delta_{34}^{\varsigma} = 0.3$$

$$\delta_{45}^{\upsilon} = 0.4 \qquad \delta_{45}^{\xi} = 0.3 \qquad \delta_{45}^{\varsigma} = 0.35$$

$$\delta_2^{\upsilon} = 0.25 \qquad \delta_2^{\xi} = 0.35 \qquad \delta_2^{\varsigma} = 0$$

$$\delta_2^{\upsilon} = 0.3 \qquad \delta_2^{\xi} = 0.4 \qquad \delta_2^{\varsigma} = 0.25$$

$$\delta_6^{\upsilon} = 0.25 \qquad \delta_6^{\xi} = 0.25 \qquad \delta_6^{\varsigma} = 0.25$$

Step 6:

−	(0,0,0)	(0,0.25,0.1)	(0.3,0.25,0.25)	(0,0,0)	(0,0,0)	(0,0,0)
(0,0,0)	−	(0,0,0)	(0,0,0)	(0,0,0)	(0.25,0,0.35)	(0.3,0.25,0.4)
(0,0.25,0.1)	(0,0,0)	−	(0.3,0.3,0.25)	(0,0,0)	(0,0,0)	(0,0,0)
(0.3,0.25,0.25)	(0,0,0)	(0.3,0.3,0.25)	−	(0.4,0.35,0.3)	(0,0,0)	(0,0,0)
(0,0,0)	(0,0,0)	(0,0,0)	(0.4,0.35,0.3)	−	(0,0,0)	(0,0,0)
(0,0,0)	(0.25,0,0.35)	(0,0,0)	(0,0,0)	(0,0,0)	−	(0.25,0.25,0.25)
(0,0,0)	(0.3,0.25,0.4)	(0,0,0)	(0,0,0)	(0,0,0)	(0.25,0.25,0.25)	−

5. **Application of key performance indicators in technology firms**

Many competitions exist in various aspects of everyday life, comparable to those seen in ecosystems. This study investigates the competition for technological innovation among major technology businesses in a fermatean neutrosophic environment. We take two things into consideration: Market share and R&D expenses. A company's market share is the total amount of sales it controls in its industry during a certain time period. The R&D Investment refers to the cash allocated for the company's research and innovation initiatives.

Market share increases represent true membership, whereas R&D spending measures non-membership. Uncertainty factors including market volatility, regulatory changes, and economic crises are measured against the level of indeterminacy membership. Data on market share and R&D investment are sourced from industry journals and business financial filings.

Leading technology companies such as Apple, Google, Microsoft, and Amazon are vying for technological superiority. Because all enterprises compete, the competition graph is complete. The membership values of the businesses (nodes) are displayed in tabular form (Table2 and Table3), whilst the membership values of edges are calculated using the following formula and provided in matrix style.

The matrix structure above depicts the competitiveness of technology businesses.

Table 2. Market share and R&D expenditures of technology firms.

Tech corporations	Market share	R&D expenditures
Apple	10	6.5
Microsoft	8.8	5.4
Google	7.2	7.5
Amazon	4	10
Meta	3.2	6.6
Samsung	2.8	4.3
Intel	2	3.6

Table 3. Normalized value of Market share and R&D expenditures of technology firms.

Tech corporations	NMS	N R&D	NMS ~NR&D
Apple	1	0.65	0.35
Microsoft	0.88	0. 54	0.34
Google	0.72	0.75	0.03
Amazon	0.4	0.10	0.6
Meta	0.32	0.66	0.34
Samsung	0.28	0.43	0.15
Intel	0.2	0.36	0.16

(1,0,1)	(0.06,0,0.055)	(0.14,0,0.05)	(0.3,0,0.175)	(0.34,0,0.005)	(0.36,0,0.11)	(0.4,0,0.145)
(0.06,0,0.055)	(1,0,1)	(0.08,0,0.105)	(0.24,0,0.23)	(0.28,0,0.06)	(0.3,0,0.055)	(0.34,0,0.09)
(0.14,0,0.05)	(0.08,0,0.105)	(1,0,1)	(0.16,0,0.125)	(0.2,0,0.045)	(0.02,0,0.085)	(0.26,0,0.195)
(0.3,0,0.175)	(0.24,0,0.23)	(0.16,0,0.125)	(1,0,1)	(0.04,0,0.17)	(0.06,0,0.285)	(0.1,0,0.32)
(0.34,0,0.005)	(0.28,0,0.06)	(0.2,0,0.045)	(0.4,0.35,0.3)	(1,0,1)	(0.02,0,0.115)	(.060,0,0.15)
(0.36,0,0.11)	(0.3,0,0.055)	(0.02,0,0.085)	(0.06,0,0.285)	(0.02,0,0.115)	(1,0,1)	(0.04,0,0.035)
(0.4,0,0.145)	(0.34,0,0.09)	(0.26,0,0.195)	(0.1,0,0.32)	(0.06,0,0.15)	(0.04,0,0.035)	(1,0,1)

The matrix structure above represents the competitiveness among tech corporations.

6. **Comparative Analysis**

The competitive environment in the technology industry is sophisticated and dynamic, demanding the adoption of current analytical tools to understand the intricate interrelationships between leading businesses. Such competitive settings have been described using Generalized Neutrosophic Competition Graphs (GNCGs), which provide a complex representation using truth, falsity, and indeterminacy memberships. GNCGs, on the other hand, have the potential to oversimplify competition through their linear combination strategy. Key Performance Indicators (KPIs) such as market share and R&D expenditures are also used to evaluate competitive performance; however, due to their quantitative nature, they frequently fail to capture the full range of dynamics, excluding indeterminate factors such as market volatility and regulatory changes. The introduction of Generalized Fermatean Neutrosophic Competition Graphs (GFNCGs) represents a substantial advance. GFNCGs use a complex algorithm to score edges between vertices that better captures the reality of competitive dynamics by allowing for uncertainty. This method provides more detailed insights for strategic planning and decision-making. Compared to GNCGs and KPIs, GFNCGs give a more comprehensive view of the competitive environment by combining quantitative and qualitative metrics. This is vital in the rapidly changing technology industry, where understanding the link between market share growth, innovation investment, and external uncertainty is essential for maintaining a competitive edge. Thus, GFNCGs are superior tools for modeling competitive scenarios in the technology sector because they provide a balanced analytical framework that captures the complexities and unpredictable nature of industrial rivalry.

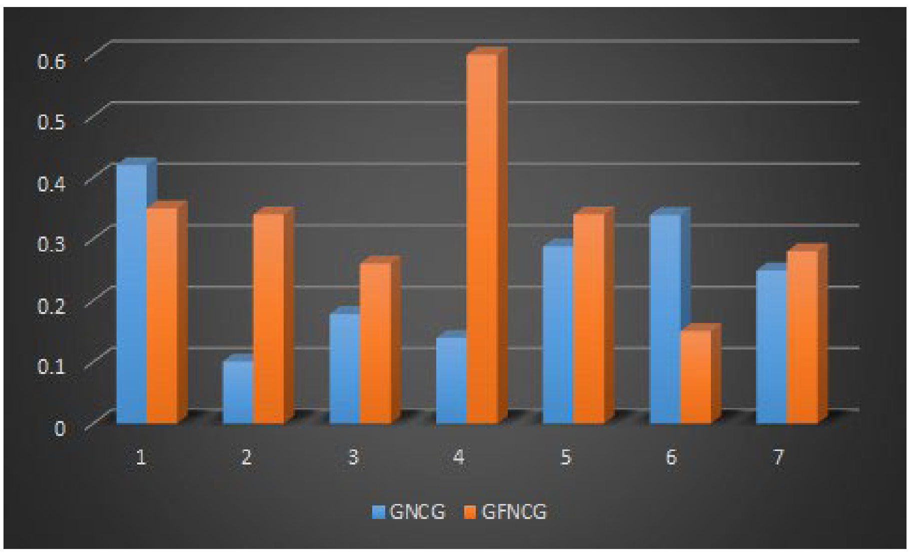


Figure 11. Comparision between GFNCG AND GNCG .

Conclusion

This work proposes a GFNCG that overcomes edge constraints. It depicts the GFNCG using a square matrix and explores concepts such as the minimal graph and competition number. In addition, the GFNCG framework is used to define a real-world application. In this application, nations' actual membership value is represented by their market share, whereas non-membership value is represented by the complement of their R&D spending. These criteria may be adjusted to capture different aspects of international competition, providing a useful perspective for studying real-world competitions. The research focuses on one-step competition, with plans to examine n-step fermatean neutrosophic competition graphs and other related concepts in the future. This study will serve as the foundation for subsequent research.

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Received: July 15, 2024. Accepted: September 07, 2024