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$\mathcal{N}g^{\#}-$ Irresolvable and Strongly $\mathcal{N}g^{\#}-$ Irresolvable Spaces in Neutrosophic Topological Spaces

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Abstract. The notion of breaking down a topological space into smaller, disjoint subsets is the basis for the topological concepts of 'resolvable space' and 'irresolvable space'. By synergizing topology and neutrosophy, we have introduced a new paradigm in neutrosophic topological spaces, represented by $\mathcal{N}g^{\#}-$ resolvable spaces, $\mathcal{N}g^{\#}-$ irresolvable spaces and strongly $\mathcal{N}g^{\#}-$ irresolvable spaces. This pioneering fusion enables the exploration of novel mathematical structures and their characterizations.

Keywords: $\mathcal{N}g^{\#}-$ closed set; $\mathcal{N}g^{\#}-$ dense set; $\mathcal{N}g^{\#}-$ Irresolvable spaces ; Strongly $\mathcal{N}g^{\#}-$ Irresolvable spaces

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1. Introduction

In our day to day life we come across uncertainties while dealing with real life problems in Science, Engineering, Business and Finance Management. Traditional mathematical methods failed to solve uncertainties. In order to overcome this difficulty, Zadeh [?] introduced the concept of fuzzy sets in 1965. In 1986, a generalization of fuzzy set was introduced by K.Atanassov [?] as intuitionistic fuzzy sets. The approaches such as fuzzy sets, intuitionistic fuzzy sets, vague sets and rough sets can be treated as Mathematical tools to avert obstacles dealing with ambiguous data. But all these approaches have their failures in solving the problems involving indeterminate and inconsistent data due to inadequacy of parametrization.

Neutrality the degree of indeterminacy as an independent concept was introduced by Florentine Smarandache [?].He defined the generalization of fuzzy sets and intuitionistic fuzzy sets as a Neutrosophic set on the three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership). Neutrosophic sets found its place into contemporary research since it helps to study indeterminacy in real life problems.

The concept of Neutrosophy has been developed into Neutrosophic topological spaces by Salama and Alblowi [?] in 2012. Additionally, they studied Neutrosophic continuity which has been shown to be quite significant in the realm of Neutrosophic topology.

Salama and Smarandache [?] initiated Neutrosophic closed sets and continuous functions.R. Dhavaseelan et.al. [?] introduced Neutrosophic generalized closed sets. Pious Missier et.al. [?], introduced the concept of $\mathcal{N}g^{\#}-$ closed and open sets in Neutrosophic Topological Spaces.

A topological space X is called resolvable if it can be expressed as the union of two or more disjoint, non-empty open subsets. In other words, there exist open sets A and B (with $A\cap B = \phi$) such that $X = A \cup B$. Otherwise, it is irresolvable. Resolvable spaces can be broken into non-overlapping open sets, whereas irresolvable spaces cannot be partitioned in a meaningful way within the given topology. Resolvable spaces and irresolvable spaces help topologists to classify and characterize topological spaces based on their "splitting" properties.(i.e.,) To distinguish between spaces that can be decomposed into disjoint open sets (resolvable) and those that cannot (irresolvable).

In this article we introduce $\mathcal{N}g^{\#}-$ resolvable spaces, and $\mathcal{N}g^{\#}-$ irresolvable spaces and strongly $\mathcal{N}g^{\#}-$ irresolvable spaces and analyse their characterizations.

2. **Preliminaries**

Definition 2.1. [?] A Neutrosophic set $\mathcal{NSA_N}$ is an object having the form $\mathcal{A_N} = \{\langle\lambda, \mu_{\mathcal{A_N}}(\lambda\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$. Here

(1) $\mu_{\mathcal{A_N}}(\lambda)-$ degree of membership
(2) $\sigma_{\mathcal{A_N}}(\lambda)-$ degree of indeterminacy
(3) $\gamma_{\mathcal{A_N}}(\lambda)-$ degree of non-membership

A Neutrosophic set $\mathcal{A_N} = \{\langle\lambda, \mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ can be identified as an ordered triple $\langle\mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle$ in $]-0, 1+[$ on $\mathcal{X_N}$.

Definition 2.2. [?] For any two Neutrosophic sets $\mathcal{A_N} = \{\langle\lambda, \mu_{\mathcal{A_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda), \gamma_{\mathcal{A_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ and $\mathcal{B_N} = \{\langle\lambda, \mu_{\mathcal{B_N}}(\lambda), \sigma_{\mathcal{B_N}}(\lambda), \gamma_{\mathcal{B_N}}(\lambda)\rangle : \lambda \in \mathcal{X_N}\}$ we have

(1) $\mathcal{A_N} \subseteq \mathcal{B_N} \iff \mu_{\mathcal{A_N}}(\lambda) \le \mu_{\mathcal{B_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda) \le \sigma_{\mathcal{B_N}}(\lambda)$ and $\gamma_{\mathcal{A_N}}(\lambda) \ge \gamma_{\mathcal{B_N}}(\lambda)$
(2) $\mathcal{A_N} \cap \mathcal{B_N} = \langle\lambda, \mu_{\mathcal{A_N}}(\lambda) \wedge \mu_{\mathcal{B_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda) \wedge \sigma_{\mathcal{B_N}}(\lambda)$ and $\gamma_{\mathcal{A_N}}(\lambda) \vee \gamma_{\mathcal{B_N}}(\lambda)\rangle$
(3) $\mathcal{A_N} \cup \mathcal{B_N} = \langle\lambda, \mu_{\mathcal{A_N}}(\lambda) \vee \mu_{\mathcal{B_N}}(\lambda), \sigma_{\mathcal{A_N}}(\lambda) \vee \sigma_{\mathcal{B_N}}(\lambda)$ and $\gamma_{\mathcal{A_N}}(\lambda) \wedge \gamma_{\mathcal{B_N}}(\lambda)\rangle$

Definition 2.3. [?] Let $\mathcal{A}_\mathcal{N} = \langle \mu_{\mathcal{A}_\mathcal{N}}(\lambda), \sigma_{\mathcal{A}_\mathcal{N}}(\lambda), \gamma_{\mathcal{A}_\mathcal{N}}(\lambda)\rangle$ be a $\mathcal{N}S$ on $\mathcal{X}_\mathcal{N}$, then the complement ${\mathcal{A}_\mathcal{N}}^c$ defined as

- ${\mathcal{A}_\mathcal{N}}^c = \{\langle \lambda, \gamma_{\mathcal{A}_\mathcal{N}}(\lambda), 1-\sigma_{\mathcal{A}_\mathcal{N}}(\lambda), \mu_{\mathcal{A}_\mathcal{N}}(\lambda)\rangle : \lambda \in \mathcal{X}_\mathcal{N}\}$

Note that for any two Neutrosophic sets $\mathcal{A}_\mathcal{N}$ and $\mathcal{B}_\mathcal{N}$,

- $(\mathcal{A}_\mathcal{N} \cup \mathcal{B}_\mathcal{N})^c = {\mathcal{A}_\mathcal{N}}^c \cap {\mathcal{B}_\mathcal{N}}^c$
- $(\mathcal{A}_\mathcal{N} \cap \mathcal{B}_\mathcal{N})^c = {\mathcal{A}_\mathcal{N}}^c \cup {\mathcal{B}_\mathcal{N}}^c$.

Definition 2.4. [?] A Neutrosophic topology ($\mathcal{NT}$) on a non-empty set $\mathcal{X}_\mathcal{N}$ is a family τ_N of Neutrosophic subsets in $\mathcal{X}_\mathcal{N}$ satisfies the following axioms:

(1) $\mathbf{0}_\mathcal{N}, \mathbf{1}_\mathcal{N} \in \tau_N$
(2) $\mathcal{R}_{N_1} \cap \mathcal{R}_{N_2} \in \tau_N$ for any $\mathcal{R}_{N_1}, R_{N_2} \in \tau_N$
(3) $\bigcup \mathcal{R}_{N_i} \in \tau_N \quad \forall \quad \mathcal{R}_{N_i} : i \in I \subseteq \tau_N$

Here the empty set $\mathbf{0}_\mathcal{N}$ and the whole set $\mathbf{1}_\mathcal{N}$ may be defined as follows:

(1) $\mathbf{0}_\mathcal{N} = \{\langle \lambda, 0, 0, 1\rangle : \lambda \in \mathcal{X}_\mathcal{N}\}$
(2) $\mathbf{1}_\mathcal{N} = \{\langle \lambda, 1, 1, 0\rangle : \lambda \in \mathcal{X}_\mathcal{N}\}$

Definition 2.5. [?] Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}_\mathcal{N}$. Then

(1) $\mathcal{N}int(\mathcal{A}_\mathcal{N}) = \bigcup\{\mathcal{G}_\mathcal{N} : \mathcal{G}_\mathcal{N}$ is a $\mathcal{NOS}$ in $\mathcal{X}_\mathcal{N}$ and $\mathcal{G}_\mathcal{N} \subseteq \mathcal{A}_\mathcal{N}\}$ is called a Neutrosophic interior of $\mathcal{A}_\mathcal{N}$.
(2) $\mathcal{N}cl(\mathcal{A}_\mathcal{N}) = \bigcap\{\mathcal{K}_\mathcal{N} : \mathcal{K}_\mathcal{N}$ is a $\mathcal{NCS}$ in $\mathcal{X}_\mathcal{N}$ and $\mathcal{A}_\mathcal{N} \subseteq \mathcal{K}_\mathcal{N}\}$ is called Neutrosophic closure of $\mathcal{A}_\mathcal{N}$.

Definition 2.6. [?] A Neutrosophic set $\mathcal{A}_\mathcal{N}$ of a $\mathcal{NTS}$ $(\mathcal{X}_\mathcal{N}, \tau_N)$ is called a neutrosophic $\mathcal{N}\alpha gCS$ if $\mathcal{N}\alpha cl(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{U}_\mathcal{N}$, whenever $\mathcal{A}_\mathcal{N} \subseteq \mathcal{U}_\mathcal{N}$ and $\mathcal{U}_\mathcal{N}$ is a $\mathcal{NOS}$ in $\mathcal{X}_\mathcal{N}$. The complement of $\mathcal{N}\alpha gCS$ is $\mathcal{N}\alpha gOS$.

Definition 2.7. [?]

A Neutrosophic set $\mathcal{A}_\mathcal{N}$ of a $\mathcal{NTS}$ $(\mathcal{X}_\mathcal{N}, \tau_N)$ is called a Neutrosophic $g^{\#}-$ closed ($\mathcal{N}g^{\#}CS$) if $\mathcal{N}cl(\mathcal{A}_\mathcal{N}) \subseteq \mathcal{Q}_\mathcal{N}$ whenever $\mathcal{A}_\mathcal{N} \subseteq \mathcal{Q}_\mathcal{N}$ and $\mathcal{Q}_\mathcal{N}$ is $\mathcal{N}\alpha gOS$ in $\mathcal{X}_\mathcal{N}$. The complement of $\mathcal{N}g^{\#}CS$ is $\mathcal{N}g^{\#}OS$.

Definition 2.8. [?] Let $\mathcal{A}_\mathcal{N}$ be a $\mathcal{NS}$ in $\mathcal{NTS}$ $\mathcal{X}_\mathcal{N}$. Then

(1) $\mathcal{N}g^{\#}int(\mathcal{A}_\mathcal{N}) = \bigcup\{\mathcal{G}_\mathcal{N} : \mathcal{G}_\mathcal{N}$ is a $\mathcal{N}g^{\#}OS$ in $\mathcal{X}_\mathcal{N}$ and $\mathcal{G}_\mathcal{N} \subseteq \mathcal{A}_\mathcal{N}\}$ is called a Neutrosophic $g^{\#}-$ interior of $\mathcal{A}_\mathcal{N}$.
(2) $\mathcal{N}g^{\#}cl(\mathcal{A}_\mathcal{N}) = \bigcap\{\mathcal{K}_\mathcal{N} : \mathcal{K}_\mathcal{N}$ is a $\mathcal{N}g^{\#}CS$ in $\mathcal{X}_\mathcal{N}$ and $\mathcal{A}_\mathcal{N} \subseteq \mathcal{K}_\mathcal{N}\}$ is called Neutrosophic $g^{\#}-$ closure of $\mathcal{A}_\mathcal{N}$.

Definition 2.9. [?] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be $\mathcal{N}g^{\#}-$ continuous function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}-$ closed set of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every neutrosophic closed set $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.10. [?] A function $f_{\mathcal{N}} : (\mathcal{X}_{\mathcal{N}}, \tau_N) \longrightarrow (\mathcal{Y}_{\mathcal{N}}, \zeta_N)$ is said to be Neutrosophic $g^{\#}-$ irresolute function if $f_{\mathcal{N}}^{-1}(\mathcal{V}_{\mathcal{N}})$ is a $\mathcal{N}g^{\#}CS$ of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ for every $\mathcal{N}g^{\#}CS$ $\mathcal{V}_{\mathcal{N}}$ of $(\mathcal{Y}_{\mathcal{N}}, \zeta_N)$.

Definition 2.11. [?] A Neutrosophic Topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called a $T_{\mathcal{N}}g^{\#}-$ space if every $\mathcal{N}g^{\#}CS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{NCS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.

Definition 2.12. [?] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called Neutrosophic dense set if there exists no $\mathcal{NCS}$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{A}_{\mathcal{N}} \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$.

Definition 2.13. [?] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called Neutrosophic nowhere dense set if there exists no non- zero $\mathcal{NOS}$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{B}_{\mathcal{N}} \subset \mathcal{N}cl(\mathcal{A}_{\mathcal{N}})$. That is $\mathcal{N}int(\mathcal{N}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$.

Definition 2.14. [?] A Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called $\mathcal{N}g^{\#}-$ dense set if there exists no $\mathcal{N}g^{\#}CS$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{A}_{\mathcal{N}} \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$. That is $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$.

3. $\mathcal{N}g^{\#}-$ **Irresolvable Spaces**

Definition 3.1. A Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called $\mathcal{N}g^{\#}-$ resolvable space if there exists a $\mathcal{N}g^{\#}-$ dense set $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathbf{1}_{\mathcal{N}}$. Otherwise $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called $\mathcal{N}g^{\#}-$ Irresolvable Space.

Example
3.2. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets, $\mathcal{M}_{\mathcal{N}_1} = \langle(0.4, 0.5, 0.4), (0.3, 0.6, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.4, 0.5, 0.4), (0.4, 0.4, 0.3)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.4, 0.4, 0.4), (0.5, 0.5, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.4, 0.6, 0.4), (0.4, 0.5, 0.5)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then, $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here, $\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}DS(\mathcal{X}_{\mathcal{N}}) = \{\mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{M}_{\mathcal{N}_3}$ is a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ and $\mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_3}^c) = \mathbf{1}_{\mathcal{N}}$. Hence, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ resolvable space.

Example 3.3. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets, $\mathcal{M}_{\mathcal{N}_1} = \langle(0.4, 0.5, 0.6), (0.4, 0.3, 0.5)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.6, 0.5, 0.4), (0.5, 0.7, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.6, 0.5, 0.3), (0.6, 0.7, 0.3)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.3, 0.5, 0.6), (0.3, 0.3, 0.6)\rangle$, $\mathcal{M}_{\mathcal{N}_5} = \langle(0.7, 0.5, 0.4), (0.6, 0.8, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_6} = \langle(0.4, 0.5, 0.7), (0.4, 0.2, 0.6)\rangle$. Now, $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathcal{M}_{\mathcal{N}_5}, \mathcal{M}_{\mathcal{N}_6}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then, $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here $\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) =$

$\{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_5}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}DS(\mathcal{X}_{\mathcal{N}}) = \{\mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_5}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{M}_{\mathcal{N}_3}$ and $\mathcal{M}_{\mathcal{N}_5}$ both are $\mathcal{N}g^{\#}-$ dense sets in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but $\mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_3}^c) = \mathcal{M}_{\mathcal{N}_2} \neq \mathbf{1}_{\mathcal{N}}, \mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_5}^c) = \mathcal{M}_{\mathcal{N}_2} \neq \mathbf{1}_{\mathcal{N}}$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is not a $\mathcal{N}g^{\#}-$ resolvable space and hence $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable space.

Theorem 3.4. *A $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ resolvable space if and only if $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ has a pair of $\mathcal{N}g^{\#}-$ dense sets $\mathcal{M}_1$ and $\mathcal{M}_2$ such that $\mathcal{M}_1 \subseteq {\mathcal{M}_2}^c$.*

Proof. Let $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ be a $\mathcal{N}g^{\#}-$ resolvable space. Suppose that for all $\mathcal{N}g^{\#}-$ dense sets $\mathcal{M}_i$ and $\mathcal{M}_j$, we have $\mathcal{M}_i \nsubseteq {\mathcal{M}_j}^c$. Then $\mathcal{M}_i \supset {\mathcal{M}_j}^c$. Then $\mathcal{N}g^{\#}cl(\mathcal{M}_i) \supset \mathcal{N}g^{\#}cl({\mathcal{M}_j}^c)$. $\implies \mathbf{1}_{\mathcal{N}} \supset \mathcal{N}g^{\#}cl({\mathcal{M}_j}^c)$. Then $\mathcal{N}g^{\#}cl({\mathcal{M}_j}^c) \neq \mathbf{1}_{\mathcal{N}}$. Also $\mathcal{M}_j \supset {\mathcal{M}_i}^c$, then $\mathcal{N}g^{\#}cl(\mathcal{M}_j) \supset \mathcal{N}g^{\#}cl({\mathcal{M}_i}^c)$. $\implies \mathbf{1}_{\mathcal{N}} \supset \mathcal{N}g^{\#}cl({\mathcal{M}_i}^c)$. Therfore, $\mathcal{N}g^{\#}cl({\mathcal{M}_i}^c) \neq \mathbf{1}_{\mathcal{N}}$. Since $\mathcal{M}_i$ is $\mathcal{N}g^{\#}-$ dense set and $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ resolvable, $\mathcal{N}g^{\#}cl(\mathcal{M}_i) = \mathbf{1}_{\mathcal{N}}$, but we got, $\mathcal{N}g^{\#}cl({\mathcal{M}_i}^c) \neq \mathbf{1}_{\mathcal{N}}$ for all Neutrosophic set $\mathcal{M}_i$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ which is a contradiction. Hence $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ has a pair of $\mathcal{N}g^{\#}-$ dense sets $\mathcal{M}_1$ and $\mathcal{M}_2$ such that $\mathcal{M}_1 \subseteq {\mathcal{M}_2}^c$.

Conversly, Suppose that the $\mathcal{NTS}$ $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ has a pair of $\mathcal{N}g^{\#}-$ dense sets $\mathcal{M}_1$ and $\mathcal{M}_2$ such that $\mathcal{M}_1 \subseteq {\mathcal{M}_2}^c$. Now Assume that $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ irresolvable space. Then for all $\mathcal{N}g^{\#}-$ dense sets $\mathcal{M}_1$ and $\mathcal{M}_2$, we have $\mathcal{N}g^{\#}cl({\mathcal{M}_1}^c) \neq \mathbf{1}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}cl({\mathcal{M}_2}^c) \neq \mathbf{1}_{\mathcal{N}}$. Which implies that there exists a $\mathcal{N}g^{\#}CS$ $\mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that ${\mathcal{M}_2}^c \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$. Now $\mathcal{M}_1 \subseteq {\mathcal{M}_2}^c \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}} \implies \mathcal{M}_1 \subset \mathcal{B}_{\mathcal{N}} \subset \mathbf{1}_{\mathcal{N}}$. That is $\mathcal{M}_1$ is not a $\mathcal{N}g^{\#}-$ dense set. Which is a contradiction. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ resolvable space. □

Theorem 3.5. *A Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ resolvable space if $\mathbf{1}_{\mathcal{N}} = \bigcup_{i=1}^{n} \mathcal{A}_i$ where $\mathcal{N}g^{\#}int(\mathcal{A}_i) = \mathbf{0}_{\mathcal{N}}$.*

Proof. Assume that $\mathbf{1}_{\mathcal{N}} = \bigcup_{i=1}^{n} \mathcal{A}_i$ where $\mathcal{N}g^{\#}int(\mathcal{A}_i) = \mathbf{0}_{\mathcal{N}}$. Which implies that $(\bigcup \mathcal{A}_i)^c = \mathbf{0}_{\mathcal{N}}$. Then $\bigcap_{i=1}^{n}(\mathcal{A}_i^c) = \mathbf{0}_{\mathcal{N}}$. Then there exists atleast two non- zero disjoint Neutrosophic sets $\mathcal{A}_i^c$ and $\mathcal{A}_j^c$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Hence $\mathcal{A}_i^c \cup \mathcal{A}_j^c \subseteq \mathbf{1}_{\mathcal{N}}$. Therefore, $\mathcal{A}_i^c \subseteq \mathcal{A}_j$ which implies that $\mathcal{N}g^{\#}cl(\mathcal{A}_i^c) \subseteq \mathcal{N}g^{\#}cl(\mathcal{A}_j)$. But $\mathcal{N}g^{\#}int(\mathcal{A}_i) = \mathbf{0}_{\mathcal{N}}$ implies that $\mathcal{N}g^{\#}cl(\mathcal{A}_i^c) = \mathbf{1}_{\mathcal{N}}$. Hence $\mathbf{1}_{\mathcal{N}} \subseteq \mathcal{N}g^{\#}cl(\mathcal{A}_j)$ which implies that $\mathcal{N}g^{\#}cl(\mathcal{A}_j) = \mathbf{1}_{\mathcal{N}}$. Also $\mathcal{N}g^{\#}int(\mathcal{A}_j) = \mathbf{0}_{\mathcal{N}}$ imples that $\mathcal{N}g^{\#}cl(\mathcal{A}_j^c) = \mathbf{1}_{\mathcal{N}}$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ has a $\mathcal{N}g^{\#}-$ dense set $\mathcal{A}_j$ such that $\mathcal{N}g^{\#}cl(\mathcal{A}_j^c) = \mathbf{1}_{\mathcal{N}}$. Hence $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ resolvable space. □

Theorem 3.6. *A Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ irresolvable space if and only if $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$ for all $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Since $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ irresolvable space, for all $\mathcal{N}g^{\#}-$ dense set $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N), \mathcal{N}g^{\#}cl({\mathcal{A}_{\mathcal{N}}}^c) \neq \mathbf{1}_{\mathcal{N}}$. Then $(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c \neq \mathbf{1}_{\mathcal{N}}$ which implies $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$.

Conversely, $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$, for all $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Suppose that $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ resolvable space. Then there exists a $\mathcal{N}g^{\#}-$ dense set $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathbf{1}_{\mathcal{N}}$. This implies that $(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}} \implies \mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Which is a contradiction. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable space. □

4. Strongly $\mathcal{N}g^{\#}-$ Irresolvable Spaces

Definition 4.1. A Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called strongly $\mathcal{N}g^{\#}-$ irresolvable space if for any $\mathcal{N}g^{\#}-$ dense set $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) = \mathbf{1}_{\mathcal{N}}$.

Example

4.2. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.6, 0.6, 0.3), (0.6, 0.6, 0.3)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.3, 0.4, 0.6), (0.3, 0.4, 0.6)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.3, 0.3, 0.7), (0.3, 0.3, 0.7)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.7, 0.7, 0.3), (0.7, 0.7, 0.3)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then, $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topology on $\mathcal{X}_{\mathcal{N}}$. Here, $\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}, \mathcal{N}g^{\#}DS(\mathcal{X}_{\mathcal{N}}) = \{\mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$. Now $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{M}_{\mathcal{N}_1})) = \mathbf{1}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{M}_{\mathcal{N}_4})) = \mathbf{1}_{\mathcal{N}}$ Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space.

Theorem 4.3. *If $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) \neq \mathbf{1}_{\mathcal{N}}$, for every Neutrosophic set $\mathcal{A}_{\mathcal{N}}$ in a strongly $\mathcal{N}g^{\#}-$ irresolvable space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$, then $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{1}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) \neq \mathbf{1}_{\mathcal{N}}$. Suppose that $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Since $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is strongly $\mathcal{N}g^{\#}-$ irresolvable space, $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$ implies that $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) = \mathbf{1}_{\mathcal{N}}$. This is a contradiction. Then it must be that $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{1}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. □

Theorem 4.4. *Let $(\mathcal{A}_{\mathcal{N}})$ be a $\mathcal{NS}$ in a strongly $\mathcal{N}g^{\#}-$ irresolvable space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. If $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$ then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}-$ nowhere dense set.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Then, $(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}$ which implies that $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathbf{1}_{\mathcal{N}}$. Since $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space, $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}{}^c)) = \mathbf{1}_{\mathcal{N}}$. $\implies \mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}$. $\implies (\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})))^c = \mathbf{1}_{\mathcal{N}}$. Which implies that, $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$. Hence proved. □

Theorem 4.5. *If $\mathcal{A}_{\mathcal{N}}$ is a $\mathcal{NS}$ in a strongly $\mathcal{N}g^{\#}-$ irresolvable space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) \neq \mathbf{0}_{\mathcal{N}}$ then $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in a strongly $\mathcal{N}g^{\#}-$ irresolvable space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) \neq \mathbf{0}_{\mathcal{N}}$. Suppose that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Now, $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = (\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}$.

$$\implies \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathbf{1}_{\mathcal{N}} \text{ (since } (\mathcal{X}_{\mathcal{N}}, \tau_N) \text{ is strongly } \mathcal{N}g^{\#} - \text{irresolvable space)}$$
$$\implies \mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}{}^c)) = \mathbf{1}_{\mathcal{N}}$$
$$\implies \mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}$$
$$\implies (\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})))^c = \mathbf{1}_{\mathcal{N}}$$
$$\implies \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$$

Which is contradiction to the hypothesis. Then it must be that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Hence proved. □

Theorem 4.6. *If $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space then $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ irresolvable space.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{NS}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ such that $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$. We claim that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$. Suppose that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$. Then $(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c = \mathbf{1}_{\mathcal{N}}$

$$\implies \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c) = \mathbf{1}_{\mathcal{N}}$$
$$\implies \mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}{}^c)) = \mathcal{N}g^{\#}int(\mathbf{1}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$$
$$\implies \mathcal{N}g^{\#}int((\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}))^c) = \mathbf{1}_{\mathcal{N}}$$
$$\implies (\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})))^c = \mathbf{1}_{\mathcal{N}}$$
$$\implies \mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) = \mathbf{0}_{\mathcal{N}}$$

This is contradiction to $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ being a strongly $\mathcal{N}g^{\#}-$ irresolvable space. Hence our assumption that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathbf{0}_{\mathcal{N}}$ is wrong. Hence we must have $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$ for all $\mathcal{N}g^{\#}-$ dense sets $\mathcal{A}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable space. □

Remark 4.7. The reverse implication of the above theorem is not true that is shown in the following example.

Example 4.8. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.4, 0.5, 0.6), (0.4, 0.3, 0.5)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.6, 0.5, 0.4), (0.5, 0.7, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.6, 0.5, 0.3), (0.6, 0.7, 0.3)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.3, 0.5, 0.6), (0.3, 0.3, 0.6)\rangle$, $\mathcal{M}_{\mathcal{N}_5} = \langle(0.7, 0.5, 0.4), (0.6, 0.8, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_6} = \langle(0.4, 0.5, 0.7), (0.4, 0.2, 0.6)\rangle$. Now, $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathcal{M}_{\mathcal{N}_5}, \mathcal{M}_{\mathcal{N}_6}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then, $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here $\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) =$

$\{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_5}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}OS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_4}, \mathcal{M}_{\mathcal{N}_6}, \mathbf{1}_{\mathcal{N}}\}$,$\mathcal{N}g^{\#}DS(\mathcal{X}_{\mathcal{N}}) = \{\mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_5}, \mathbf{1}_{\mathcal{N}}\}$. Here $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable space. But $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{M}_{\mathcal{N}_3})) \neq \mathbf{1}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{M}_{\mathcal{N}_5})) \neq \mathbf{1}_{\mathcal{N}}$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is not strongly $\mathcal{N}g^{\#}-$ irresolvable space.

Definition 4.9. A Neutrosophic topological space $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is called maximal $\mathcal{N}g^{\#}-$ irresolvable if $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable and every $\mathcal{N}g^{\#}-$ dense set of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ open set.

Example

4.10. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.4, 0.4, 0.5), (0.4, 0.5, 0.5)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.5, 0.6, 0.4), (0.5, 0.5, 0.4)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.7, 0.7, 0.3), (0.8, 0.8, 0.2)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.3, 0.3, 0.7), (0.2, 0.2, 0.8)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here, $\mathcal{N}g^{\#}CS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}OS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}DS(\mathcal{X}_{\mathcal{N}}) = \{\mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_3}, \mathbf{1}_{\mathcal{N}}\}$. Here, $\mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_1}^c) \neq \mathbf{1}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}cl(\mathcal{M}_{\mathcal{N}_3}^c) \neq \mathbf{1}_{\mathcal{N}}$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable space. Also, every $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}OS$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a maximal $\mathcal{N}g^{\#}-$ irresolvable space.

Theorem 4.11. *If $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a maximal $\mathcal{N}g^{\#}-$ irresolvable space then $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ be a $\mathcal{N}g^{\#}-$ dense set in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$. Since $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a maximal $\mathcal{N}g^{\#}-$ irresolvable space, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ irresolvable space and every $\mathcal{N}g^{\#}-$ dense set of $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is $\mathcal{N}g^{\#}-$ open set. Then $\mathcal{A}_{\mathcal{N}}$ is $\mathcal{N}g^{\#}OS$ which implies that $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) = \mathcal{A}_{\mathcal{N}}$. Now $\mathcal{N}g^{\#}cl(\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}})) = \mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$. Hence $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space. □

Remark 4.12. Converse of the above theorem need not be true.

Example

4.13. Let $\mathcal{X}_{\mathcal{N}} = \{p, q\}$. Consider the Neutrosophic sets $\mathcal{M}_{\mathcal{N}_1} = \langle(0.6, 0.6, 0.3), (0.6, 0.6, 0.3)\rangle$, $\mathcal{M}_{\mathcal{N}_2} = \langle(0.3, 0.4, 0.6), (0.3, 0.4, 0.6)\rangle$, $\mathcal{M}_{\mathcal{N}_3} = \langle(0.3, 0.3, 0.7), (0.3, 0.3, 0.7)\rangle$, $\mathcal{M}_{\mathcal{N}_4} = \langle(0.7, 0.7, 0.3), (0.7, 0.7, 0.3)\rangle$. Now $(\mathcal{X}_{\mathcal{N}}, \tau_N) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_2}, \mathcal{M}_{\mathcal{N}_3}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$ is Neutrosophic topological space. Then $\tau_N = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$ is $\mathcal{NT}$ on $\mathcal{X}_{\mathcal{N}}$. Here $\mathcal{N}g^{\#}OS(\mathcal{X}_{\mathcal{N}}) = \{\mathbf{0}_{\mathcal{N}}, \mathcal{M}_{\mathcal{N}_1}, \mathbf{1}_{\mathcal{N}}\}$, $\mathcal{N}g^{\#}DS(\mathcal{X}_{\mathcal{N}}) = \{\mathcal{M}_{\mathcal{N}_1}, \mathcal{M}_{\mathcal{N}_4}, \mathbf{1}_{\mathcal{N}}\}$. Then $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space. Here, $\mathcal{M}_{\mathcal{N}_4}$ is a $\mathcal{N}g^{\#}DS$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ but not a $\mathcal{N}g^{\#}OS$. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is not a maximal $\mathcal{N}g^{\#}-$irresolvable space.

Theorem 4.14. *Let $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ be a strongly $\mathcal{N}g^{\#}-$ irresolvable space. Then $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \subseteq (\mathcal{N}g^{\#}int(\mathcal{B}_{\mathcal{N}}))^c$ for any two $\mathcal{N}g^{\#}-$ dense sets $\mathcal{A}_{\mathcal{N}}, \mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$.*

Proof. Let $\mathcal{A}_{\mathcal{N}}$ and $\mathcal{B}_{\mathcal{N}}$ be any two $\mathcal{N}g^{\#}-$ dense sets in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. Then $\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}}) = \mathbf{1}_{\mathcal{N}}$ which implies that $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{A}_{\mathcal{N}})) = \mathbf{1}_{\mathcal{N}} \neq \mathbf{0}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}int(\mathcal{N}g^{\#}cl(\mathcal{B}_{\mathcal{N}})) = \mathbf{1}_{\mathcal{N}} \neq \mathbf{0}_{\mathcal{N}}$. Since $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a strongly $\mathcal{N}g^{\#}-$ irresolvable space, by Theorem **??**, we have $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$ and $\mathcal{N}g^{\#}int(\mathcal{B}_{\mathcal{N}}) \neq \mathbf{0}_{\mathcal{N}}$. We know that every strongly $\mathcal{N}g^{\#}-$ irresolvable space is $\mathcal{N}g^{\#}-$ irresolvable space. Therefore, $(\mathcal{X}_{\mathcal{N}}, \tau_N)$ is a $\mathcal{N}g^{\#}-$ irresolvable space. Then by Theorem **??**, we get $\mathcal{A}_{\mathcal{N}} \subseteq {\mathcal{B}_{\mathcal{N}}}^c$. Now $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \subseteq \mathcal{A}_{\mathcal{N}} \subseteq {\mathcal{B}_{\mathcal{N}}}^c \subseteq (\mathcal{N}g^{\#}int(\mathcal{B}_{\mathcal{N}}))^c$. Therefore, $\mathcal{N}g^{\#}int(\mathcal{A}_{\mathcal{N}}) \subseteq (\mathcal{N}g^{\#}int(\mathcal{B}_{\mathcal{N}}))^c$ for any two $\mathcal{N}g^{\#}-$ dense sets $\mathcal{A}_{\mathcal{N}}, \mathcal{B}_{\mathcal{N}}$ in $(\mathcal{X}_{\mathcal{N}}, \tau_N)$. □

Conclusion:

Our work combines the concepts of topology and neutrosophy, resulting in the development of $\mathcal{N}g^{\#}-$ resolvable and irresolvable spaces. We derived the maximal $\mathcal{N}g^{\#}-$ irresolvable space , a stronger form of $\mathcal{N}g^{\#}-$ irresolvable space which is also $\mathcal{N}g^{\#}-$ irresolvable space with the property that every $\mathcal{N}g^{\#}-$ dense set is $\mathcal{N}g^{\#}-$ open.Further, we proved that a Strongly $\mathcal{N}g^{\#}-$ irresolvable space is open heriditarily irresolvable.This article allows researchers to probe the decomposability of topological spaces and uncover their underlying structure,which promises to reveal new insights and theoretical applications in Neutrosophic topological spaces.

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