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Plithogenic Forest Hypersoft Sets in Plithogenic Contradiction Based Multi-Criteria Decision Making

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Abstract: Plithogenic based decision making approaches are more versatile and accommodative with regard to attribute handling This paper introduces the concepts of Plithogenic Forest hypersoft set (PFHS) and develops a Plithogenic centered decision-making model with PFHS representations. The Plithogenic method of devising decisions based on contradictions is integrated with the newly introduced representations of PFHS to develop a robust decision-making technique to deal with attributes and sub-attributive values at a larger scale. The integrated method proposed in this work is applied to a decision-making problem of site selection for establishing manufacturing plants. The core attributes are identified and the respective Plithogenic forest hypersoft sets are constructed with the possible sub-attributes. In this case, each of the core attributes itself forms a Plithogenic tree hypersoft set representations with several sub-attribute values, the alternatives are subjected to each of the criteria to determine the optimal ranking in specific to the criteria. Also, aggregate score values are determined to obtain a more comprehensive ranking. The concept of PFHS shall be integrated with other decision-making methods to evolve novel methods of decision-making.

Keywords: Plithogenic Forest Hypersoft sets, attributes, sub-attribute values, decision-making, site selection.

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1. Introduction

Smarandache [1] introduced the concept of hypersoft sets as an extension of soft sets to deal with attributes values subjected to each of the attributes say A1, A2,..An. The hypersoft set representations of the form Ai1, Aj2,..Akn are more compatible and comprehensive as they deal with various attribute values at a time whereas the soft sets deal with only a single attribute value at an instance.

The hypersoft sets shall be primarily classified into fuzzy, intuitionistic, neutrosophic based on the nature of the values indicating the magnitude of association between the attribute values and the elements of universe of discourse. The hypersoft sets are applied in several decision-making circumstances especially in ranking kind of multi-criteria decision-making problems. Researchers prefer these hypersoft sets of different types to handle the scenario involving several criteria or the attribute in designing optimal solution. Researchers have also discussed different forms of hypersoft sets such as Single-valued, multi-valued hypersoft sets[2], Bipolar Hypersoft sets[3], Picture Hypersoft sets[4], convex and concave hypersoft sets[5], N-hypersoft sets[6], bijective hypersoft sets[7] and many other as a means of extending the efficacy of hypersoft sets in data handling. Smarandache[8] extended hypersoft sets to Plithogenic hypersoft sets (PHS) in which the degree of appurtenance is presented to each of the attribute values with respect to the elements of the universe of discourse. Smarandache[20] has sketched out a vivid picture between Plithogenic soft sets and Plithogenic hypersoft sets. Plithogenic hypersoft sets are also applied in decision making to develop a more comprehensive solution to the decision-making problems. Researchers have developed PHS based decision models in different domains, To mention a few significant works, Martin and Smarandache[9] presented the applications of combined plithogenic hypersoft sets. Martin et al [10, 13] explored the applications of extended plithogenic hypersoft sets in Covid-19 decision making. Priya et al[11,12] induced the plithogenic cognitive analysis with combined connection. Martin and Smarandache[14] leveraged the notion of concentric plithogenic hypergraph embedded with Plithogenic hypersoft sets in decision making. Ahmad et al[15,17] formulated a multi-criteria decision-making model using plithogenic hypersoft sets. Rana et al[16,18] introduced plithogenic fuzzy whole hypersoft sets and generalized plithogenic whole hypersoft sets and applied multi-attribute decision making diagnostic models. Majid et al[19] formulated a decision model for site selection using plithogenic multipolar fuzzy hypersoft sets. These recent contributions substantiate the proficiency of plithogenic hypersoft sets in making optimal decisions.

Smarandache[21] also introduced few types of soft sets such as indeterm soft sets, indeterm hypersoft sets and tree soft sets. In the indeterm hypersoft sets, either any of the attributes deal with indeterminate values. In case of tree soft sets, the attributes that are considered form a tree structure with root attribute at level 0 and branches indicating the attribute values and attribute sub-values in the subsequent levels. However, in considering the attribute values, the tree soft set representations reflect tree hypersoft sets. This uniqueness of tree soft set has motivated the authors to evolve the concept of Plithogenic forest hypersoft sets (PFHS) which is a union of several Plithogenic tree soft sets (PTSS). The proposed notion of PFHS based representations is employed in making decisions on site selection on integrating with contradiction based Plithogenic decision method.

The remaining contents of the paper are structured into different sections as follows. Section 2 presents the conceptualization of Plithogenic Forest Hypersoft sets. Section 3 describes the decision-making model framework applied in this research work. Section 4 applies the model to the decision-making problem of site selection of manufacturing plants. Section 5 discusses the results and the last section concludes the work.

2. Theoretical Development of Plithogenic Forest Hypersoft Sets (PFHS)

This section presents the conceptualization of Forest Hypersoft sets and then describes the extension of the same to PFHS.

Let U be the universe of discourse, H be the non-empty subset of U, A be the set of attributes

Each of the attributes has different levels

Level 1 be the sub attribute values

Level 2 be the sub-sub attribute values:

Level n be the n-sub attribute values

Each of the attributes forms a Tree soft sets and all these tree soft sets together form a forest hypersoft sets.

The Forest Hypersoft Set shall be defined as G : P(Forest (A)) → P(H)

Where Forest (A) = {Tree (A)} and Tree (A) = {Ai1 | i1= 1, 2, ... }

Let us first discuss the construction of forest hypersoft sets with a simple example. Let us consider a decision-making problem on supplier selection based on different attributes say A1,A2,A3. In this case, the attributes A1, A2 and A3 forms the root level. Each of the attributes has different levels where each level indicates the sub attribute values. Thus, each attribute with its respective sets of sub attribute values together forms a tree soft set. In this case three tree soft sets are obtained for three different attributes. The union of these three tree soft sets together form a forest hypersoft sets.

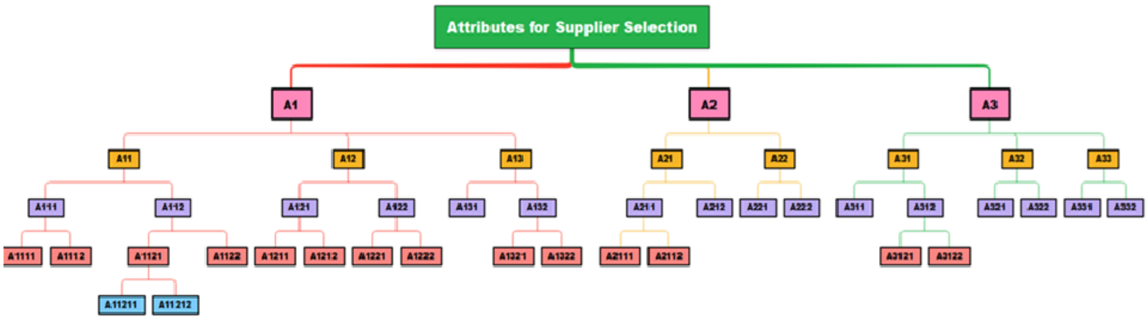

Fig.1. Plithogenic Forest Hypersoft sets

From the above fig.1., the attributes A1,A2 and A3 form the root level say L0. The next level L1 is subjected to each of the attributes which consists of the attribute values say for $A1$, the attribute sub values are A11, A12, A13 and for A2 it is, A21, A22 are the attribute values and then for A3 it is A31, A32 and A33. The level L2, consists of sub - sub – attribute values (sub_2 – attribute values) say for A11, the A111 and A1112 are the respective sub-attribute values however they are the sub_2 – attribute values of A1. Similarly, the attribute values A1111 and A1112 are the sub attribute values of A111 and sub_2 – attribute values of A11 and sub_3 – attribute values of A1. Also, A11211 and A11212 are the sub-attribute values of A1121, sub_2 – attribute values of A112, sub_3 – attribute values of A11 and sub_4 – attribute values of A1. A similar kind of discussion shall be made for the attributes A2 and A3.

If suppose the attribute A1 is considered for ranking the suppliers then, the attribute values from each of the sub$_n$ attribute values has to be chosen and this will be of the form F (A1112 × A11212 × A1211 × A1221 × A131 × A1322), this expression is derived from the representations of tree soft sets. If the decision maker considers the attribute A1, then the attribute values say A11, A12 and A13 have to be considered. In this case, each of the attribute values form a tree structure and hence ultimately at the end only the attribute values represented in last levels shall be taken for decision-making.

3. Classification of Forest Hypersoft sets

The forest hypersoft sets shall be classified generally as fuzzy forest hypersoft sets, intuitionistic forest hypersoft sets and neutrosophic hypersoft sets based on the membership values representing the degree of satisfaction, the alternatives make with the attribute values taken for consideration.

Example

Let the set of suppliers be {S1,S2,..Sn} and let us consider a set say H = {S1,S2,S3,S4}. Then let us consider the attribute values A1112 ,A11212 , A1211, A1221, A131 and A1322. Then Forest Hypersoft set is of the form F (A1112 × A11212 × A1211 × A1221 × A131 × A1322) = { S1 (0.2),S2 (0.5), S3 (0.5), S4 (0.7)} is termed as fuzzy forest hypersoft set.

F (A1112 × A11212 × A1211 × A1221 × A131 × A1322) = { S1 (0.2,0.7),S2 (0.5,0.4), S3 (0.5,0.3), S4 (0.7,0.2)} is termed as intuitionistic forest hypersoft set.

F (A1112 × A11212 × A1211 × A1221 × A131 × A1322) = { S1 (0.2,0.1,0.6),S2 (0.5,,0.2,0.4), S3 (0.5,0.3,0.4), S4 (0.7,0.2,0.1)} is termed as neutrosophic forest hypersoft set.

However, in case of Plithogenic forest hypersoft sets, the dominant attribute values are initially assigned and the contradiction degrees between the attribute values are determined. Then the degree of appurtenance of the alternative with respect to the dominant attribute values are determined and the contradiction degree are also considered in making decisions. For simplification, the plithogenic forest hypersoft sets are represented in a tabular form. Also, the Plithogenic forest hypersoft sets shall be classified based on their degree of appurtenance.

Table 1. Classification of Plithogenic forest hypersoft sets

Crisp	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	1	1	1	1	1	1
S2	1	1	1	1	1	1
S3	1	1	1	1	1	1
S4	1	1	1	1	1	1
S5	1	1	1	1	1	1
Fuzzy	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	0.2	0.4	0.5	0.3	0.7	0.8
S2	0.1	0.7	0.9	0.4	0.6	0.4
S3	0.5	0.6	0.3	0.2	0.7	0.1

S4	0.4	0.6	0.7	0.6	0.4	0.5
S5	0.6	0.4	0.2	0.7	0.5	0.8
Intuitionistic	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	(0.2, 0.8)	(0.4, 0.6)	(0.5, 0.5)	(0.3, 0.7)	(0.7, 0.3)	(0.8, 0.2)
S2	(0.1, 0.9)	(0.7, 0.3)	(0.9, 0.1)	(0.4, 0.6)	(0.6, 0.4)	(0.4, 0.6)
S3	(0.1, 0.9)	(0.7, 0.3)	(0.9, 0.1)	(0.4, 0.6)	(0.6, 0.4)	(0.4, 0.6)
S4	(0.4, 0.6)	(0.6, 0.4)	(0.7, 0.3)	(0.6, 0.4)	(0.4, 0.6)	(0.5, 0.5)
S5	(0.6, 0.4)	(0.4, 0.6)	(0.2, 0.8)	(0.7, 0.3)	(0.5, 0.5)	(0.8, 0.2)
Neutrosophic	**A1112**	**A11212**	**A1211**	**A1221**	**A131**	**A1322**
S1	(0.2,0.2,0.7)	(0.4,0.1,0.5)	(0.5,0.1,0.4)	(0.3,0.2,0.6)	(0.7,0.1,0.2)	(0.8,0.01,0.1)
S2	(0.1,0.15,0.8)	(0.7,0.20,0.2)	(0.9,0.1,0.0)	(0.4,0.1,0.5)	(0.6,0.2,0.3)	(0.4, 0.1, 0.5)
S3	(0.5, 0.1, 0.4)	(0.6, 0.2, 0.3)	(0.3,0.1,0.6)	(0.2,0.1,0.7)	(0.7,0.1,0.2)	(0.1, 0.1, 0.8)
S4	(0.4, 0.1, 0.5)	(0.6, 0.1, 0.3)	(0.7,0.2,0.2)	(0.6,0.1,0.3)	(0.4,0.1,0.5)	(0.5, 0.1, 0.4)
S5	(0.6, 0.1, 0.3)	(0.4, 0.1, 0.5)	(0.2,0.1,0.7)	(0.7,0.1,0.2)	(0.5,0.1,0.4)	(0.8, 0.1, 0.1)

In the above table 1., the values in each cell indicate the degrees of appurtenance of the suppliers with each of the dominant attribute values. The above table demonstrates the classifications of Plithogenic forest hypersoft sets into crisp, fuzzy, intuitionistic and neutrosophic based on the appurtenance values. In other representations of Forest Hypersoft sets the appurtenance degree reflect all the attribute values whereas in case of Plithogenic Forest Hypersoft sets, the appurtenance degree is considered for each of the attribute values.

Modelling Framework of Plithogenic Forest Hypersoft sets based Decision Making [22]

This section outlines the working procedure of the Plithogenic contradiction-based decision method with the representations of Plithogenic Forest Hypersoft sets.

Step 1: The decision making ecosystem is well defined by determining the alternatives say S1, S2,..Sn and attributes with sub_n attribute values. The dominant attribute values are identified and classified as benefit and non-benefit.

Step 2: The initial contradiction matrix considering the alternatives and the dominant attribute values is constructed from the attribute matrix with each cell representing the attribute value of the alternatives. The qualitative attribute matrix is then converted to the contradiction matrix by calculating the degree of contradiction existing between the attribute value of the alternative and the dominant attribute value.

	A_{1i}	A_{2j}	…..	…..	A_{nh}
S_1	A_{11}	A_{23}	…..	…..	A_{n4}
S_2	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
S_n	A_{14}	A_{21}	…..	…..	A_{n2}

The above matrix representation is the attribute matrix which is constructed by considering the attribute values possessed by the alternatives in par with the dominant attribute values. The respective contradiction matrix is drawn by considering the degree of contradiction between the attribute values in the matrix and the dominant attribute values.

	A_{1i}	A_{2j}	…..	…..	A_{nh}
S_1	$C(A_{11}, A_{1i})$	$C(A_{23}, A_{2j})$	…..	…..	$C(A_{n4}, A_{nh})$
S_2	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
S_n	$C(A_{14}, A_{1i})$	$C(A_{21}, A_{2j})$	…..	…..	$C(A_{n2}, A_{nh})$

The above is the contradiction matrix obtained from the above attribute matrix, where each of the cell values represents the contradiction degrees between the dominant attribute values and the actual attribute value of the alternatives

Step 3: The weighted contradiction matrix is calculated by multiplying the attribute value weights with the contradiction degrees.

	A_{1i}	A_{2j}	…..	…..	A_{nh}
S_1	$w_1C(A_{11},A_{1i})$	$w_2C(A_{23}, A_{2j})$	…..	…..	$w_nC(A_{n4},A_{nh})$
S_2	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
:	…..	…..	…..	…..	…..
S_n	$w_1C(A_{14},A_{1i})$	$w_2C(A_{21}, A_{2j})$	…..	…..	$w_nC(A_{n2},A_{nh})$

Step 4: The cumulative score values of the alternatives with respect to the benefit nature of attribute values and the cost nature of attribute values are determined. The cumulative score values of the benefit nature of attribute values are denoted by B_q and the cumulative score values of the cost nature of attribute values are denoted by C_k.

Step 5: The alternatives are ranked based on the differences between these benefit and non-benefit attribute values i.e B_q - $C_k$and the highest rankings are assigned to the alternatives with maximum differences of values.

The above working procedure shall be characterized into three major phased process. The first phase comprises defining decision making ecosystem, construction of attribute matrix and contradiction matrix. The second phase consists of steps 3 and 4 and the last phase consists of step 5.

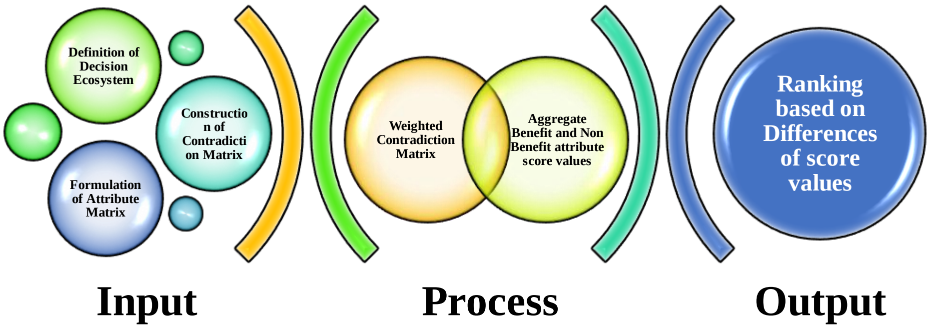

Fig.2. Process of three major phased

4. Application of the Proposed Modeling Framework in site selection

This section presents the modality of applying Plithogenic contradictions-based decision making with Plithogenic hypersoft sets in site selection for establishing manufacturing plants say P1,P2,P3,P4,P5. The core attributes considered in general are presented in Fig.3.

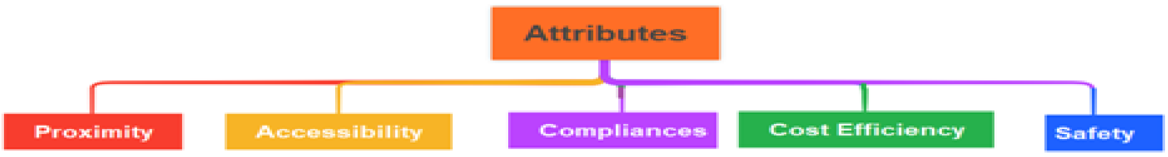

Fig.3. Classification of Attributes

The core attributes occupy the root level of the forest and each of the attributes takes a form of tree hypersoft sets and it is presented as follows.

Firstly, the attribute Proximity is considered. The following figure 4, represents the tree representations of the attribute Proximity and its sub attribute values at different levels.

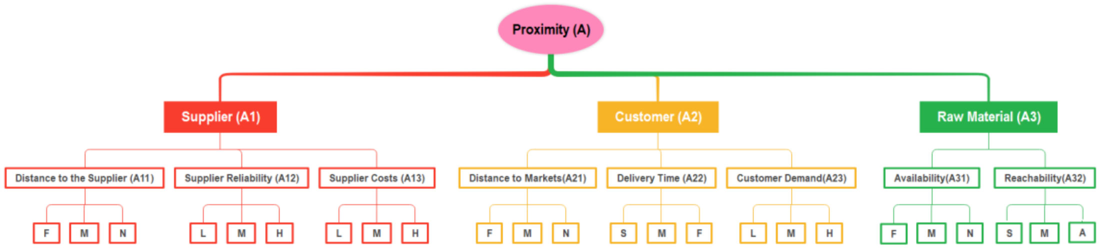

Fig.4.Proximity

The description of the attributes in the above fig.4. is sketched out in the following table 2.

Table 2. Description of the attributes

Attribute	**Sub-Attribute values**	**Sub2 Attribute Values (Sub-sub Attribute Values)**	**Sub3Attribute Values (Sub-sub – sub–Attribute Values)**
Proximity (A)	Supplier (A1)	Distance to the Supplier (A11)	Far (A111) Moderate (A112) **Near (A113)**
		Supplier Reliability (A12)	Low (A121) Moderate (A122) **High (A123)**
		Supplier Costs (A13)	**Low (A131)** Moderate (A132) High (A133)
	Customer (A2)	Distance to Markets (A21)	Far (A211) Moderate (A212) **Near (A213)**
		Delivery Time (A22)	Slow (A221) Moderate (A222) **Fast (A223)**
		Customer Demand (A23)	Low (A231) Moderate (A232) **High (A233)**
	Raw Material (A3)	Proximity to Sources (A31)	Far (A311) Moderate (A312) **Near (A313)**
		Raw material Availability (A32)	Scare (A321) Moderate (A322) **Abundant (A323)**

If the criteria proximity is alone considered the dominant sub-sub-sub attribute values have to be considered which are {Near, High, Low, Near, Fast, High, Near, Abundant}

Let us label it as P(A1111× A1123×A1131×A1213×A1223×A1233×A1313× A1323)

Table 3. Attribute matrix

Attributive Values	P1	P2	P3	P4	P5
A1133	Far	Far	Moderate	Near	Near
A1233	Low	Moderate	High	Low	Low
A1311	Moderate	Low	Moderate	High	Low
A2133	Near	Moderate	Fast	Moderate	Moderate
A2233	Moderate	Fast	Fast	Slow	Slow
A2333	High	Low	Low	Moderate	High
A3133	Scarce	Abundant	Moderate	Abundant	Scarce
A3233	Moderate	Near	Far	Far	Far

The above table 3, represents the attribute matrix.

The contradiction degrees with respect to the dominant attribute values are presented in Table 4.

Table 4. Contradiction value

Dominant Attribute Value	Contradiction Degree
A113	C(A111,A113) =2/3, C(A112,A113) =1/3
A123	C(A121,A123) =2/3, C(A122,A123) =1/3
A131	C(A132,A131) =1/3, C(A133,A131) =2/3
A213	C(A211,A213) =2/3, C(A212, A213)= 1/3
A223	C(A221,A223) =2/3, C(A222,A223) =1/3
A233	C(A231,A233) =2/3, C(A232, A233) =1/3
A313	C(A311, A313)=2/3, C(A312,A313)=1/3
A323	C(A321,A323) =2/3, C(A322,A323)= 1/3

Based on the contradiction values presented in the above Table 4, the respective contradiction matrix is as follows.

Table 5. Contradiction matrix

Attributive Values	P1	P2	P3	P4	P5
A1133 (C)	2/3	2/3	1/3	0	0

Attributive Values	P1	P2	P3	P4	P5
A1233 (B)	2/3	1/3	0	2/3	2/3
A1311 (C)	1/3	0	1/3	2/3	0
A2133 (C)	0	1/3	2/3	1/3	1/3
A2233 (C)	1/3	0	0	2/3	2/3
A2333 (B)	0	2/3	2/3	1/3	0
A3133 (C)	2/3	0	1/3	0	2/3
A3233 (B)	1/3	0	2/3	2/3	2/3

In the above matrix, the contradiction values of the attribute values with respect to the dominant attribute values are presented in the contradiction matrix. The weighted contradiction matrix is determined by considering equal weightage to all the dominant attribute values.

Table 6. Weighted contradiction matrix

Alternatives	A1133 (C)	A1233 (B)	A1311 (C)	A2133 (C)	A2233 (C)	A2333 (B)	A3133 (C)	A3233 (B)
P1	0.08333	0.08333	0.041667	0	0.041667	0	0.08333	0.041667
P2	0.08333	0.041667	0	0.041667	0	0.08333	0	0
P3	0.041667	0	0.041667	0.08333	0	0.08333	0.04167	0.08333
P4	0	0.08333	0.08333	0.041667	0.08333	0.04167	0	0.08333
P5	0	0.08333	0	0.041667	0.08333	0	0.08333	0.08333

By using step 4 and step 5, the score values of the alternatives are determined and presented in Table 7.

Table 7. Differences in Score values

Alternatives	B_A	C_A	$B_A - C_A$
P1	0.124997	0.249994	-0.124997
P2	0.125	0.124997	3E-06
P3	0.16666	0.208337	-0.04168
P4	0.20833	0.208327	3E-06
P5	0.16666	0.208327	-0.04167

Secondly, the attribute Accessibility is considered. The following figure 5. represents the tree representations of the attribute Accessibility and its sub attribute values at different levels.

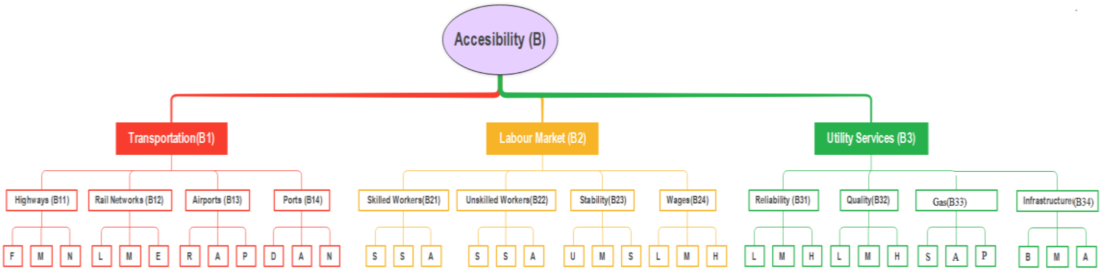

Fig.5. Accessibility

The description of the attributes in the above fig.5. is sketched out in the following table 8.

Table 8. Description of the attributes

Attribute	**Sub-Attribute values**	**Sub2 Attribute Values (Sub-sub–Attribute Values)**	**Sub3Attribute Values (Sub-sub – sub–Attribute Values)**
Accessibility (B)	Transportation (B1)	Proximity to Highways (B11)	Far (B111) Moderate (B112) **Near (B113)**
		Access to Rail Networks (B12)	Limited (B121) Moderate (B122) **Extensive (B123)**
		Access to Airports (B13)	Remote (B131) Accessible (B132) **Proximate (B133)**
		Access to Ports (B14)	Distant (B141) Accessible (B142) **Nearby (B143)**
	Labour Market (B2)	Availability to Skilled Workers (B21)	Scare (B211) Sufficient (B212) **Abundant (B213)**
		Availability to Unskilled Workers (B22)	Scare (B221) Sufficient (B222) **Abundant (B223)**
		Labor Market Stability (B23)	Unstable (B231) Moderate (B232) **Stable (B233)**
		Wage Labour (B24)	**Low (B241)** Moderate (B242) High (B243)

	Utility Services (B3)	Reliability of Electricity (B31)	Unreliable (B311) Moderate (B312) **Reliable (B313)**
		Availability and Quality of Water (B32)	Limited (B321) Adequate (B322) **Excellent (B323)**
		Availability of Gas (B33)	Scare(B331) Available (B332) **Plentiful (B333)**
		Telecommunications Infrastructure (B34)	Basic (B341) Moderate (B342) **Advanced(B343)**

If the criteria Accessibility is alone considered the dominant (sub_3) sub-sub-sub attribute values to be considered are {Near, Extensive, Proximate, Nearby, Abundant, Stable, Low, Reliable, Excellent, Plentiful, Advanced}

Let us consider the representation of the form

P(B1133xB1233xB1333xB1433xB2133xB2233xB233xB2411xB313xB3233xB3333xB3433)

The attribute matrix with respect to the attribute Accessibility is presented in Table 9.

Table 9. Attribute matrix

Attributive Values	P1	P2	P3	P4	P5
B1133	Near	Far	Moderate	Near	Far
B1233	Limited	Moderate	Limited	Extensive	Moderate
B1333	Accessible	Proximate	Proximate	Remote	Accessible
B1433	Nearby	Accessible	Distant	Nearby	Accessible
B2133	Scare	Abundant	Sufficient	Abundant	Scare
B2233	Sufficient	Scare	Scare	Sufficient	Abundant
B2333	Stable	Unstable	Stable	Unstable	Moderate
B2411	Moderate	Low	High	Moderate	High
B3133	Reliable	Moderate	Unreliable	Moderate	Unreliable
B3233	Adequate	Excellent	Limited	Adequate	Excellent
B3333	Plentiful	Available	Scare	Plentiful	Scare
B3433	Basic	Advanced	Basic	Moderate	Advanced

The contradiction degrees with respect to the dominant attribute values are presented in Table 10.

Table 10. Contradiction value

Dominant Attribute Value	**Contradiction Degree**
B113	C(B111, B113) = 2/3, C(B112, B113) =1/3
B123	C(B121, B123) = 2/3, C(B122, B123) =1/3
B133	C(B131, B133) =2/3, C(B132, B133) =1/3
B143	C(B141, B143) = 2/3, C(B142, B143) =1/3
B213	C(B211, B213) = 2/3, C(B212, B213) =1/3
B223	C(B221, B223) = 2/3, C(B222, B223) =1/3
B233	C(B231, B233) = 2/3, C(B232, B233) =1/3
B241	C(B242, B241) =1/3, C(B243, B241) =2/3
B313	C(B311, B313) =2/3, C(B312, B313) =1/3
B323	C(B321, B323) =2/3, C(B322, B323) =1/3
B333	C(B331, B333) = 2/3, C(B332, B333) =1/3
B343	C(B341, B343) =2/3, C(B412, B343) =1/3

The respective contradiction matrix is as follows and presened in Table 11.

Table 11. Contradiction matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
B1133 (C)	0	2/3	1/3	0	2/3
B1233(C)	2/3	1/3	2/3	0	1/3
B1333 (C)	1/3	0	0	2/3	1/3
B1433 (C)	0	1/3	2/3	0	1/3
B2133(B)	2/3	0	1/3	0	2/3
B2233(B)	1/3	2/3	2/3	1/3	0
B2333(B)	0	2/3	0	2/3	1/3
B2411(C)	1/3	0	2/3	1/3	2/3
B3133(B)	0	1/3	2/3	1/3	2/3
B3233(B)	1/3	0	2/3	1/3	0
B3333(B)	0	1/3	2/3	0	2/3
B3433(B)	2/3	0	2/3	1/3	0

The weighted contradiction matrix is as follows

Table 12. Weighted contradiction matrix

Alternatives	B113 (C)	B123 (C)	B133 (C)	B143 (C)	B213 (B)	B223 (B)	B233 (B)	B241 (C)	B313 (B)	B323 (B)	B333 (B)	B343 (B)
P1	0	0.05556	0.027778	0	0.05556	0.027778	0	0.027778	0	0.027778	0	0.05556
P2	0.05556	0.027778	0	0.027778	0	0.05556	0.05556	0	0.027778	0	0.027778	0
P3	0.027778	0.05556	0	0.05556	0.027778	0.05556	0	0.05556	0.05556	0.05556	0.05556	0.05556
P4	0	0	0.05556	0	0	0.027778	0.05556	0.027778	0.027778	0.027778	0	0.027778
P5	0.05556	0.027778	0.027778	0.027778	0.05556	0	0.027778	0.05556	0.05556	0	0.05556	0

By following the similar procedure, the score values are presented in Table 13

Table 13. Difference in Score values

Alternatives	B_B	C_B	B_B- C_B
P1	0.111116	0.166676	0.05556
P2	0.111116	0.166676	0.05556
P3	0.194458	0.305578	0.11112
P4	0.083338	0.166672	0.083334
P5	0.194454	0.194458	4E-06

Thirdly the attribute Compliances is taken into account. Fig.6., represents the tree representations of the attribute

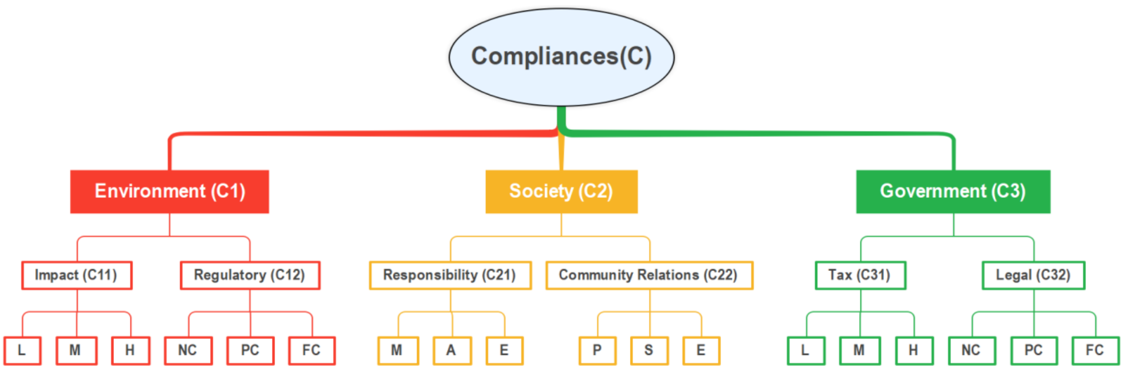

Fig.6. Compliances

The description of the attributes in the above fig.6. is sketched out in the following table 14.

Table 14. Description of attributes

Attribute	Sub-Attribute values	Sub2 Attribute Values (Sub-sub Attribute Values)	Sub3Attribute Values (Sub-sub – sub–Attribute Values)
Compliances (C)	Environment (C1)	Environmental Impact (C11)	**Low (C111)** Moderate (C112) High (C113)
		Regulatory Compliance (C12)	Non-Compliant (C121) Partially Compliant (C122) **Fully Compliant (C123)**
	Society (C2)	Social Responsibility (C21)	Minimal (C211) Adequate (C212) **Extensive(C213)**
		Community Relations (C22)	Poor (C221) Satisfactory (C222) **Excellent (C223)**
	Government (C3)	Tax Compliance (C31)	Low(C311) Moderate (C312), **High (C313)**
		Legal Compliance (C32)	Non- Compliant (C321) Partially Compliant (C322) **Fully Compliant (C323)**

If the criteria Compliances is alone considered the dominant sub-sub-sub attribute values to be considered are {Low, Fully Compliant, Extensive, Excellent, High, Fully Compliant}

Let us consider the representation of the form P(C1111xC1233xC2133xC2233xC3133xC3233)

The attribute matrix with respect to the attribute Compliances is presented in Table 15.

Table 15. Attribute matrix

Attributive Values	P1	P2	P3	P4	P5
C1111	High	Low	Moderate	High	Moderate
C1233	Partially Compliant	Non-Compliant	Non-Compliant	Partially Compliant	Fully Compliant
C2133	Extensive	Adequate	Minimal	Adequate	Minimal
C2233	Poor	Satisfactory	Excellent	Satisfactory	Excellent

Attributive Values	P1	P2	P3	P4	P5
C3133	Moderate	High	Low	Moderate	High
C3233	Non-Compliant	Partially Compliant	Fully Compliant	Non-Compliant	Fully Compliant

The contradiction degrees with respect to the dominant attribute values are presented in Table 16.

Table 16. Contradiction value

Dominant Attribute Value	Contradiction Degree
C111	C(C112, C111) = 1/3, C(C113, C111) = 2/3
C123	C(C121, C123) = 2/3, C(C122, C123) = 1/3
C213	C(C211, C213) = 2/3, C(C212 ,C213) = 1/3
C223	C(C221, C223)= 2/3, C(C222, C223) = 1/3
C313	C(C311,C313) = 2/3, C(C312, C313) = 1/3
C323	C(C321, C323) = 2/3 , C(C322, C323) = 1/3

The respective contradiction matrix is as follows and presented in Table 17.

Table 17. Contradiction matrix

Attributive Values	P1	P2	P3	P4	P5
C1111(C)	2/3	0	1/3	2/3	1/3
C1233(B)	1/3	2/3	2/3	1/3	0
C2133(B)	0	1/3	2/3	1/3	2/3
C2233(B)	2/3	1/3	0	1/3	0
C3133(C)	1/3	0	2/3	1/3	0
C3233(C)	2/3	1/3	0	2/3	0

The weighted contradiction matrix is as follows,

Table 18. Weighted contradiction matrix

Alternatives	C1111(C)	C1233(B)	C2133(B)	C2233(B)	C3133(C)	C3233(C)
P1	0.11111	0.055556	0	0.11111	0.055556	0.11111
P2	0	0.11111	0.055556	0.055556	0	0.055556

Alternatives	C1111(C)	C1233(B)	C2133(B)	C2233(B)	C3133(C)	C3233(C)
P3	0.055556	0.11111	0.11111	0	0.11111	0
P4	0.11111	0.055556	0.055556	0.055556	0.055556	0.11111
P5	0.055556	0	0.11111	0	0	0

By following the similar procedure, the score values are obtained and presented in Table 19.

Table 19. Difference in Score values

Alternatives	**Bc**	**Cc**	**Bc- Cc**
P1	0.166666	0.277776	-0.11111
P2	0.222222	0.055556	0.166666
P3	0.22222	0.166666	0.055554
P4	0.166668	0.277776	-0.11111
P5	0.11111	0.055556	0.055554

Fourthly the attribute Cost Efficiency is taken into account. Fig.7. represents the tree representations of the attribute

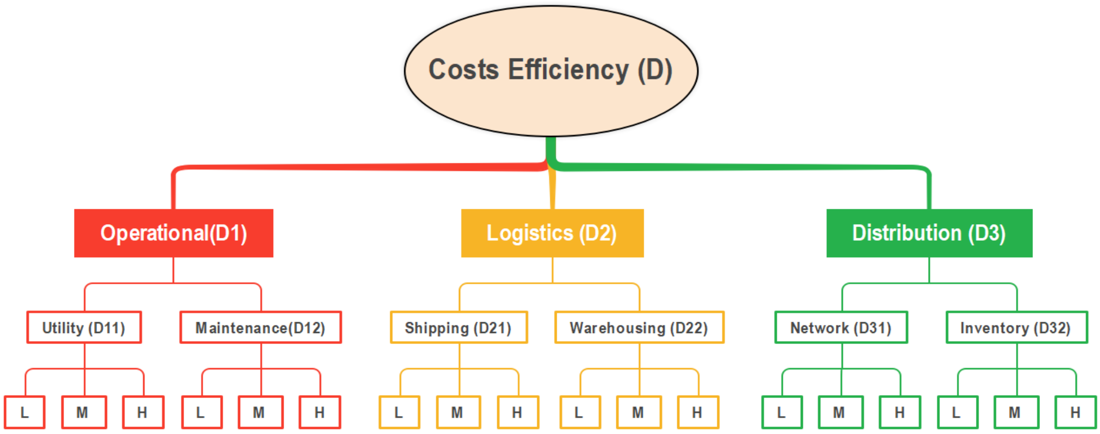

Fig.7. Cost Efficiency

The description of the attributes in the above fig.7. is sketched out in the following table 20.

Table 20. Description of Attributes

Attribute	**Sub-Attribute values**	**Sub2 Attribute Values (Sub-sub Attribute Values)**	**Sub3Attribute Values (Sub-sub – sub–Attribute Values)**
Costs Efficiency (D)	Operation (D1)	Utility Costs(D11)	**Affordable (D111)** Moderate (D112) Expensive (D113)
		Maintenance Costs (D12)	**Low (D121)** Moderate (D122) High (D123)
	Logistics (D2)	Shipping and Freight Costs (D21)	**Economical (D211)** Moderate (D212) Expensive (D213)
		Ware housing Costs (D22)	**Low (D221)** Moderate (D222) High (D223)
	Distribution (D3)	Distribution Network Costs (D31)	**Low-Cost (D311)** Moderate (D312) High-cost (D313)
		Inventory Holding Costs (D32)	**Low (D321)** Moderate (D322) High (D323)

If the criteria Cost Efficiency is alone considered the dominant sub-sub-sub attribute values to be considered are {Affordable, Low, Economical, Low, Low- Cost, Low}

Let us consider the representation of the form P(D1111xD1211xD2111xD2211xD3111xD3211)

The attribute matrix with respect to the attribute Cost Efficiency is presented in Table 21.

Table 21. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
D1111	Affordable	Moderate	Expensive	Affordable	Moderate
D1211	Moderate	High	Low	High	Low
D2111	Expensive	Economical	Moderate	Moderate	Expensive
D2211	Low	Moderate	High	Moderate	High
D3111	Moderate	Low-cost	Moderate	High-cost	Low-cost
D3211	High	Moderate	Low	Low	High

The contradiction degrees with respect to the dominant attribute values are presented in Table 22.

Table 22. Contradiction value

Dominant Attribute Value	Contradiction Degree
D111	C(D112, D111) = 1/3, C(D113, D111) = 2/3
D121	C(D122,D121) = 1/3 , C(D123, D121) =2/3
D211	C(D212, D211) = 1/3, C(D213, D211) =2/3
D221	C(D222, D221) = 1/3 , C(D223, D221) =2/3
D311	C(D312, D311) = 1/3 , C(D313, D311) = 2/3
D321	C(D322, D321) = 1/3 , C(D323, D321) = 2/3

The respective contradiction matrix obtained as follows

Table 23. Contradiction matrix

Attributive Values	P1	P2	P3	P4	P5
D1111(C)	0	1/3	2/3	0	1/3
D1211(B)	1/3	2/3	0	2/3	0
D2111(C)	2/3	0	1/3	1/3	2/3
D2211(C)	0	1/3	2/3	1/3	2/3
D3111(C)	1/3	0	1/3	2/3	0
D3211(B)	2/3	1/3	0	0	2/3

The weighted contradiction matrix is

Table 24. Weighted contradiction matrix

Alternatives	D1111(C)	D1211(B)	D2111(C)	D2211(C)	D3111(C)	D3211(B)
P1	0	0.05556	0.11111	0	0.05556	0.11111
P2	0.05556	0.11111	0	0.05556	0	0.05556
P3	0.11111	0	0.05556	0.11111	0.05556	0
P4	0	0.11111	0.05556	0.05556	0.11111	0
P5	0.05556	0	0.11111	0.11111	0	0.11111

By following the similar procedure, the score values are obtained and presented in Table 25.

Table 25. Difference in Score values

Alternatives	B_D	C_D	B_D- C_D
P1	0.16667	0.16667	0
P2	0.16667	0.11112	0.05555
P3	0	0.33334	-0.33334
P4	0.11111	0.22223	-0.11112
P5	0.11111	0.27778	-0.16667

Finally, the attribute Safety is considered. The diagrammatic representation is presented in Fig.8.

Fig.8. Safety

The description of the attributes in the above fig.8. is sketched out in the following table 26.

Table 26. Description of Attributes

Attribute	Sub-Attribute values	Sub2 Attribute Values (Sub-sub Attribute Values)	Sub 3 Attribute Values (Sub-sub – sub–Attribute Values)
Safety (E)	Community (E1)	Community Safety (E11)	Low(E111) Moderate (E112) **High (E113)**
		Natural Disaster Risk (E12)	**Low (E121)** Moderate (E122) High (E123)
	Risk (E2)	Risk Assessment (E21)	**Low (E211)** Moderate (E212)

			High (E213)
		Emergency Response (E22)	Inadequate (E221) Adequate (E222) **Robust (E223)**
	Quality (E3)	Work Environment quality (E31)	Poor (E311) Acceptable (E312) **Excellent (E313)**
		Health and Safety (E32)	Basic (E321) Safety (E322) **Comprehensive (E323)**

If the criteria Compliances is alone considered the, respective dominant sub-sub-sub attribute values to be considered are {Affordable, Low, Economical, Low, Low- Cost, Low}

Let us consider a representation of the form P(E1133xE1211xE2111xE2233xE3133xE3233)

Table 27. Attribute matrix

Attributive Values	**P1**	**P2**	**P3**	**P4**	**P5**
E1133	Moderate	High	Low	High	Low
E1211	Low	Moderate	High	Low	Moderate
E2111	High	High	Low	Moderate	Moderate
E2233	Adequate	Robust	Adequate	Inadequate	Robust
E3133	Excellent	Acceptable	Excellent	Poor	Acceptable
E3233	Basic	Safety	Basic	Safety	Comprehensive

The contradiction degrees with respect to the dominant attribute values are presented in Table 28.

Table 28. Contradiction value

Dominant Attribute Value	**Contradiction Degree**
E113	C(E111, E113) = 2/3, C(E112, E113) = 1/3
E121	C(E122, E121) = 1/3, C(E123, E121) = 2/3
E211	C(E212,E211) = 1/3 , C(E213, E211) = 2/3
E223	C(E221, E223) = 2/3, C(E222, E223) = 1/3
E313	C(E311, E313) = 2/3, C(E312, E313) = 1/3
E323	C(E321, E323) = 2/3, C(E322, E323) = 1/3

By using the contradiction values, the contradiction matrix is as follows and presented in Table 29.

Table 29. Contradiction matrix

Attributive Values	P1	P2	P3	P4	P5
E1133(B)	1/3	0	2/3	0	2/3
E1211(C)	0	1/3	2/3	0	1/3
E2111(C)	2/3	2/3	0	1/3	1/3
E2233(C)	1/3	0	1/3	2/3	0
E3133(C)	0	1/3	0	2/3	1/3
E3233(B)	2/3	1/3	2/3	1/3	0

The respective weighted contradiction matrix is as follows,

Table 30. Weighted contradiction matrix

Alternatives	E1133(B)	E1211(C)	E2111(C)	E2233(C)	E3133(C)	E3233(B)
P1	0.05556	0	0.1111	0.05556	0	0.1111
P2	0	0.05556	0.1111	0	0.05556	0.05556
P3	0.1111	0.1111	0	0.05556	0	0.1111
P4	0	0	0.05556	0.1111	0.1111	0.05556
P5	0.1111	0.05556	0.05556	0	0.05556	0

By following the similar procedure, the score values are obtained and presented in Table 31.

Table 31. Difference in Score values

Alternatives	B_E	C_E	B_E- C_E
P1	0.16666	0.16666	0
P2	0.05556	0.22222	-0.16666
P3	0.2222	0.16666	0.05554
P4	0.05556	0.27776	-0.2222
P5	0.1111	0.16668	-0.05558

From the Table [7,13,19,25,31] the overall score values shall be determined and presented in Table 32.

Table 32. Overall score value

Alternatives	Cumulative Score Values
P1	-0.18057

Alternatives	Cumulative Score Values
P2	0.111119
P3	-0.1528
P4	-0.36109
P5	-0.20836

5. Results and Discussion

The final score values of the alternatives with respect to each of the core attributes and the aggregate score values of the alternatives are determined from the Tables respectively.

The ranks of the alternatives are presented in the Table 33.

Table 33. Ranks of the alternatives

Alternatives	Ranking Results Based on the Core Attributes and Aggregate Score Values					
	Proximity	Accessibility	Compliance	Cost Efficiency	Safety	Aggregate
P1	5	4	5	3	3	5
P2	1	4	1	1	5	1
P3	4	1	3	5	2	2
P4	1	2	5	4	4	4
P5	3	3	3	2	1	3

The graphical representation of the scores of the alternatives with respect to each of the attributes and the aggregate measures is presented as follows

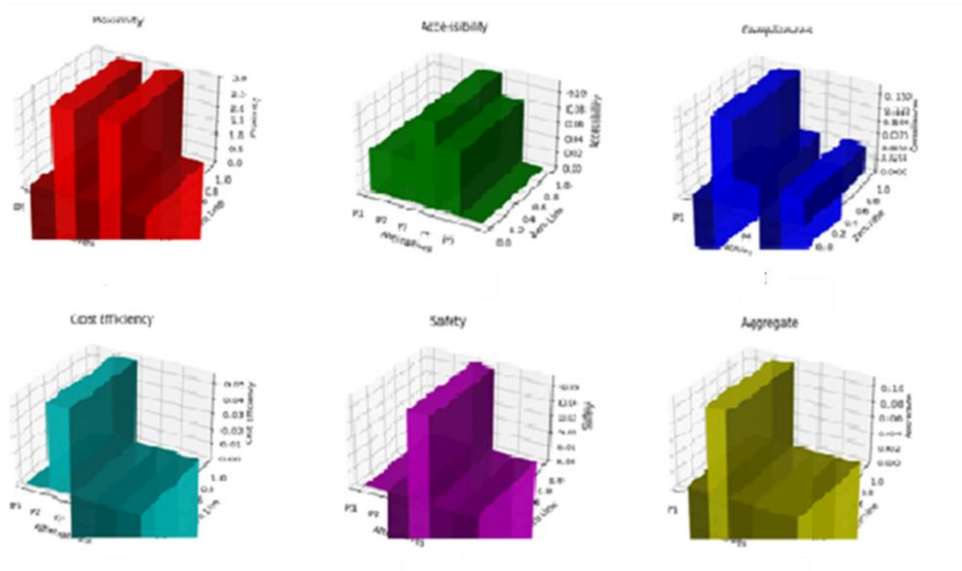

Fig.9. Graphical representation of Scores

From the ranking results presented in the table 33, the following inferences are obtained,

- P2 consistently performs well across most attributes, especially in compliance, cost efficiency, and overall aggregate score.
- P1 consistently ranks low in most attributes, particularly in proximity and aggregate score.
- P3 shows a strong performance in accessibility but is the worst in cost efficiency.
- P4 ranks well in proximity and accessibility but poorly in compliance and aggregate score.
- P5 excels in safety and has a balanced performance across other attributes, making it a solid all-around option.

Thus Plithogenic Forest Hypersoft sets are effectively applied in ranking the alternatives and these sets facilitates in choosing the alternatives based on attributes and sub_3 attribute values. The intervention of PFHS assists in making intense decisions by laying deep examination of the attributes. In other decision methods, the attributes are considered in shallow sense, however in this research work, the attributes are considered in a deeper manner.

Conclusion

This research work introduces the notion of Forest Hypersoft sets and Plithogenic Forest Hypersoft sets. The conceptualizations and classifications are well presented with suitable illustrations. The applications of Plithogenic Forest Hypersoft sets are sketched out with special reference to site selection of the manufacturing plants. The results are comprehensive in nature with the implications of Plithogenic based Forest Hypersoft sets. This decision approach shall be extended to different decision circumstances by augmenting with other decision methods.

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