



Neutrosophic-Based Enhanced Framework for Multi-Attribute Decision-Making Using Single-Valued Neutrosophic Sets in Evaluating Quality of University General Education Courses

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Abstract: The quality evaluation of university general education courses is a process that comprehensively assesses aspects such as teaching effectiveness, content design, teacher qualifications, and student feedback. Its purpose is to ensure that the courses effectively enhance students' overall competencies, such as critical thinking, communication skills, and social responsibility. Through a scientific evaluation system, course design can be optimized, teaching methods improved, and overall educational quality enhanced, fostering students' all-round development. The quality evaluation of university general education courses is useful MADM. The single-valued neutrosophic sets (SVNSs) is useful approach to administrate fuzzy information during the quality evaluation of university general education courses. Currently, the Aczel-Alsina operations and power geometric (PG) approach was administrated to put forward the multipleattribute decision-making (MADM). In this study, the single-valued neutrosophic number Aczel-Alsina power geometric (SVNNAAPG) approach is administrated in light with Aczel-Alsina operations and PG approach and SVNSs. The SVNNAAPG approach is administrated for MADM. Finally, numerical example for quality evaluation of university general education courses is administrated to conduct the SVNNAAPG approach. The major contributions of this work are tackled: (1) Aczel-Alsina operations and PG approach are enhanced under SVNSs; (2) SVNNAAPG approach is administrated in light with Aczel-Alsina operations and PG approach; (3) SVNNAAPG approach is tackled for MADM with SVNSs; (4) Finally, numerical example for quality evaluation of university general education courses is administrated to conduct the SVNNAAPG approach.

Keywords: multiple-attribute decision-making; single-valued neutrosophic sets (SVNSs); power geometric (PG); quality evaluation of university general education courses

1. Introduction

1.1. Background and Motivation

The evaluation of the quality of general education courses in universities is a systematic, comprehensive, and scientific process aimed at improving course quality and promoting the all-round development of students' overall abilities. The evaluation focuses on various aspects, including course design, teaching implementation, teaching effectiveness, and student feedback. First, the evaluation of course design looks at whether the course objectives are clear, the content is rich, the structure is reasonable, and whether it meets the actual needs and developmental requirements of students. Second, the evaluation of teaching implementation examines the teaching methods, attitudes, use of resources, and classroom management. Teachers should adopt diverse teaching methods, stimulate students' interest and initiative in learning, and emphasize the integration of theory and practice. The evaluation of teaching effectiveness is measured through students' learning outcomes, knowledge acquisition, skill enhancement, and personal development. This can be assessed through exams, assignments, project presentations, and practical operations. Lastly, student feedback is collected through surveys, interviews, and discussions to gather students' opinions and suggestions on the courses, understand their satisfaction levels, and identify areas for improvement. In summary, the evaluation of general education course quality in universities should be student-centered, considering multiple factors comprehensively. It should adopt scientific and reasonable evaluation methods, continuously improve course quality, and promote students' holistic development. Zhao [1] explored the construction of university general education curriculum systems, particularly focusing on the training of preservice teachers in basic education. Zhao discussed several issues present in the design of general education curricula, such as randomness in course selection, unclear objectives, and little relevance to teacher training. The study emphasized the need to clarify the role of general education in teacher training, improve course management, and build a high-quality teaching workforce to better equip future teachers with comprehensive skills. Wang, Tang and Zhang [2] examined the reform of practical teaching systems for electronic information courses under the framework of general education. Using the case of Henan University of Technology, the authors emphasized the importance of constructing a comprehensive practical teaching system. They argued that reforms in teaching methods and evaluation systems, as well as the integration of curriculum groups, were crucial for enhancing students' engagement in practical activities and improving overall teaching effectiveness. Ma [3] investigated the design of general education courses in universities. Drawing inspiration from the "Harvard General Education Red Book" and the undergraduate course reforms in the U.S., Ma analyzed the implications of these developments for Chinese universities. The study aimed to promote a more comprehensive, specific, and efficient approach to general education curriculum design in China, focusing on the cultivation of students' overall competencies. Yang [4] explored the integration of MOOCs (Massive Open Online Courses) into university general education. Using the case of the course "Ten Lectures on Chinese and Foreign Cultural Spirit" from Yangtze University, the study highlighted how MOOCs could align with the goals of general education. Yang argued that the interactive and expansive nature of MOOCs made them suitable for fostering research-based and autonomous learning, aligning well with the broader goals of general education. Zu and Feng [5] focused on the construction of general education curriculum systems in the context of emerging engineering disciplines ("New Engineering"). They pointed out that there was insufficient integration between the general education curriculum and the needs of new engineering disciplines. The study called for strategies to bridge this gap, promoting the development of general education to meet the evolving demands of engineering education. Yan and Zhou [6] conducted a study on the influence of general education on students' health-related lifestyles. They examined the "Five-Classroom Teaching" fitness course at Shandong Modern College to assess how general education reforms

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could positively impact students' exercise habits and overall health. The study observed that students were increasingly disengaged from physical activities due to digital immersion and proposed that well-structured general education courses could help instill healthier lifestyles. Hao [7] discussed the general education curriculum design for property management majors in universities. Hao emphasized the importance of cultivating interdisciplinary talents for the modern service industry, where comprehensive competencies are highly valued. The study highlighted the need for integrating ideological and political education into the curriculum, as well as focusing on practical skills that would prepare students for real-world challenges in property management. Gao [8] explored the goals and implementation pathways of general education within the ideological and political courses in universities. Gao argued that these courses play a crucial role in shaping future builders and successors of socialist causes, and that general education should focus not only on political literacy but also on humanistic and scientific competencies. The study proposed that ability education should take precedence over knowledge-based education to cultivate well-rounded individuals. Zeng [9] examined the optimization and innovation of college employment guidance courses under the framework of general education. Zeng analyzed the challenges faced by current employment guidance curricula and proposed strategies for improving and innovating these courses to better prepare students for career development. The study emphasized that optimizing these courses was essential for achieving higher employment rates for graduates. Gao et al. [10] analyzed the construction logic, current situation, and practical strategies for general education courses in local universities. The study identified several challenges in the current design and implementation of general education courses, such as the lack of a scientific curriculum system and inadequate teaching models. The authors proposed strategies for building a more robust general education system, including optimizing course evaluation mechanisms and strengthening implementation guarantees.

1.2. Objectives of the Study

MADM involves evaluating and selecting alternatives based on multiple criteria or attributes[11-14]. Decision-makers must consider the importance and weight of each attribute to determine the optimal solution [15-17]. This approach is often used in complex situations like resource allocation, project evaluation, and strategic planning, requiring effective handling of uncertainty and fuzziness to ensure rational and fair decision outcomes [18-24]. The quality evaluation of university general education courses is useful MADM. The SVNSs [25] is useful approach to administrate fuzzy information during the quality evaluation of university general education courses. Currently, the Aczel-Alsina operations [26, 27] and power geometric (PG) approach [28, 29] was administrated to put forward the MADM. Furthermore, many approaches administrated the Aczel-Alsina operations [30, 31] and PG approach [28, 29] to put up with the MADM, respectively. Until now, no or few techniques have been tackled on Aczel-Alsina operations and PG approach under SVNSs. In this study, the combination of Single-Valued Neutrosophic MADM with Aczel-Alsina operations and the power geometric method offers significant advantages. Firstly, SVNSs effectively handle uncertainty by providing a three-dimensional approach to fuzzy information, which is crucial for evaluating complex and subjective teaching quality. Secondly, Aczel-Alsina operations flexibly combine information, enhancing decision-making accuracy for diverse educational evaluation scenarios. The power geometric method ensures that all factors are reasonably considered and integrated through weighted averaging, resulting in comprehensive and fair outcomes. Additionally, this method is highly adaptable, allowing adjustments based on changing market and educational needs, thus providing a dynamic evaluation tool. These advantages make this approach highly valuable for assessing business English teaching quality.

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Therefore, in this study, the SVNNAAPG approach is administrated in light with Aczel-Alsina operations and PG approach and SVNSs. The SVNNAAPG approach is administrated for MADM. Finally, numerical example for quality evaluation of university general education courses is administrated to conduct the SVNNAAPG approach. The major contributions of this work are tackled: (1) Aczel-Alsina operations and PG approach are enhanced under SVNSs; (2) SVNNAAPG approach is administrated in light with Aczel-Alsina operations and PG approach; (3) SVNNAAPG approach is tackled for MADM with SVNSs; (4) Finally, numerical example for quality evaluation of university general education courses is administrated to conduct the SVNNAAPG approach.

1.3. Structure of the Paper

The research structure of this work is put forward. In Sect. 2, the SVNSs is put forward. In Sect. 3, SVNNAAPG approach is administrated in light with Aczel-Alsina operations and PG approach. In Sect. 4, SVNNAAPG approach is tackled for MADM with SVNSs. Sect. 5 puts forward the numerical example for quality evaluation of university general education courses through comparative analysis. Some remarks are put forward in Sect. 6.

2. Preliminaries

Wang et al. [25] resolved the SVNSs **Definition 1 [25]**. The SVNSs is resolved:

$$RA = \left\{ \left(\theta, RT_{A}\left(\theta\right), RI_{A}\left(\theta\right), RF_{A}\left(\theta\right)\right) \middle| \theta \in X \right\}$$

$$\tag{1}$$

where $RT_{A}(\theta), RI_{A}(\theta), RF_{A}(\theta)$ is truth-membership (TM), indeterminacy-membership (IM) and falsity-membership (FM), $RT_{A}(\theta), RI_{A}(\theta), RF_{A}(\theta) \in [0,1], \ 0 \le RT_{A}(\theta) + RI_{A}(\theta) + RF_{A}(\theta) \le 3$.

The SVNN is structured as $RA = (RT_A, RI_A, RF_A)$, where $RT_A \in [0,1], RI_A \in [0,1], RF_A \in [0,1]$, and $0 \le RT_A + RI_A + RF_A \le 3$.

Definition 2 [32]. Let $RA = (RT_A, RI_A, RF_A)$ and $RB = (RT_B, RI_B, RF_B)$, the score value is resolved:

$$RSV(RA) = \frac{\left(2 + RT_A - RI_A - RF_A\right)}{3}, \quad OSV(OA) \in [0,1].$$
⁽²⁾

$$RSV(RB) = \frac{\left(2 + RT_B - RI_B - RF_B\right)}{3}, \qquad RSV(RA) \in [0,1].$$
(3)

Definition 3[32]. Let $RA = (RT_A, RI_A, RF_A)$ and $RB = (RT_B, RI_B, RF_B)$, the accuracy value is resolved:

$$RAV(RA) = RT_A - RF_A, RAV(RA) \in [-1,1] .$$
⁽⁴⁾

$$RAV(RA) = RT_A - RF_A, RAV(RA) \in [-1,1] .$$
⁽⁵⁾

Peng et al.[32] resolved the order issues.

Definition 4[32]. Let
$$RA = (RT_A, RI_A, RF_A)$$
, $RB = (RT_B, RI_B, RF_B)$, let

$$RSV(RA) = \frac{\left(2 + RT_A - RI_A - RF_A\right)}{3} \quad \text{and} \quad RSV(RB) = \frac{\left(2 + RT_B - RI_B - RF_B\right)}{3} \quad \text{, and let}$$

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 $RAV(RA) = RT_A - RF_A$ and $RAV(RA) = RT_A - RF_A$, if RSV(RA) < RSV(RB), RA < RB; if RSV(RA) = RSV(RB), (1) if RAV(RA) = RAV(RB), RA = RB; (2) if RAV(RA) > RAV(RB), RA < RB.

Definition 5[25, 33]. Let $RA = (RT_A, RI_A, RF_A)$ and $RB = (RT_B, RI_B, RF_B)$, $r\theta \ge 1, r\lambda > 0$, the Aczel-Alsina operations among with SVNNs are administrated:

$$(1) RA \oplus RB = \begin{pmatrix} 1 - e^{-\left(\left(-\ln(1-RT_{A})\right)^{r\theta} + \left(-\ln(1-RT_{B})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(\left(-\ln(RI_{A})\right)^{r\theta} + \left(-\ln(RI_{B})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(\left(-\ln(RT_{A})\right)^{r\theta} + \left(-\ln(RT_{B})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(\left(-\ln(1-RT_{A})\right)^{r\theta} + \left(-\ln(1-RT_{B})\right)^{r\theta}\right)^{1/r\theta}}, \\ 1 - e^{-\left(\left(-\ln(1-RT_{A})\right)^{r\theta} + \left(-\ln(1-RT_{B})\right)^{r\theta}\right)^{1/r\theta}}, \\ 1 - e^{-\left(\left(-\ln(1-RT_{A})\right)^{r\theta} + \left(-\ln(1-RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ 1 - e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ 1 - e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{1/r\theta}}, \\ e^{-\left(r\lambda\left(-\ln(RT_{A})\right)^{1/r\theta}}, \\ e^{-\left(r$$

3. SNNAAPG operator

Yong et al. [34] and Ashraf et al. [33] administrated the SVNNAAWG operator.

Definition 6. Let $RA_i = (RT_i, RI_i, RF_i)(i = 1, 2, ..., n)$ be the SVNNs with weight $rw_i = (rw_1, rw_2, ..., rw_n)^T$, $\sum_{i=1}^n rw_i = 1$, $r\theta \ge 1$. If

$$SVNNAAWG_{rw} \left(RA_{1}, RA_{2}, \dots, RA_{n}\right) = \bigotimes_{i=1}^{n} \left(RA_{i}\right)^{rw_{i}}$$

$$= \begin{pmatrix} e^{-\left(\sum_{i=1}^{n} rw_{i}\left(-\ln\left(1-RT_{j}\right)\right)^{r\theta}\right)^{1/r\theta}}, \\ e^{-\left(\sum_{i=1}^{n} rw_{i}\left(-\ln\left(RI_{j}\right)\right)^{r\theta}\right)^{1/r\theta}}, \\ 1-e^{-\left(\sum_{i=1}^{n} rw_{i}\left(-\ln\left(RI_{j}\right)\right)^{r\theta}\right)^{1/r\theta}}, 1-e^{-\left(\sum_{i=1}^{n} rw_{i}\left(-\ln\left(RF_{j}\right)\right)^{r\theta}\right)^{1/r\theta}} \end{pmatrix}$$

$$(6)$$

The SVNNAAWG has three properties.

Property 1. (idempotency). If $RA_i = RA = (RT, RI, RF)$,

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$$SVNNAAWG_{rw}(RA_1, RA_2, \cdots, RA_n) = RA$$
⁽⁷⁾

Property2. (Monotonicity). Let $RA_i = (RT_{A_i}, RI_{A_i}, RF_{A_i})$, $RB_i = (RT_{B_i}, RI_{B_i}, RF_{B_i})$. If $RT_{A_i} \leq RT_{B_i}, RI_{A_i} \geq RI_{B_i}, RF_{A_i} \geq RF_{B_i}$ holds for all i, then

$$SVNNAAWG_{rw} (RA_1, RA_2, \cdots, RA_n)$$

$$\leq SVNNAAWG_{rw} (RB_1, RB_2, \cdots, RB_n)$$
(8)

Property 3 (Boundedness). Let $RA_i = (RT_{A_i}, RI_{A_i}, RF_{A_i})$. If

 $RA^+ = (\max_i(RT_i), \min_i(RI_i), \min_i(RF_i)), RA^- = (\min_i(RT_i), \max_i(RI_i), \max_i(RF_i)), \text{then}$

$$RA^{-} \leq \text{SVNNAAWG}_{nv} \left(RA_{1}, RA_{2}, \cdots, RA_{n} \right) \leq RA^{+}$$
(9)

Then, the SVNNAAPG operator is resolved on SVNNAAWG technique and PG technique [28, 29].

Definition 7. Let $RA_i = (RT_i, RI_i, RF_i)$ (i = 1, 2, 3, ..., n) be SVNNs, $r\theta \ge 1$. If

$$SVNNAAPG(RA_1, RA_2, \cdots, RA_n) = \bigotimes_{i=1}^n (RA_i)^{(\pi + RT(RA_i)) / \sum_{i=1}^n (\pi + RT(RA_i))}$$
(10)

1 ...

where $\pi \in R, \pi \ge 0$, $RT(RA_a) = \sum_{\substack{j=1 \ a \ne j}}^m Sup(RA_a, RA_j)$, $Sup(RA_a, RA_j)$ is support for RA_a from

 $RA_{j}, \text{ with serval conditions: (1) } Sup(RA_{a}, RA_{b}) \in [0,1]; (2) Sup(RA_{b}, RA_{a}) = Sup(RA_{a}, RA_{b}); (3)$ $Sup(RA_{a}, RA_{b}) \geq Sup(RA_{s}, RA_{t}), \text{ if } d(RA_{a}, RA_{b}) \leq d(RA_{s}, RA_{t}), \text{ where } d \text{ is distance measure.}$

The Theorem 2 is resolved.

Theorem 2. Let $RA_i = (RT_i, RI_i, RF_i)(i = 1, 2, ..., n)$ be the SVNNs, $r\theta \ge 1$. If

$$SVNNAAPG(RA_{1}, RA_{2}, \dots, RA_{n}) = \bigotimes_{i=1}^{n} (RA_{i})^{(\pi+RT(RA_{i}))} / \sum_{i=1}^{n} (\pi+RT(RA_{i})) / \sum_{i=1}^{n} (\pi+RT(RA_{i$$

where $\pi \in R, \pi \ge 0$, $RT(RA_a) = \sum_{\substack{j=1 \ a \ne j}}^m Sup(RA_a, RA_j)$, $Sup(RA_a, RA_j)$ is decision support for RA_a

from RA_i , with given decision conditions: (1) $Sup(RA_a, RA_b) \in [0,1]$; (2)

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$$Sup(RA_b, RA_a) = Sup(RA_a, RA_b)$$
; (3) $Sup(RA_a, RA_b) \ge Sup(RA_s, RA_t)$, if

 $d(RA_a, RA_b) \leq d(RA_s, RA_t)$, where *d* is the distance information.

Proof:

(a) Let i = 2, we have:

$$\begin{split} & \text{SVNNAAPG}\left(RA_{1}, RA_{2}\right) \\ &= \left(\left(RA_{1}\right)^{\left(\pi + RT(RA_{1})\right) \left(\sum_{i=1}^{2} \left(\pi + RT(RA_{i})\right)}{\left(\sum_{i=1}^{2} \left(\pi + RT(RA_{i})\right)\right)^{(r)}} \right)^{1/r\theta}} \otimes \left(RA_{2}\right)^{\left(\pi + RT(RA_{1})\right) \left(\sum_{i=1}^{2} \left(\pi + RT(RA_{i})\right)\right)} \right) \\ &= \left(e^{-\left(\left(\frac{\pi + RT(RA_{1})}{\sum_{i=1}^{2} \left(\pi + RT(RA_{i})\right)\right)} \left(-\ln(1 - RT_{1})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\left(\frac{\pi + RT(RA_{1})}{\sum_{i=1}^{2} \left(\pi + RT(RA_{i})\right)\right)} \left(-\ln(RI_{1})\right)^{(r)}}\right)^{1/r\theta}}, \\ &= \left(e^{-\left(\frac{\left(\pi + RT(RA_{1})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{1})\right)} \left(-\ln(1 - RT_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RI_{2})\right)^{(r)}}\right)^{1/r\theta}}, \\ &= \left(e^{-\left(\frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RI_{2})\right)^{(r)}}\right)^{1/r\theta}}, \\ &= \left(e^{-\left(\frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, \\ &= \left(e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{(r)}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum_{i=1}^{2} \left(\pi + RT(RA_{2})\right)} \left(-\ln(RF_{2})\right)^{1/r\theta}}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum(\pi + RT(RA_{2}))} \left(-\ln(RF_{2})\right)^{1/r\theta}}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum(\pi + RT(RA_{2})} \left(-\ln(RF_{2})\right)^{1/r\theta}}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum(\pi + RT(RA_{2})} \left(-\ln(RF_{2})\right)^{1/r\theta}}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum(\pi + RT(RA_{2})} \left(-\ln(RF_{2})\right)^{1/r\theta}}}\right)^{1/r\theta}}, 1 - e^{-\left(\sum_{i=1}^{2} \frac{\left(\pi + RT(RA_{2})\right)}{\sum(\pi + RT(RA_{2})} \left($$

If Eq. (11) hold for i = k, we have:

$$\begin{aligned} & \text{SVNNAAPG}(RA_{1}, RA_{2}, \dots, RA_{k}) = \bigotimes_{i=1}^{k} (RA_{i})^{(\pi + RT(RA_{i}))} / \sum_{i=1}^{n} (\pi + RT(RA_{i})) \\ & = \begin{pmatrix} -\left(\sum_{i=1}^{k} \frac{(\pi + RT(RA_{i}))}{\sum_{i=1}^{k} (\pi + RT(RA_{i}))} (-\ln(1 - RT_{j}))^{r\theta} \right)^{1/r\theta} \\ e & , \\ & -\left(\sum_{i=1}^{k} \frac{(\pi + RT(RA_{i}))}{\sum_{i=1}^{k} (\pi + RT(RA_{i}))} (-\ln(RI_{j}))^{r\theta} \right)^{1/r\theta} \\ & , \\ & 1 - e & -\left(\sum_{i=1}^{k} \frac{(\pi + RT(RA_{i}))}{\sum_{i=1}^{k} (\pi + RT(RA_{i}))} (-\ln(RI_{j}))^{r\theta} \right)^{1/r\theta} \\ & , \\ & , 1 - e & -\left(\sum_{i=1}^{k} \frac{(\pi + RT(RA_{i}))}{\sum_{i=1}^{k} (\pi + RT(RA_{i}))} (-\ln(RF_{j}))^{r\theta} \right)^{1/r\theta} \end{pmatrix} \end{aligned}$$

(b) Let i = k + 1. From Definition 5 and Eq. (11), we obtain

$$\begin{split} & \text{SVNNAAPG}\left(OA_{1}, OA_{2}, \cdots, OA_{k}, OA_{k+1}\right) \\ &= \bigotimes_{i=1}^{k} \left(RA_{i}\right)^{\left(\pi + RT(RA_{i})\right)} \left(\sum_{i=1}^{k+1} \left(\pi + RT(RA_{i})\right)\right)} \otimes \left(RA_{k+1}\right)^{\left(\pi + RT(RA_{k+1})\right)} \left(\sum_{i=1}^{k+1} \left(\pi + RT(RA_{i})\right)\right)} \left(-\ln(1 - RT_{j})\right)^{r\theta}\right)^{1/r\theta} \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k} \frac{\left(\pi + RT(RA_{i})\right)}{\sum_{i=1}^{k} \left(\pi + RT(RA_{i})\right)} \left(-\ln(1 - RT_{j})\right)^{r\theta}\right)^{1/r\theta} \\ -\left(\sum_{i=1}^{k} \frac{\left(\pi + RT(RA_{i})\right)}{\sum_{i=1}^{k} \left(\pi + RT(RA_{i})\right)} \left(-\ln(RI_{j})\right)^{r\theta}\right)^{1/r\theta} \\ ,1 - e^{-\left(\sum_{i=1}^{k} \frac{\left(\pi + RT(RA_{k+1})\right)}{\sum_{i=1}^{k} \left(\pi + RT(RA_{k})\right)} \left(-\ln(1 - RT_{k+1})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\frac{\left(\pi + RT(RA_{k+1})\right)}{\sum_{i=1}^{k} \left(\pi + RT(RA_{k})\right)} \left(-\ln(0I_{k+1})\right)^{r\theta}\right)^{1/r\theta} \\ ,1 - e^{-\left(\sum_{k=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(0I_{k+1})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{k=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\left(\pi + RT(RA_{k})\right)} \left(-\ln(0I_{k+1})\right)^{r\theta}\right)^{1/r\theta} \\ ,1 - e^{-\left(\sum_{k=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}\right)^{1/r\theta} \\ ,1 - e^{-\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &= \begin{pmatrix} -\left(\sum_{i=1}^{k+1} \frac{\left(1 + RT(RA_{k})\right)}{\sum_{i=1}^{k+1} \left(1 + RT(RA_{k})\right)} \left(-\ln(RI_{j})\right)^{r\theta}}\right)^{1/r\theta} \\ &, \\ &=$$

In light with (a), (b), and (c), it could be known that Eq. (11) holds any *i*. The SVNNAAPG has serval properties.

Property 4 (idempotency). If $RA_i = RA = (RT, RI, RF)$,

$$SVNNAAPG(RA_1, RA_2, \cdots, RA_n) = RA$$
⁽¹¹⁾

Property 5 (Monotonicity). Let $RA_i = (RT_{A_i}, RI_{A_i}, RF_{A_i})$, $RB_i = (RT_{B_i}, RI_{B_i}, RF_{B_i})$. If $RT_{A_i} \leq RT_{B_i}, RI_{A_i} \geq RI_{B_i}, RF_{A_i} \geq RF_{B_i}$ holds for all i, then

$$SVNNAAPG(RA_{1}, RA_{2}, \dots, RA_{n})$$

$$\leq SVNNAAPG(RB_{1}, RB_{2}, \dots, RB_{n})$$
(12)

Property 6 (Boundedness). Let $RA_i = (RT_{A_i}, RI_{A_i}, RF_{A_i})$. If

$$RA^{+} = (\max_{i}(RT_{i}), \min_{i}(RI_{i}), \min_{i}(RF_{i})), RA^{-} = (\min_{i}(RT_{i}), \max_{i}(RI_{i}), \max_{i}(RF_{i})), \text{ then}$$
$$RA^{-} \leq \text{SVNNAAPG}\left(RA_{1}, RA_{2}, \cdots, RA_{n}\right) \leq RA^{+}$$
(13)

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4. Methodology

Then, the SVNNAAPG technique is resolved for MADM with SVNSs. Let $RZ = \{RZ_1, RZ_2, ..., RZ_n\}$ be attributes. Let $RP = \{RP_1, RP_2, ..., RP_m\}$ be alternatives. $RQ = (RQ_{ij})_{m \times n} = (RT_{ij}, RI_{ij}, RF_{ij})_{m \times n}$ is the SVNN-matrix. The SVNNAAPG approach is resolved for MADM.

Step 1. Put forward the SVNN-matrix $RQ = (RQ_{ij})_{m \times n} = (RT_{ij}, RI_{ij}, RF_{ij})_{m \times n}$.

$$RQ = \begin{bmatrix} RQ_{ij} \end{bmatrix}_{m \times n} = \begin{bmatrix} RQ_{11} & RQ_{12} & \dots & RQ_{1n} \\ RQ_{21} & RQ_{22} & \dots & RQ_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ RQ_{m1} & RQ_{m2} & \dots & RQ_{mn} \end{bmatrix}$$
(14)

$$RQ = \begin{bmatrix} RQ_{ij} \end{bmatrix}_{m \times n} = (RT_{ij}, RI_{ij}, RF_{ij})_{m \times n}$$

$$= \begin{bmatrix} (RT_{11}, RI_{11}, RF_{11}) & (RT_{12}, RI_{1}, RF_{12}) & \dots & (RT_{1n}, RI_{1n}, RF_{1n}) \\ (RT_{21}, RI_{21}, RF_{21}) & (RT_{22}, RI_{22}, RF_{22}) & \dots & (RT_{2n}, RI_{2n}, RF_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (RT_{m1}, RI_{m1}, RF_{m1}) & (RT_{m2}, RI_{m2}, RF_{m2}) & \dots & (RT_{mn}, RI_{mn}, RF_{mn}) \end{bmatrix}$$

$$(15)$$

$$(15)$$

$$(RT_{m1}, RI_{m1}, RF_{m1}) = (RT_{m2}, RI_{m2}, RF_{m2}) = (RT_{m1}, RI_{m2}, RF_{m2}) = (RT_{m1}, RT_{m2}, RF_{m2}) = (RT_{m1}, RT_{m2}, RF_{m2}) = (RT_{m2}, RT_{m2}, RT_{m2}, RF_{m2}) = (RT_{m2}, RT_{m2}, RT_{m2}, RF_{m2}) = (RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}, RT_{m2}) = (RT_{m2}, RT_{m2}, RT_{m2}$$

Step 2. Normalize the
$$RQ = (RQ_{ij})_{m \times n} = (RT_{ij}, RI_{ij}, RF_{ij})_{m \times n}$$
 to
 $NRQ = [NRQ_{ij}]_{m \times n} = (NRT_{ij}, NRI_{ij}, NRF_{ij})_{m \times n}$.
 $NRQ_{ij} = (NRT_{ij}, NRI_{ij}, NRF_{ij})$
 $= \begin{cases} (RT_{ij}, RI_{ij}, RF_{ij}), RZ_{j} \text{ is a benefit criterion} \\ (RF_{ij}, 1 - RI_{ij}, RT_{ij}), RZ_{j} \text{ is a cost criterion} \end{cases}$
(16)

Step 3. In light with $NRQ = [NRQ_{ij}]_{m \times n} = (NRT_{ij}, NRI_{ij}, NRF_{ij})_{m \times n}$, $NRQ = [NRQ_{ij}]_{m \times n} = (NRT_{ij}, NRI_{ij}, NRF_{ij})_{m \times n}$ with SVNNAAPG to get the SVNNs $NRQ_i = (NRT_i, NRI_i, NRF_i)(i = 1, 2, \dots, m)$:

$$NRQ_{i} = SVNNAAPG(NRQ_{i1}, NRQ_{i2}, \dots, NRQ_{in})$$

$$= \bigotimes_{j=1}^{n} \left(NRQ_{ij} \right)^{\left(\pi + RT(NRQ_{ij})\right)} \left(\sum_{j=1}^{n} (\pi + RT(NRQ_{ij})) \left(-\ln(1 - NRT_{ij}) \right)^{r\theta} \right)^{1/r\theta},$$

$$= \begin{pmatrix} -\left[\sum_{i=1}^{n} \frac{(\pi + RT(NRQ_{ij}))}{\sum_{j=1}^{n} (\pi + RT(NRQ_{ij}))} \left(-\ln(1 - NRT_{ij}) \right)^{r\theta} \right]^{1/r\theta},$$

$$-\left[\sum_{j=1}^{n} \frac{(\pi + RT(NRQ_{ij}))}{\sum_{j=1}^{n} (\pi + RT(NRQ_{ij}))} \left(-\ln(NRI_{ij}) \right)^{r\theta} \right]^{1/r\theta},$$

$$1 - e^{-\left[\sum_{j=1}^{n} \frac{(\pi + RT(NRQ_{ij}))}{\sum_{j=1}^{n} (\pi + RT(NRQ_{ij}))} \left(-\ln(NRF_{ij}) \right)^{r\theta} \right]^{1/r\theta}},$$

$$(17)$$

Step 4. Construct the $RSV(NRQ_i)$, $RAV(NRQ_i)(i=1,2,\cdots,m)$.

$$RSV(NRQ_i) = \frac{\left(2 + NRT_i - NRI_i - NRF_i\right)}{3},$$

$$RAV(NRQ_i) = NRT_i - NRF_i$$
(18)

Step 5. Rank the alternatives through $RSV(NRQ_i)$, $RAV(NRQ_i)$.

5. Applications

5.1. Case Study Description

The quality evaluation of university general education courses is a systematic and comprehensive assessment process for courses designed to develop students' overall competencies. These courses aim to cultivate skills such as critical thinking, communication, teamwork, and social responsibility. The quality of these courses not only directly impacts students' learning experiences and skill development but also plays a crucial role in the overall educational standards of the university. Hence, a scientific and reasonable course evaluation system is essential to ensure and enhance the quality of general education. First, the core of course quality evaluation lies in assessing teaching effectiveness. By evaluating the achievement of course objectives, students' learning outcomes, the design and use of teaching materials, and the scientific and cutting-edge nature of the content, one can determine whether the course truly meets the goal of fostering students' overall competencies. Additionally, student engagement and satisfaction are important indicators. Students' classroom performance, feedback, and learning experience surveys provide a direct reflection of how well the course is received and its actual impact on students. Second, the quality of the instructor is a key factor influencing the quality of the course. The teacher's professional competence, innovation in teaching methods, and ability to interact with students all significantly affect the course. Therefore, course quality evaluation should also include assessments of the instructor's teaching ability, attitude, and methods to ensure that teachers can use effective teaching strategies to engage students and promote their holistic development. Third, the design and arrangement of the course content are also central to the evaluation. General education courses should be broadly applicable and diverse, catering to students from different academic backgrounds and interests. Additionally, the course should integrate theory with practice, ensuring sufficient theoretical depth while enhancing students' practical abilities and

problem-solving skills through hands-on activities. Finally, establishing a course quality evaluation system should involve multi-perspective assessments, including peer reviews and external expert evaluations, to ensure objectivity and comprehensiveness. These multi-dimensional evaluations help identify areas for improvement and provide a foundation for course enhancement. In summary, the quality evaluation of university general education courses is a complex and multi-layered task. It not only assesses the courses themselves but also provides feedback on the broader educational system. Through scientific and systematic evaluation, courses can be continuously optimized, teaching quality improved, and the ultimate goal of cultivating highly qualified talents achieved. The quality evaluation of university general education courses is useful MADM. In this paper, numerical example for quality evaluation of university general education courses is resolved through employing SVNNAAPG approach. Five colleges are evaluated with four attributes: $(1)RZ_1$ is Teaching Effectiveness-This indicator assesses whether the course has achieved its intended teaching objectives, including students' knowledge acquisition, improvement of overall competencies, and the development of critical thinking, communication, and problem-solving skills; $(2)RZ_2$ is Instructor Quality-This evaluates the instructor's professional expertise, innovation in teaching methods, classroom management, and interaction with students, ensuring that the instructor can effectively guide students and stimulate their interest in learning; $(3)RZ_3$ is Course Content Design-This indicator focuses on the rationality and relevance of the course content, including the integration of theory and practice, whether the content meets students' needs, and its adaptability to students from different academic backgrounds.; ④RZ4 is Student Feedback and Engagement-By analyzing students' classroom performance, attendance, engagement, and feedback, this indicator evaluates student participation and satisfaction with the course, reflecting the course's actual teaching effectiveness and the overall learning experience.

5.2. Application of the Framework to Quality Evaluation

Then, the SVNNAAPG approach is put forward quality evaluation of university general education courses under SVNNs.

Step 1. Implement the SVNN-matrix $RQ = (RQ_{ij})_{5\times4}$ as in Table 1 in light with statistical analysis.

	RZ_1	RZ_2	RZ_3	RZ_4
\mathbf{RP}_1	(0.36, 0.23, 0.35)	(0.31, 0.24, 0.36)	(0.34, 0.62, 0.36)	(0.28, 0.34, 0.39)
RP ₂	(0.33, 0.16, 0.62)	(0.62, 0.23, 0.28)	(0.39, 0.35, 0.63)	(0.32, 0.36, 0.38)
RP ₃	(0.61, 0.35, 0.63)	(0.64, 0.29, 0.61)	(0.31, 0.26, 0.63)	(0.67, 0.16, 0.34)
RP ₄	(0.69, 0.24, 0.42)	(0.65, 0.36, 0.34)	(0.62, 0.61, 0.38)	(0.69, 0.34, 0.47)
RP ₅	(0.64, 0.28, 0.67)	(0.68, 0.27, 0.62)	(0.62, 0.27, 0.67)	(0.37, 0.26, 0.42)

Table 1. SVININ IIIauitz	Table	e 1. S	VNN	matrix
	Table	1 C	VNN	motri

Step 2. Normalize $RQ = (RQ_{ij})_{5\times 4}$ to $NRQ = [NRQ_{ij}]_{5\times 4}$ (See the Table 2).

Table 2. The NRQ matrix				
	RZ_1	RZ_2	RZ_3	RZ_4
\mathbf{RP}_1	(0.36, 0.23, 0.35)	(0.31, 0.24, 0.36)	(0.34, 0.62, 0.36)	(0.28, 0.34, 0.39)
RP ₂	(0.33, 0.16, 0.62)	(0.62, 0.23, 0.28)	(0.39, 0.35, 0.63)	(0.32, 0.36, 0.38)
RP ₃	(0.61, 0.35, 0.63)	(0.64, 0.29, 0.61)	(0.31, 0.26, 0.63)	(0.67, 0.16, 0.34)
RP ₄	(0.69, 0.24, 0.42)	(0.65, 0.36, 0.34)	(0.62, 0.61, 0.38)	(0.69, 0.34, 0.47)
RP ₅	(0.64, 0.28, 0.67)	(0.68, 0.27, 0.62)	(0.62, 0.27, 0.67)	(0.37, 0.26, 0.42)

Step 3. Put forward the NRQ_i (i = 1, 2, 3, 4, 5) with SVNNAAPG approach (Table 3).

Alternatives	NRQ_i
RP ₁	(0.4015, 0.3324, 0.2961)
\mathbb{RP}_2	(0.7165,0.2213,0.3658)
RP ₃	(0.5053, 0.5327, 0.2985)
RP ₄	(0.4645,0.5031,0.3332)
RP ₅	(0.5214,0.4325,0.3101)

Table 3. The <i>NRQ</i> , by SVNNAAPG operator ($\theta = 2, \pi = 2$	Table 3. The NRQ_i by	SVNNAAPG	operator ($\theta = 2, \pi = 2$
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Step 4. Obtain $RSV(NRQ_i)(i=1,2,3,4,5)$ (See Table 4).

Table 4. The $RSV(NRQ_i)$

Alternatives	$RSV(NRQ_i)$	Order
RP_1	0.6918	5
RP ₂	0.7777	1
RP ₃	0.7175	4
RP ₄	0.7138	3
RP ₅	0.7179	2

Step 5. From Table 4, the order is $RP_2 > RP_5 > RP_3 > RP_4 > RP_1$, and the best colleges is XP_2 .

5.3. Influence analysis

To administrate the effects for final results according to parameters of SVNNAAPG approach, the results are administrated in Tables 5-6 and Figure 1.

θ	$RSV(RP_1)$	$RSV(RP_2)$	$RSV(RP_3)$	$RSV(RP_4)$	$RSV(RP_5)$
1	0.4890	0.6159	0.5157	0.5259	0.5383
2	0.6918	0.7777	0.7175	0.7138	0.7179
3	0.7476	0.8075	0.7689	0.7529	0.7715
4	0.7689	0.8155	0.7883	0.7648	0.7946
5	0.7792	0.8188	0.7983	0.7697	0.8072
6	0.7852	0.8204	0.8047	0.7724	0.8149
7	0.7891	0.8216	0.8091	0.7742	0.8200
8	0.7920	0.8226	0.8126	0.7757	0.8237
9	0.7941	0.8234	0.8154	0.7769	0.8266
10	0.7959	0.8244	0.8178	0.7778	0.8288

Table 5. Parameter values for SVNNAAPG approx	ach
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Table 6. Order for SVNNAAPG approach

	11	
r heta	Order	
1	$RP_2 > RP_5 > RP_4 > RP_3 > RP_1$	
2	$RP_2 > RP_5 > RP_3 > RP_4 > RP_1$	
3	$RP_2 > RP_5 > RP_3 > RP_4 > RP_1$	
4	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$	
5	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$	
6	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$	
7	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$	

8	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$
9	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$
10	$RP_2 > RP_5 > RP_3 > RP_1 > RP_4$





It could be seen from Table 5-6 when different parameter values are resolved, the order is slightly different.

5.4. Comparative analysis

The SVNNAAPG approach is compared with SVNNWA approach and SVNNWG approach [32], SVNN-EDAS technique [35] and SVNN-CODAS technique[36] and. The results are resolved in Table 7.

Approaches	Order
SVNNWA technique [32]	$RP_2 > RP_5 > RP_4 > RP_3 > RP_1$
SVNNWG technique [32]	$RP_2 > RP_5 > RP_3 > RP_4 > RP_1$
SVNN-EDAS technique [35]	$RP_2 > RP_5 > RP_3 > RP_4 > RP_1$
SVNN-CODAS technique [36]	$RP_2 > RP_5 > RP_3 > RP_4 > RP_1$
SVNNAAPG technique	$RP_2 > RP_5 > RP_3 > RP_4 > RP_1$

Table 7. Results for different approaches

Obtained from Table 7, it is administrated that the order of these approaches is slightly different, however, the optimal business English college is RP_2 and the worst college is RP_1 .

5.5. Discussion analysis

The SVNNAAPG approach presents distinct advantages and some drawbacks in multi-attribute decision-making, particularly in complex scenarios such as evaluating business English teaching quality. Some advantages for SVNNAAPG approach are outlined: (1) Effective handling of uncertainty: The SVNNAAPG method uses SVNSs to manage complex fuzzy and uncertain information. This three-dimensional representation captures the truth, falsity, and indeterminacy of information, allowing decision-makers to better understand and manage complexity. This capability enhances the accuracy and reliability of evaluations, helping to make more precise judgments, especially in subjective assessments. (2) High flexibility: The method incorporates Aczel-Alsina operations, providing the ability to flexibly adjust the weights of various factors. This flexibility allows decision-makers to tailor the evaluation model to specific needs and environmental conditions, adapting to different assessment scenarios. This is particularly important in educational quality assessments where conditions and priorities may change over time. (3) Comprehensiveness and inclusiveness: The introduction of the power geometric method allows SVNNAAPG to perform weighted averaging of multiple attributes, ensuring that all relevant factors are reasonably considered in the decision-making process. This comprehensiveness not only ensures fairness but also balances different factors, making the evaluation results more objective and credible.

6. Conclusion

6.1. Summary of Key Findings

The quality evaluation of university general education courses is a crucial aspect of MADM. In this study, the SVNNAAPG approach is applied, leveraging Aczel-Alsina operations and the PG approach within the framework of SVNSs. This method is specifically designed for MADM applications, aiming to enhance the accuracy and reliability of the evaluation process. A numerical example is provided to illustrate the application of the SVNNAAPG approach in evaluating business English teaching quality. The major contributions of this work are highlighted as follows: (1) Enhancement of Aczel-Alsina operations and PG approach under SVNSs: The study enhances the Aczel-Alsina operations and the PG approach within the context of SVNSs. These operations and methods are particularly effective in handling uncertainty and vagueness, which are inherent in decision-making processes. By applying these enhanced operations, the study aims to improve the precision and robustness of the evaluation. (2) Administration of the SVNNAAPG approach: The SVNNAAPG approach is systematically applied, integrating Aczel-Alsina operations and the PG approach. This integration ensures a comprehensive evaluation framework that can address the complexities associated with quality evaluation of university general education courses. (3) Application of SVNNAAPG for MADM with SVNSs: The SVNNAAPG approach is tailored for MADM scenarios involving SVNSs. SVNSs are adept at representing uncertain and imprecise information, making them suitable for evaluating qualitative aspects of teaching quality. This application demonstrates the method's capability to handle complex decision-making environments effectively. (4) Validation through numerical example: A numerical example is employed to validate the SVNNAAPG approach in the context of quality evaluation of university general education courses. This example showcases the practical application and potential benefits of the method, highlighting its effectiveness in real-world scenarios. The example provides a step-by-step implementation, demonstrating how the SVNNAAPG approach can be utilized to derive meaningful insights and support decision-making.

In conclusion, the application of the SVNNAAPG approach in quality evaluation of university general education courses offers significant improvements in the accuracy and reliability of the evaluation process. By enhancing Aczel-Alsina operations and the PG approach within SVNSs, the study provides a robust

framework for MADM. The numerical example further validates the method's practical applicability, making it a valuable tool for educators and decision-makers in the field of university general education courses. This approach not only addresses the inherent uncertainties in teaching quality evaluation but also offers a systematic method for making informed decisions.

6.1. Future Research Directions

In light of the above analysis, this study faces several challenges. First, the complexity of the SVNNAAPG method might make it difficult for educators and administrators to use, particularly those without a strong mathematical background. This could limit the widespread adoption of the method in the quality evaluation of university general education courses. Additionally, the research may rely on a limited dataset, which could affect its applicability across diverse educational contexts. Another challenge is that the model may not be dynamic enough to keep pace with rapid changes in the demands of education and student needs, potentially reducing its effectiveness over time. Looking ahead, there are several promising directions for further research. One important step is to simplify the model, making it more user-friendly and practical for a broader audience of educators and administrators. Developing intuitive software tools could assist non-technical users in applying the method more easily. Furthermore, it is crucial to validate the method internationally by testing it in various cultural and educational settings to ensure its wider applicability and effectiveness. Lastly, enhancing the model's adaptability to changes in educational policies, student expectations, and technological advancements would be beneficial. By regularly updating the model and incorporating machine learning, it could adjust to new trends and requirements in real-time. These efforts could significantly improve the quality evaluation of university general education courses and ensure its long-term relevance and utility.

Acknowledgment

The work was supported by the Research on the Dynamic Development Trajectory of College Counselors' Work Effectiveness and Its Influence Mechanism Based on Students' Evaluation, 2024 Higher Education Scientific Research Planning Project of the Chinese Society of Higher Education under Grant No. 24FD0416.

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Received: June 26, 2024. Accepted: Oct 13, 2024