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On the basics of neutrosophic homotopy theory

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Abstract. In this paper, we introduce and study the concept of neutrosophic homotopic functions using topological properties. Also, we obtain some properties of neutrosophic homotopic functions and concept of neutrosophic homotopy.

Keywords: Neutrosophic sets, neutrosophic homotopic function, equivalence relation.

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1. Introduction

Zadeh [4] introduced the degree of membership/truth (t) in 1965 and defined the fuzzy set. As a generalization of fuzzy sets, Atanassov [1] introduced the degree of nonmembership/falsehood (f) in 1986 and defined the intuitionistic fuzzy set. Smarandache proposed the term "neutrosophic" because "neutrosophic" etymologically comes from "neutrosophic" [French neutre, Latin neuter, neutral, and Greek sophia, skill/wisdom] which means knowledge of neutral thought, and this third/neutral represents the main distinction between "fuzzy / intuitionistic" logic/set and "neutrosophic" logic/set, that is, the included middle component, that is, the neutral/indeterminate/unknown part (besides the truth"/membership" and falsehood"/nonmembership" components that both appear in fuzzy logic/set). Smarandache introduced the degree of indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set on three components (t, i, f) = (truth, indeterminacy, falsehood). The concept of neutrosophic set developed by Smarandache [2, 3] is a more general platform which extends the concepts of the classic set and fuzzy set, intuitionistic fuzzy set and interval valued intuitionistic fuzzy set. Neutrosophic set theory is applied to various part (refer to the site http://fs.gallup.unm.edu/neutrosophy.htm). In this paper, we introduced and studied the concept of neutrosophic homotopic functions using topological properties. Also, we obtained some properties of neutrosophic homotopic functions and concept of neutrosophic homotopy.

2. Preliminaries

Definition 2.1. Let A be a nonempty set. The neutrosophic set [2] on A is defined to be a structure

$$A := \{\langle x, \mu(x), \gamma(x), \psi(x)\rangle \mid x \in A\}, \tag{1}$$

where $\mu : A \to [0,1]$ is a truth membership function, $\gamma : A \to [0,1]$ is an indeterminate membership function, and $\psi : A \to [0,1]$ is a false membership function. The neutrosophic fuzzy set in (1) is simply denoted by $A = (\mu_A, \gamma_A, \psi_A)$.

For any neutrosophic set A of X and any $t, s, r \in [0,1]$, the (t,s,p)-cut of A is defined as $A_{t,r,s} = \{x \in X : \mu(x) \geq t \wedge \gamma(x) \leq s \wedge \psi(x) \leq p\}$.

Lemma 2.2. *A neutrosophic equivalence relation R on a set X is a relation that is reflexive, symmetric, and transitive:*

1. *Reflexive: $R_T(x,x) = 1$, $R_I(x,x) = 0$ and $R_F(x,x) = 0$ for all $x \in X$*
2. *Symmetric: $R(x,y) = R(y,x)$ for all $x, y \in X$*
3. *Transitive: $R \circ R \subseteq R$.*

Let R be an equivalence relation on A and $\langle x_{t,s,p}, y_{m,n,q}\rangle \in R$, $x, y \in X$, then we say $x_{t,s,p}$ equivalent to $y_{m,n,q}$ or $x_{t,s,p}$ and $y_{m,n,q}$ are equivalent. For any $x_{t,s,p} \in A$, using $(x_{t,s,p})_R = \cup\{y_{m,n,q} : \langle x_{t,s,p}, y_{m,n,q}\rangle \in R\}$. This is called R-equivalence class of $x_{t,s,p}$ and simply denoted by $(x_{t,s,p})$. Now, we put $A/B = \{(x_{t,s,p}) : x_{t,s,p} \in A\}$. We denoted, $R^{-1} = \cup\{\lambda R_\lambda^{-1} : t, s, p \in [0,1]\}$, it is inverse relation of R. If, $R = \cup\{\langle x_{t,s,p}, y_{m,n,q}\rangle : x, y \in X\}$, then $R^{-1} = \cup\{\langle y_{m,n,q}, x_{t,s,p}\rangle : x, y \in X\}$ and $(R^{-1})^{-1} = R$.

3. Basics of neutrosophic homotopy theory

Definition 3.1. Let (A, τ_1) and (B, τ_2) be neutrosophic topological spaces and $f, g : A \to B$ are neutrosophic continuous functions. Then f is neutrosophic homotope to g if there exists a neutrosophic continuous function $F : A \times I \to B$ such that for every $t \in I$, $F_{t,s,p}(x, 0) = f_{t,s,p}(x)$, $F_{t,s,p}(x, 1) = g_{t,s,p}(x)$, $F_{t,s,p}(x, (m,n,q)) = f_{t,s,p}^{m,n,q}(x)$. If f and g are neutrosophic homotopic functions, we write $f \sim g$.

Proposition 3.2. *It is clear that every continuous function homotopic to itself.*

Definition 3.3. Let (A, τ_1) and (B, τ_2) be neutrosophic topological spaces. Let $X_0 \subset X$ and there exists functions $f, g : X \to Y$ such that the condition for every $x_0 \in X_0$, $f(x_0) = g(x_0)$ is satisfied. Then f is neutrosophic homotopic to g relative to x_0 and written by $f \sim g/X_0$, if there exists a function $F : A \times I \to B$ such that the following conditions hold:

(1) $F_{t,s,p}(x, 0) = f(x)$, $F_{t,s,p}(x, 1) = g(x)$, for every $x \in X$,
(2) $F_{t,s,p}(x_0, (m, n, q)) = f(x_0) = g(x_0)$, for every $x_0 \in X_0$.

Theorem 3.4. *Neutrosophic homotopy relation is neutrosophic equivaliance relation.*

Proof. Now that for every continuous function f, $f \sim f$. Assume that $f \sim g$, then there exists a function F such that the conditions of the Definition are satisfied. If we rewrite the function F as $F'(x, (t, s, p)) = F(x, 1 - t)$, then we get $g \sim f$. The distributive condition is clear. □

Definition 3.5. Let X and Y be topological spaces and A and B are neutrosophic topological spaces on X and Y, respectively. Let $f, g \subset A \times B$ neutrosophic continuous and $f \sim g$. If $|img| = 1$, then it is called that f is homotopic to arbitrary.

Definition 3.6. X is contractibility or X may deformated to a point if the identity definition on X is homotopic to arbitrary.

Theorem 3.7. *Let X be a neutrosophic topological space and Y can be deformated to a point then all of the neutrosophic continuous function $f : A \to B$ is homotopic to arbitrary.*

Proof. B may deformated neutrosophic topological space then for every $t, s, p \in I$, $B_{t,s,p}$ may deformated topological subspace. Therefore there exists a function $g : B \to B$ such that for $y_0 \in B_{t,s,p}$ arbitrary, $g_{t,s,p}(y) = y_0$ such that $1_{B_{t,s,p}} : B_{t,s,p} \to B_{t,s,p}$ is homotopic to $g_{t,s,p}$, that is, $1_{B_{t,s,p}} \sim g_{t,s,p}$. Therefore, there exists a continuous function $f_{t,s,p} : B_{t,s,p} \times I^2 \to B_{t,s,p}$ such that for every $y \in Y$, $F_{t,s,p}(y, 0) = 1_{B_{t,s,p}}(y, 0) = 1_{B_{t,s,p}}(y), F_{t,s,p}(y, 1) = g_{t,s,p}(y)$. We assume that, $f : A_{t,s,p} \to B_{t,s,p}$ neutrosophic continuous function. We define that neutrosophic function $G_{t,s,p} : A_{t,s,p} \times I^2 \to B_{t,s,p}$ as $G_{t,s,p}(x, (m, n, q)) = F_{t,s,p}(f_{t,s,p}(x), (m, n, q))$. It is clear that $G_{t,s,p}$ is continuous function for every $t, s, p \in I$ and $G_{t,s}(x, 0) = F_{t,s,p}(f_{t,s,p}(x), 0) = f_{t,s,p}(x)$, $G_{t,s,p}(x, 1) = F_{t,s,p}(f_{t,s,p}(x), 1) = y_0$. Thus f is homotopic to arbitrary for every $t, s, p \in I$. Therefore f is neutrosophic homotopic to arbitrary. □

Theorem 3.8. *Let $A \in NS(X)$, $B \in NS(Y)$, $C \in NS(Z)$ be neutrosophic topological spaces and $f \subset A \times B$, $g \subset B \times C$ be neutrosophic continuous functions. If $g \sim h$, then $g \circ f$ and $h \circ f$ are neutrosophic continuous and $g \circ f \sim h \circ f$.*

Proof. If $g \sim h$, then there exists a neutrosophic continuous function F such that $F_{t,s,p}(y, 0) = g_{t,s,p}$ and $F_{t,s,p}(y, 1) = f_{t,s,p}$ for every $t, s, p \in I$. Let us define a function G with respect to F such that $G_{t,s,p}(x,(m, n, q)) = F_{t,s,p}(f(x),(m, n, q))$ for every $t, s, p \in I$. It is clear that G is a neutrosophic continuous function. However, $G_{t,s,p}(x, 0) = F_{t,s,p}(f_{t,s,p}(x), 0) = g_{t,s,p}f_{t,s,p}$ and $G_{t,s,p}(x, 1) = F_{t,s,p}(f_{t,s,p}(x), 1) = h_{t,s,p}f_{t,s,p}$ for every $t, s, p \in I$, too. Therefore, $g_{t,s,p}f_{t,s,p} \sim h_{t,s,p}f_{t,s,p}$, for every $t, s, p \in I$ thus $gf \sim hf$. □

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