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Grey Relational Analysis Framework with 2-Tuple Linguistic Neutrosophic Set for Building Material Supplier Selection

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Abstract: Choosing building materials suppliers is crucial as it directly affects the quality, cost, and timeline of a project. Quality suppliers ensure the reliability and consistency of materials, reducing construction risks. The right suppliers offer competitive pricing and flexible delivery schedules, helping control budgets and optimize resource use. Additionally, suppliers' professional support and technical services can enhance construction efficiency and quality. By carefully selecting suppliers, project teams can establish long-term partnerships, improving overall construction standards and market competitiveness. Thus, supplier selection is a key factor in the success of construction projects. The process of selecting and applying building material suppliers (BMSs) is a multiple-attribute group decision-making (MAGDM) method. This study constructs the 2-tuple linguistic neutrosophic number combined grey relational analysis (2TLNN-CGRA) approach, which integrates the grey relational analysis (GRA) and 2-tuple linguistic neutrosophic sets (2TLNNSs). To validate the method, a numerical example for building material supplier selection is provided, accompanied by comparative studies to highlight the effectiveness of the 2TLNN-CGRA approach. The key contributions of this research include (1) the utilization of the 2TLNN-CGRA approach to address MAGDM under 2TLNSs conditions; (2) the incorporation of both 2TLNN Hamming distance (2TLNNHD) and 2TLNN Euclidean distance (2TLNNED) within the 2TLNSs framework in the 2TLNN-CGRA approach; (3) the demonstration of the method through a numerical example for supplier selection; and (4) a series of efficient comparisons with other existing decision-making approaches.

Keywords: Multiple-attribute group decision-making (MAGDM); 2-tuple linguistic neutrosophic sets (2TLNNSs); 2TLNN-CGRA approach; building material suppliers selection

1. Introduction

The construction sector plays a pivotal role in stimulating economic development and serves as a key driver of economic expansion. In today's era, marked by rapid technological progress and increasing economic globalization, China's construction market has evolved into a dynamic arena characterized by significant potential, lucrative opportunities, and robust growth [1, 2]. Its impact is profound, not only in terms of its direct contributions to economic value within the sector but also through its influence on the expansion of more than a dozen related industries. Predominantly made up of small and medium-sized enterprises (SMEs), which account for over 90% of its composition, the construction industry is integral to both urban and rural development in China [3]. These SMEs play a vital role in redistributing the rural labor force towards urban centers, thus driving significant socio-economic changes. Unlike the manufacturing sector, which also operates on an order-based production model, the construction industry handles a diverse range of products. Each construction project is subject to meticulous phases of planning, design, bidding, and procurement, and is uniquely bound by specific constraints on time, quality, and cost [4, 5]. The construction industry is distinguished by several unique features: it operates in specific, unchangeable locations where both production and consumption occur, requiring tailored design and construction due to the diversity and variability of each site, as opposed to standardized mass production. A project typically cannot commence without the direct involvement of the property owner. Furthermore, the sector is labor-intensive, demanding a substantial human workforce. It also exhibits long production timelines and large-scale projects that are marked by phased execution and operational fluidity[6, 7] and a production process involving multiple interdependent sub-projects and departments, requiring close collaboration to ensure seamless progress, highlighting the systematic nature of the industry [8, 9]. As competition in the construction materials market intensifies, no single enterprise can cover the entire lifecycle of a construction project independently. Collaboration across various systems is essential in the phases of design, procurement, construction, and operations. Consequently, to maintain cost-effectiveness, boost competitiveness, and enhance management skills, construction firms need to focus on choosing the most appropriate supplier teams. Construction materials typically make up about 60% of the overall project expenses, underscoring the importance of optimizing supply chain efficiency and the strategic utilization of limited site space, which are key to the industry's swift progress. The way engineering materials are categorized and the procurement strategies implemented by construction companies play a pivotal role. Creating a robust supplier management system and selecting suppliers that align with the firm's strategic goals are critical for managing procurement expenses effectively and sustaining long-term cost effectiveness, thereby maximizing company profits [10]. Supplier management, as part of a company's overall strategy, plays a crucial role. Material costs, which make up 50-60% of the total project cost, have a direct impact on profitability, with a 1% reduction in procurement costs leading to a 5-10% increase in operating profits. Given the importance of material costs, strengthening cost control in this area is vital for securing profit margins [11]. Effective supplier management provides construction companies with critical insights, greatly benefiting production management, investment decisions, and mergers. High-quality supplier management enables companies to quickly source superior products and services at competitive prices, significantly contributing to profitability [12]. Moreover, securing supply during shortages of essential raw materials or services provides a competitive edge. To maintain this advantage, companies must continuously optimize their supplier management systems, driving strategic growth in the supply chain field.

MAGDM is a structured approach used to tackle complex decision problems involving multiple criteria and stakeholders [13, 14]. This methodology facilitates a systematic evaluation of various alternatives based on a set of predefined attributes, allowing a group of decision-makers to reach a consensus or informed decision. MAGDM is particularly useful in scenarios where decisions require balancing diverse perspectives and interests, as it incorporates inputs from all participants, ensuring a comprehensive assessment [15-18]. The process typically involves several key steps: defining the problem and objectives, identifying decision-makers and stakeholders, selecting relevant attributes, gathering and processing data, and finally, applying an appropriate decision-making model to aggregate preferences and rank the alternatives[19-23]. Techniques like TOPSIS [24] and GRA [25] are commonly used within this framework. MAGDM is widely applicable in fields ranging from business and engineering to public policy and healthcare, where decisions impact multiple people and operational facets [26, 27]. The approach not only supports a democratic decision-making process but also enhances the legitimacy and acceptance of the final decision by involving all key stakeholders [28, 29]. The evaluation and selection of BMSs utilize a MAGDM approach. The 2TLNNSs [30-33] serve as effective approach for handling uncertain data during the BMSs evaluation and selection. Deng [34] established the GRA approach. In comparison to other MADM approaches, the GRA approach [35-42] is adept at managing the shape similarity between each option from the positive ideal alternative (PIA) and negative ideal alternative (NIA). In this study, the 2TLNN-CGRA approach is developed, integrating the principles of GRA approach and 2TLNNSs. A numerical example specifically for BMS selection has been created, and several distinct comparisons have been conducted to validate the effectiveness of 2TLNN-CGRA approach. The primary research objectives and motivations are outlined: (1) 2TLNN-CGRA approach is constructed to address MAGDM challenges using 2TLNSs; (2) The 2TLNN-CGRA approach is constructed simultaneously in line with 2TLNNHD and 2TLNNED; (3)A numerical example for BMSs evaluation and selection is presented to demonstrate the application of the 2TLNN-CGRA approach; (4) Comparative analysis is conducted with several decision-making approaches to highlight the advantages of the new method.

The fundamental structure of this research is outlined as follows: Section 2 details the construction of the 2TLNNSs. In Section 3, the 2TLNN-CGRA method is developed to address the MAGDM. Section 4 presents a numerical example for evaluating and selecting BMSs. Finally, Section 5 summarizes the conclusions.

2. Preliminaries

Wang et al. [31] formulated the 2TLNSs.

Definition 1 [31]. Let $r\theta_1, r\theta_2, \ldots, r\theta_R$ be linguistic term sets (LTSs), and $r\theta = \{rl_0 = exceedingly\ bad, rl_1 = very\ bad, rl_2 = bad, rl_3 = medium, rl_4 = well, rl_5 = very\ well, rl_6 = exceedingly\ well\}$, then the 2TLNSs is formulated:

$$r\theta = \left\langle (rl_t, ra,), (rl_i, rb), (rl_f, rc) \right\rangle \tag{1}$$

with 2-tuple linguistic information $(rl_t, ra,), (rl_i, rb), (rl_f, rc)$ is membership, indeterminacy and non-membership and $0 \le \Delta^{-1}(rl_t, ra,) + \Delta^{-1}(rl_i, rb) + \Delta^{-1}(rl_f, rc) \le 3R$.

Definition 2[31]. Let $r\theta_1 = \left\langle (rl_{t_1}, ra_1,), (rl_{i_1}, rb_1), (rl_{f_1}, rc_1) \right\rangle$, $r\theta_2 = \left\langle (rl_{t_2}, ra_2,), (rl_{i_2}, rb_2), (rl_{f_2}, rc_2) \right\rangle$ be 2-tuple linguistic neutrosophic number (2TLNN), $\lambda > 0$, the novel operational laws are formulated:

$$(1)\ r\theta_1 \oplus r\theta_2 = \left\{ \begin{array}{l} \Delta\left(R\left(\frac{\Delta^{-1}(rl_{t_1}, ra_1,)}{R} + \frac{\Delta^{-1}(rl_{t_2}, ra_2,)}{R} - \frac{\Delta^{-1}(rl_{t_1}, ra_1,)}{R} \cdot \frac{\Delta^{-1}(rl_{t_2}, ra_2,)}{R} \right) \right), \\ \Delta\left(R\left(\frac{\Delta^{-1}(rl_{i_1}, rb_1)}{R} \cdot \frac{\Delta^{-1}(rl_{i_2}, rb_2)}{R} \right) \right), \Delta\left(R\left(\frac{\Delta^{-1}(rl_{f_1}, rc_1)}{R} \cdot \frac{\Delta^{-1}(rl_{f_2}, rc_2)}{R} \right) \right) \end{array} \right\}; \tag{2}$$

$$(2)\ r\theta_1 \otimes r\theta_2 = \left\{ \begin{array}{l} \Delta\left(R\left(\frac{\Delta^{-1}(rl_{t_1}, ra_1,)}{R} + \frac{\Delta^{-1}(rl_{t_2}, ra_2,)}{R} \right) \right), \\ \Delta\left(R\left(\frac{\Delta^{-1}(rl_{i_1}, rb_1)}{R} + \frac{\Delta^{-1}(rl_{i_2}, rb_2)}{R} - \frac{\Delta^{-1}(rl_{i_1}, rb_1)}{R} \cdot \frac{\Delta^{-1}(rl_{i_2}, rb_2)}{R} \right) \right), \\ \Delta\left(R\left(\frac{\Delta^{-1}(rl_{f_1}, rc_1)}{R} + \frac{\Delta^{-1}(rl_{f_2}, rc_2)}{R} - \frac{\Delta^{-1}(rl_{f_1}, rc_1)}{R} \cdot \frac{\Delta^{-1}(rl_{f_2}, rc_2)}{R} \right) \right) \end{array} \right\}; \quad (3)$$

$$(3)\ \lambda r\theta_1 = \left\{ \Delta\left(R\left(1 - \left(1 - \frac{\Delta^{-1}(rl_{t_1}, ra_1,)}{R} \right)^{\lambda} \right) \right), \Delta\left(R\left(\frac{\Delta^{-1}(rl_{i_1}, rb_1)}{R} \right)^{\lambda} \right), \Delta\left(R\left(\frac{\Delta^{-1}(rl_{f_1}, rc_1)}{R} \right)^{\lambda} \right) \right\}; \quad (4)$$

(4)

$$(r\theta_1)^{\lambda} = \left\{ \Delta\left(R\left(\frac{\Delta^{-1}(rl_{t_1}, ra_1,)}{R} \right)^{\lambda} \right), \Delta\left(R\left(1 - \left(1 - \frac{\Delta^{-1}(rl_{i_1}, rb_1)}{R} \right)^{\lambda} \right) \right), \Delta\left(R\left(1 - \left(1 - \frac{\Delta^{-1}(rl_{f_1}, rc_1)}{R} \right)^{\lambda} \right) \right) \right\}. \quad (5)$$

Wang et al. [31] formulated the score-value technique (SVT) and accuracy-value technique (AVT) for 2TLNNs.

Definition 3[31]**.** Let $r\theta_1 = \langle (rl_{t_1}, ra_1,), (rl_{i_1}, rb_1), (rl_{f_1}, rc_1) \rangle$, $r\theta_2 = \langle (rl_{t_2}, ra_2,), (rl_{i_2}, rb_2), (rl_{f_2}, rc_2) \rangle$, the SVF and AVF are formulated:

$$SVT(r\theta_1) = \frac{\begin{pmatrix} 2R + \Delta^{-1}(rl_{t_1}, ra_1,) \\ -\Delta^{-1}(rl_{i_1}, rb_1) - \Delta^{-1}(rl_{f_1}, rc_1) \end{pmatrix}}{3R}, \quad SVT(r\theta_1) \in [0,1] \quad (6)$$

$$SVT(r\theta_2) = \frac{\begin{pmatrix} 2R + \Delta^{-1}(rl_{t_2}, ra_2,) \\ -\Delta^{-1}(rl_{i_2}, rb_2) - \Delta^{-1}(rl_{f_2}, rc_2) \end{pmatrix}}{3R}, \quad SVT(r\theta_2) \in [0,1] \quad (7)$$

$$AVT(r\theta_1) = \frac{1}{2R} \begin{pmatrix} R + \Delta^{-1}(rl_{t_1}, ra_1,) \\ -\Delta^{-1}(rl_{f_1}, rc_1) \end{pmatrix}, \quad AVT(r\theta_1) \in [0,1] \quad (8)$$

$$AVT(r\theta_2) = \frac{1}{2R} \begin{pmatrix} R + \Delta^{-1}(rl_{t_2}, ra_2,) \\ -\Delta^{-1}(rl_{f_2}, rc_2) \end{pmatrix}, \quad AVT(r\theta_2) \in [0,1] \quad (9)$$

For two $r\theta_1 = \langle (rl_{t_1}, ra_1,), (rl_{i_1}, rb_1), (rl_{f_1}, rc_1) \rangle$ and $r\theta_2 = \langle (rl_{t_2}, ra_2,), (rl_{i_2}, rb_2), (rl_{f_2}, rc_2) \rangle$, the order is formulated: (1) $if\ SVT(r\theta_1) \prec SVT(r\theta_2), r\theta_1 \prec r\theta_2;$ (2) $if\ SVT(r\theta_1) = SVT(r\theta_2), AVT(r\theta_1) \prec AVT(r\theta_2),$ $r\theta_1 \prec r\theta_2;$ $if\ SVT(r\theta_1) = SVT(r\theta_2),\ AVT(r\theta_1) = AVT(r\theta_2),\ r\theta_1 = r\theta_2.$

The 2TLNNWA approach [31] is formulated.

Definition 4[31]. Let $r\theta_j = \left\langle \left(rl_{t_j}, ra_j,\right), \left(rl_{i_j}, rb_j\right), \left(rl_{f_j}, rc_j\right) \right\rangle$ be 2TLNNs, 2TLNNWA approach is formulated:

$$\begin{aligned}
&\text{2TLNNWA}\left(r\theta_1, r\theta_2, \ldots, r\theta_n\right) \\
&= rw_1 r\theta_1 \oplus rw_2 r\theta_2 \ldots \oplus rw_n r\theta_n = \mathop{\oplus}_{j=1}^{n} rw_j r\theta_j \\
&= \begin{pmatrix} \Delta\left(R\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta^{-1}\left(rl_{t_j}, ra_j,\right)}{R}\right)^{rw_j}\right)\right), \\ \Delta\left(R\prod_{j=1}^{n}\left(\frac{\Delta^{-1}\left(rl_{i_j}, rb_j\right)}{R}\right)^{rw_j}\right), \\ \Delta\left(R\prod_{j=1}^{n}\left(\frac{\Delta^{-1}\left(rl_{f_j}, rc_j\right)}{R}\right)^{rw_j}\right) \end{pmatrix}
\end{aligned} \tag{10}$$

with rw_j is weight value of $r\theta_j, \sum_{j=1}^{n} rw_j = 1.$

Then the 2TLNNHD (2TLNN Hamming distance) and 2TLNNED (2TLNN Euclidean distance) [30] is formulated.

Definition 5[30]. Let $r\theta_1 = \left\langle \left(rl_{t_1}, ra_1,\right), \left(rl_{i_1}, rb_1\right), \left(rl_{f_1}, rc_1\right) \right\rangle$, $r\theta_2 = \left\langle \left(rl_{t_2}, ra_2,\right), \left(rl_{i_2}, rb_2\right), \left(rl_{f_2}, rc_2\right) \right\rangle$, then 2TLNNHD and 2TLNNED are formulated:

$$2TLNNHD\left(r\theta_1, r\theta_2\right) = \frac{1}{3}\begin{pmatrix} \left|\frac{\Delta^{-1}\left(rl_{t_1}, ra_1,\right) - \Delta^{-1}\left(rl_{t_2}, ra_2,\right)}{R}\right| \\ + \left|\frac{\Delta^{-1}\left(rl_{i_1}, rb_1\right) - \Delta^{-1}\left(rl_{i_2}, rb_2\right)}{R}\right| \\ + \left|\frac{\Delta^{-1}\left(rl_{f_1}, rc_1\right) - \Delta^{-1}\left(rl_{f_2}, rc_2\right)}{R}\right| \end{pmatrix} \tag{11}$$

$$2TLNNED\left(r\theta_1, r\theta_2\right) = \sqrt{\frac{1}{3}\begin{pmatrix} \left|\frac{\Delta^{-1}\left(rl_{t_1}, ra_1,\right) - \Delta^{-1}\left(rl_{t_2}, ra_2,\right)}{R}\right|^2 \\ + \left|\frac{\Delta^{-1}\left(rl_{i_1}, rb_1\right) - \Delta^{-1}\left(rl_{i_2}, rb_2\right)}{R}\right|^2 \\ + \left|\frac{\Delta^{-1}\left(rl_{f_1}, rc_1\right) - \Delta^{-1}\left(rl_{f_2}, rc_2\right)}{R}\right|^2 \end{pmatrix}} \tag{12}$$

3. 2TLNN-CGRA approach for MAGDM with 2TLNSs

The 2TLNN-CGRA is formulated for MAGDM. Let alternatives $RA=\left(RA_1,RA_2,\ldots,RA_m\right)$ and attributes $RB=\left(RB_1,RB_2,\ldots,RB_m\right)$ with weight values $rw=\left(rw_1,rw_2,\cdots,rw_n\right)$ and experts $RC=\left(RC_1,RC_2,\ldots,RC_m\right)$ with weight $r\omega=\left(r\omega_1,r\omega_2,\cdots,r\omega_q\right)$, the 2TLNN-CGRA approach are formulated for MAGDM.

Step 1. Formulate the 2TLNN-matrix $RM=\left[RM_{ij}^{(t)}\right]_{m\times n}$:

$$RM^{(t)}=\left[RM_{ij}^{(t)}\right]_{m\times n}=\begin{matrix} \\ RA_1 \\ RA_2 \\ \vdots \\ RA_m \end{matrix}\begin{matrix} \begin{matrix} RB_1 & RB_2 & \ldots & RB_n \end{matrix} \\ \begin{bmatrix} RM_{11}^{(t)} & RM_{12}^{(t)} & \ldots & RM_{1n}^{(t)} \\ RM_{21}^{(t)} & RM_{22}^{(t)} & \ldots & RM_{2n}^{(t)} \\ \vdots & \vdots & \vdots & \vdots \\ RM_{m1}^{(t)} & RM_{m2}^{(t)} & \ldots & RM_{mn}^{(t)} \end{bmatrix} \end{matrix} \tag{13}$$

where $RM_{ij}^{(t)}=\left(\left(rl_{t_{ij}}^{(t)},ra_{ij}^{(t)},\right),\left(rl_{i_{ij}}^{(t)},rb_{ij}^{(t)},\right),\left(rl_{f_{ij}}^{(t)},rc_{ij}^{(t)},\right)\right)$ is 2TLNNs.

Step 2. Formulate the overall 2TLNN-matrix $RM=\left[RM_{ij}\right]_{m\times n}$:

$$RM=\left[RM_{ij}\right]_{m\times n}=\begin{matrix} \\ RA_1 \\ RA_2 \\ \vdots \\ RA_m \end{matrix}\begin{matrix} \begin{matrix} RB_1 & RB_2 & \ldots & RB_n \end{matrix} \\ \begin{bmatrix} RM_{11} & RM_{12} & \ldots & RM_{1n} \\ RM_{21} & RM_{22} & \ldots & RM_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ RM_{m1} & RM_{m2} & \ldots & RM_{mn} \end{bmatrix} \end{matrix} \tag{14}$$

In line with the 2TLNNWA approach, the $RM=\left[RM_{ij}\right]_{m\times n}$ is formulated:

$$\begin{aligned} RM_{ij}&=\left(\left(rl_{t_{ij}},ra_{ij},\right),\left(rl_{i_{ij}},rb_{ij},\right),\left(rl_{f_{ij}},rc_{ij},\right)\right) \\ &=r\omega_1 RM_{ij}^{(1)}\oplus r\omega_2 RM_{ij}^{(2)}\oplus\cdots\oplus r\omega_q RM_{ij}^{(q)} \\ &=\left\{\begin{array}{l} \Delta\left(R\left(1-\prod_{t=1}^{q}\left(1-\frac{\Delta^{-1}\left(rl_{t_{ij}}^{(t)},ra_{ij}^{(t)},\right)}{R}\right)^{r\omega_t}\right)\right), \\ \Delta\left(R\prod_{t=1}^{q}\left(\frac{\Delta^{-1}\left(rl_{i_{ij}}^{(t)},rb_{ij}^{(t)},\right)}{R}\right)^{r\omega_t}\right), \\ \Delta\left(R\prod_{t=1}^{q}\left(\frac{\Delta^{-1}\left(rl_{f_{ij}}^{(t)},rc_{ij}^{(t)},\right)}{R}\right)^{r\omega_t}\right) \end{array}\right\} \end{aligned} \tag{15}$$

Step 3. Formulate the normalized 2TLNN-matrix $NRM = \left[NRM_{ij} \right]_{m\times n}$ [43].

For benefit attributes:

$$NRM_{ij} = NRM_{ij} = \left(\left(rl^{n}_{t_{ij}}, ra^{n}_{ij}, \right), \left(rl^{n}_{i_{ij}}, rb^{n}_{ij}, \right), \left(rl^{n}_{f_{ij}}, rc^{n}_{ij}, \right) \right)$$
$$= \left(\left(rl_{t_{ij}}, ra_{ij}, \right), \left(rl_{i_{ij}}, rb_{ij}, \right), \left(rl_{f_{ij}}, rc_{ij}, \right) \right) \tag{16}$$

For cost attributes:

$$NRM_{ij} = NRM_{ij}$$
$$= \left(\left(rl^{n}_{t_{ij}}, ra^{n}_{ij}, \right), \left(rl^{n}_{i_{ij}}, rb^{n}_{ij}, \right), \left(rl^{n}_{f_{ij}}, rc^{n}_{ij}, \right) \right) \tag{17}$$
$$= \left(\begin{array}{l} \Delta\left(R - \Delta^{-1}\left(rl_{t_{ij}}, ra_{ij}, \right) \right), \Delta\left(R - \Delta^{-1}\left(rl_{i_{ij}}, rb_{ij}, \right) \right), \\ \Delta\left(R - \Delta^{-1}\left(rl_{f_{ij}}, rc_{ij}, \right) \right) \end{array} \right)$$

Step 4. Formulate the weight through entropy.

The weight is vital for MAGDM [44-48]. Entropy [49] is formulated for weight. The normalized matrix (NM) is Formulated:

$$NM_{ij} = \frac{1}{2} \left(\begin{array}{l} \dfrac{\left(SVT\left(\left(s\vartheta^{N}_{t_{ij}}, sx^{N}_{ij} \right), \left(s\vartheta^{N}_{i_{ij}}, sy^{N}_{ij} \right), \left(s\vartheta^{N}_{f_{ij}}, sz^{N}_{ij} \right) \right) + 0.5 \right)}{\sum_{i=1}^{m} \left(SVT\left(\left(s\vartheta^{N}_{t_{ij}}, sx^{N}_{ij} \right), \left(s\vartheta^{N}_{i_{ij}}, sy^{N}_{ij} \right), \left(s\vartheta^{N}_{f_{ij}}, sz^{N}_{ij} \right) \right) + 0.5 \right)} \\ + \dfrac{\left(AVT\left(\left(s\vartheta^{N}_{t_{ij}}, sx^{N}_{ij} \right), \left(s\vartheta^{N}_{i_{ij}}, sy^{N}_{ij} \right), \left(s\vartheta^{N}_{f_{ij}}, sz^{N}_{ij} \right) \right) + 0.5 \right)}{\sum_{i=1}^{m} \left(AVT\left(\left(s\vartheta^{N}_{t_{ij}}, sx^{N}_{ij} \right), \left(s\vartheta^{N}_{i_{ij}}, sy^{N}_{ij} \right), \left(s\vartheta^{N}_{f_{ij}}, sz^{N}_{ij} \right) \right) + 0.5 \right)} \end{array} \right) \tag{18}$$

The uncertain Shannon entropy (USE) is Formulated:

$$USE_{j} = -\frac{1}{\ln m} \sum_{i=1}^{m} NM_{ij} \ln NM_{ij} \tag{19}$$

and $NM_{ij} \ln NM_{ij} = 0$ if $NLNDM_{ij} = 0$.

The weight numbers are Formulated:

$$rw_{j} = \frac{1 - USE_{j}}{\sum_{j=1}^{n} \left(1 - USE_{j} \right)} \tag{20}$$

Step 5. Formulate the 2TLNNPIA (2TLNN positive ideal alternative) and 2TLNNNIA (2TLNN negative ideal alternative)[43]:

$$2TLNNPIA = \left\{ 2TLNNPIA_{j} \right\} \tag{21}$$

$$2TLNNNIA = \left\{ 2TLNNNIA_{j} \right\} \tag{22}$$

$$2TLNNPIA_{j} = \left(\left(rl^{n+}_{t_{ij}}, ra^{n+}_{ij}, \right), \left(rl^{n+}_{i_{ij}}, rb^{n+}_{ij}, \right), \left(rl^{n+}_{f_{ij}}, rc^{n+}_{ij} \right) \right) \tag{23}$$

$$2TLNNNIA_j = \left(\left(rl_{t_{ij}}^{n-}, ra_{ij}^{n-},\right),\left(rl_{i_{ij}}^{n-}, rb_{ij}^{n-},\right),\left(rl_{f_{ij}}^{n-}, rc_{ij}^{n-}\right)\right) \quad (24)$$

$$SVF\left(\left(rl_{t_{ij}}^{n+}, ra_{ij}^{n+},\right),\left(rl_{i_{ij}}^{n+}, rb_{ij}^{n+},\right),\left(rl_{f_{ij}}^{n+}, rc_{ij}^{n+}\right)\right) = \max_i SVF\left(\left(rl_{t_{ij}}^{n}, ra_{ij}^{n},\right),\left(rl_{i_{ij}}^{n}, rb_{ij}^{n},\right),\left(rl_{f_{ij}}^{n}, rc_{ij}^{n},\right)\right) \quad (25)$$

$$SVF\left(\left(rl_{t_{ij}}^{n-}, ra_{ij}^{n-},\right),\left(rl_{i_{ij}}^{n-}, rb_{ij}^{n-},\right),\left(rl_{f_{ij}}^{n-}, rc_{ij}^{n-}\right)\right) = \min_i SVF\left(\left(rl_{t_{ij}}^{n}, ra_{ij}^{n},\right),\left(rl_{i_{ij}}^{n}, rb_{ij}^{n},\right),\left(rl_{f_{ij}}^{n}, rc_{ij}^{n},\right)\right) \quad (26)$$

Step 6. Formulate the CGRC (combined grey rational coefficients) between 2TLNNPIA and 2TLNNNIA:

$$2TLNNPIA\left(CGRC_{ij}\right) = \frac{1}{2}\left(\frac{\left(\begin{array}{l}\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} 2TLNNHD\left(NRM_{ij}, 2TLNNPIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNHD\left(NRM_{ij}, 2TLNNPIA_j\right)\end{array}\right)}{\left(\begin{array}{l}2TLNNHD\left(NRM_{ij}, 2TLNNPIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNHD\left(NRM_{ij}, 2TLNNPIA_j\right)\end{array}\right)} + \frac{\left(\begin{array}{l}\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} 2TLNNED\left(NRM_{ij}, 2TLNNPIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNED\left(NRM_{ij}, 2TLNNPIA_j\right)\end{array}\right)}{\left(\begin{array}{l}2TLNNED\left(NRM_{ij}, 2TLNNPIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNED\left(NRM_{ij}, 2TLNNPIA_j\right)\end{array}\right)}\right) \quad (27)$$

$$2TLNNNIA\left(CGRC_{ij}\right) = \frac{1}{2}\left(\frac{\left(\begin{array}{l}\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} 2TLNNHD\left(NRM_{ij}, 2TLNNNIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNHD\left(NRM_{ij}, 2TLNNNIA_j\right)\end{array}\right)}{\left(\begin{array}{l}2TLNNHD\left(NRM_{ij}, 2TLNNNIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNHD\left(NRM_{ij}, 2TLNNNIA_j\right)\end{array}\right)} + \frac{\left(\begin{array}{l}\min\limits_{1\le i\le m}\min\limits_{1\le j\le n} 2TLNNED\left(NRM_{ij}, 2TLNNNIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNED\left(NRM_{ij}, 2TLNNNIA_j\right)\end{array}\right)}{\left(\begin{array}{l}2TLNNED\left(NRM_{ij}, 2TLNNNIA_j\right)\\ +\rho\max\limits_{1\le i\le m}\max\limits_{1\le j\le n} 2TLNNED\left(NRM_{ij}, 2TLNNNIA_j\right)\end{array}\right)}\right) \quad (28)$$

Step 7. Formulate the CGRD (combined grey relation degree) between 2TLNNPIS and 2TLNNNIS:

$$2TLNNPIA\left(CGRD_i\right) = \sum_{j=1}^{n} rw_j\, 2TLNNPIA\left(CGRC_{ij}\right) \quad (29)$$

$$2TLNNNIA\left(CGRD_i\right) = \sum_{j=1}^{n} rw_j\, 2TLNNNIA\left(CGRC_{ij}\right) \quad (30)$$

Step 8. Formulate the 2TLNNCRRD (2TLNN combined relative relational degree):

$$2TLNNCRRD_i = \frac{2TLNNPIA\left(CGRD_i\right)}{2TLNNNIA\left(CGRD_i\right) + 2TLNNPIA\left(CGRD_i\right)} \quad (31)$$

Step 9. Formulate the optimal choice with largest 2TLNNCRRD.

4. Numerical example and comparative analysis

4.1. Numerical example

Choosing building materials suppliers is a complex and critical process that plays a vital role in the success of construction projects. This process affects not only the quality and cost of the project but also the construction timeline and final outcome. Firstly, understanding market trends is the initial step in selecting suppliers. Market research helps in grasping the basic information about different suppliers, including their product types, service scope, and market reputation. This allows the project team to narrow down choices and focus on candidates that meet project needs. Establishing good communication channels is crucial. Building a transparent and open communication relationship with potential suppliers helps in better understanding their capabilities and service levels. Through communication, you can assess the supplier's responsiveness and problem-solving abilities, which are critical for smooth project implementation. Building long-term partnerships is another important factor in supplier selection. Establishing stable relationships with suppliers not only helps in securing preferential services and conditions in future projects but also enhances the stability and reliability of the supply chain. Long-term partnerships often mean deeper trust and better coordination, which are important guarantees for successful project execution. Additionally, considering the supplier's innovation capability and technical support is an important aspect of the selection process. In the construction industry, continuous innovation in technology and materials can lead to more efficient and environmentally friendly solutions. Choosing suppliers with strong R&D capabilities and technical support can introduce the latest technologies and materials to the project, enhancing its overall competitiveness. Finally, the supplier's financial stability and corporate culture are also worth considering. A supplier with good financial health and a positive corporate culture is generally more reliable in fulfilling contracts and handling unexpected situations. Whether their corporate culture aligns with the project team's values can also influence the cooperation experience and project implementation. In summary, selecting building materials suppliers is a process that requires comprehensive consideration. Through market research, effective communication, building long-term partnerships, evaluating innovation capabilities, and assessing financial stability and culture, project teams can make more informed choices. The BMSs evaluation and selection is the MAGDM. Five BMSs $RA_i \left(i = 1, 2, 3, \cdots, 5\right)$ are evaluated through four attributes (See Table 1):

Table 1. Four attributes for BMSs selection

Attribute	Description
Material Quality-RB_1	Ensure that the supplier provides materials meeting industry standards and project requirements for safety and durability.
Delivery Capability-RB_2	Evaluate the supplier's timeliness and reliability in delivery to ensure the project progresses as planned without delays.
Cost Effectiveness-RB_3	Analyze the supplier's price competitiveness and payment terms to control the project budget and optimize resource allocation.

Attribute	**Description**
Technical Support and Service-RB_4	Consider the technical support and after-sales service provided by the supplier for professional assistance and problem-solving during construction.

Five BMSs $RA_i\left(i=1,2,3,\cdots,5\right)$ are assessed through 2TLNNs by three experts with weight values $r\omega=\left(1/3,1/3,1/3\right)$. The 2TLNN-CGRA approach is formulated for BMSs selection.

Step 1. Formulate the 2TLNN-matrix $RM=\left[RM_{ij}^{(t)}\right]_{5\times 4}\left(t=1,2,3\right)$ (See Table 1-3).

Table 1. 2TLNNs from the first expert

	RB_1	RB_2
RA_1	$((rl_2,0), (rl_4, 0), (rl_3, 0))$	$((rl_1, 0), (rl_3, 0), (rl_2, 0))$
RA_2	$((rl_4, 0), (rl_1, 0), (rl_2, 0))$	$((rl_5, 0), (rl_3, 0), (rl_4, 0))$
RA_3	$((rl_4, 0), (rl_2, 0), (rl_5, 0))$	$((rl_3, 0), (rl_2, 0), (rl_1, 0))$
RA_4	$((rl_2, 0), (rl_4, 0), (rl_1, 0))$	$((rl_4, 0), (rl_5, 0), (rl_2, 0))$
RA_5	$((rl_3, 0), (rl_1, 0), (rl_5, 0))$	$((rl_2, 0), (rl_2, 0), (rl_4, 0))$

	RB_3	RB_4
RA_1	$((rl_4, 0), (rl_2, 0), (rl_3, 0))$	$((rl_2, 0), (rl_3, 0), (rl_1, 0))$
RA_2	$((rl_3, 0), (rl_4, 0), (rl_3, 0))$	$((rl_4, 0), (rl_5, 0), (rl_4, 0))$
RA_3	$((rl_2, 0), (rl_1, 0), (rl_5, 0))$	$((rl_1, 0), (rl_1, 0), (rl_4, 0))$
RA_4	$((rl_4, 0), (rl_3, 0), (rl_1, 0))$	$((rl_2, 0), (rl_3, 0), (rl_4, 0))$
RA_5	$((rl_5, 0), (rl_2, 0), (rl_1, 0))$	$((rl_5, 0), (rl_1, 0), (rl_1, 0))$

Table 2. 2TLNNs through a second expert

	RB_1	RB_2
RA_1	$((rl_5, 0), (rl_2, 0), (rl_1, 0))$	$((rl_2, 0), (rl_4, 0), (rl_3, 0))$
RA_2	$((rl_4, 0), (rl_3, 0), (rl_1, 0))$	$((rl_4,0), (rl_2,0), (rl_1,0))$
RA_3	$((rl_1, 0), (rl_2, 0), (rl_4, 0))$	$((rl_2, 0), (rl_4, 0), (rl_5, 0))$
RA_4	$((rl_5, 0), (rl_2, 0), (rl_3, 0))$	$((rl_2, 0), (rl_4, 0), (rl_3, 0))$

RA_5	((rl_2, 0), (rl_1, 0), (rl_5, 0))	((rl_2, 0), (rl_3, 0), (rl_4, 0))

	RB_3	RB_4
RA_1	((rl_2, 0), (rl_4, 0), (rl_5, 0))	((rl_3, 0), (rl_2, 0), (rl_5, 0))
RA_2	((rl_5,0), (rl_2,0), (rl_3,0))	((rl_5,0), (rl_2,0), (rl_3,0))
RA_3	((rl_5, 0), (rl_1, 0), (rl_2, 0))	((rl_4, 0), (rl_3, 0), (rl_2, 0))
RA_4	((rl_1, 0), (rl_5, 0), (rl_2, 0))	((rl_3, 0), (rl_2, 0), (rl_5, 0))
RA_5	((rl_2, 0), (rl_3, 0), (rl_4, 0))	((rl_2, 0), (rl_2, 0), (rl_3, 0))

Table 3. 2TLNNs through third expert

	RB_1	RB_2
RA_1	((rl_3, 0), (rl_2, 0), (rl_5, 0))	((rl_1, 0), (rl_3, 0), (rl_4, 0))
RA_2	((rl_2, 0), (rl_4, 0), (rl_1, 0))	((rl_3, 0), (rl_5, 0), (rl_2, 0))
RA_3	((rl_2, 0), (rl_3, 0), (rl_4, 0))	((rl_4, 0), (rl_2, 0), (rl_1, 0))
RA_4	((rl_3, 0), (rl_4, 0), (rl_2, 0))	((rl_3, 0), (rl_5, 0), (rl_2, 0))
RA_5	((rl_4, 0), (rl_1, 0), (rl_2, 0))	((rl_5, 0), (rl_3, 0), (rl_2, 0))

	RB_3	RB_4
RA_1	((rl_5, 0), (rl_4, 0), (rl_4, 0))	((rl_2 ,0), (rl_5, 0), (rl_4, 0))
RA_2	((rl_5, 0), (rl_3, 0), (rl_4, 0))	((rl_1, 0), (rl_4, 0), (rl_3, 0))
RA_3	((rl_1, 0), (rl_2, 0), (rl_4, 0))	((rl_4, 0), (rl_2, 0), (rl_3, 0))
RA_4	((rl_2, 0), (rl_1, 0), (rl_4, 0))	((rl_1, 0), (rl_3, 0), (rl_5, 0))
RA_5	((rl_4, 0), (rl_1, 0), (rl_2, 0))	((rl_5, 0), (rl_2, 0), (rl_1, 0))

Step 2. The invited experts weights are $r\omega = \left(1/3, 1/3, 1/3\right)$, the $RM = \left[RM_{ij}\right]_{5\times4}$ is formulated in Table 4.

Table 4. The $RM = \left[RM_{ij}\right]_{5\times4}$

	RB_1	RB_2
RA_1	((rl_4, 0.1723), (rl_2, -0.3489), (rl_3, 0.2847))	((rl_2, -0.4912), (rl_4, 0.2378), (rl_5, -0.3912))
RA_2	((rl_1, -0.2891), (rl_3, 0.4356), (rl_5, -0.1237))	((rl_2, 0.2983), (rl_4, -0.1874), (rl_3, 0.4223))
RA_3	((rl_5, 0.3924), (rl_2, -0.2678), (rl_1, 0.1579))	((rl_2, -0.4371), (rl_3, 0.3145), (rl_4, -0.2748))

RA_4	((rl_2 , -0.4213), (rl_5, 0.2085), (rl_4, -0.3692))	((rl_2, 0.4568), (rl_4, -0.4532), (rl_3, 0.1567))
RA_5	((rl_1, .1453), (rl_4, -0.4765), (rl_3, 0.3294))	((rl_4, -0.2139), (rl_2, 0.0921), (rl_1, -0.3485))
	RB_4	RB_3
RA_1	((rl_2, 0.1245), (rl_4, -0.3478), (rl_5, 0.2987))	((rl_1, -0.4123), (rl_2, 0.1509), (rl_4, -0.2745))
RA_2	((rl_2, -0.2134), (rl_4, 0.3891), (rl_3, -0.1765))	((rl_5, 0.2354), (rl_2, -0.4321), (rl_3, 0.0987))
RA_3	((rl_2, 0.3012), (rl_3, -0.2894), (rl_4, 0.4832))	((rl_2, -0.1456), (rl_1, 0.2678), (rl_5, -0.3999))
RA_4	((rl_2, -0.4798), (rl_4, 0.1056), (rl_3, -0.3698))	((rl_5, 0.4589), (rl_2, -0.2143), (rl_1, 0.3123))
RA_5	((rl_4, 0.2123), (rl_2, -0.4512), (rl_1, 0.1769))	((rl_4, -0.3245), (rl_3, 0.0912), (rl_1, -0.1876))

Step 3. If all attributes are benefit-oriented, the $NRM = \left[NRM_{ij} \right]_{5\times4}$ (Table 5) is same to $RM = \left[RM_{ij} \right]_{5\times4}$.

Table 5. The $NRM = \left[NRM_{ij} \right]_{5\times4}$

	RB_1	RB_2
RA_1	((rl_4, 0.1723), (rl_2, -0.3489), (rl_3, 0.2847))	((rl_2, -0.4912), (rl_4, 0.2378), (rl_5, -0.3912))
RA_2	((rl_1, -0.2891), (rl_3, 0.4356), (rl_5, -0.1237))	((rl_2, 0.2983), (rl_4, -0.1874), (rl_3, 0.4223))
RA_3	((rl_5, 0.3924), (rl_2, -0.2678), (rl_1, 0.1579))	((rl_2, -0.4371), (rl_3, 0.3145), (rl_4, -0.2748))
RA_4	((rl_2 , -0.4213), (rl_5, 0.2085), (rl_4, -0.3692))	((rl_2, 0.4568), (rl_4, -0.4532), (rl_3, 0.1567))
RA_5	((rl_1, .1453), (rl_4, -0.4765), (rl_3, 0.3294))	((rl_4, -0.2139), (rl_2, 0.0921), (rl_1, -0.3485))
	RB_4	RB_3
RA_1	((rl_2, 0.1245), (rl_4, -0.3478), (rl_5, 0.2987))	((rl_1, -0.4123), (rl_2, 0.1509), (rl_4, -0.2745))
RA_2	((rl_2, -0.2134), (rl_4, 0.3891), (rl_3, -0.1765))	((rl_5, 0.2354), (rl_2, -0.4321), (rl_3, 0.0987))
RA_3	((rl_2, 0.3012), (rl_3, -0.2894), (rl_4, 0.4832))	((rl_2, -0.1456), (rl_1, 0.2678), (rl_5, -0.3999))
RA_4	((rl_2, -0.4798), (rl_4, 0.1056), (rl_3, -0.3698))	((rl_5, 0.4589), (rl_2, -0.2143), (rl_1, 0.3123))
RA_5	((rl_4, 0.2123), (rl_2, -0.4512), (rl_1, 0.1769))	((rl_4, -0.3245), (rl_3, 0.0912), (rl_1, -0.1876))

Step 4. Formulate the weight numbers through entropy.

$$rw = (0.1755, 0.3267, 0.2748, 0.2230).$$

Step 5. Generate 2TLNNPIA and 2TLNNNIA (Table 6).

Table 6. The 2TLNNPIA and 2TLNNNIA

	RB_1	RB_2
2TLNNPIA	((rl_5, 0.3924), (rl_2, -0.2678), (rl_1, 0.1579))	((rl_4, -0.2139), (rl_2, 0.0921), (rl_1, -0.3485))

2TLNNNIA	((rl_1, -0.2891), (rl_3, 0.4356), (rl_5, -0.1237))	((rl_2, -0.4912), (rl_4, 0.2378), (rl_5, -0.3912))
	RB_3	RB_4
2TLNNPIA	((rl_4, 0.2123), (rl_2, -0.4512), (rl_1, 0.1769))	((rl_5, 0.4589), (rl_2, -0.2143), (rl_1, 0.3123))
2TLNNNIA	((rl_2, -0.4798), (rl_4, 0.1056), (rl_3, -0.3698))	((rl_1, -0.4123), (rl_2, 0.1509), (rl_4, -0.2745))

Step 6. Generate $2TLNNPIA\left(CGRC_{ij}\right)$, $2TLNNNIA\left(CGRC_{ij}\right)$ (Table 7-8).

Table 7. The $2TLNNPIA\left(CGRC_{ij}\right)$

Alternatives	RB_1	RB_2	RB_3	RB_4
RA_1	0.7255	0.5444	0.4065	0.5754
RA_2	0.5497	0.4589	0.5614	0.7396
RA_3	1.0000	0.8125	0.5614	0.7856
RA_4	0.7540	0.4589	0.5922	1.0000
RA_5	0.5164	1.0000	1.0000	0.7179

Table 8. The $2TLNNNIA\left(CGRC_{ij}\right)$

Alternatives	RB_1	RB_2	RB_3	RB_4
RA_1	0.7398	1.0000	0.5176	1.0000
RA_2	1.0000	0.6348	0.5834	0.8492
RA_3	0.6580	0.6121	0.4359	0.5090
RA_4	0.6504	0.8357	1.0000	0.4552
RA_5	0.6356	0.7525	0.8088	0.5291

Step 7. Generate $2TLNNPIA\left(CGRD_i\right), 2TLNNNIA\left(CGRD_i\right)$ (Table 9):

Table 9. The $2TLNNPIA\left(CGRD_i\right), 2TLNNNIA\left(CGRD_i\right)$

	$2TLNNPIA\left(CGRD_i\right)$	$2TLNNNIA\left(CGRD_i\right)$
RA_1	0.5452	0.8218
RA_2	0.5656	0.7326
RA_3	0.7704	0.5488
RA_4	0.6680	0.7635
RA_5	0.8522	0.6976

Step 8. Generate the defined $2TLNNCRRD_i$ (Table 10).

Table 10. The $2TLNNCRRD_i$

	$2TLNNCRRD_i$	Order
RA_1	0.3988	5
RA_2	0.4357	4
RA_3	0.5840	1
RA_4	0.4666	3
RA_5	0.5499	2

Step 9. From the $2TLNNCRRD_i$, the order is formulated: $RA_3 > RA_5 > RA_4 > RA_1 > RA_1$ and RA_3 is the optimal BMS.

4.2. Comparative analysis

The 2TLNN-CGRA approach is compared with the 2TLNNWA decision approach [31], 2TLNNWG decision approach [31], 2TLNN-MABAC decision approach [50], 2TLNN-CODAS decision approach [51], 2TLNN-CLVA decision approach [52] and 2TLNN-TODIM decision approach [30]. The derived comparative results are formulated in Table 11.

Table 17. Order for different approaches

Approaches	Order
2TLNNWA approach [31]	$RA_3 > RA_5 > RA_4 > RA_1 > RA_1$
2TLNNWG approach [31]	$RA_3 > RA_5 > RA_1 > RA_4 > RA_1$
2TLNN-MABAC approach [50]	$RA_3 > RA_5 > RA_4 > RA_1 > RA_1$
2TLNN-CODAS approach [51]	$RA_3 > RA_5 > RA_4 > RA_1 > RA_1$
2TLNN-CLVA approach [52]	$RA_3 > RA_5 > RA_4 > RA_1 > RA_1$
2TLNN-TODIM approach [30]	$RA_3 > RA_5 > RA_1 > RA_4 > RA_1$
2TLNN-CGRA approach	$RA_3 > RA_5 > RA_4 > RA_1 > RA_1$

Through comparative analysis, it was found that the ranking produced by the 2TLNN-CGRA approach aligns with that of several other methods, including the 2TLNNWA approach, 2TLNNWG approach, 2TLNN-MABAC approach, 2TLNN-CODAS approach, 2TLNN-CLVA approach, and 2TLNN-TODIM approach. However, there are minor discrepancies in the rankings between the 2TLNN-CGRA method and both the 2TLNNWG and 2TLNN-TODIM approaches. Despite these slight variations, all methods consistently identified the same optimal and least desirable BMS. This consistency across different methodologies confirms the effectiveness of the 2TLNN-CGRA approach.

5. Conclusion

The purpose of selecting building materials suppliers is to ensure the overall success and quality of a project. By choosing reliable suppliers, high-quality and consistent materials can be guaranteed, enhancing the safety and durability of the construction. This helps reduce risks during the building process and avoids delays and additional costs due to material issues. Additionally, selecting competitive supplier aids in budget control by negotiating better prices and terms. The delivery capability and service quality of suppliers are also crucial factors to ensure the project progresses on schedule. Establishing long-term partnerships can enhance supply chain efficiency and stability, laying a solid foundation for future projects. Thus, carefully selecting suppliers is a key step in achieving project goals and maintaining market competitiveness. The selection of BMSs utilizes the MAGDM framework. In this research, the 2TLNN-CGRA method was developed, drawing on traditional GRA combined with 2TLNNs. A numerical example was crafted to select BMSs, and various comparative analyses were conducted to assess the validity of the 2TLNN-CGRA method. The key contributions of this study are outlined: (1) The 2TLNN-CGRA method is applied to address MAGDM challenges using 2TLNNs; (2) It integrates the techniques of 2TLNNHD and 2TLNNED within the framework of 2TLNNs; (3) A practical example for selecting BMSs showcases the application of the 2TLNN-CGRA method; (4) Comparative analyses are performed with various established decision-making methods to demonstrate the method's effectiveness. This paper highlights the 2TLNN-CGRA method's ability to capture shape similarity in MAGDM contexts, utilizing both 2TLNNHD and 2TLNNED within the 2TLNN framework. However, it also notes a limitation: the 2TLNN-CGRA method does not fully address consensus-building issues within MAGDM.

In the study of the 2TLNN-CGRA approach, several potential shortcomings have been identified which suggest areas for improvement. At the same time, these issues provide clear directions for further research that could enhance the method's applicability and utility in practical scenarios: (1)The complexity of the 2TLNN-CGRA method poses a significant barrier to its broad adoption, particularly for decision-makers who lack advanced training in statistics and decision theory. This complexity not only makes it difficult to understand but also challenging to implement effectively in real-world situations where quick and comprehensible decision-making tools are needed. To address this, future research could focus on simplifying the method. One promising direction is the development of a web-based decision support system that utilizes a user-friendly graphical interface. This system would allow users to input data easily and manage the complex calculations automatically, thereby making the method more accessible to a wider audience. (2) The method's high data requirements can be a limitation, especially in environments where precise and consistent linguistic evaluations are hard to come by. To overcome this, more practical case studies across various industries and project scales should be undertaken. These studies should aim to apply the method in diverse cultural and economic contexts to better understand its flexibility and effectiveness under different conditions. Such research would not only test the method's adaptability but also help refine it to better meet the diverse needs of global users. (3) The validation of the 2TLNN-CGRA method, primarily through idealized numerical examples, may not adequately reflect the complex and dynamic nature of real-world scenarios. Therefore, a more comprehensive comparison with other decision-making methods currently in use, particularly those employed in supply chain management and building materials supplier selection, would be valuable. Evaluating the 2TLNN-CGRA method against these established methods could reveal insights into its efficiency, cost-effectiveness, and ease of use, providing a more rounded understanding of its potential and limitations.

Through addressing these flaws and pursuing the suggested research directions, the 2TLNN-CGRA method could be significantly enhanced, making it not only more practical and user-friendly but also a more robust tool for decision-making in various fields. This evolution is crucial for advancing decision theory and its application in areas requiring nuanced and multifaceted analysis.

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