NSS

Neutrosophic Sets and Systems, Vol. 73, 2024

University of New Mexico

Neutrosophic Rhotrices for Improved Diagnostic Accuracythrough Score Rhotrix Computation

Ananda Priya B[1], Gnanachandra P[2*] Saeid Jafari[3], and Florentin Smarandache[4]

[1]Research Scholar, Ayya Nadar Janaki Ammal College (Affliated to Madurai Kamraj University); priyaraghul331@gmail.com

[2]Ayya Nadar Janaki Ammal College (Affliated to Madurai Kamraj University); pgchandra07@gmail.com

[3]Mathematical and Physical Science Foundation, 4200 Slagelse, Denmark; jafaripersia@gmail.com

[4]Department of Mathematics, University of New Mexico, Gallup, NM, USA; smarand@unm.edu

[*]Correspondence: pgchandra07@gmail.com

Abstract. We introduce neutrosophic rhotrices, which serve as a novel extension of neutrosophic matrices. The primary objective is to establish the foundational structure for neutrosophic rhotrices and to define crucial operations that can be performed on them. We explore the concept of neutrosophic rhotrices in depth, outlining the fundamental operations necessary for their effective manipulation. Furthermore, we investigate the essential properties of neutrosophic rhotrices utilizing these newly established operations. In addition, we provide an algorithm designed to enhance decision-making processes in medical diagnostics, supported by an illustrative example to clarify its application.

Keywords: Neutrosophic Rhotrix, Rhotrix, Heart based Multiplication, Trace

---

In 2003, Ajibade [1] introduced a method for representing arrays of numbers in a rhomboidal shape, which he named rhotrix and developed the structure of an n-dimensional rhotrix to fall between the $(n-1)\times(n-1)$ - dimensional matrix and the $n \times n$ - dimensional matrix. Ajibade [10] also proposed the first multiplication method for rhotrices, called heart-based multiplication. In 2004, B. Sani [2] introduced another multiplication method for rhotrices, known as row-column multiplication, which is similar to traditional matrix multiplication. These two methods are the primary techniques for rhotrix multiplication, though various other multiplications can be defined, none of which generate algebraic structures.

Sani [3] also introduced a technique for transforming a rhotrix into a matrix form, referred to as a coupled matrix. In 2008, A. O. Isere [4] expanded on Ajibade's work by introducing the concept of even-dimensional rhotrix, since Ajibade's rhotrix is always of odd dimension. Additionally, numerous authors have contributed to the development of rhotrix theory, exploring concepts such as rhotrix groups [7, 11], rhotrix rings [5], rhotrix vector spaces [6], and the application of rhotrices in various fields, including cryptography [12–14].

The concept of neutrosophy was introduced by Florentin Smarandache [8, 16] in the 1990s. Neutrosophic logic extends classical and fuzzy logic by considering three components: truth (T), indeterminacy (I), and falsity (F). This triad is utilized to handle real-world problems where information is incomplete, inconsistent, or uncertain.

Kandasamy and Smarandache [17–19] expanded on this concept by introducing neutrosophic algebraic structures, including neutrosophic fields, vector spaces, groups, and rings. In linear algebra, matrices are essential for understanding vector spaces and linear transformations, prompting the creation of neutrosophic matrices. A neutrosophic matrix is an extension of the classical matrix concept, incorporating neutrosophic logic, which deals with indeterminacy. Recently, Mohammad Abobala and et al. [15] investigated the algebraic properties of these matrices, such as diagonalization, invertibility, determinants, and their algebraic representations through linear transformations. Neutrosophic matrices have found applications in various fields, particularly in dealing with uncertain, inconsistent, and incomplete information. Mamoni Dhar [9], introduced neutrosophic soft matrices and also a score matrix addressing patient-symptoms and symptoms-disease neutrosophic soft matrices is also proposed to aid in decision-making. Recently, the authors [20] introduced fuzzy rhotrices and its application in decision making of medical diagnostics has been studied.

We seek to introduce the concept of neutrosophic rhotrices and establish the fundamental operations required for their manipulation. We will define the basic operations and furthermore, conduct an in-depth examination of the fundamental properties of neutrosophic rhotrices, utilizing the newly defined operations to explore their characteristics. Additionally, an algorithm is proposed for medical diagnosis using neutrosophic rhotrices accompanied with an illustrative example.

Let n be positive odd integer. A rhotrix of size n is represented as

$$\left\langle \begin{array}{ccccccccccc} & & & & & a_1 & & & & & \\ & & & & a_2 & a_3 & a_4 & & & & \\ & & & .. & .. & .. & .. & .. & & & \\ & & .. & .. & .. & .. & .. & .. & .. & & \\ a_{\{\frac{t+1}{2}\}-\frac{n}{2}} & & .. & .. & .. & a_{\frac{t+1}{2}} & .. & .. & .. & & a_{\{\frac{t+1}{2}\}+\frac{n}{2}} \\ & & .. & .. & .. & .. & .. & .. & .. & & \\ & & & .. & .. & .. & .. & .. & & & \\ & & & & a_{t-3} & a_{t-2} & a_{t-1} & & & & \\ & & & & & a_t & & & & & \end{array} \right\rangle,$$

where $t = \frac{n^2+1}{2}$. In th above representation, the entries in red represents the first row, entries in blue represents first column, and entries in purple represents the diagonal. Here the major horizontal axis is $a_{\{\frac{t+1}{2}\}-\frac{n}{2}} \cdots a_{\frac{t+1}{2}} \cdots a_{\{\frac{t+1}{2}\}+\frac{n}{2}}$ and the major vertical axis is $a_1\ a_3\ \cdots\ a_{\frac{t+1}{2}}\ \cdots\ a_{t-2}\ a_t$. Heart of rhotrix is the entry intersecting both major horizontal axis and major vertical axis of the rhotrix and that is $a_{\frac{t+1}{2}}$.

Heart-based Multiplication, as introduced by A. O. Ajibade, involves multiplying each entry of the first rhotrix by the heart of the second rhotrix, and then adding the product of the corresponding entry of the second rhotrix with the heart of the first rhotrix. In addition, the process of adding two rhotrices is done by summing each corresponding entry. To transpose a rhotrix, one flips its entries over the major vertical axis, keeping the entries on this axis unchanged.

1. Neutrosophic Rhotrix

Within this section, we lay the groundwork by presenting neutrosophic rhotrices and elucidating fundamental operations, namely neutrosophic rhotrix addition and heart-based multiplication. Subsequently, we analyze the algebraic attributes of neutrosophic rhotrices under these operations, with a specific emphasis on exploring their trace properties.

Definition 1.1. A ***neutrosophic rhotrix*** is defined as a rhotrix with each of the entries are from a neutrosophic set $R = [(r_i^T, r_i^I, r_i^F)]$, where r_i^T, r_i^I, r_i^F characterized by a membership function $\mu_R : F \to [0,1]$, indeterminate function $\mathcal{I}_R : F \to [0,1]$, falsity function $\mathcal{F}_R : F \to [0,1]$ respectively and F is any field.

Suppose $R_1 = \left\langle \begin{array}{ccc} & r_1 & \\ r_2 & r_3 & r_4 \\ & r_5 & \end{array} \right\rangle$ is a rhotrix in $R_n(F)$, then the neutrosophic rhotrix is represented as $(R_1)_N = \left\langle \begin{matrix} & (\mu_R(r_1), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) & \\ (\mu_R(r_2), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) & (\mu_R(r_3), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) & (\mu_R(r_4), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) \\ & (\mu_R(r_5), \mathcal{I}_R(r_1), \mathcal{F}_R(r_1)) & \end{matrix} \right\rangle$

In simpler terms, it can be written as,

$$(R_1)_N = \left\langle \begin{matrix} & (r_1^T, r_1^I, r_1^F) & \\ (r_2^T, r_2^I, r_2^F) & (r_3^T, r_3^I, r_3^F) & (r_4^T, r_4^I, r_4^F) \\ & (r_5^T, r_5^I, r_5^F) & \end{matrix} \right\rangle$$

Definition 1.2. We define the **operations on neutrosophic rhotrices** as follows:
(i) The addition of two neutrosophic rhotrices is possible only if they are of the same size. Addition of two neutrosophic rhotrices $'A'_N$ and $'B'_N$ is defined as:

$$\text{The } i^{th} \text{ entry of } A_N + B_N = (max\{a_i^T, b_i^T\}, max\{a_i^I, b_i^I\}, min\{a_i^F, b_i^F\}), \text{ for all } i.$$

That is, for rhotrices $A_N = \left\langle \begin{matrix} & (a_1^T, a_1^I, a_1^F) & \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) & \end{matrix} \right\rangle$ of size 3, and $B_N = \left\langle \begin{matrix} & (b_1^T, b_1^I, b_1^F) & \\ (b_2^T, b_2^I, b_2^F) & (b_3^T, b_3^I, b_3^F) & (b_4^T, b_4^I, b_4^F) \\ & (b_5^T, b_5^I, b_5^F) & \end{matrix} \right\rangle$ of size 3,

$$A_N + B_N = \left\langle \begin{matrix} & \begin{matrix}(max\{a_1^T, b_1^T\}, max\{a_1^I, b_1^I\}, \\ min\{a_1^F, b_1^F\})\end{matrix} & \\ \begin{matrix}(max\{a_2^T, b_2^T\}, max\{a_2^I, b_2^I\} \\ , min\{a_2^F, b_2^F\})\end{matrix} & \begin{matrix}(max\{a_3^T, b_3^T\}, max\{a_3^I, b_3^I\} \\ , min\{a_3^F, b_3^F\})\end{matrix} & \begin{matrix}(max\{a_4^T, b_4^T\}, max\{a_4^I, b_4^I\} \\ , min\{a_4^F, b_4^F\})\end{matrix} \\ & \begin{matrix}(max\{a_5^T, b_5^T\}, max\{a_5^I, b_5^I\}, \\ min\{a_5^F, b_5^F\})\end{matrix} & \end{matrix} \right\rangle$$

(ii) The heart based neutrosophic multiplication of two neutrosophic rhotrices is possible only if they are of the same size. Multiplication of two neutrosophic rhotrices 'A' and 'B' is defined as:

$$\begin{aligned} \text{The } i^{th} \text{ entry of } A_N \circ B_N = (& max\{min(a_i^T, e_b^T), min(b_i^T, e_a^T)\}, max\{min(a_i^I, e_b^I), min(b_i^I, e_a^I)\}, \\ & min\{max(a_i^F, e_b^F), max(b_i^F, e_a^F)\}), \text{ for all } i, \text{ except the } heart \end{aligned}$$

$$Heart \text{ of } A_N \circ B_N = (min\{e_a, e_b\}, min\{e_a, e_b\}, max\{e_a, e_b\})$$

Definition 1.3 (Trace of Neutrosophic Rhotrix)**.** The trace of a neutrosophic rhotrix of size 'n' is adding the entries in major vertical axis and it is denoted as $Tr(.)$.

That is, for a rhotrix $A_N = \left\langle \begin{matrix} & (a_1^T, a_1^I, a_1^F) & \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) & \end{matrix} \right\rangle$ of size 3,

$Tr(A_N) = (max\{a_1^T, a_3^T, a_5^T\}, max\{a_1^I, a_3^I, a_5^I\}, min\{a_1^F, a_3^F, a_5^F\})$

Theorem 1.4. *For any neutrosophic rhotrices with the neutrosophic addition operation, the following axioms holds:*

- $(A_N + B_N) + C_N = A_N + (B_N + C_N)$ *(Associativity),*
- $A_N + O = A_N$, *where* $O = \left\langle \begin{matrix} & (0,0,1) & \\ (0,0,1) & (0,0,1) & (0,0,1) \\ & (0,0,1) & \end{matrix} \right\rangle$ *(Additive Identity),*
- $A_N + B_N = B_N + A_N$ *(Commutativity), where* A_N, B_N, *and* C_N *are any neutrosophic rhotrices*

Proof. Let $A_N, B_N,$ and C_N of size 3 be any three neutrosophic rhotrices . Then,
$A_N + (B_N + C_N) = (A_N + B_N) + C_N =$

$$\left\langle \begin{matrix} & \begin{matrix}(max\{a_1^T, b_1^T, c_1^T\}, max\{a_1^I, b_1^I, c_1^I\}, \\ min\{a_1^F, b_1^F, c_1^F\})\end{matrix} & \\ \begin{matrix}(max\{a_2^T, b_2^T, c_2^T\}, max\{a_2^I, b_2^I, c_2^I\}, \\ min\{a_2^F, b_2^F, c_2^F\})\end{matrix} & \begin{matrix}(max\{a_3^T, b_3^T, c_3^T\}, max\{a_3^I, b_3^I, c_3^I\}, \\ min\{a_3^F, b_3^F, c_3^F\})\end{matrix} & \begin{matrix}(max\{a_4^T, b_4^T, c_4^T\}, max\{a_4^I, b_4^I, c_4^I\}, \\ min\{a_4^F, b_4^F, c_4^F\})\end{matrix} \\ & \begin{matrix}(max\{a_5^T, b_5^T, c_5^T\}, max\{a_5^I, b_5^I, c_5^I\}, \\ min\{a_5^F, b_5^F, c_5^F\})\end{matrix} & \end{matrix} \right\rangle$$

Suppose $O = \left\langle \begin{matrix} & (x_1^T, x_1^I, x_1^F) & \\ (x_2^T, x_2^I, x_2^F) & (x_3^T, x_3^I, x_3^F) & (x_4^T, x_4^I, x_4^F) \\ & (x_5^T, x_5^I, x_5^F) & \end{matrix} \right\rangle$. For $A_N + O = A_N$,

$$\left\langle \begin{matrix} & \begin{matrix}(max\{a_1^T, x_1^T\}, max\{a_1^I, x_1^I\}, \\ min\{a_1^F, x_1^F\})\end{matrix} & \\ \begin{matrix}(max\{a_2^T, x_2^T\}, max\{a_2^I, x_2^I\}, \\ min\{a_2^F, x_2^F\})\end{matrix} & \begin{matrix}(max\{a_3^T, x_3^T\}, max\{a_3^I, x_3^I\}, \\ min\{a_3^F, x_3^F\})\end{matrix} & \begin{matrix}(max\{a_4^T, x_4^T\}, max\{a_4^I, x_4^I\}, \\ min\{a_4^F, x_4^F\})\end{matrix} \\ & \begin{matrix}(max\{a_5^T, x_5^T\}, max\{a_5^I, x_5^I\}, \\ min\{a_5^F, x_5^F\})\end{matrix} & \end{matrix} \right\rangle$$

$$= \left\langle \begin{matrix} & (a_1^T, a_1^I, a_1^F) & \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) & \end{matrix} \right\rangle$$

Then, $max\{a_i^T, x_i^T\} = a_i^T, max\{a_i^I, x_i^I\} = a_i^I and min\{a_i^F, x_i^F\} = a_i^F$, for all i, which implies $x_i^T = x_i^I = 0, x_i^F = 1$ for all i. Therefore, the additive identity for rhotrix size 3 is $\left\langle \begin{matrix} & (0,0,1) & \\ (0,0,1) & (0,0,1) & (0,0,1) \\ & (0,0,1) & \end{matrix} \right\rangle$. In general, the additive identity for rhotrix size 'n' is a rhotrix with all its entries as $(0,0,1)$.

Since each entry of $A_N + B_N = (max\{a_i^T, b_i^T\}, max\{a_i^I, b_i^I\}, min\{a_i^F, b_i^F\})$ and $max\{a_i, b_i\} =$

$max\{b_i, a_i\}$, and $min\{a_i, b_i\} = min\{b_i, a_i\}$ it is obvious that the commutative axiom holds for neutrosophic addition. □

Theorem 1.5. *The heart based multiplication operation among any neutrosophic rhotrices holds the following axioms:*

- $(A_N \circ B_N) \circ C_N = A_N \circ (B_N \circ C_N)$ *(Associativity),*
- $A_N \circ I = A_N$, *where* $I = \left\langle \begin{matrix} & (0,0,1) & \\ (0,0,1) & (1,1,0) & (0,0,1) \\ & (0,0,1) & \end{matrix} \right\rangle$ *(Multiplicative Identity),*
- $A_N \circ B_N = B_N \circ A_N$ *(Commutativity), where* A_N, B_N, *and* C_N *are any neutrosophic rhotrices*

Proof. Let us consider any three neutrosophic rhotrices $A_N, B_N,$ and C_N of size 3, and let $e_a, e_b,$ and e_c be the entries of heart of the rhotrices $A, B,$ and C, respectively. Then, the i^{th} entry of $(A_N \circ B_N) \circ C_N$

$$= \begin{cases} \Big(max\{min(a_i^T, e_b^T, e_c^T), min(b_i^T, e_b^T, e_c^T), min(c_i^T, e_b^T, e_c^T)\}, \\ max\{min(a_i^I, e_b^I, e_c^I), min(b_i^I, e_b^I, e_c^I), min(c_i^I, e_b^I, e_c^I)\}, \\ min\{max(a_i^F, e_b^F, e_c^F), max(b_i^F, e_b^F, e_c^F), max(c_i^F, e_b^F, e_c^F)\} \Big) & , \forall i, \text{ except } heart \\ \Big(min\{e_a^T, e_b^T, e_c^T\}, min\{e_a^I, e_b^I, e_c^I\}, max\{e_a^F, e_b^F, e_c^F\} \Big) & , for\ heart \end{cases}$$

$= i^{th}$ entry of $A_N \circ (B_N \circ C_N)$

Suppose $I_N = \left\langle \begin{matrix} & (x_1^T, x_1^I, x_1^F) & \\ (x_2^T, x_2^I, x_2^F) & (x_3^T, x_3^I, x_3^F) & (x_4^T, x_4^I, x_4^F) \\ & (x_5^T, x_5^I, x_5^F) & \end{matrix} \right\rangle$

For $A_N \circ I_N = A_N$,

The i^{th} entry of $A_N \circ I_N = \begin{cases} \Big(max\{min(a_i^T, x_3^T), min(x_i^T, e_a^T)\}, \\ max\{min(a_i^I, x_3^I), min(x_i^I, e_a^I)\}, \\ min\{max(a_i^F, x_3^F), max(x_i^F, e_a^F)\} \Big) & , \forall i, \text{ except the } heart \\ (min\{e_a^T, x_3^T\}, min\{e_a^I, x_3^I\}, max\{e_a^F, x_3^F\}) & , for\ heart \end{cases}$

Then, $min\{e_a^T, x_3^T\} = e_a^T$, implies $x_3^T = 1$ and similarly $x_3^I = 1$. Also, $max\{e_a^F, x_3^F\} = e_a^F$, implies $x_3^F = 0$. Since $x_3^T = 1$ and $max\{min(a_i^T, x_3^T), min(x_i^T, e_a^T)\} = a_i^T$, $x_i^T = 0$. Similarly, $x_i^I = 0$ and $x_i^F = 1$

Therefore, the additive identity for rhotrix size 3 is $I = \left\langle \begin{matrix} & (0,0,1) & \\ (0,0,1) & (1,1,0) & (0,0,1) \\ & (0,0,1) & \end{matrix} \right\rangle$. In general, the multiplicative identity for rhotrix size 'n' is a rhotrix of size 'n'with all entries (0,0,1) and heart of the rhotrix as (1,1,0).

Since the heart based rhotrix multiplication is commutative, it is obvious that the commutative axiom holds for heart based neutrosophic multiplication. □

Theorem 1.6. *Heart based multiplication of neutrosophic rhotrices is distributive with respect to addition of neutrosophic rhotrices. That is,* $A_N \circ (B_N + C_N) = A_N \circ B_N + A_N \circ C_N$, *where* A_N, B_N, C_N *are neutrosophic rhotrices.*

Proof. Let $A_N = \left\langle \begin{matrix} & (a_1^T, a_1^I, a_1^F) & \\ (a_2^T, a_2^I, a_2^F) & (e_a^T, e_a^I, e_a^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) & \end{matrix} \right\rangle$,

$B_N = \left\langle \begin{matrix} & (b_1^T, b_1^I, b_1^F) & \\ (b_2^T, b_2^I, b_2^F) & (e_b^T, e_b^I, e_b^F) & (b_4^T, b_4^I, b_4^F) \\ & (b_5^T, b_5^I, b_5^F) & \end{matrix} \right\rangle$ be neutrosophic rhotrices.

Then, the i^{th} entry of

$$\begin{aligned} A_N \circ (B_N + C_N) =& \Big(max\Big\{ min(a_i^T, max\{e_b^T, e_c^T\}), min(e_a^T, max\{b_i^T, c_i^T\}) \Big\}, max\Big\{ min(a_i^I, max\{e_b^I, e_c^I\}), \\ & min(e_a^I, max\{b_i^I, c_i^I\}) \Big\}, min\Big\{ max(a_i^F, min\{e_b^F, e_c^F\}), max(a_i^F, max\{e_b^F, e_c^F\}) \Big\} \Big) \\ =& \Big(max\{ min(a_i^T, e_b^T, e_c^T), min(b_i^T, e_a^T, e_c^T), min(c_i^T, e_a^T, e_b^T) \}, max\{ min(a_i^I, e_b^I, e_c^I), \\ & min(b_i^I, e_a^I, e_c^I), min(c_i^I, e_a^I, e_b^I) \}, min\{ min(a_i^F, e_b^F, e_c^F), min(b_i^F, e_a^F, e_c^F), \\ & min(c_i^F, e_a^F, e_b^F) \} \Big) \\ =& i^{th} \text{ entry of } (A_N \circ B_N) + (A_N \circ C_N) \end{aligned}$$

Therefore, $A_N \circ (B_N + C_N) = (A_N \circ B_N) + (A_N \circ C_N)$. This means that, we have proved, for any rhotrices of size 3, distributive property holds. In similar manner, we also can prove, distributive axiom holds for any rhotrices of size 'n'. □

Theorem 1.7. *For two neutrosophic rhotrices* A_N *and* B_N *and any scalar* λ *such that* $0 \leq \lambda \leq 1$, *the following properties hold:*

(i) $Tr(A_N + B_N) = Tr(A_N) + Tr(B_N)$

(ii) $Tr(\lambda A_N) = \lambda Tr_f(A_N)$

(iii) $Tr(A_N) = Tr(A_N^T)$

Proof. Consider two neutrosophic rhotrices of size 3,

$$A_N = \left\langle \begin{array}{ccc} & (a_1^T, a_1^I, a_1^F) & \\ (a_2^T, a_2^I, a_2^F) & (a_3^T, a_3^I, a_3^F) & (a_4^T, a_4^I, a_4^F) \\ & (a_5^T, a_5^I, a_5^F) & \end{array} \right\rangle \text{ and}$$

$$B_N = \left\langle \begin{array}{ccc} & (b_1^T, b_1^I, b_1^F) & \\ (b_2^T, b_2^I, b_2^F) & (b_3^T, b_3^I, e_b^F) & (b_4^T, b_4^I, b_4^F) \\ & (b_5^T, b_5^I, b_5^F) & \end{array} \right\rangle.$$

(i) Since the maximum operation is associative, we have the following:

$$\begin{aligned} Tr(A_N + B_N) &= (max\{max(a_1^T, b_1^T), max(a_3^T, b_3^T), max(a_5^T, b_5^T)\}, max\{max(a_1^I, b_1^I), max(a_3^I, b_3^I), \\ &\quad max(a_5^I, b_5^I)\}, min\{min(a_1^F, b_1^F), min(a_3^F, b_3^F), min(a_5^F, b_5^F)\}) \\ &= (max\{max(a_1^T, a_3^T, a_5^T), max(b_1^T, b_3^T, b_5^T)\}, max\{max(a_1^I, a_3^I, a_5^I), \\ &\quad max(b_1^I, b_3^I, b_5^I)\}, min\{min(a_1^F, a_3^F, a_5^F), min(b_1^F, b_3^F, b_5^F)\}) \\ &= Tr_f(A_N) + Tr_f(B_N) \end{aligned}$$

Therefore, $Tr(A_N + B_N) = Tr(A_N) + Tr(B_N)$, for any neutrosophic rhotrices of size 'n'.

(ii) Now, $Tr(\lambda A_N) = (max\{min(\lambda, a_1^T), min(\lambda, a_3^T), min(\lambda, a_5^T)\}, max\{min(\lambda, a_1^I), min(\lambda, a_3^I), min(\lambda, a_5^I)\}, min\{max(\lambda, a_1^F), max(\lambda, a_3^F), max(\lambda, a_5^F)\}) = (min\{\lambda, max(a_1^T, a_3^T, a_5^T)\}), min\{\lambda, max(a_1^I, a_3^I, a_5^I)\}, max\{\lambda, min(a_1^F, a_3^F, a_5^F)\}$ (∵ (A∧B)∨(A∧C)∨(A∧D)=A∧(B∨C∨D), where ∧ and ∨ represents *min* and *max* operation resp.) and $\lambda Tr_f(A_f) = (min\{\lambda, max(a_1^T, a_3^T, a_5^T)\}), min\{\lambda, max(a_1^I, a_3^I, a_5^I)\}, max\{\lambda, min(a_1^F, a_3^F, a_5^F)\}$. Therefore, $Tr(\lambda A_N) = \lambda Tr_f(A_N)$.

(iii) Since there is no changes in the major horizontal axis in A_N^T and A_N, $Tr(A_N) = Tr(A_N^T)$.

□

2. Elevating Diagnostic Precision with Neutrosophic Rhotrices and Fuzzy rhotrices

In the context of decision-making, when using matrices for medical diagnosis, if an individual does not exhibit a particular symptom, the value corresponding to that symptom is set to zero, and computations proceed accordingly, accounting for the absence of that symptom. However, when applying rhotrices, individuals can be categorized based on the number of symptoms they exhibit. For example, patients with three symptoms can be grouped together, as can those with two symptoms, which simplifies the overall diagnostic process by creating more manageable groups.

Method 1 (Using Neutrosophic Rhotrices)

In this method, we incorporate neutrosophic rhotrices into medical diagnosis, this framework advances healthcare by effectively managing uncertainty, indeterminacy, and imprecision.

Let S represent the set of symptoms associated with certain diseases, D denote the set of diseases, and P refer to the set of patients.

Step 1: Construct a symptom-disease neutrosophic rhotrix $A = [a_i j]$ of size n (=number of symptoms and diseases) in the following manner.
Here, $s_i d_j^T$ represents the membership value indicating how much symptom s_i contributes to the occurrence of disease d_j, $s_i d_j^I$ represents the membership value indicating how much symptom s_i contributes to the indeterminacy of disease d_j, and $s_i d_j^F$ represents the membership value indicating how much symptom s_i contributes to the falsity of disease d_j.Thus, the symptom-disease neutrosophic rhotrix A can be formed as

$$\left\langle \begin{array}{ccccc} & & (s_1d_1^T, s_1d_1^I, s_1d_1^F) & & \\ & (s_3d_1^T, s_3d_1^I, s_3d_1^F) & (s_2d_2^T, s_2d_2^I, s_2d_2^F) & (s_1d_3^T, s_1d_3^I, s_1d_3^F) & \\ (s_5d_1^T, s_5d_1^I, s_5d_1^F) & (s_4d_2^T, s_4d_2^I, s_4d_2^F) & (s_3d_3^T, s_3d_3^I, s_3d_3^F) & (s_2d_4^T, s_2d_4^I, s_2d_4^F) & (s_1d_5^T, s_1d_5^I, s_1d_5^F) \\ & (s_5d_3^T, s_5d_3^I, s_5d_3^F) & (s_4d_4^T, s_4d_4^I, s_4d_4^F) & (s_3d_5^T, s_3d_5^I, s_3d_5^F) & \\ & & (s_5d_5^T, s_5d_5^I, s_5d_5^F) & & \end{array} \right\rangle$$

Step 2: Construct a patient-symptom neutrosophic matrix $B = [b_i j]$ of size n (=number of symptoms and patients) in the following manner.

Here, $p_i s_j^T$ represents the membership value indicating how much the patient p_i suffers from the symptom s_j, $p_i s_j^I$ represents the membership value indicating how much the patient p_i contributes to the indeterminacy of symptom s_j, and $p_i s_j^F$ represents the membership value indicating how much the patient p_i contributes to the falsity of the symptom s_j.Thus, the symptom-disease neutrosophic rhotrix B can be formed as

$$\left\langle \begin{array}{ccccc} & & (p_1s_1^T, p_1s_1^T, p_1s_1^T) & & \\ & (p_3s_1^T, p_3s_1^T, p_3s_1^T) & (p_2s_2^T, p_2s_2^I, p_2s_2^F) & (p_1s_3^T, p_1s_3^I, p_1s_3^F) & \\ (p_5s_1^T, p_5s_1^I, p_5s_1^F) & (p_4s_2^T, p_4s_2^I, p_4s_2^F) & (p_3s_3^T, p_3s_3^I, p_3s_3^F) & (p_2s_4^T, p_2s_4^I, p_2s_4^F) & (p_1s_5^T, p_1s_5^I, p_1s_5^F) \\ & (p_5s_3^T, p_5s_3^I, p_5s_3^F) & (p_4s_4^T, p_4s_4^I, p_4s_4^F) & (p_3s_5^T, p_3s_5^I, p_3s_5^F) & \\ & & (p_5s_5^T, p_5s_5^I, p_5s_5^F) & & \end{array} \right\rangle$$

Step 3: Compute the rhotrix C by performing heart-based neutrosophic multiplication between rhotrices A and B, where the operation $\circ$ denotes the heart based neutrosophic multiplication.

Step 4: Construct the complement rhotrices A^c and B^c for rhotrices A and B respectively, both having the same dimensions. Afterward, calculate the composition rhotrix D by applying the heart-based neutrosophic multiplication $\circ$ to the complement matrices A^c and B^c.

Step 5: Transform the rhotrix A and B to A_S and B_S by adding the truth and indeterminacy values, then subtracting the falsity value for each neutrosophic number in the rhotrix A and

B.

Step 6: Derive the score rhotrix M by applying the minimum operator, denoted as $(-)$, to the rhotrices A_S and B_S. This results in $M = A_S(-)B_S$.
This will be used to analyze and interpret the relationships or differences between the elements under study.

Illustrative Analysis: Assume there are five symptoms and five diseases. Out of these, three symptoms (s_1, s_3, s_5) are associated with three specific diseases (d_1, d_3, d_5), while the remaining two symptoms (s_2, s_4) correspond to two other diseases (d_2, d_4).

Let us, for illustration, take symptom disease neutrosophic rhotrix as

$$A = \left\langle \begin{array}{ccccc} & & (0.2,0.4,0.6) & & \\ & (0.5,0.1,0.3) & (0.2,0.5,0.2) & (0.4,0.4,0.1) & \\ (0.9,0.6,0.3) & (0.2,0.9,0.1) & (0.1,0.7,0.4) & (0.2,0.6,0.3) & (0.1,0.5,0.4) \\ & (0.3,0.4,0.8) & (0.7,0.9,0.1) & (0.5,0.5,0.5) & \\ & & (0.1,0.4,0.1) & & \end{array} \right\rangle$$

Consider a scenario with five symptoms and five diseases. Three patients (p_1, p_3, p_5) experience three symptoms (s_1, s_3, s_5), while the other two patients (p_2, p_4) exhibit the remaining two symptoms (s_2, s_4).

Let us, for illustration, take the patient symptom neutrosophic rhotrix as

$$B = \left\langle \begin{array}{ccccc} & & (0.3,0.4,0.1) & & \\ & (0.2,0.1,0.2) & (0.1,0.9,0.4) & (0.7,0.3,0.1) & \\ (0.4,0.3,0.3) & (0.7,0.1,0.3) & (0.4,0.1,0.3) & (0.7,0.7,0.7) & (0.4,0.2,0.7) \\ & (0.4,0.4,0.1) & (0.3,0.4,0.7) & (0.9,0.9,0.9) & \\ & & (0.4,0.4,0.2) & & \end{array} \right\rangle$$

For the above illustration, we heart based multiply the two rhotrices A and B and get

$$C = \left\langle \begin{array}{ccccc} & & (0.2,0.4,0.4) & & \\ & (0.4,0.1,0.3) & (0.2,0.7,0.3) & (0.4,0.3,0.3) & \\ (0.4,0.3,0.3) & (0.2,0.1,0.3) & (0.1,0.1,0.3) & (0.2,0.7,0.3) & (0.1,0.2,0.4) \\ & (0.3,0.4,0.4) & (0.4,0.4,0.3) & (0.4,0.7,0.5) & \\ & & (0.1,0.4,0.3) & & \end{array} \right\rangle$$

By using,

$$A^c = \left\langle \begin{array}{ccccc} & & (0.8, 0.6, 0.4) & & \\ & (0.5, 0.9, 0.7) & (0.8, 0.5, 0.8) & (0.6, 0.6, 0.9) & \\ (0.1, 0.4, 0.7) & (0.8, 0.1, 0.9) & (0.9, 0.3, 0.6) & (0.8, 0.4, 0.7) & (0.9, 0.5, 0.6) \\ & (0.7, 0.6, 0.2) & (0.3, 0.1, 0.9) & (0.5, 0.5, 0.5) & \\ & & (0.9, 0.6, 0.9) & & \end{array} \right\rangle \text{ and}$$

$$B^c = \left\langle \begin{array}{ccccc} & & (0.7, 0.6, 0.9) & & \\ & (0.8, 0.9, 0.8) & (0.9, 0.1, 0.6) & (0.3, 0.7, 0.9) & \\ (0.6, 0.7, 0.7) & (0.3, 0.9, 0.7) & (0.6, 0.9, 0.7) & (0.3, 0.3, 0.3) & (0.6, 0.8, 0.3) \\ & (0.6, 0.6, 0.9) & (0.7, 0.6, 0.3) & (0.1, 0.1, 0.1) & \\ & & (0.6, 0.6, 0.8) & & \end{array} \right\rangle, \text{ we get } D =$$

$$\left\langle \begin{array}{ccccc} & & (0.7, 0.6, 0.7) & & \\ & (0.8, 0.9, 0.7) & (0.9, 0.5, 0.6) & (0.6, 0.6, 0.9) & \\ (0.6, 0.4, 0.7) & (0.6, 0.3, 0.7) & (0.6, 0.3, 0.7) & (0.6, 0.4, 0.6) & (0.6, 0.5, 0.6) \\ & (0.6, 0.6, 0.7) & (0.7, 0.3, 0.6) & (0.5, 0.5, 0.6) & \\ & & (0.6, 0.6, 0.8) & & \end{array} \right\rangle$$

Then, by the aforementioned algorithm, we transform A to

$$A_S = \left\langle \begin{array}{ccccc} & & 0.2 & & \\ & 0.2 & 0.6 & 0.4 & \\ 0.4 & 0 & -0.1 & 0.6 & -0.1 \\ & 0.3 & 0.5 & 0.6 & \\ & & 0.2 & & \end{array} \right\rangle$$

and B to B_S= $\left\langle \begin{array}{ccccc} & & 0.6 & & \\ & 1 & 0.8 & 0.3 & \\ 0.3 & 0.2 & 0.2 & 0.4 & 0.5 \\ & 0.5 & 0.4 & 0.4 & \\ & & 0.4 & & \end{array} \right\rangle$.

Then, the score rhotrix thus obtained is, $\left\langle \begin{array}{ccccc} & & -0.4 & & \\ & -0.8 & -0.2 & 0.1 & \\ 0.1 & -0.2 & -0.3 & 0.2 & -0.6 \\ & -0.8 & 0.1 & 0.2 & \\ & & -0.2 & & \end{array} \right\rangle$

We can observe that, p_1 is affected by d_3, p_2 is affected by d_4, p_3 is affected by d_5, p_4 is affected by d_4 and p_5 is affected by d_1, by finding maximum of each row.

Method 2 (Using Fuzzy Rhotrices)

In optimizing diagnostics with fuzzy rhotrices, let S represent the set of symptoms associated with certain diseases, D denote the set of diseases, and P refer to the set of patients.

Step 1: Construct a symptom-disease fuzzy rhotrix $A = [a_ij]$ of size n (=number of symptoms and diseases) in the following manner, based on the following example. Consider there are 5 symptoms and 5 diseases, in which 3 symptoms(s_1, s_3, s_5) can indicate 3 diseases (d_1, d_3, d_5) and another 2 symptoms (s_2, s_4) indicate another 2 diseases (d_2, d_4).Here, s_id_j represents the membership value indicating how much symptom s_i contributes to the occurrence of disease d_j.

Then the symptom-disease fuzzy rhotrix A can be formed as $A = \left\langle \begin{array}{ccccc} & & s_1d_1 & & \\ & s_3d_1 & s_2d_2 & s_1d_3 & \\ s_5d_1 & s_4d_2 & s_3d_3 & s_2d_4 & s_1d_5 \\ & s_5d_3 & s_4d_4 & s_3d_5 & \\ & & s_5d_5 & & \end{array} \right\rangle$

Suppose, for illustration, $A = \left\langle \begin{array}{ccccc} & & 0.2 & & \\ & 0.5 & 0.7 & 0.4 & \\ 0.6 & 0.1 & 0.9 & 0.6 & 0.4 \\ & 0.2 & 0.1 & 0.4 & \\ & & 0.5 & & \end{array} \right\rangle$

Step 2: Construct a patient-symptom fuzzy matrix $B = [b_ij]$ of size n (=number of symptoms and patients) in the following manner, based on the following example. Consider there are 5 symptoms and 5 diseases, in which 3 patients (p_1, p_3, p_5) has 3 symptoms(s_1, s_3, s_5) and another 2 patients (p_2, p_4) indicate another 2 symptoms (s_2, s_4).

Here, p_is_j represents the membership value indicating how much a patient p_i have affected by the symptom s_j.

Then

the patient-symptom fuzzy rhotrix can be formed as $B = \left\langle \begin{array}{ccccc} & & p_1s_1 & & \\ & p_3s_1 & p_2s_2 & p_1s_3 & \\ p_5s_1 & p_4s_2 & p_3s_3 & p_2s_4 & p_1s_5 \\ & p_5s_3 & p_4s_4 & p_3s_5 & \\ & & p_5s_5 & & \end{array} \right\rangle$

Suppose, for illustration, $B = \left\langle \begin{array}{ccccc} & & 0.5 & & \\ & 0.7 & 0.9 & 0.8 & \\ 0.2 & 0.5 & 0.9 & 0.2 & 0.3 \\ & 0.4 & 0.5 & 0.1 & \\ & & 0.2 & & \end{array} \right\rangle$

Step 3: Evaluate $C = A \circ B$, where $\circ$ is heart based fuzzy multiplication

For the above illustration, $C = \left\langle \begin{array}{ccccc} & & 0.5 & & \\ & 0.7 & 0.9 & 0.8 & \\ 0.6 & 0.5 & 0.9 & 0.6 & 0.4 \\ & 0.4 & 0.5 & 0.4 & \\ & & 0.5 & & \end{array} \right\rangle$

Step 4: Build the complement matrix A^c and B^c of A and B of size n and form the composition matrix D by computing $A^c \circ B^c$.

Then, $D = \left\langle \begin{array}{ccccc} & & 0.1 & & \\ & 0.1 & 0.1 & 0.1 & \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ & 0.1 & 0.1 & 0.1 & \\ & & 0.1 & & \end{array} \right\rangle$

Step 5: Compute $M = C(-)D$, where (-) denotes min operator Then, M for the above illustration is $\left\langle \begin{array}{ccccc} & & 0.4 & & \\ & 0.6 & 0.8 & 0.7 & \\ 0.5 & 0.4 & 0.8 & 0.5 & 0.3 \\ & 0.3 & 0.4 & 0.3 & \\ & & 0.4 & & \end{array} \right\rangle$

Step 6: Calculating the relativity values and form the comparison matrix

Then, the comparison matrix for the above illustration is $\left\langle \begin{array}{ccccc} & & 0 & & \\ & -0.143 & 0 & 0.143 & \\ 0.4 & -0.2 & 0 & 0.2 & -0.4 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle$,

We can observe that, p_1 is affected by d_3, p_2 is affected by d_4, p_3 is affected by d_3 and d_5, p_4 is affected by d_4 and p_5 is affected by d_1, by finding maximum of each row.

Comparative Analysis: The method applied for medical diagnostics using fuzzy rhotrices, observe that it only deals with truthfulness and do not explicitly address indeterminacy, which limits their ability to manage situations where medical data is incomplete or contradictory. But, our current algorithm uses neutrosophic rhotrices which are designed to handle three types of uncertainties: truth, indeterminacy, and falsity. In medical diagnosis, this means that a symptom can be partially present (truth), uncertain (indeterminacy), or absent (falsity), offering a richer framework for capturing ambiguity in symptoms.

Neutrosophic rhotrices, due to their sophisticated structure, provide significant advantages in decision-making processes, especially in situations where data can be organized into distinct categories or groups. For instance, in the field of medical diagnostics, neutrosophic rhotrices can facilitate the classification of patients based on their symptoms. This grouping allows healthcare professionals to make more precise and personalized decisions regarding diagnosis and treatment.

Compared to neutrosophic matrices, computations in neutrosophic rhotrices are much simpler. Rhotrices reduce computational complexity by handling multi-dimensional data in a more streamlined manner, enabling faster execution of operations like matrix multiplication and inversion. This efficiency is particularly valuable when dealing with large datasets in medical diagnosis.

In summary, the use of neutrosophic rhotrices in medical diagnostics not only streamlines the decision-making process but also promotes a more informed and personalized approach to patient care.

Conclusion

We introduced neutrosophic rhotrices, a novel extension of neutrosophic matrices, designed to effectively manage the complexities of uncertainty, indeterminacy, and falsity in decision-making processes, particularly in the context of medical diagnostics. By extending the traditional matrix structure with an additional dimension, neutrosophic rhotrices provide a more flexible and comprehensive framework for representing and processing medical data where ambiguity is inherent. Further, we explored the basic properties of neutrosophic rhotrices, such as their algebraic properties and their computational simplicity compared to neutrosophic matrices. Additionally, we compared the medical diagnostic method using neutrosophic rhotrices with that of fuzzy rhotrices and found that neutrosophic rhotrices provide significantly better results in the decision-making process.

References

1. AO Ajibade, The concept of rhotrix in mathematical enrichment, *International Journal of Mathematical Education in Science and Technology*, 34(2):175-179, 2003
2. B Sani, An alternative method for multiplication of rhotrices, *International Journal of Mathematical Education in Science and Technology*, 35(5):777-781, 2004
3. B Sani, Conversion of a rhotrix to a 'coupled matrix', *International Journal of Mathematical Education in Science and Technology*, 39(2):244-249, 2008
4. A O Isere, Even dimensional rhotrix, *Notes on Number Theory and Discrete Mathematics*, 24(2):125-133, 20

5. A Mohammed. The non-commutative full rhotrix ring and its subrings, *Science World Journal*, 13(2):24–36, 2018
6. Abdulhadi Aminu, Rhotrix vector spaces, *International Journal of Mathematical Education in Science and Technology*, 41(4):531–538, 2010
7. A Mohammed and UE Okon, On subgroups of non-commutative general rhotrix group, *Notes on Number Theory and Discrete Mathematics*, 22(2):72–90, 2016
8. F. Smarandache , Neutrosophic set - a generalization of the intuitionistic fuzzy set, Granular Computing, 2006 IEEE International Conference, 38 – 42,2006,DOI:10.1109/GRC.2006.1635754.
9. Dhar, M., Neutrosophic soft matrices and its application in medical diagnosis. Journal of fuzzy extension and application, 2 (1), 23-32, 2021.
10. Absalom, Ezugwu E and Ajibade, Abiodun O and Mohammed, A, Generalization of heart-oriented rhotrix multiplication and its algorithm implementation, *International Journal of Computer Applications*, Vol. 975,pg. no.8887, 2011
11. Tudunkaya SM, An Extension of Certain Construction of Finite Groups, *Nigerian Journal of Scientific Research*,vol. 17(3), 367-371, 2018
12. Bhuvaneshwari R and Kaladevi V, Energy Of Chemical Graphs With Adjacency Rhotrix, *European Journal of Molecular & Clinical Medicine*, vol. 7(08),2020
13. P Gnanachandra, B Ananda Priya and Seenivasan M(2023), AI-Driven Innovations in Cryptography: Enhancing Key Generation and Security, *E3S Web Conferences*, 399
14. Ananda Priya B, Gnanachandra P, Seenivasan M, Classification of the Commutative General Rhotrix Group: An Algebraic Approach, 6th International Conference on Mathematical Modelling, Applied Analysis and Computation-2023.
15. Mohammad Abobala, On the Representation of Neutrosophic Matrices by Neutrosophic Linear Transformations, Journal of Mathematics, 2021, DOI: 10.1155/2021/5591576.
16. F. Smarandache, A geometric interpretation of the neutrosophic set — A generalization of the intuitionistic fuzzy set Granular Computing (GrC), 2011 IEEE International Conference , 602 – 606, 2011, DOI: 10.1109/GRC.2011.6122665.
17. W.B. Vasantha Kandasamy & F. Smarandache. Neutrosophic Super Matrices and Quasi Super Matrices. *Ohio: Zip Publishing*, 2012.
18. WB, Vasantha Kandasamy, Ilanthenral Kandasamy, and Florentin Smarandache, Neutrosophic quadruple vector spaces and their properties, *Mathematics*, 7(8), 2019, 758.
19. Vasantha Kandasamy W.B,Ilanthenral Kandasamy and Florentin Smarandache, Semi-Idempotents in Neutrosophic Rings, *Mathematics*, 2019, 7(6), 507, https://doi.org/10.3390/math7060507
20. Ananda Priya, B. Gnanachandra P., Harnessing Fuzzy Rhotrices for Accurate Medical Diagnosis through Symptom – Disease Rhotrices, Preprint

Received: July 19, 2024. Accepted: September 07, 2024