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A generalized hybrid distance measure for SVNS with an application in medical diagnosis

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Abstract: A generalized hybrid distance measure of a single value neutrosophic set (SVNS) is proposed to analyze the patient's factors and diseases. Since Neutrosophic Set (NS) may communicate contradictory and ambiguous information, it is a crucial and useful tool for modelling uncertainty information. A distance measure for NS information is a key tool that is always employed in many approaches including medical diagnostics. Several distance measures in NS can be found in the literature, however, only a few of them proposed hybrid techniques. This paper aims to develop a new distance measure by considering exponential function and to define new hybrid distance measures for NS. The work also includes the formulation of the properties and application in the medical diagnosis for the validity phase. This study examines the possibility of cardiac problems in pregnant women. The eight factors considered in this study are age, obesity, smoking, family pathological history, personal pathological history, electrocardiogram, ultrasound, and functional class. The generalized hybrid distance measure has been used to discuss the factors of six different diseases, present consistent results.

Keywords: Neutrosophic set; distance measure; hybrid distance measure; medical diagnosis

1. Introduction

A neutrosophic set (NS) is a generalization of the concept of a fuzzy set (FS) and intuitionistic fuzzy set (IFS), which allows for the inclusion of uncertainty, indeterminacy, and vagueness. This effective mathematical model was proposed by [1]. An extension of the neutrosophic set is the single-valued neutrosophic set (SVNS)[2]. The SVNS is a generalization of the classic set, FS, and IFS [3]. The IFS has difficulties when it comes to communicating decision-related information. It can be viewed as a level of membership and a level of non-membership that is unable to deal with all kinds of doubts and contradictory information. In order to manage uncertain and inconsistent information, SVNS includes three terms: truth-membership, indeterminacy-membership, and falsity-membership. Over the years, a lot of information regarding the model measure for the SVNS model has been introduced and discussed. For instance, correlation coefficients, similarity measures, entropy measures, distance measures, and inclusion measures.

The similarity measure can be described as a crucial method in determining the degree of similarity between two objects. The numerous similarity measures can be represented into two categories which are crisp and neutrosophic similarity measures. The crisp similarity measures can be said to be real valued functions and neutrosophic similarity measures can be considered as the level of similarity, considering the similarity among the three membership values one by one. Because of that, the neutrosophic similarity acts towards a single valued neutrosophic number as a measure of the similarity. In multiple criteria decision making, to determine the differences between the criteria, similarity measure of neutrosophic set can be used [4 - 7]. According to [8], similarity measures are very important in many applications with various fields of decision making such as pattern recognition, image thresholding, and multicriteria decision making.

A hybrid similarity measure is a metric that combines two or more similarity measures to produce a single similarity value. A generalized hybrid similarity measure of a neutrosophic set is a metric that considers the different aspects of uncertainty, indeterminacy, and vagueness inherent in a neutrosophic set, and combines them to produce a single similarity value. New hybrid distance-based similarity measures are studied by [9] and [10]. Mondal et al [9] proposed hybrid binary logarithm similarity measure for dealing indeterminacy in decision making situations. Meanwhile, Ulucay et al [10] introduced a new hybrid distance-based similarity measure for refined rough neutrosophic set and the method is applied in medical diagnosis for some diseases.

Beside similarity measure, the distance measure also can be used to perform a similar task. Distance value represents the degree of dissimilarity between two sets. [11] developed a distance measure that improves decision-making flexibility under uncertainty by integrating soft expert systems with SVNSs. Zeng et al [12] developed a new distance measure between IFSs and applied it in pattern recognition. For SVNS, [13] produced a novel distance measure for SVNS and applied to multicriteria group decision-making problem. [14] integrated the distance measure with entropy weight measure for application in medical diagnosis.

The SVNS is important in modelling theory. Thus, it can be used in real scientific and technical applications. Due to the growing amount of information accessible from contemporary medical equipment, medical diagnosis contains a lot of incomplete, uncertain, imprecise, and inconsistent information, which is essential information about medical diagnosis problems [15]. Symptoms usually involves a lot of incomplete, ambiguous, and inconsistent information for a disease. According to [16], in medical diagnosis problems, symptoms and data inspection of diseases may be changed in different time intervals. This leads to the question of whether the use of a single inspection period can make it possible to conclude that a particular patient has a particular disease or not. Sometimes symptoms of different illnesses can appear in a person being treated. [17 – 18] applied the theory of SVNS to medical diagnosis. Studies using clustering approaches have indicated that the hybrid technique has enhanced the precision of disease diagnosis based on patient symptoms [18]. Besides SVNS, [19] applied the theory of rough neutrosophic set to medical diagnosis. [20] investigated a unique multi-parameter similarity measure for interval-valued neutrosophic sets (IVNSs) and it provides stable results and improves diagnostic accuracy by taking symptom interactions into account. In medical diagnostics, a bi-parametric discriminant measure addresses decision-making uncertainty and provides numerical examples to demonstrate its usefulness, allowing for effective symptomatic diagnosis [21].

The purposes of this study are to formulate a novelty distance measure of SVNS, which is the extension of [12] and to construct a new hybrid distance measure. The work starts by extending an existing distance measure from [12] to create a new one for SVNS. To increase the precision of the distance calculation between two NS, a hybrid distance measure is created by combining the recently created distance measure with the several existing distance measures for NS. The objective of the hybrid distance measure would be to better capture the complexities and nuances of the relationship between two NSs. The details of the hybrid distance measure would depend on the specific requirements and applications for which it is being developed. Several factors were taken into consideration when developing the hybrid distance measure, such as the importance of imprecision and uncertainty as well as the particular needs of the applications it would be used for. Eventually, by exploiting the increased accuracy in processing neutrosophic data, the generalized hybrid distance measure will be used to address issues related to medical diagnosis.

The remainder of the paper is structured as follows. Section two contains preliminary definition for some key terms, while section three introduces new definitions of a novel distance measure, and the hybrid distance measure is introduced in section four. Section five focuses on the application of new distance measures in medical diagnosis. Finally, section six is the conclusion of the present study.

2. Preliminaries

2.1. Neutrosophic Set

Definition 2.1.1: Neutrosophic Set [1]

Consider a space of points (objects) denoted by X, with each individual element in X represented by x. Within this context, the neutrosophic set $NS(A)$ can be understood as an object that takes a specific form $A = \left[\left(x, T_A(x), I_A(x), F_A(x)\right) : x \in X\right]$, where the functions $T, I, F : X \to]^-0, 1^+[$ define respectively the truth-membership function, an indeterminacy membership function and a falsity-membership function of the element $x \in X$ to the set A with the condition :

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+.$$

The function $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$.

Example: Truth (*T*), Indeterminacy (*I*), and Falsity (*F*) values in a NS are usually expressed as ranges, providing flexibility in expressing uncertainty. Let's look at a NS A, which stands for a collection of tall students. The ranges for each student *x* are the Truth (*T*(x)), Indeterminacy (*I*(x)), and Falsity (*F*(x)) of being tall.

For a student x:
$T(x)$ = [0.7,0.9] (The student's height ranges from 70% to 90%).
$I(x)$ = [0.1, 0.2] (There is a 10% to 20% margin of error regarding the student's height),
$F(x)$ = [0.1, 0.3] (The student is not tall by 10% to 30%).
This indicates that although there is some uncertainty and a small amount of falsity, student x is generally regarded as tall.

Definition 2.1.2: Single Value Neutrosophic Set [2]

Consider a space of points (objects) denoted by X, with each individual element in X represented by x. A single-valued neutrosophic set $A\left(SVNS(A)\right)$ is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in $X, T_A(x), I_A(x), F_A(x) \in [0,1]$. A SVNS can be written as

$$A = \{\langle x\ ;\ T_A(x), I_A(x), F_A(x)\rangle : x \in X\} \tag{1}$$

Example: A SVNS simplifies the neutrosophic set by giving truth, indeterminacy, and falsity fixed values (rather than ranges) within the interval [0,1]. Let's look at a SVNS A, which represents the percentage of exam takers who fall into the pass category.

For a student y:
$T(y)$ = 0.8 (A student is 80% likely to have passed)
$I(y)$ = 0.1 (There is 10% uncertainty over the student's passing)
$F(y)$ = 0.2 (The student is 20% likely to have failed)
In this instance, the fixed numbers give a clear picture of the student's exam result's validity, uncertainty, and falsity. As a result, the student is passed the exam

2.2 Distance Measure

One mathematical tool for calculating the distance between two neutrosophic sets is the distance measure for that sets. Depending on the intended use, a variety of distance measures, including Hamming distance, Hausdroff distance, Euclidean distance, and more generic distance functions, can be defined.

Definition 2.2.1: Several Distance Measures for SVNS
Let $X = \{x_1, x_2, \ldots, x_n\}$ be the universe of discourse. Let $A = \{x_i, T_A(x_i), I_A(x_i), F_A(x_i)) : x_i \in X\}$ and $B = \{x_i, T_B(x_i), I_B(x_i), F_B(x_i)) : x_i \in X\}$ be two neutrosophic sets. Several distance measures between two SVNS, A and B can be defined as:

1. Normalized Hamming distance measure [5]:

$$d_M^N(A,B) = \frac{1}{3n}\sum_{i=1}^{n}(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|) \tag{2}$$

2. Normalized Euclidean distance measure [5]:

$$d_E^N(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}((T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)} \tag{3}$$

3. Extended Hausdroff distance metric [5]:

$$d_S^N(A,B) = \frac{1}{n}\sum_{i=1}^{n} max(|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|) \tag{4}$$

4. Mustapha et al [17]:

$$d_{NM}^N(A,B) = \frac{2}{n}\sum_{i=1}^{n}\frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\};}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}} \tag{5}$$

5. Ye and Zhang [6]:

$$d_{YZ}^N(A,B) = 1 - \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\min(T_A(x_i), T_B(x_i))}{\max(T_A(x_i), T_B(x_i))} + \frac{\min(I_A(x_i), I_B(x_i))}{\max(I_A(x_i), I_B(x_i))} + \frac{\min(F_A(x_i), F_B(x_i))}{\max(F_A(x_i), F_B(x_i))}\right) \tag{6}$$

6. Ren et al [7]:

$$d_{Ren}^{N}(A,B)=\frac{1}{2n}\sum_{i=1}^{n}\left(\frac{(T_A(x_i)-T_B(x_i))^2}{2+T_A(x_i)+T_B(x_i)}+\frac{(I_A(x_i)-I_B(x_i))^2}{2+I_A(x_i)+I_B(x_i)}+\frac{(F_A(x_i)-F_B(x_i))^2}{2+F_A(x_i)+F_B(x_i)}+|m_A(x_i)-m_B(x_i)|\right), \quad (7)$$

where $m_j(x_i)=\frac{1+T_j(x_i)-F_j(x_i)}{2}, j=A,B$

7. Binary Logarithm [9]:

$$BL(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}log_2(2-\left(\frac{1}{3}\left(T_A(x_i)-T_B(x_i)\right)+\left(I_A(x_i)-I_B(x_i)\right)+\left(F_A(x_i)-F_B(x_i)\right)\right) \quad (8)$$

3. New Distance Measure for Single Value Neutrosophic Set

In this section, a novel approach for determining the distance measure between SVNS is introduced. It is extended from the work by [12], which incorporates exponential distance to minimize any potential loss of information.

Definition 3.1: Let $X=\{x_1,x_2,\ldots,x_n\}$ be the universe of discourse. Let $A=\{x_i, T_A(x_i),I_A(x_i),F_A(x_i)):x_i\in X\}$ and $B=\{x_i, T_B(x_i),I_B(x_i),F_B(x_i)): x_i \in X\}$ be two neutrosophic sets. A distance measure between two SVNS, A and B can be defined as:

$$d_{New}^{N}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\left(d_{exp}(A,B)\times d_{tif}(A,B)\right) \quad (9)$$

where $d_{exp}(A,B)=e^{|T_A(x_i)-T_B(x_i)|-1}$, $d_{tif}(A,B)=\frac{1}{3}\sum_{i=1}^{n}(|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)|)$, $d_{exp}(A,B)$ denotes the exponential distance measure, which is determined by the difference of the ith membership degree between A and B. and $d_{tif}(A,B)$ denotes the average including absolute value of ith membership degree, intermediate degree and false degree between A and B.

Theorem 3.1: $d_{New}^{N}(A,B)$ is the distance measure between SVNS A and B.

Proposition:

$(S1)$ $0\le d_{New}^{N}(A,B)\le 1$

$(S2)$ $d_{New}^{N}(A,B)=d_{New}^{N}(B,A)$

$(S3)$ $d_{New}^{N}(A,B)=0$ if and only if $A=B$

$(S4)$ For $A\subseteq B\subseteq C$, then we have $d_{New}^{N}(A,C)\ge d_{New}^{N}(A,B)$ and $d_{New}^{N}(A,C)\ge d_{New}^{N}(B,C)$

Proof(S1):

Suppose $T_A(x_i),T_B(x_i),I_A(x_i),I_B(x_i),F_A(x_i)$ and $F_B(x_i)\in[0,1]$, thus, we have

$$|T_A(x_i)-T_B(x_i)|\in[0,1]\Rightarrow|T_A(x_i)-T_B(x_i)|-1\in[-1,0]$$

$$\Rightarrow e^{|T_A(x_i)-T_B(x_i)|-1}\in[e^{-1},e^{0}]\subseteq[0,1]$$

$\therefore d_{exp}(A,B) \in [0,1]$.

$$|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| \in [0,3]$$
$$\Rightarrow \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} \in [0,1]$$

$\therefore d_{tif}(A,B) \in [0,1]$.

Hence,

$d_{exp}(A,B) \times d_{tif}(A,B) \in [0,1] \Rightarrow d^N_{New}(A,B) \in [0,1]$.

Proof(*S2*):

$d^N_{New}(A,B) = d^N_{New}(B,A)$ is obvious.

Proof(*S3*):

$d^N_{New}(A,B) = 0$ if and only if $A = B$;

Sufficiency: If $d^N_{New}(A,B) = 0$, then for every x_i, we have $d_{exp}(A,B) = 0$, which means

$$\frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} = 0$$
$$\Rightarrow |T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)| = 0$$

Then,

$$|T_A(x_i) - T_B(x_i)| = 0$$
$$|I_A(x_i) - I_B(x_i)| = 0$$
$$|F_A(x_i) - F_B(x_i)| = 0$$

Hence, we can have $T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i)\ and\ F_A(x_i) = F_B(x_i)$ for every x_i. That means *A*=*B*.

Necessity: If *A*=*B*, then it means for every x_i, we have $T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i)\ and\ F_A(x_i) = F_B(x_i)$, thus

$$d_{tif}(A,B) = \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3} = 0$$

$$\Rightarrow d^N_{New}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(d_{exp}(A,B) \times d_{tif}(A,B)\right) = 0$$

Proof(*S4*):

For $A \subseteq B \subseteq C$, then we have $d^N_{New}(A,C) \geq d^N_{New}(A,B)$ and $d^N_{New}(A,C) \geq d^N_{New}(B,C)$.

If $A \subseteq B \subseteq C$, we have

$$T_A(x_i) \leq T_B(x_i) \leq T_C(x_i)$$
$$I_A(x_i) \geq I_B(x_i) \geq I_C(x_i)$$
$$F_A(x_i) \geq F_B(x_i) \geq F_C(x_i)$$

Then, we get

$$|T_A(x_i) - T_C(x_i)| \geq |T_A(x_i) - T_B(x_i)|$$
$$|T_A(x_i) - T_C(x_i)| \geq |T_B(x_i) - T_A(x_i)|$$
$$|I_A(x_i) - I_C(x_i)| \geq |I_A(x_i) - I_B(x_i)|$$
$$|I_A(x_i) - I_C(x_i)| \geq |I_B(x_i) - I_A(x_i)|$$
$$|F_A(x_i) - F_C(x_i)| \geq |F_A(x_i) - F_B(x_i)|$$
$$|F_A(x_i) - F_C(x_i)| \geq |F_B(x_i) - F_A(x_i)|$$

Further, we can get

$$e^{|T_A(x_i) - T_C(x_i)| - 1} \geq e^{|T_A(x_i) - T_B(x_i)| - 1}$$

$$\frac{|T_A(x_i) - T_C(x_i)| + |I_A(x_i) - I_C(x_i)| + |F_A(x_i) - F_C(x_i)|}{3} \geq \frac{|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|}{3}$$

That is

$$d_{tif}(A, C) \geq d_{tif}(A, B)$$
$$d_{exp}(A, C) \geq d_{exp}(A, B)$$

Thus, we have $d_{New}^N(A, C) \geq d_{New}^N(A, B)$.
Similarly, we have $d_{New}^N(A, C) \geq d_{New}^N(B, C)$.
The theorem proved completely.

4. New Hybrid Distance Measure for SVNS

Hybrid formulas combine many methods or techniques, often coming from multiple disciplines or areas of expertise. Hybrid approaches utilize the benefits of each technique to get results that are more accurate than those obtained by using only one strategy. Assessing persistent similarities with greater accuracy should be the outcome of incorporating different similarity measures into a hybrid formula. Moreover, hybrid formulas are often more adaptable and dependable in a variety of datasets and situations. Because they can handle a wider range of input data types and attributes, they are suitable for a multitude of applications.

Definition 4.1: Hybrid vector distance measures of SVNS
Let $A = [T_A(x_i), I_A(x_i), F_A(x_i)]$ and $B = [T_B(x_i), I_B(x_i), F_B(x_i)]$ be two SVNSs in a universe of discourse $X = \{x_1, x_2, \dots, x_n\}$. Then, the hybrid distance measure of SVNSs in the vector space is defined as follows:

$$Hyb(A, B) = \tau\big(d(A, B)\big) + (1 - \tau)d_{New}^N(A, B)) \quad (10)$$

where $0 \leq \tau \leq 1$ and $d(A, B)$ is replaced with $BL\ (A, B), d_{NM}^N(A, B), d_M^N(A, B), d_E^N(A, B),$ $d_S^N(A, B), d_{YZ}^N(A, B)$ and $d_{Ren}^N(A, B)$ in Equations (2-8) for various hybrid distance measures.

Theorem 4.1 The formulation of properties for new hybrid distance measure of SVNS
The generalized hybrid distance measure $Hyb(A, B)$ for neutrosophic set A and B satisfies the following properties. For proving purpose, let $d(A, B)$ is defined as binary logarithm distance(Equation (8)), then Equation (10) becomes

$Hyb(A, B) = \tau(BL(A, B) + (1 - \tau)d_{New}^N(A, B))$ with $0 \leq \tau \leq 1$

Proposition:

$(S1)$ $0 \leq Hyb(A,B) \leq 1$

$(S2)$ $Hyb(A,B) = Hyb(B,A)$

$(S3)$ $Hyb(A,B) = 0$ if and only if $A = B$

$(S4)$ For $A \subseteq B \subseteq C$, then we have $Hyb(A,C) \geq Hyb(A,B)$ and $Hyb(A,C) \geq Hyb(B,C)$

Proof$(S1)$:

Suppose $BL(A,B)$ and $d^N_{New}(A,B) \in [0,1]$, thus, we have

$$0 \leq BL(A,B) + d^N_{New}(A,B) \leq 2.$$

$With\ 0 \leq \tau \leq 1,$

$$0 \leq \tau(BL(A,B) + (1-\tau)d^N_{New}(A,B)) \leq 1$$

Hence, the hybrid distance measure of SVNS is within $[0,1]$. Thus $0 \leq Hyb(A,B) \leq 1$.

Proof$(S2)$:

$Hyb(A,B) = Hyb(B,A)$ is obvious.

Proof$(S3)$:

For any two distances $BL(A,B)$ and $d^N_{New}(A,B)$, if $A = B$, then we have $BL(A,A)$ and $d^N_{New}(A,A)$. According to the proof of $d^N_{New}(A,B)$ shown in Section 3.1 and the proof of $BL(A,B)$ in [9], we have

$$BL(A,A) = 0 \text{ and } d^N_{New}(A,A) = 0,$$

$with\ 0 \leq \tau \leq 1,$

$$\tau(BL(A,A) + (1-\tau)d^N_{New}(A,A)) = 0 + 0 = 0$$

Similarly, if $A = B$

Hence, $Hyb(A,B) = 0$ if and only if $A = B$.

Proof$(S4)$:

For $A \subseteq B \subseteq C$, according to the proof of $d^N_{New}(A,B)$ shown in Section 3.1 and the proof of $BL(A,B)$ in [6], we have $d^N_{New}(A,C) \geq d^N_{New}(A,B)$, $d^N_{New}(A,C) \geq d^N_{New}(B,C)$, $BL(A,C) \geq BL(A,B)$ and $BL(A,C) \geq BL(B,C)$. Then, we have

$$BL(A,C) + d^N_{New}(A,C) \geq BL(A,B) + d^N_{New}(A,B).$$

With $0 \leq \tau \leq 1$,

$$\tau BL(A,C) + (1-\tau)d^N_{New}(A,C)) \geq \tau BL(A,B) + (1-\tau)d^N_{New}(A,B).$$

Thus, we have $Hyb(A,C) \geq Hyb(A,B)$.

Similarly, we have $Hyb(A,C) \geq Hyb(B,C)$.

The theorem was proved completely. The other hybrid distance measures can be proven similarly.

5. Application of new hybrid distance measure in medical diagnosis

Since the physical and hormonal changes that take place during pregnancy can put the heart under additional stress, pregnancy can be a high-risk time for women with heart disease. Heart failure, irregular heart rhythm, and blood clots are just a few of the pregnancy issues that may be more likely to occur in women who already have heart disease. In extreme situations, a pregnant woman with heart problems may potentially have a heart attack. All things considered, it's critical for pregnant women who have heart disease to be aware of the potential hazards involved and to seek medical advice from a specialist to decide the best course of action.

In this study, the data is collected from [22]. The following eight symptoms are commonly used to assess the level of risk for a pregnant woman with heart disease by doctors or medical officers.

Age: Older women are at higher risk of developing heart disease, and pregnancy can increase this risk.
Obesity: Women who are overweight or obese have an increased risk of developing heart disease, and this risk is further increased during pregnancy.
Smoking: Smoking increases the risk of heart disease and should be avoided by pregnant women with heart disease.
Family Pathological History: A family history of heart disease can indicate an increased risk of heart disease in pregnant women.
Personal Pathological History: A personal history of heart disease or heart-related conditions can increase the risk of complications during pregnancy.
Electrocardiogram (ECG): An ECG can detect heart disease and help determine the risk to the mother and fetus during pregnancy.
Ultrasound: An ultrasound can help detect heart problems and monitor the fetus during pregnancy.
Functional Class: The functional class of a woman's heart disease can help determine the risk of complications during pregnancy and guide management decisions.

Table 1. The relation between a woman and factors

Factor	(T,I,F)
A	(0.6,0.50,0.37)
OB	(1.0,0.25,0.60)
TAB	(1.0,0.00,0.00)
FPH	(1.0,0.00,0.00)
PPH	(1.0,0.00,0.00)
ECG	(0.5,0.30,0.40)
ECO	(0.5,0.30,0.40)
FH	(0.7,0.15,1.00)

The risks of pregnancy can be increased in a woman *W*, with major factor I={age(A), obesity (OB), smoking (TAB), personal pathological history PPH), family pathological history (FPH), electrocardiogram (ECG), ultrasound (ECO), functional class (FH)}. The diagnoses listed in the set are D = {obstruction at exit (OEX), obstruction at entry (OEN), rhythm disorders (RD), conduction disorder (CDS), congenital disease (CD), genetic disease (GD)} can affect pregnant women. These diseases can complicate pregnancy, delivery, and the postpartum period and have negative effects on both the mother and the fetus. OEX and OEN can make it more difficult for the fetus to be born and may necessitate a caesarean section. In addition to increasing the risk of difficulties during pregnancy and delivery, RD and CDS can lead to irregular heartbeats. CD and GD can cause congenital anomalies or genetic abnormalities in the fetus, which can impact its development and survival.

Pregnant women need to be examined and monitored by a healthcare provider if they have any of these conditions, to minimize the risk of complications and ensure the best possible outcome for both the mother and the fetus. The data from [22] are shown in Tables 1 and 2 which represent the relation between women and symptoms and the relation between symptoms and diagnoses, respectively.

Table 2. The relation between factors and diseases

	A	**OB**	**TAB**	**FPH**
OEX	(0.9,0.2,0.3)	(0.2,0.6,0.4)	(0.9,0.1,0.3)	(0.2,0.2,0.3)
OEN	(0.2,0.2,0.4)	(0.3,0.5,0.6)	(0.8,0.1,0.2)	(0.2,0.4,0.1)
RD	(0.8,0.3,0.2)	(0.9,0.2,0.3)	(0.9,0.2,0.3)	(0.2,0.1,0.1)
CDS	(0.9,0.2,0.3)	(0.8,0.2,0.1)	(0.9,0.2,0.3)	(0.3,0.6,0.5)
CD	(0.9,0.1,0.3)	(0.9,0.2,0.2)	(0.8,0.1,0.2)	(0.2,0.8,0.1)
GD	(0.9,0.2,0.3)	(0.8,0.2,0.3)	(0.2,0.1,0.4)	(0.8,0.2,0.3)
	PPH	**ECG**	**ECO**	**FH**
OEX	(0.9,0.1,0.2)	(0.8,0.2,0.3)	(0.7,0.2,0.2)	(0.9,0.2,0.1)
OEN	(0.8,0.2,0.3)	(0.7,0.2,0.3)	(0.8,0.2,0.3)	(0.9,0.2,0.3)
RD	(0.9,0.2,0.3)	(0.2,0.2,0.4)	(0.2,0.8,0.5)	(0.9,0.2,0.3)
CDS	(0.9,0.2,0.2)	(0.2,0.5,0.6)	(0.9,0.2,0.2)	(0.9,0.1,0.3)
CD	(0.2,0.5,0.5)	(0.9,0.2,0.2)	(0.8,0.2,0.3)	(0.9,0.2,0.2)
GD	(0.9,0.2,0.3)	(0.3,0.8,0.6)	(0.2,0.1,0.5)	(0.8,0.2,0.3)

The generalized hybrid distance measure is defined according to different values of τ as stated below:

i) If $\tau = 0$, the hybrid distance measure reduced to the new formulated distance measure (Equation 9).

ii) If $\tau = 1$, the hybrid distance measure is generalized to either Hamming, Hausdroff, Euclidean, Mustapha et al [17], Ye & Zhang [6], Ren et al [7] or Binary Logarithm (Mondal et al [9]) measure (Equations (2-8)).

iii) If $\tau = 0.5$, the hybrid distance measure is the mean distance measure between two distance measures.

iv) If $\tau = 0.1$ and $\tau = 0.9$, the hybrid distance measure is defined.

Table 3. The new distance measure for a woman W with diseases ($\tau = 0$)

Diseases	New Distance Measure
OEX	0.167808
OEN	0.157324
RD	0.088902
CDS	0.127549
CD	0.22461
GD	0.107783

Table 4. The new hybrid distance measures for a woman W with diseases when $\tau = 1$

	Hamming	Hausdorff	Euclidean	Mustapha et al (2021)	Ye & Zhang (2014)	Binary Logarithm (Mondal et al (2018))	Ren et al (2019)
OEX	0.260833	0.475	0.349929	0.379077	0.545701	0.205562	0.201525
OEN	0.254167	0.45875	0.331461	0.377367	0.532337	0.19821	0.198151
RD	0.225	0.425	0.296564	0.343536	0.514688	0.173466	0.163107
CDS	0.275	0.425	0.33434	0.397885	0.543531	0.217922	0.198624
CD	0.3125	0.5125	0.403774	0.432617	0.563323	0.252555	0.244957
GD	0.254167	0.4375	0.314431	0.380204	0.539364	0.197605	0.178489

The results of the hybrid distance measures, which were applied to analyze pregnant women's symptoms and forecast their propensity to develop specific diseases while pregnant, are shown in Tables 3–7. The results of the analysis using various parameter values are shown in the tables. The results from the hybrid distance measures are considered adequate for medical diagnostics despite the variances in parameter values. This shows that the suggested strategy is solid and capable of offering trustworthy information on the patient's condition.

Table 5. The new hybrid distance measures for a woman W with diseases when τ=0.5

	Hamming	Hausdorff	Euclidean	Mustapha et al (2021)	Ye & Zhang (2014)	Binary Logarithm (Mondal et al (2018))	Ren et al (2019)
OEX	0.214321	0.321404	0.258868	0.273443	0.356755	0.186685	0.184666
OEN	0.205745	0.308037	0.244393	0.267346	0.344831	0.177767	0.177737
RD	0.156951	0.256951	0.192733	0.216219	0.301795	0.131184	0.126004
CDS	0.201274	0.276274	0.230944	0.262717	0.33554	0.172735	0.163086
CD	0.268555	0.368555	0.314192	0.328613	0.393966	0.238583	0.234784
GD	0.180975	0.272642	0.211107	0.243993	0.323574	0.152694	0.143136

It is clear from looking at the results shown in the Tables 3-7 that the pregnant woman has a very high likelihood of developing RD throughout her pregnancy. Her symptoms exhibit important signs and patterns that are consistent with RD according to the hybrid distance measurements. Therefore, doctors could begin treating this patient early in order to prevent difficulties for the mother and the unborn child. Pre-existing heart problems, hypertension, and obesity all raise a woman's risk of getting an RD during pregnancy.

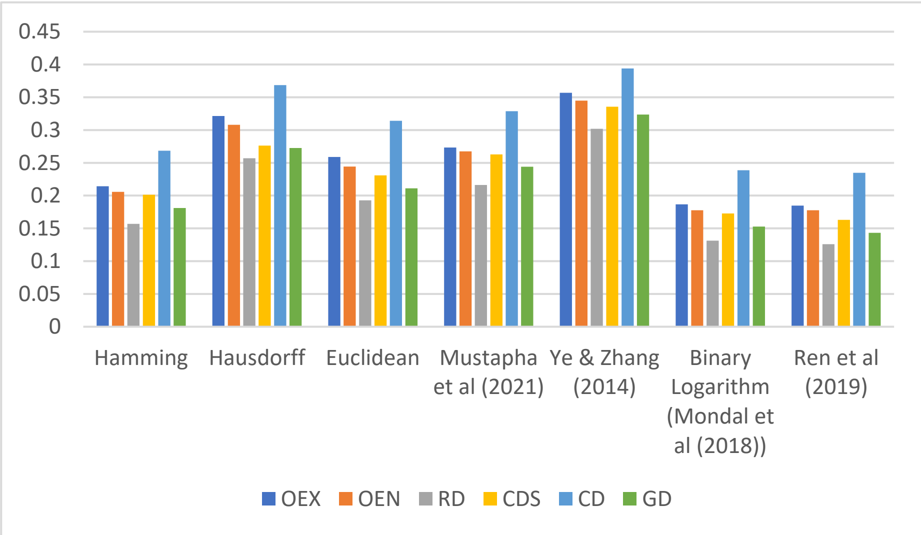

Figure 1. The new hybrid distance measures for a woman *W* with diseases when τ=0.5

Table 6. The new hybrid distance measures for a woman *W* with diseases when τ=0.9

	Hamming	**Hausdorff**	**Euclidean**	**Mustapha et al (2021)**	**Ye & Zhang (2014)**	**Binary Logarithm (Mondal et al (2018))**	**Ren et al (2019)**
OEX	0.251531	0.444281	0.331717	0.35795	0.507912	0.171584	0.198153
OEN	0.244482	0.428607	0.314048	0.355363	0.494836	0.161412	0.194068
RD	0.21139	0.39139	0.275797	0.318072	0.472109	0.097358	0.155687
CDS	0.260255	0.395255	0.313661	0.370852	0.501933	0.136586	0.191516
CD	0.303711	0.483711	0.385857	0.411816	0.529451	0.227404	0.242923
GD	0.239528	0.404528	0.293766	0.352961	0.496206	0.116765	0.171419

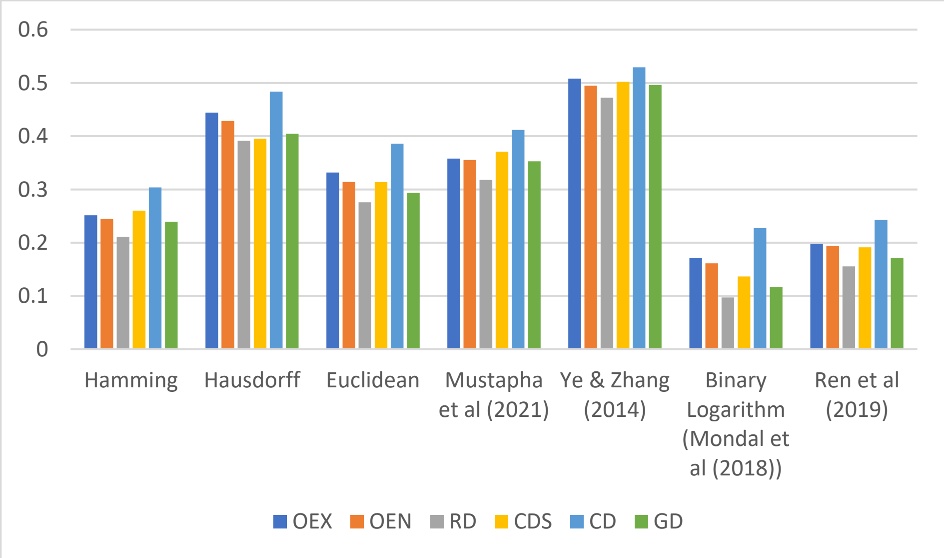

Figure 2. The new hybrid distance measures for a woman *W* with diseases when τ=0.9

Table 7. The new hybrid distance measures for a woman *W* with diseases when τ=0.1

	Hamming	Hausdorff	Euclidean	Mustapha et al (2021)	Ye & Zhang (2014)	Binary Logarithm (Mondal et al (2018))	Ren et al (2019)
OEX	0.177111	0.198527	0.18602	0.188935	0.205598	0.201787	0.17118
OEN	0.167008	0.187466	0.174738	0.179328	0.194825	0.194121	0.161406
RD	0.102511	0.122511	0.109668	0.114365	0.13148	0.165009	0.096322
CDS	0.142294	0.157294	0.148228	0.154582	0.169147	0.208885	0.134656
CD	0.233399	0.253399	0.242526	0.245411	0.258481	0.249761	0.226645
GD	0.122421	0.140755	0.128448	0.135025	0.150941	0.188623	0.114854

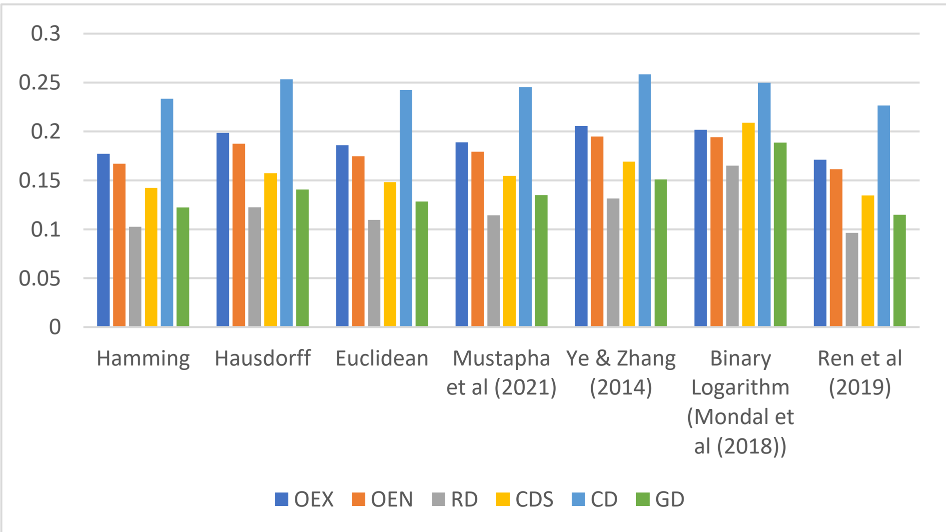

Figure 3. The new hybrid distance measures for a woman *W* with diseases when τ=0.1

As shown in Tables 5-7, new distance measure proposed in our study demonstrates its contribution to the analysis of RD compared to other distance measures, even when considering various values of the parameter, τ. The results obtained also presented clearly in Figures 1-3. This indicates that our new distance measure is particularly effective in capturing the relevant characteristics and patterns associated with RD, making it a valuable tool for diagnosing and predicting this condition.

In the analysis, we observed that the shortest distance measure is highly influenced by the hybridization between the new distance measure and [7] when τ is set to 0.1. This finding suggests that the combination of our new distance measure with Ren et al [7] yields the most accurate representation of RD symptoms, resulting in a distance measure that is closest to zero and indicating a higher likelihood of RD.

Furthermore, the second-lowest distance measure is predominantly influenced by the hybridization between the new distance measure and Binary Logarithm with a τ value of 0.1. This indicates that this particular combination captures important aspects of RD symptoms, contributing to a relatively low distance measure and suggesting a significant association with RD. In addition to the above observations, the results presented in Tables 3-7 and Figures 1-3 also highlight the relatively lower significance of CD compared to other diseases for the pregnant woman under consideration.

6. Conclusions

In conclusion, the proposed generalized hybrid distance measure of a NS provides a valuable tool for the analysis of patients's symptoms and diseases. This approach considers the inherent uncertainty and indeterminacy associated with medical data, allowing for a more comprehensive and accurate assessment of the patient's condition. By incorporating both the distance measure and the neutrosophic set theory, this hybrid approach addresses the limitations of traditional distance measures and offers a more nuanced representation of the patient's symptoms.

The utilization of this hybrid distance measure contributes significantly to the field of medical diagnosis and treatment. It enables healthcare professionals to make decisions based on a more comprehensive understanding of the patient's symptoms and diseases, leading to improved accuracy in diagnoses and tailored treatment plans.

Furthermore, the proposed approach has the potential for broader applications beyond medical analysis. Its ability to handle uncertainty and indeterminacy makes it suitable for decision-making processes in various domains where imprecise or incomplete information is common.

In conclusion, the generalized hybrid distance measure of a neutrosophic set offers a promising approach for analyzing patient symptoms and diseases, providing a robust framework for medical practitioners and researchers to improve diagnostic accuracy and optimize treatment outcomes.

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