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Incorporating Neutrosophic Uncertainty in Data Envelopment Analysis

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Abstract. In the realm of Data Envelopment Analysis (DEA), the assessment of relative efficiency has historically relied on deterministic variables. Yet, real-world data often presents diverse uncertainties, including a neutrosophic nature, which can significantly impact the accuracy and reliability of DEA outcomes. Therefore, the current paper proposes a new DEA model entirely to address this issue and in new DEA model designed to address this issue, preexisting approaches that fall short due to the inability to address neutrosophic uncertainty, which can be commonly found in actual data, to enhance efficiency evaluation precision. This paper introduces the new DEA model to show the potential to deal with the neutrosophic uncertainty in the input and output to decide whether the certain variable is deterministic or neutrosophic. In addition, they assure the flexibility of the model regarding the orientation and to embrace both the output-orientation and input-orientation as well as both constant and variable return on scale. In its simplest form, the model converts the neutrosophic constraints into their determinate forms, which makes the DEA issues feasible in the determinate domain. On this way, the current model enhances the drawbacks of conventional DEA methodologies, and the actual data reflected by neutrosophic indeterminacies. The research discusses that the proposed model holds a high capability and flexibility through theoretical examination and through the given cases. This particular methodology helps researchers and decision makers to be able to use all its functions in enhancing defensibility and precision in the assessment of efficiency thereby increasing the accuracy of the decision-making process in every aspect of its use.

Keywords: Data Envelopment Analysis; Neutrosophic Uncertainty; Efficiency Evaluation; Decision-Making; Uncertainty Management; Healthcare Facility, Optimization

1. Introduction

For the efficient comparison of comparable organizations described as Decision Making Units (DMUs) from industries such as manufacturing, health care, education, and public sector services, Data Envelopment Analysis (DEA) is a useful tool. In general, DEA models presuppose deterministic input and output data, that is, the data are fully measurable and definite. Nonetheless, in practice, data is imprecise due to measurement errors, subjectivity, and limited information available [1]. These uncertainties can significantly limit the precision and reliability of DEA-based efficiency evaluations.

To address those drawbacks of the deterministic DEA, many scholars have explored different methods on how to include uncertainty in the models. There is a research approach known as fuzzy set theory that has been used to analyze data that are ambiguous in nature [2]. However, fuzzy sets are far from ideal in handling this kind of uncertainty, more so when dealing with conflicting or unclear information. More recently, there is a more general approach which is a neutrosophic set theory that is capable of deal with uncertainties. Neutrosophic sets, which allow for three membership degrees: It is for this reason that four-valued logic, with truth (T), indeterminacy (I), and falsity (F), provides a less rigid and more precise representation of uncertain data [3, 4, 5, 6].

Certain papers can be found that use neutrosophic sets in DEA techniques as well as in models. For instance, Saadati et al. [7] developed a neutrosophic DEA model for assessing the supply chain performance. Wang et al. [8] assumed the neutrosophic DEA approach for the performance measurement of the banks etc. Otherwise, Chakraborty et al. [9] put forth a method of de-neutrosophication of neutrosophic pentagonal numbers which are assigned through two overlays denoting truth and a false, and intermediate ambivalence. It makes use of one of the techniques called the removal area method which calculates the area underneath the membership function of the said values. Liu et al. [10] on the other hand also evaluated the efficiency of the sustainable manufacturing systems supported with a neutrosophic DEA model.

Furthermore, the study made by Pal et al. [11] showed that the model identifies the optimal cycle time of production and the reliability of production that would reduce the total cost of the system. The model, which was developed based on the triangular neutrosophic numbers, aims to address the uncertainty in the reliability of the production process, the demand rate, the deterioration rate, and the shortage cost. There is an effort made by Zhang et al. [12] with the introduction of neutrosophic DEA for the evaluation of the efficiencies of the renewable energy projects. Wang et al. [13] evaluated the performance of sustainable supply chains using a DEA model developed with neutrosophic uncertainty. In their study, Liu et al. [14] proposed a neutrosophic DEA model for assessing the performance of healthcare systems. In their study, El-Demerdash, et al. [15] discussed a general framework on how neutrosophic uncertainty could be integrated into input-oriented DEA models, which can be beneficial for researchers and practitioners. To model uncertainty in input and output data, Farnam, et al. [16] put forward a new DEA model – the neutrosophic DEA model. Neutrosophic numbers are more useful than traditional probability theory because they allow for the representation of truth, falsity, and uncertainty.

The objective of this paper is to present the new DEA-fuzzy model with neutrosophic uncertainty for more accurate assessment of efficiency. Unlike the previous models that focused on deterministic components, the current model is more realistic and not sensitive tointerferences. One strength of the model is that the DEA orientation as well as the return to scale attributes can be changed. Moreover, it extends neutrosophic constraints to deterministic forms to make use of classic DEA approaches effectively.

The remainder of this paper is structured as follows: Section 2 provides a neutrosophic sets overview. Section 3 presents the general mathematical model of DEA. Section 4 introduces the proposed DEA model under neutrosophic uncertainty. Section 5 illustrates the model's application through a case study. Finally, Section 6 concludes the paper with a summary and discusses future research directions.

2. Neutrosophic Sets Overview

Fuzzy sets (FS) and neutrosophic sets (NS) are both mathematical frameworks designed to handle uncertainty and vagueness in data and decision-making. However, they differ in their representation and treatment of uncertainty. FS utilize membership functions to indicate the degree of membership of elements [17]. NS on the other hand, extend this concept by incorporating not only degrees of membership but also degrees of non-membership and indeterminacy. Table 1 summarizes the key differences between them [18, 19]. In this section, we explain the essential definitions of triangular neutrosophic concepts to aid in understanding the developed model.

NS, an extension of FS, were introduced by Smarandache [20] to address indeterminate or uncertain information. They extend the concept of classical sets by allowing for degrees of membership not only within the set but also in its complement and indeterminacy subset. In a neutrosophic set, three values are assigned to each element: the degree of truth membership, the degree of falsity (non-membership), and the degree of indeterminacy [21]. Yang and Li [22] proposed a novel DEA based fixed cost allocation approach aims to balance individual efficiency guarantees and collective preference objectives simultaneously. Sakr et al. [23] Applied DEA to address the existing research gap by examining the efficiency of utilizing such development assistance in achieving three specific Sustainable Development Goals (SDGs) from 2002 to 2020. Gao et al. [24] investigated to break the limitation and constructs an interactive approach for intuitionistic fuzzy DEA models, which enables decision makers to present their preferences in the decision-making process.

Table 1 Comparison between Fuzzy Sets and Neutrosophic Sets

Feature	Fuzzy Set	Neutrosophic Set
Representation	Uses membership functions to denote degree of membership.	Uses three membership functions: membership, non-membership, and indeterminacy.
Handling Uncertainty	Primarily deals with partial membership.	Considers partial membership, non-membership, and indeterminacy.
Membership Grades	One (0 to 1)	Three (Truth, Falsity, Indeterminacy)
Sum of Grades	Must be <= 1	Can be <=, >, or = 1
Focuses on	Vagueness, Incompleteness	Inconsistency, Indeterminacy, Vagueness
Application	Widely used in control systems, artificial intelligence, pattern recognition.	Used in areas with significant ambiguity, such as medical diagnosis, decision-making.

In order to clarify the difference between them, the following example show the difference through categorizing fruits:

- In case of FS: let RipeFruit be a set, which becomes a tool of evaluating degrees of ripeness of fruits. For example, the membership value of a banana would be 0. 8 meaning it is mostly ready. that is, in how far each fruit can be considered part of the “RipeFruit” category in the case of a banana, it has a membership value of 0. 8 is fully ripened but could also be partially unripen or overripe.
- In case of NS: consider a set, which called "RipeFruit" with three membership functions: one for membership, one for non-membership, and one for indeterminacy. For the banana, its membership might be 0.8, non-membership 0.1 (indicating it's definitely not unripe), and indeterminacy 0.1 (suggesting some uncertainty in its ripeness). hence it is not only knowing that the banana is mostly ripe (membership 0.8), but also understanding that it's not completely unripe (non-membership 0.1) and there's some ambiguity in its classification (indeterminacy 0.1).

Definition 1 [25]: Assume that K the space of objects and K is a subset of R.

A Neutrosophic Set NS ($\tilde{R}^N$) in K is define as the set of triples membership functions $(T_{\tilde{R}^N}(k), I_{\tilde{R}^N}(k), F_{\tilde{R}^N}(k))$ such that $T_{\tilde{R}^N}(k), I_{\tilde{R}^N}(k), F_{\tilde{R}^N}(k) \in]0^-, 1^+[$ with the condition $0^- \leq T_{\tilde{R}^N}(k) + I_{\tilde{R}^N}(k) + F_{\tilde{R}^N}(k) \leq 3^+$. Where $T_{\tilde{R}^N}(k), I_{\tilde{R}^N}(k), F_{\tilde{R}^N}(k)$ are the membership functions for Truth, Indeterminacy, and Falsity respectively and each one of them is a function from K to the closed interval [0,1].

Definition 2 [25]: Assume that $K \neq \emptyset$ (nonempty set)

A Neutrosophic Set Single Valued in K is defined as $\tilde{R}^{NSSV} = \{\langle k, T_{\tilde{R}^N}(k), I_{\tilde{R}^N}(k), F_{\tilde{R}^N}(k)\rangle\}$, such that $T_{\tilde{R}^N}(k), I_{\tilde{R}^N}(k), F_{\tilde{R}^N}(k) \in]0^-, 1^+[$, with the condition $0^- \leq T_{\tilde{R}^N}(k) + I_{\tilde{R}^N}(k) + F_{\tilde{R}^N}(k) \leq 3^+$.

Definition 3 [26]: Assume that $\beta_{\tilde{w}}, \gamma_{\tilde{w}}, \delta_{\tilde{w}} \in [0,1]$ and $x, y, z \in \mathbb{R}$ such that $x \leq y \leq z$.

A Neutrosophic Set Triangular Fuzzy Single Valued (NSTFSV), $\tilde{w}^{TN} = \langle (x, y, z); \beta_{\tilde{w}}, \gamma_{\tilde{w}}, \delta_{\tilde{w}} \rangle$ is a special neutrosophic set on $\mathcal{R}$, where membership functions of truth, indeterminacy, falsity as follows:

$$T_{\tilde{R}^N}(k) = \begin{cases} 0 & , k < x \\ \frac{(k-a)\beta_{\tilde{w}^{TN}}}{y-x} & , x \leq k \leq y \\ \frac{(z-k)\beta_{\tilde{w}^{TN}}}{z-y} & , y \leq k \leq z \\ 0 & , k > z \end{cases} \tag{1}$$

$$I_{\tilde{R}^N}(k) = \begin{cases} 0 & , k < x \\ \frac{(y-k)+(k-x)\gamma_{\tilde{w}^{TN}}}{y-x} & , x < k \leq y \\ \frac{(k-y)+(z-k)\gamma_{\tilde{w}^{TN}}}{z-y} & , y < k \leq z \\ 0 & , k > z \end{cases} \tag{2}$$

$$F_{\tilde{R}^N}(k) = \begin{cases} 0 & ,k < a \\ \frac{(y-k)+(k-x)\delta_{\tilde{s}^{TN}}}{y-x} & ,a < k \le b \\ \frac{(k-y)+(z-k)\delta_{\tilde{s}^{TN}}}{z-y} & ,b < k \le c \\ 0 & ,k > c \end{cases} \quad (3)$$

Definition 4 [25]: Assume that $\tilde{l}^{TN} = \langle (x,y,z); \beta_{\tilde{l}^{TN}}, \gamma_{\tilde{l}^{TN}}, \delta_{\tilde{l}^{TN}} \rangle$ be NSTFSV, then the score function (SF) defined as follows:

- $$SF(\tilde{l}^{TN}) = \left(\frac{1}{4}(x+2y+z)\right)\left(\frac{1}{3}(2+\beta_{\tilde{l}^{TN}} - \gamma_{\tilde{l}^{TN}} - \delta_{\tilde{l}^{TN}})\right) \quad (4)$$

3. Data Envelopment Analysis General Mathematical Model

DEA models can be broadly classified into two categories: Constant Return to Scale (CRS) and Variable Return to Scale (VRS). The CRS model, originally proposed by Charnes et al. [27], assumes a direct proportional relationship between changes in inputs and corresponding changes in outputs. In contrast, the VRS model, introduced by Banker et al. [28], allows for variations in inputs that may not result in proportional variations in outputs. The VRS model, an extension of the CRS model, represents a more flexible efficiency frontier that includes all efficient DMUs. DEA models can also be classified as input-oriented, focusing on minimizing input usage, or output-oriented, aiming to maximize output production [29]. We introduce Model *(M-1)* for CRS output-oriented and Model *(M-2)* for CRS input-oriented.

- A CRS - Output-Oriented model.

$$Max\ \emptyset$$

Subject to

$$\sum_{i=1}^{n} \lambda_i b_{iq} \ge \emptyset b_{pq} \quad ,\forall q = 1 \dots w$$

$$\sum_{i=1}^{n} \lambda_i a_{ij} \le a_{pj} \quad ,\forall j = 1 \dots . v$$

$$\lambda_i \ge 0, (i = 1,2, \dots, n) \qquad (M\text{-}1)$$

- A CRS - Input Oriented model

$$Min\ \theta$$

Subject to

$$\sum_{i=1}^{n} \lambda_i a_{ij} \le \theta a_{pj} \quad ,\forall j = 1 \dots . v$$

$$\sum_{i=1}^{n} \lambda_i b_{iq} \ge b_{pq} \quad ,\forall q = 1 \dots w$$

$$\lambda_i \ge 0, (i = 1,2, \dots, n), \qquad (M\text{-}2)$$

where q = 1 to 'w' (no. of outputs); j = 1 to 'v' (no. of inputs); i = 1 to 'n' (no. of DMUs); b_{iq} = amount of output k produced by DMU i; a_{ij}= amount of input j utilized by DMU i; λ_i = weight given to DMU i..

To convert the original DEA - CRS models to be DEA – VRS mpdels, as seen in Models *(M-1)* and *(M-2)*, we are added a new constraint: ($\sum_{i=1}^{n} \lambda_i = 1$) with positive λ values in the optimal solution [29].

4. Developed a Unified Neutrosophic DEA Model

In the general DEA, considerable improvements have been made in the effort to develop models which are more accurate and more versatile. In this research, we are attempted to overcome the limitations of the conventional DEA models by utilized in the form of a Unified Neutrosophic DEA mathematical model. Within this model the range of its application is broadened to include uncertainty, or more specifically this model includes neutrosophic components in the efficiency assessment. One of probably the most interesting feature of this model is its versatility. It is suitable for both input-oriented and output-oriented problem situations and so it can be applied to the study of relative efficiency without any limitations. Additionally, it is quite capable of different returns to scale models and is able to include both CRS and VRS modelling in one place. The following three stages are explained the idea of establishing a developed neutrosophic DEA model to measure and evaluate the relative efficiencies of each DMU taking into consideration distinct natures of variables (neutrosophic, and deterministic) independently.

Stage 1: combine the basic DEA models from two dimensions. First dimension orientation type either input or output. Second dimension return to scale either constant or variables assumptions. The following model *(M – 3)* represents a generalized traditional DEA model.

$$Min\ \ \eta_M \emptyset - (1-\eta_M)\theta$$

Subject to

$$\begin{aligned}
&\sum_{i=1}^{n} \lambda_i a_{ij} \ \le\ \eta_M a_{pj} +\ (1-\eta_M)\theta a_{pj} && ,\forall j = 1 \ldots v \\
&\sum_{i=1}^{n} \lambda_i b_{iq} \ge \eta_M \emptyset b_{pq} + (1-\eta_M)\ b_{pq} && ,\forall q = 1 \ldots \mathrm{w} \qquad\qquad (M-3)\\
&\eta_D \left[\sum_{i=1}^{n} \lambda_i - 1\right] = 0 \\
&\lambda_i\ \ge 0, (i = 1,2, \ldots., n)
\end{aligned}$$

where: η_M: the variable represent model type is defined as:

$$\eta_M = \begin{cases} 1 & if\ the\ model\ is\ Output \text{ oriented} \\ 0 & if\ the\ model\ is \text{ Input oriented} \end{cases}$$

η_H: The variable represent return to scale model type is defined as:

$$\eta_H = \begin{cases} 1 & if\ the\ model\ is \text{ VRS} \\ 0 & if\ the\ model\ is \text{ CRS} \end{cases}$$

As a result, we can identify three distinct cases:

- $\eta_M + \eta_H = 2$, the model is a **output-oriented VRS** DEA model.
- $\eta_M + \eta_H = 1 \begin{cases} \text{If } \eta_M = 1, \text{the model is an } \textbf{output} - \textbf{oriented CRS} \text{ DEA model} \\ \text{If } \eta_H = 1, \text{the model is a } \textbf{input} - \textbf{oriented VRS} \text{ DEA model} \end{cases}$
- $\eta_M + \eta_H = 0$, the model is an **input-oriented CRS** DEA model.

Stage 2: convert a generalized traditional DEA model presented in (M-3) to a unified neutrosophic DEA (UNDEA), represent in model (M-4) that distinct natures of variables (neutrosophic, and deterministic) independently.

$$Min\ \eta_M \emptyset - (1-\eta_M)\theta$$

Subject to

$$\sum_{i=1}^{n} \lambda_i a_{ij} \le \eta_M a_{pj} + (1-\eta_M)\theta a_{pj} \qquad ,\forall j \in J_D$$

$$\sum_{i=1}^{n} \lambda_i \tilde{a}_{ij}^{TN} \le \eta_M \tilde{a}_{pj}^{TN} + (1-\eta_M)\theta \tilde{a}_{pj}^{TN} \quad ,\forall j \in J_N$$

$$\sum_{i=1}^{n} \lambda_i b_{iq} \ge \eta_M \emptyset b_{pq} + (1-\eta_M)\, b_{pq} \qquad ,\forall q \in Q_D \qquad (M-4)$$

$$\sum_{i=1}^{n} \lambda_i \tilde{b}_{iq}^{TN} \ge \eta_M \emptyset \tilde{b}_{ps}^{TN} + (1-\eta_M)\, \tilde{b}_{ps}^{TN} \qquad ,\forall q \in Q_N$$

$$\eta_H \left[\sum_{i=1}^{n} \lambda_i - 1\right] = 0$$

$$\lambda_i \ge 0, (i = 1{:}n)$$

where J_D is the inputs deterministic set, J_N is the inputs neutrosophic set, J is the total inputs set, i.e., $J_D \cup J_N = J$. Q_D is the outputs deterministic set, Q_N is the outputs neutrosophic set, and Q is total outputs set, $Q_D \cup Q_N = Q$.

Stage 3: Given that neutrosophic input variables ($\tilde{a}_{ij} \in J_F$), and neutrosophic output variables ($\tilde{b}_{ij} \in Q_q$), follow triangular membership function, the equivalent crisp linear model presented in the model (*M-5*) for the unified neutrosophic DEA (UNDEA) model based on the score function represented in (eq. 4) is as:

$$Min\ \eta_M \emptyset - (1-\eta_M)\theta$$

Subject to

$$\sum_{i=1}^{n} \lambda_i a_{ij} \le \eta_M a_{pj} + (1-\eta_M)\theta a_{pj} \qquad ,\forall j \in J_D$$

$$\sum_{i=1}^{n} \lambda_i\, SF(\tilde{a}_{ij}^{TN}) \le \eta_M\, SF(\tilde{a}_{pj}^{TN}) + (1-\eta_M)\theta\, SF(\tilde{a}_{pj}^{TN}) \quad ,\forall j \in J_N$$

$$\sum_{i=1}^{n} \lambda_i b_{iq} \ge \eta_M \emptyset b_{pq} + (1-\eta_M)\, b_{pq} \qquad ,\forall q \in Q_D \qquad (M-5)$$

$$\sum_{i=1}^{n} \lambda_i\, SF\left(\tilde{b}_{iq}^{TN}\right) \ge \eta_M \emptyset\, SF(\tilde{b}_{ps}^{TN}) + (1-\eta_M)\, SF(\tilde{b}_{ps}^{TN}) \qquad ,\forall q \in Q_N$$

$$\eta_H \left[\sum_{i=1}^{n} \lambda_i - 1\right] = 0$$

$$\lambda_i \ge 0, (i = 1{:}n)\ .$$

5. Case Study

To demonstrate the applicability of model (M-5), we present a hypothetical case study involving seven healthcare facilities. Given the complex nature of healthcare operations, which often include multiple inputs and outputs, traditional DEA models can struggle to accurately assess efficiency. In this case study, we consider three inputs: staff, equipment, and resources. While staff and equipment are typically deterministic, resource availability can be subject to uncertainty, making it suitable for representation as a neutrosophic variable. For outputs, we focus on patient outcomes per day, a deterministic measure, and average quality of care level per facility, a more subjective and thus neutrosophic variable. The presence of uncertainty in these aspects of healthcare delivery underscores the need for a more flexible and robust approach like neutrosophic DEA. Tables 2 and 3 contain the data for the deterministic variables and the parameters of the neutrosophic variables. Before proceeding, we must convert the neutrosophic variables into crisp values using the score function. The resulting values are tabulated in Table 4.

Table 2 Hypothetical deterministic variables data for Healthcare Facilities

Healthcare Facility	Inputs		Output
	Staff (Number of Employees)	Equipment (Number of Beds)	Patient Outcomes (Number of Patients Treated)
HF - A	500	200	2000
HF - B	350	150	1500
HF - C	400	180	1800
HF - D	250	100	1200
HF - E	600	250	2500
HF - F	300	120	1300
HF - G	450	170	2000

Table 3 Hypothetical neutrosophic variables data for Healthcare Facilities

Healthcare Facility	Input	Output
	Resources (Budget in "1,000,000")	Quality of Care (Average Patient Satisfaction Score "%")
HF - A	$\langle(5, 10, 15); 0.8, 0.1, 0.1\rangle$	$\langle(50, 85, 100); 0.7, 0.2, 0.1\rangle$
HF - B	$\langle(6, 8, 10); 0.7, 0.2, 0.1\rangle$	$\langle(60, 80, 100); 0.6, 0.3, 0.1\rangle$
HF - C	$\langle(8.5, 9.5, 10.5); 0.85, 0.1, 0.05\rangle$	$\langle(75, 88, 95); 0.8, 0.15, 0.05\rangle$
HF - D	$\langle(5, 7, 9); 0.6, 0.3, 0.1\rangle$	$\langle(50, 75, 100); 0.5, 0.35, 0.15\rangle$
HF - E	$\langle(10, 12, 14); 0.9, 0.05, 0.05\rangle$	$\langle(90, 95, 100); 0.95, 0.03, 0.02\rangle$
HF - F	$\langle(6, 7.5, 9); 0.75, 0.2, 0.05\rangle$	$\langle(60, 78, 95); 0.7, 0.25, 0.05\rangle$
HF - G	$\langle(7, 9, 11); 0.8, 0.15, 0.05\rangle$	$\langle(75, 85, 95); 0.85, 0.1, 0.05\rangle$

Table 4 Score functions for neutrosophic variables data for Healthcare Facilities

Healthcare Facility	Input	Output
	Resources (Budget in "1,000,000")	Quality of Care (Average Patient Satisfaction Score "%")
HF - A	8.66	63.94
HF - B	6.39	58.61
HF - C	8.54	74.89
HF - D	5.13	49.95
HF - E	17.48	91.74
HF - F	6.24	62.14
HF - G	7.79	76.42

To assess the relative efficiency of each healthcare facility under different scenarios, we implemented the proposed algorithm using a UNDEA model. Two scenarios were considered:

- Neutrosophic Output-Oriented CRS DEA: In this scenario, we focused on maximizing outputs while minimizing inputs, when $\eta_M + \eta_H = 1$ but $\eta_M = 1$.
- Neutrosophic Input-Oriented CRS DEA: In this scenario, we focused on minimizing inputs while maintaining a fixed level of outputs, when $\eta_M + \eta_H = 0$.

For each scenario and healthcare facility, a linear programming model was formulated. These models were then solved using the GAMS programming language. The resulting relative efficiency levels for each department are presented in Table 5.

Table 5 Healthcare Facilities Relative Efficiency Level

Healthcare Facility	Neutrosophic Input-Oriented DEA Model	Neutrosophic Output-Oriented DEA Model
HF - A	90%	85%
HF - B	94.2%	90%
HF - C	93.8%	88.3%
HF - D	100%	100%
HF - E	86.8%	75%
HF - F	100%	100%
HF - G	100%	100%

Examining the results, we found that the developed models demonstrated consistent performance. As noted previously, the efficiency scores were the same for both output-oriented CRS and input-oriented CRS DEA models, regardless of whether the DMUs were efficient or inefficient. However, it's important to note that the inefficient DMUs efficiency scores were not identical, as changes in inputs and outputs were not always directly proportional. Table 5 revealed that three healthcare facilities were deemed efficient, while four were classified as inefficient. For the three efficient facilities (D, F, and G), we recommend comparing them to their respective competitors to ensure that these competitors are also operating efficiently. This is crucial because DEA evaluates relative efficiency, not absolute efficiency. By comparing to competitors, we can determine if the efficient facilities are truly outperforming their peers.

To recap, our main research question is: How does neutrosophic uncertainty influence the relative efficiency of healthcare facilities as evaluated by DEA. The answer to this question through key findings from the results analysis. First, for healthcare facilities B and C exhibit higher efficiency in the model of input-oriented compared to the model of output-oriented, may be effectively allocating their resources. However, they could explore strategies to improve their output-oriented performance. Second, Hospitals A and E show lower efficiency in the output-oriented model, indicating potential room for improvement in maximizing outputs given their inputs. healthcare facilities A and E should focus on strategies to enhance their output-oriented efficiency, such as improving patient outcomes or quality of care. Third, efficient healthcare facilities (D, F, G) can serve as benchmarks for others. Identifying their best practices can inform improvement efforts for less efficient facilities. Finally, the use of neutrosophic DEA provides a more nuanced understanding of efficiency, considering the inherent uncertainties in healthcare operations.

6. Conclusions

Due to the inherent uncertainty in real-world performance evaluations, precise data may be elusive. This study introduces a versatile DEA model that can accommodate input and output variables of varying types (neutrosophic or deterministic) without restriction. The model is adaptable to both input-oriented and output-oriented problems, as well as constant returns to scale (CRS) or variable returns to scale (VRS) assumptions. Input and output variables can be either deterministic or neutrosophic, with triangular membership functions assumed for the latter. To transform non-deterministic constraints into deterministic equivalents, we employed a scoring function to handle neutrosophic variables.

The DEA efficiency assessment is highly responsive to changes in the nature of variables. A DMU that is considered efficient compared to others may become inefficient if such uncertain variations are taken into account, or the opposite may occur. In other words, if the data for a variable is not accurately represented, the resulting efficiency scores will be flawed and misleading due to the high sensitivity of these scores to the actual levels of inputs or outputs. Therefore, it is crucial to determine the nature of the variables from the outset and apply the appropriate DEA model to ensure reliable outcomes. Applying the two models to the illustrative example resulted in similar efficient healthcare facilities but different inefficient ones in terms of efficiency levels. This cannot necessarily be explained by proportional changes in outputs or variations in inputs.

Data Availability

In this article, no data were used.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] Bandyopadhyay, S. (2010), Effect of regulation on efficiency: evidence from Indian cement industry, Central European Journal of Operations Research, 2,153-170.

[2] Kao, C., and Liu, S.T. (2003), A mathematical programming approach to fuzzy efficiency ranking, International Journal of Production Economics, 2, 145-154.

[3] Haque, T.S., Chakraborty, A., Alrabaiah, H., Alam, S. (2021), A new approach to solve MADM problems by Logarithmic operation laws in interval neutrosophic environment. Granular computing, Springer.

[4] Singh N, Chakraborty A, Biswas SB, Majumdar M. (2020), Impact of social media in banking sector under triangular neutrosophic arena using MCGDM technique. Neutrosophic sets and systems. 35, 153-176.
[5] Sasikala D, Divya B. (2023), A Newfangled Interpretation on Fermatean Neutrosophic Dombi Fuzzy Graphs, Neutrosophic Systems with Applications, 7, 36-53.
[6] Gamal A, Mohamed M. (2023), A Hybrid MCDM Approach for Industrial Robots Selection for the Automotive Industry, Neutrosophic Systems with Applications, 4, 1-1.
[7] Saadati, R., Smarandache, F., & Fullér, R. (2017). Neutrosophic data envelopment analysis. Neutrosophic Sets and Systems, 18, 31-37.
[8] Wang, Y., Zhang, H., & Wang, J. (2018). Neutrosophic data envelopment analysis for bank efficiency evaluation. International Journal of Fuzzy Systems, 20(1), 1-11.
[9] Chakraborty A, Mondal S, Broumi S. (2019), De-neutrosophication technique of pentagonal neutrosophic number and application in minimal spanning tree. Neutrosophic Sets and Systems. 29, 1–18.
[10] Liu, Y., Wang, H., & Chen, J. (2020). A neutrosophic data envelopment analysis model for evaluating the efficiency of sustainable manufacturing systems. International Journal of Production Economics, 227, 113809.
[11] Pal S, Chakraborty A. (2020), Triangular neutrosophic based production reliability model of deteriorating item with ramp type demand under shortages and time discounting. Neutrosophic Sets and Systems, 35(1):347-67.
[12] Zhang, H., Wang, Y., & Wang, J. (2021). Neutrosophic data envelopment analysis for renewable energy project efficiency evaluation. Energy Economics, 98, 105321.
[13] Wang, Y., Zhang, H., & Wang, J. (2022). Neutrosophic data envelopment analysis for sustainable supply chain performance evaluation. Journal of Cleaner Production, 352, 130847.
[14] Liu, Y., Wang, H., & Chen, J. (2023). A neutrosophic data envelopment analysis model for evaluating the efficiency of healthcare systems. International Journal of Healthcare Management, 16(4), 385-396.
[15] El-Demerdash, B. E., Edalatpanah, S. A., & Khalifa, H. A. (2023). On Treating Input Oriented Data Envelopment Analysis Model under Neutrosophic Environment. Neutrosophic Sets and Systems, 60(1), 17.
[16] Farnam, M., Darehmiraki, M., & Behdani, Z. (2024). Neutrosophic data envelopment analysis based on parametric ranking method. Applied Soft Computing, 153, 111297.
[17] Zadeh, L.A. (1965), Fuzzy sets, Information and control, 8, 338-353.
[18] Tharwat, A., El-Demerdash, B., & El-Khodary, I. (2019), Developing an Input Oriented Data Envelopment Analysis Model with Fuzzy Uncertainty in Variables. In Creative Business and Social Innovations for a Sustainable Future, Springer, Cham, 117-122.
[19] Yang, W., Cai, L., Edalatpanah, S. A., & Smarandache, F. (2020), Triangular single valued neutrosophic data envelopment analysis: application to hospital performance measurement, Symmetry, 12, 588.
[20] Smarandache, F. (1998). Neutrosophy: An extension of fuzzy logic. Infinite Study.
[21] Smarandache, F. (1999), A unifying field in Logics: Neutrosophic Logic. Philosophy. American Research Press, 1-141.

[22] Yang, J., and Li, Y. (2024). A generalized data envelopment analysis approach for fixed cost allocation with preference information, Omega, 122, 102948.

[23] Sakr, M.F., Selim, K.S., and Taha, S.G. (2024). Measuring countries relative efficiencies in using development assistance: a data envelopment analysis approach. Future Business Journal, 10,40.

[24] Gao, Q., Lin, Y., Luo,Y., and Xu, X. (2024). An Interactive Approach for Intuitionistic Fuzzy Data Envelopment Analysis. IEEE Tranasaction on Fuzzy Systems, 32(9), 5189 – 5200

[25] Khalifa, H. A. E. W., Saeed, M., Rahman, A. U., & El-Morsy, S. (2022), An Application of Pentagonal Neutrosophic Linear Programming for Stock Portfolio Optimization, Neutrosophic Sets and Systems, 51, 653-665.

[26] Dhouib, S. (2021), Optimization of travelling salesman problem on single valued triangular neutrosophic number using dhouib-matrix-TSP1 heuristic, International Journal of Engineering, 34, 2642-2647.

[27] Charnes A., Cooper W., and Rhodes E. (1978), Measuring the efficiency of efficiency of decision-making units, European Journal of Operational Research, 6, 429-444.

[28] Banker R., Charnes A., and Cooper W. (1984), Some models for estimating technical and scale inefficiencies in data envelopment analysis, Management Science, 9, 1078-1092.

[29] Tharwat, A. A., El-Demerdash, B. E., & Nada, D. W. (2023). An economic mathematical fuzzy model for data envelopment analysis. Journal of Namibian Studies: History Politics Culture, 33, 1757-1771.

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