



## Solving the shortest path Problem in an interval-valued Neutrosophic Pythagorean environment using an enhanced A\* search algorithm

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**Abstract:** The A\* search algorithm is widely utilized to evaluate the shortest path in a given network. However, in a traditional A\* search algorithm, the nodes are assumed to have crisp values, i.e., a single value. This assumption may not hold in many real-world scenarios where uncertainty or ambiguity is involved. In such cases, an interval-valued Neutrosophic Pythagorean (IVNP) environment can provide a more sound and accurate representation. Interval-valued Neutrosophic Pythagorean sets (IVNPS) are an effective way to model vague and imprecise data, which is prevalent in executive problems. These sets provide a more flexible way to capture uncertainty by allowing the values of nodes in the graph to vary within certain intervals rather than having fixed values. This interval representation can effectively handle imprecise or incomplete information and is a powerful tool in executive processes. In this research paper, we proposed an improved A\* search algorithm that takes advantage of the interval-valued neutrosophic Pythagorean environment. This algorithm aims to evaluate the shortest path in a graph under uncertainty and ambiguity. The proposed algorithm incorporates the IVNPS theory into the A\* search framework to handle the uncertainty in node values and edge weights. It utilizes the concept of neutrosophic Pythagorean distance to calculate the heuristic function and make informed decisions on the next node to expand.

### Keywords:

A\* search algorithm; heuristic function; interval-valued Neutrosophic Pythagorean number.

## 1. Introduction:

In a neutrosophic environment, uncertainty and indeterminacy play an important part. The conception of a neutrosophic set (NS), first suggested by Smarandache in 1995, and allows for the representation of indeterminacy in the form of three components, i.e., truth, indeterminacy, and false membership. Thus, as a result, numerous papers [1–10] have been available in the area of uncertainty environment.

Various researchers have provided different work on SPP in various environments. Ahuja et al. [11] suggested a different kind of distributed heap as an efficient technique for finding the SPP of the graph. Yang et al. [12] demonstrated a diagram-theoretic method for calculating the right line. Zheng and Ibarra [13] showed how to evaluate the single-basis shortest possible route problem using the logarithmic sequence of 'n'. Arsham [14] investigated the shortest path problem's robustness. Tzoreff [15] investigated an unconnected SPP with different group path lengths.

Zhang and Lin [17] proposed the reverse SPP. Samaranadache and Vasantha proposed a fundamental neutrosophic mathematical structure in their paper [18], and it can also be applied to uncertain and neutrosophic models. Zwick and Roditty [19] obtained some findings related to valuable types of the SPP. Desaulniers and Irnich [20] suggested SPP with holdup forces. Jowers and Buckley [21] used fuzzy logic to present SPP. Said broumi [22] solved the SPP problem using ACO. Turner [23] successfully developed an algebraic method for solving a set of SPP on both normal and acyclic graphs. Deng et al. [24] suggested a new method for ambiguous SPP.

Broumi et al. [25] proposed a new idea to conduct a study on features that include verification and representations. Harish and Nancy [26] suggested an updated score function for use in the executive process. For address the issue of neutrosophic decision-making with insufficient weight support, Sahin and Liu [27] optimized the use of variations. Broumi et al. [28, 29] suggested a novel idea to evaluate the SPP. Hu and Sotirov [30] suggested semi-definite training for the rectangular SPP, and the QSPP was solved with some arithmetic operations using the division algorithm. Leitert and Dragan [31] solved SPP with little difficulty. Zhang et al. [32] suggested a new idea to solve SPP with scattered uncertainties.

The purpose of this study is to (i) improve the A\* search method for finding the shortest path problem over neutrosophic interval-valued Pythagorean numbers; (ii) find the heuristic values and the best one for each node in IVNPS; (iii) find the good things about A\* search in terms of its features, adaptability, and quick selections; and (iv) figure out the suggested approach at each step and find the results.

This research paper arranges the remaining sections in the following order: In Section 2, we talk about why we're doing this. Section-3 discuss the fundamental concepts of neutrosophic sets, intuitionistic neutrosophic sets, neutrosophic Pythagorean sets, neutrosophic interval-valued Pythagorean sets, and the various operations applicable to them. In Section-4, the A\* search algorithm and the heuristic function are defined and explained. Section-5 provides a mathematical explanation for determining

the most efficient route using the IVNPS, a heuristic computational method, and a modified A\* Search algorithm. Section-6 presents a numerical illustration of the process for calculating the IVNPS in neutrosophic situations. Section-7 conducted a comparative analysis of the shortest path using various networks and parameters, while also highlighting the benefits of the proposed methodology. Section-8 states the outcome of the work.

**2. Motivation:**

- In this chapter, we proposed an improved A\* search algorithm. This algorithm objectives to evaluate the shortest path in a graph under uncertainty and ambiguity. The proposed algorithm incorporates the IVNPS theory into the A\* search framework to handle the uncertainty in node values and edge weights. It utilizes the concept of neutrosophic Pythagorean distance to calculate the heuristic function and make informed decisions on the next node to expand.
- In this chapter, our aim is to evaluate the minimum cost between the initial vertex and the final vertex.
- Furthermore, we illustrate one numerical example using an algorithm.

**3. Preliminaries:**

This section of the paper presents an analysis of the fundamental concepts and definitions of the different sets.

**Definition-3.1 [33]**

A Neutrosophic set  $\hat{S}$  in a universal set  $\hat{X}$  is defined as  $\hat{S} = \{ \langle \hat{x}: \hat{T}_{\hat{S}}(\hat{x}), \hat{I}_{\hat{S}}(\hat{x}), \hat{F}_{\hat{S}}(\hat{x}) \rangle : \hat{x} \in \hat{X} \}$ . Here  $\hat{T}_{\hat{S}}(\hat{x}), \hat{I}_{\hat{S}}(\hat{x}), \hat{F}_{\hat{S}}(\hat{x})$  defined the three membership degrees and  $\hat{x} \in \hat{S}$  to the set A.

Three membership degrees is written as

$$0^+ \leq \{ (\hat{T}_{\hat{S}}(\hat{x}) + \hat{I}_{\hat{S}}(\hat{x}) + \hat{F}_{\hat{S}}(\hat{x})) \} \leq 3^+ \text{-----}$$

(1)

**Definition-3.2[35]**

A Pythagorean neutrosophic set  $\hat{S}$  in a universal set  $\hat{X}$  is defined as  $\hat{S} = \{ \langle \hat{x}: \hat{T}_{\hat{S}}(\hat{x}), \hat{I}_{\hat{S}}(\hat{x}), \hat{F}_{\hat{S}}(\hat{x}) \rangle : \hat{x} \in \hat{X}, \text{ where } \hat{T}_{\hat{S}}(\hat{x}), \hat{I}_{\hat{S}}(\hat{x}) \text{ are dependent, and}$

$\widehat{F}_{\widehat{S}}(\widehat{x})$  independent component., for all  $\widehat{x} \in \widehat{X}$ , with the condition  $0^+ \leq \{(T_{\widehat{S}}(\widehat{x}))^2 + (I_{\widehat{S}}(\widehat{x}))^2 + (F_{\widehat{S}}(\widehat{x}))^2 \leq 2\}$ .

**Definition-3.3[36]**

An Interval-valued neutrosophic Pythagorean set  $\widehat{S}$  in a universal set  $\widehat{X}$  is defined as

$$\widehat{S} = \{ \widehat{X} < , [ \left( \frac{\widehat{T}_{\widehat{S}}^L + \widehat{T}_{\widehat{S}}^U}{2} \right)^2 + \left( \frac{\widehat{I}_{\widehat{S}}^L + \widehat{I}_{\widehat{S}}^U}{2} \right)^2 + \left( \frac{\widehat{F}_{\widehat{S}}^L + \widehat{F}_{\widehat{S}}^U}{2} \right)^2 ] \leq 2$$

Where  $[\widehat{T}_{\widehat{S}}^L, \widehat{T}_{\widehat{S}}^U], [\widehat{I}_{\widehat{S}}^L, \widehat{I}_{\widehat{S}}^U], [F_{\widehat{A}}^L, F_{\widehat{A}}^U]$  represents the inferior and superior bound of the three membership degrees.

**Definition-3.4[36]**

Let  $\widehat{A}_1 = \langle w, [T_{\widehat{A}_1}^L, T_{\widehat{A}_1}^U], [I_{\widehat{A}_1}^L, I_{\widehat{A}_1}^U], [F_{\widehat{A}_1}^L, F_{\widehat{A}_1}^U] \rangle$  and

$\widehat{A}_2 = \langle v, [T_{\widehat{A}_2}^L, T_{\widehat{A}_2}^U], [I_{\widehat{A}_2}^L, I_{\widehat{A}_2}^U], [F_{\widehat{A}_2}^L, F_{\widehat{A}_2}^U] \rangle$  be two neutrosophic interval-valued Pythagorean set, then sum of two set is defined as

$$\begin{aligned} \widehat{A}_1 + \widehat{A}_2 = \{ \langle w + v, [T_{\widehat{A}_1}^L + T_{\widehat{A}_2}^L - T_{\widehat{A}_1}^L T_{\widehat{A}_2}^L, T_{\widehat{A}_1}^U + T_{\widehat{A}_2}^U - \\ T_{\widehat{A}_1}^L T_{\widehat{A}_2}^U], [I_{\widehat{A}_1}^L I_{\widehat{A}_2}^L, I_{\widehat{A}_1}^U I_{\widehat{A}_2}^U], \\ [F_{\widehat{A}_1}^L F_{\widehat{A}_2}^L, F_{\widehat{A}_1}^U F_{\widehat{A}_2}^U] \rangle \} \end{aligned}$$

$$\begin{aligned} \widehat{A}_1 \widehat{A}_2 = \{ \langle wv, [T_{\widehat{A}_1}^L T_{\widehat{A}_2}^L, T_{\widehat{A}_1}^L T_{\widehat{A}_2}^U], [I_{\widehat{A}_1}^L + I_{\widehat{A}_2}^L - I_{\widehat{A}_1}^L I_{\widehat{A}_2}^L, I_{\widehat{A}_1}^U + I_{\widehat{A}_2}^U \\ - I_{\widehat{A}_1}^U I_{\widehat{A}_2}^U], [F_{\widehat{A}_1}^L + F_{\widehat{A}_2}^L - F_{\widehat{A}_2}^L F_{\widehat{A}_1}^L, F_{\widehat{A}_1}^U + F_{\widehat{A}_2}^U \\ - F_{\widehat{A}_2}^U F_{\widehat{A}_1}^U] \rangle \} \end{aligned}$$

**Definition-3.5[36]**

Let  $\hat{S} = \{ \langle w, [T_{\hat{S}}^L, T_{\hat{S}}^U], [I_{\hat{S}}^L, I_{\hat{S}}^U], [F_{\hat{S}}^L, F_{\hat{S}}^U] \rangle \mid w \in W \}$

Let us assume  $T_{\hat{S}}^L = \alpha, T_{\hat{S}}^U = \beta, I_{\hat{S}}^L = \gamma, I_{\hat{S}}^U = \delta, F_{\hat{S}}^L = \sigma, F_{\hat{S}}^U = \rho$  then the improved IVNP score function is expressed as

$$S(\hat{S}) = \frac{(\alpha^2 - \gamma^2 - \sigma^2)(1 + \sqrt{1 - \alpha^2 - \gamma^2 - \sigma^2}) + (\beta^2 - \delta^2 - \rho^2)(1 + \sqrt{1 - \beta^2 - \delta^2 - \rho^2})}{2}$$

#### 4. A\* search algorithm

In computer science and machine learning, the A\* search technique is a famous way to identify the most efficient route between any two given nodes in a network. We call it "A\*" because it adds up the heuristic function to evaluate the path from initial node (g) to final node (h). The A\* algorithm uses an arrangement of best-first search and Dijkstra's algorithm to efficiently explore the search gap and evaluate the optimal path. It guarantees to get the shortest path providing the heuristic function satisfies certain conditions, such as being admissible (never overestimating the actual cost) and consistent (always satisfying the triangle inequality).

$$\hat{f}_{ij} = \hat{g}_{ij} + \hat{h}_{ij}$$

The parameter  $\hat{f}_{ij}$  is utilized to determine the most efficient route from each specific node to the other node.

The parameter  $\hat{g}_{ij}$  is utilized to calculate the expense of transitioning from each moment of one node to another node.

The parameter  $\hat{h}_{ij}$  represents the heuristic, which is the estimated cost between the initial node and the final node.

#### 4.1 The neutrosophic Pythagorean SPP with interval values

The implementation of the neutrosophic interval-valued Pythagorean technique could be used for formulating the problem of determining the shortest route in a connected network.

##### Variables:

Let  $d(u, v)$  represent the shortest path (SP) between two corresponding nodes  $u$  and  $v$ .

Let  $x(u, v)$  represent the membership degree of the path that is shortest between  $u$  and  $v$  at a node.

The non-membership degree of the shortest path (SP) from node  $u$  to node  $v$  is indicated by the symbol  $y(u, v)$ .

Let  $z(u, v)$  represent the degree of indeterminacy of the shortest path (SP) connecting node  $u$  and  $v$ .

**Objective:**

Reduce the total length of all the path's edges or arc lengths i.e., minimize  $\sum_{(u, v) \in E} d(u, v)$ .

**Constraints:**

For each edge  $(u, v)$ ,  $d(u, v) = \sqrt{(x(u, v))^2 - (y(u, v))^2 + z(u, v)}$ .

For each vertex  $u$ , the membership degrees  $(x(u, v))$ , non-membership degrees  $(y(u, v))$ , and indeterminacy degrees  $(z(u, v))$  satisfy the neutrosophic Pythagorean equation:

$$(x(u, v))^2 + (y(u, v))^2 + (z(u, v))^2 = 1.$$

For each vertex  $u$ ,  $\sum_{(u, v) \in E} (x(u, v))^2 = 1$ .

The three variables  $x(u, v)$ ,  $y(u, v)$ , and  $z(u, v)$  detain the uncertainty related to this path, and this formulation guarantees that the value of  $d(u, v)$  represents the SP length from node  $u$  to  $v$ . The goal function aims to reduce the overall length of every edge in the path. The constraints guarantee that the neutrosophic Pythagorean equation is satisfied by the membership, non-membership, and indeterminacy degrees, and that the membership degrees for all outgoing edges from each vertex sum up to one.

**4.2 Theorem:**

If  $A^*$  is optimal under an Interval-valued Neutrosophic Pythagorean environment, then  $h \sim j$  is permissible with an Interval-valued Neutrosophic Pythagorean cost

**Proof**

To prove this statement by using the contradiction method, we assume that  $A^*$  is optimal under an Interval-valued Neutrosophic Pythagorean environment, but  $h \sim j$  is not admissible with an

Interval-valued Neutrosophic Pythagorean cost. We will then show that this assumption leads to a contradiction, which proves that the original statement is true.

Let's denote the optimal solution as  $A^*$ , and its cost as  $Cost(A^*)$ .

Now, we assume that  $h \sim j$  is not admissible with an Interval-valued Neutrosophic Pythagorean cost. This means that there exists another solution,  $B$ , such that  $Cost(B) < Cost(h \sim j)$ .

Since  $A^*$  is optimal, we have:

$$Cost(A^*) \leq Cost(B)$$

But since  $h \sim j$  is not admissible with an Interval-valued Neutrosophic Pythagorean cost, we also have:

$$Cost(h \sim j) \leq Cost(B)$$

Combining these two inequalities, we have:

$$Cost(A^*) \leq Cost(h \sim j)$$

However, since  $A^*$  is optimal, it should have the lowest possible cost. Therefore, we can write:

$$Cost(A^*) \leq Cost(h \sim j) \leq Cost(A^*)$$

This implies that  $Cost(A^*) = Cost(h \sim j)$ .

But this contradicts our assumption that  $h \sim j$  is not admissible with an Interval-valued Neutrosophic Pythagorean cost. If  $h \sim j$  is not admissible, it means that there exists another solution  $B$  with a lower cost. But from our contradiction, we know that  $Cost(A^*) = Cost(h \sim j)$ .

Thus, our assumption that  $h \sim j$  is not admissible leads to a contradiction. Therefore, we can conclude that if  $A^*$  is optimal under an Interval-valued Neutrosophic Pythagorean environment, and then  $h \sim j$  is admissible with an Interval-valued Neutrosophic Pythagorean cost.

### 4.3 Calculation of heuristic costs

The heuristic value is a cost calculates approximately from every single node to the destination node. The approximate cost can be determined in several of manner, such as Hamming distance, Manhattan distance, Euler distance, and others. A reversal search strategy based on breadth first was used to evaluate the heuristic values in this case.

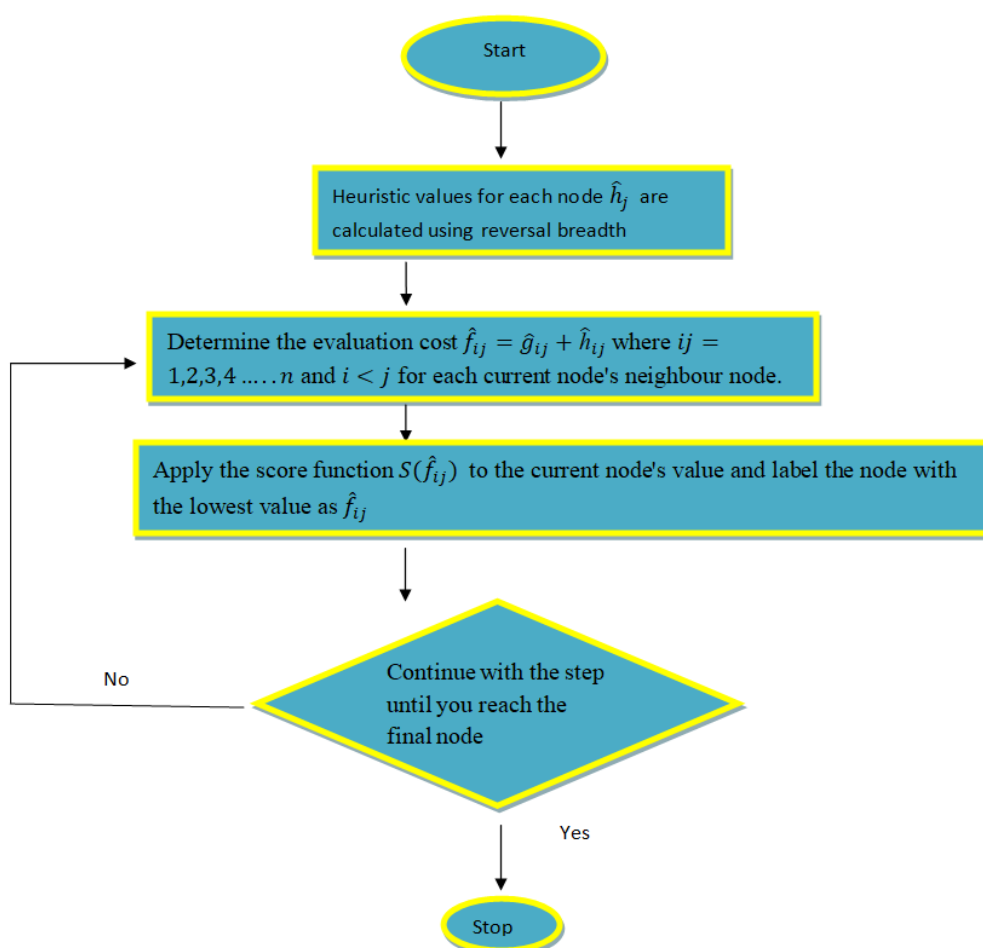
#### 4.3.1 The Heuristic Cost Algorithm

Step 1: Initiate with the targeted node and set the node value  $\hat{h}_j$  to 0, where  $j = n$ . The

computation is carried out in the opposite order.

Step 2: Add the real cost values from the current node to the values from the possible precursor node. Using the score equation, compare the values, Then select the value that is the smallest as a heuristic value and label it  $\hat{h}_j$ .

Step 3: Continue with the procedure until you come to the last node.



**Fig-1: The idea for the algorithm is represented by a flow chart diagram.**



## 5. Proposed algorithm

This section presents a method for determining the most efficient route between initial to final node using a heuristic methodology. This proposed algorithm is used for classification.

Step-1: Select the node that originated to be the starting node and label the initial node as node 1.

Step-2: Determine the estimate cost  $\hat{f}_{ij} = \hat{g}_{ij} + \hat{h}_{ij}$  where  $ij = 1,2,3,4 \dots n$  and  $i < j$  for each current node's neighbor node.

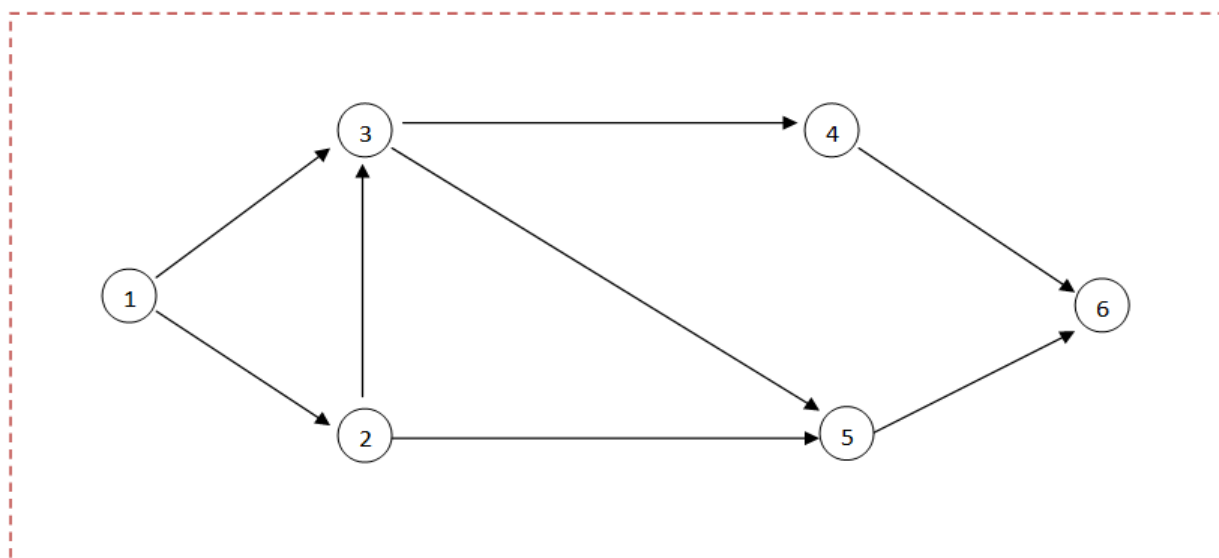
Step-3: Utilize the score function  $S(\hat{f}_{ij})$  to the existing node's value and then label the node according to the lowest value as  $\hat{f}_{ij}$ .

Step-4: Continue with the labeled nodes' successors  $\hat{f}_{ij}$  and determine the assessment cost. Step 3 should be repeated to get the smallest amount value and connect the node.

Step -5: Continue the procedure until you arrive at node n, i.e. destination node.

Step- 6: The path with the least distance is calculated through the combination of each and every one of the nodes discovered in the preceding steps (Fig. 1).

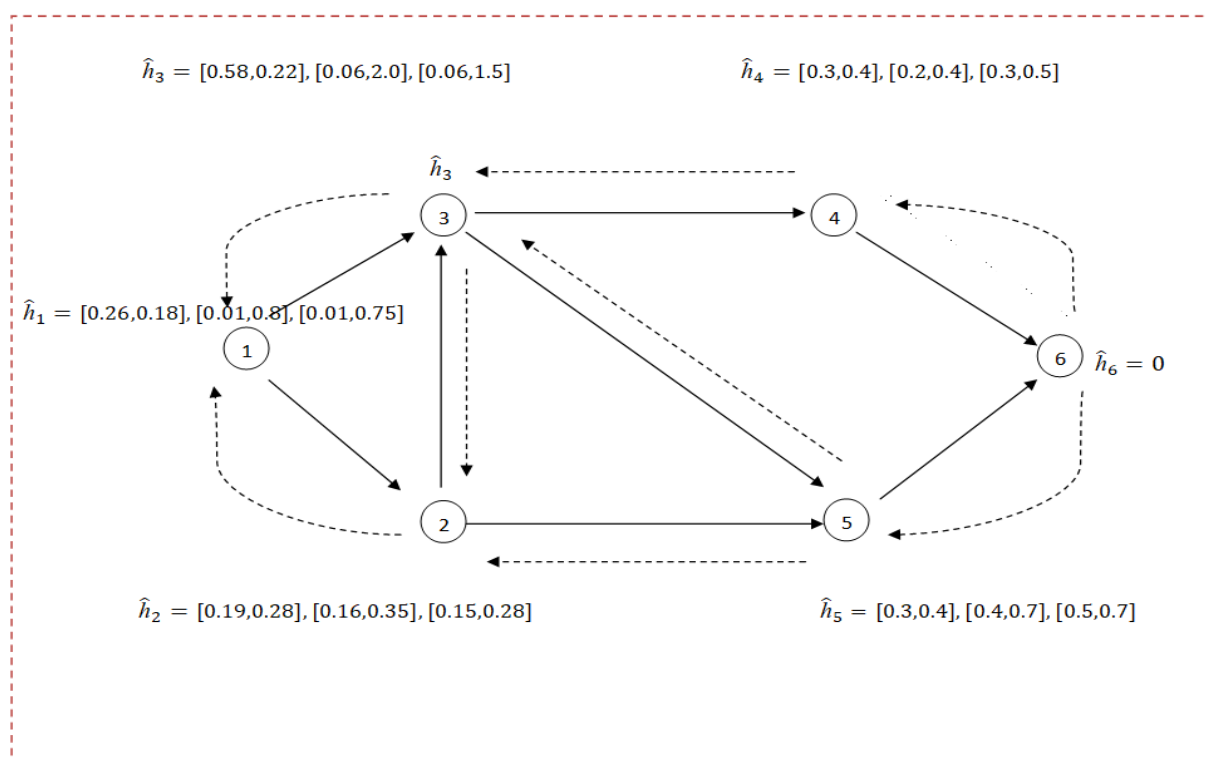
## 6. Numerical Example:



**Fig-2: IVNP Network**

**Table-1: Arc Weights**

Arc	Neutrosophic Pythagorean cost with interval values
1-2	$\langle [0.4,0.6],[0.3,0.5],[0.3,0.4] \rangle$
1-3	$\langle [0.4,0.5],[0.3,0.4],[0.3,0.5] \rangle$
2-3	$\langle [0.5,0.6],[0.4,0.5],[0.3,0.4] \rangle$
2-5	$\langle [0.5,0.6],[0.3,0.5],[0.3,0.4] \rangle$
3-4	$\langle [0.4,0.6],[0.3,0.5],[0.2,0.3] \rangle$
3-5	$\langle [0.3,0.5],[0.4,0.5],[0.5,0.6] \rangle$
4-6	$\langle [0.3,0.4],[0.2,0.4],[0.3,0.5] \rangle$
5-6	$\langle [0.3,0.4],[0.4,0.7],[0.5,0.7] \rangle$



**Fig-3: Heuristic Calculation of IVNP Network**

$$\hat{f}_{12} = \hat{g}_{12} + \hat{h}_2$$

$$\hat{f}_{12} = ([0.4,0.6], [0.3,0.5], [0.3,0.4]) + ([0.19,0.28], [0.16,0.35], [0.15,0.28])$$

$$\hat{f}_{12} = ([0.52,0.71], [0.04,0.17], [0.04,0.11])$$

$$S(\hat{f}_{12}) = 0.34$$

$$\hat{f}_{13} = \hat{g}_{13} + \hat{h}_3$$

$$\hat{f}_{13} = ([0.4,0.5], [0.3,0.4], [0.3,0.5]) + ([0.58,0.22], [0.06,2], [0.06,1.5])$$

$$\hat{f}_{13} = ([0.26,0.18], [0.01,0.08], [0.01,0.75])$$

$$S(\hat{f}_{13}) = 0.28$$

From  $S(\hat{f}_{12})$  &  $S(\hat{f}_{13})$  minimum is  $S(\hat{f}_{13})$  which value is 0.28. Hence label from 1 to 3

### Step-3

Now we visit node-4 and Node-5 from successor's node 1 to 3 , and then carry out step-2 according to the proposed.

$$\hat{f}_{34} = \hat{g}_{34} + \hat{h}_4$$

$$\hat{f}_{34} = ([0.4,0.6], [0.3,0.5], [0.2,0.3]) + ([0.3,0.4], [0.2,0.4], [0.3,0.5])$$

$$\hat{f}_{34} = [0.13,0.28], [0.06,2.0], [0.06,1.5]$$

$$S(\hat{f}_{34}) = 0.47$$

$$\hat{f}_{35} = \hat{g}_{35} + \hat{h}_5$$

$$\hat{f}_{35} = ([0.3,0.5], [0.4,0.5], [0.5,0.6]) + ([0.3,0.4], [0.4,0.7], [0.5,0.7])$$

$$\hat{f}_{35} = [0.09,0.21], [0.16,0.35], [0.25,0.36],$$

$$S(\hat{f}_{34}) = 0.43$$

From  $S(\hat{f}_{34})$  &  $S(\hat{f}_{35})$  minimum is  $S(\hat{f}_{35})$  which value is 0.43. Hence label from  $1 \rightarrow 3 \rightarrow 5$

**Step 4:** Only node 6 serves as the successor of node 5, so continue with step-2 of the algorithm to determine its estimated cost.

$$\hat{f}_{56} = \hat{g}_{56} + \hat{h}_6$$

$$\hat{f}_{56} = ([0.3,0.4], [0.4,0.7], [0.5,0.7]) + 0$$

$$\hat{f}_{56} = ([0.3,0.4], [0.4,0.7], [0.5,0.7])$$

$$S(\hat{f}_{56}) = 0.45$$

The shortest path can be identified by adding all of the nodes in the above steps  $\hat{f}_{56}, 1 \rightarrow 3 \rightarrow 5 \rightarrow 6$  with the minimum Cost is 0.45

### 7. Comparison study:

This section involves the evaluation of our algorithm using alternative methodologies. [37] and [38].

Optimal route with varying network configurations	Path(route)	Shortest path cost
Enayattabar M. et al. [37]	$1 \rightarrow 2 \rightarrow 5 \rightarrow 6$	0.95
Jan N. et al. [38]	$1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$	----
Our proposed	$1 \rightarrow 3 \rightarrow 5 \rightarrow 6$	0.45

**Table-2: Shortest path with different network**

### 8. Conclusion:

This research paper examines the benefits of using the interval-valued Neutrosophic Pythagorean Number to determine the Shortest Path. This method integrates uncertainty by utilizing the interval-valued Neutrosophic Pythagorean Number edge weight in an interval-valued Neutrosophic

Pythagorean environment. The outcome demonstrates the usefulness and efficiency of the algorithm in calculating the shortest path under an IVNP environment. Compared to the traditional A\* algorithm, our approach provides more accurate and robust solutions, considering the uncertainty and imprecision in the graph. The suggested approach can be used in real life applications in logistics management, communication systems, and many other network optimization issues that are expressed as shortest-path issues.

### Conflicts of Interest:

The authors declare no conflicts of interest.

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