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Intelligent Framework for Regional Tourism Ecological Efficiency Evaluation under High-Quality Development with Single-Valued Neutrosophic Information

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Abstract: The evaluation of regional tourism ecological efficiency (RTEE) under high-quality development aims to measure how the tourism industry promotes economic growth while minimizing resource consumption and environmental impact. By analyzing economic output per unit of carbon emissions, it assesses the sustainability and environmental performance of the tourism sector. This evaluation method considers resource utilization, energy efficiency, and ecological protection, helping decision-makers optimize tourism development models, achieve a balance between economic and ecological benefits, and promote the transition of tourism towards a low-carbon, green, and high-quality direction. The RTEE evaluation under high-quality development is MADM. The single-valued neutrosophic number generalized power geometric BM (SVNNGPGBM) technique is put forward and MADM decision techniques are put forward in line with SVNNGPGBM technique and power geometric (PG) technique. Finally, numerical example about RTEE evaluation under high-quality development and comparative analysis was put forward the SVNNGPGBM technique.

Keywords: MADM; SVNSs; GGBM technique; ecological efficiency evaluation

1. Introduction

The evaluation of regional tourism ecological efficiency under the context of high-quality development holds profound strategic significance. Firstly, it provides a scientific basis for balancing the economic benefits of the tourism industry with ecological protection. By quantifying environmental impacts such as resource consumption, carbon emissions, and waste management, this evaluation reveals whether the tourism sector is reducing its ecological burden while achieving economic growth. Such assessments not only help identify efficient, low-consumption tourism development models but also support decision-making for optimizing resource allocation, thereby promoting the green transformation and sustainable development of the tourism industry.

Secondly, the evaluation offers crucial insights for policymakers and businesses in formulating environmentally friendly policies and strategies. Governments can leverage the evaluation results to implement targeted policies that encourage low-carbon, eco-friendly tourism practices, avoiding the inefficient "pollute first, clean up later" development model. Simultaneously, businesses can adjust their operational strategies based on the evaluation outcomes to improve resource efficiency and environmental awareness, thereby enhancing their market competitiveness. Furthermore, regional tourism ecological evaluation helps foster ecological collaboration between regions, driving a win-win situation for both economic growth and environmental protection. By guiding regions to prioritize ecological conservation in tourism development, the evaluation system encourages green competition and cooperation, improving overall ecological efficiency and supporting the achievement of a green, low-carbon, and sustainable high-quality economic growth, ultimately enhancing regional competitiveness and sustainable development capacity.

In today's increasingly fierce market competition, enterprises or individuals often face various decision-making [1-3]. As information technology continues to advance rapidly, individuals are encountering increasingly complex decision-making environments [4-7]. The complexity of the decision-making environment makes it increasingly unlikely for individual decision-makers to make multi-attribute decisions. In real life, it is often necessary to evaluate and choose from numerous alternative options (candidates, objects, etc.). Especially for major decision-making problems, it is necessary for experts from different fields to leverage each person's professional advantages, evaluate the solutions from different aspects, and then select or rank the solutions, which is commonly known as MADM problems[8-12]. Figure 1 shows the factors which affect the decisions' results.

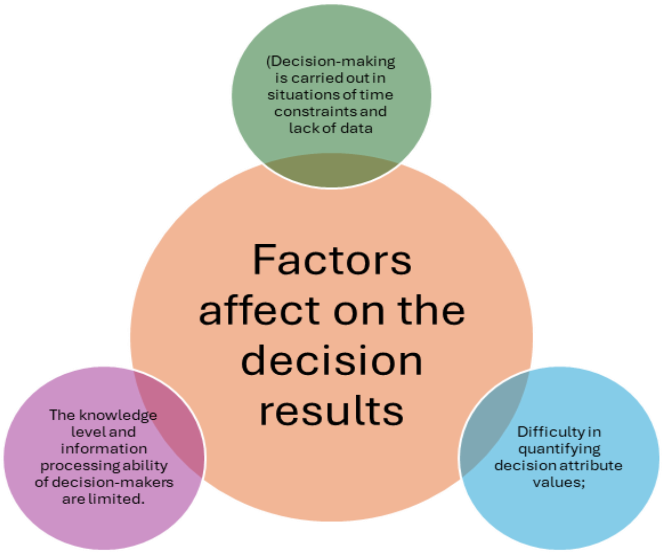

Figure 1. Factors that affect the decisions' results.

The complexity of the decision-making environment makes it sometimes difficult for decision experts to use precise numerical values to represent the evaluation information of solutions, while using a single value set to express the evaluation information is very convenient and feasible[13-18]. Therefore, the study of MADM problems based on single valued intermediate intelligence sets not only has significant theoretical significance for the development of management science, but also has significant practical significance for solving many complex decision-making problems.

The main contributions of this study are outlined:

- The SVNNGPGBM technique is constructed.
- The maximizing deviation technique is proposed to handle weight.
- The SVNNGPGBM technique is applied to MADM.
- A numerical example for RTEE evaluation under high-quality development is demonstrated using the SVNNGPGBM technique.

To conduct so, The SVNSs was constructed on Sec. 2. The SVNNGPGBM technique is conducted on Sec. 3. MADM approach based on SVNNGPGBM technique is conducted in Sec. 4. A numerical example for RTEE evaluation under high-quality development is conducted on Sec. 5. Conclusion is conducted for Sec. 6.

2. Literature review

The evaluation of regional tourism ecological efficiency under high-quality development aims to assess the balance between economic growth and ecological protection in the tourism industry. The evaluation focuses on the equilibrium between resource utilization, environmental impact, and economic benefits, emphasizing the concepts of green and sustainable development. By analyzing environmental indicators such as energy consumption, carbon emissions, and waste management, alongside tourism output, it reveals whether regional tourism effectively reduces environmental burdens while achieving economic growth. This evaluation system provides a scientific basis for policymakers to optimize resource allocation, enhance ecological efficiency, and promote the green transformation and sustainable development of the tourism industry, thus supporting high-quality regional economic growth. The research on tourism ecological efficiency (TEE) has evolved significantly over the years, focusing on various regions and methodologies. In 2016, Yao, Chen, Yin and Li [19] conducted an empirical analysis on the TEE of Hainan Province, using a tourism ecological efficiency model to calculate the efficiency for different tourism activities. They concluded that optimizing the tourism model with "short distances, long stays, and high consumption" can improve TEE, while the opposite would reduce it. In 2018, Gao [20] studied the spatial effects of TEE in Jiangsu Province, using tools like MaxDEA and OpenGeoda to measure efficiency across 13 cities. The study revealed significant spatial imbalances in TEE and suggested policies for improvement. Lin and Lin [21] evaluated the eco-efficiency of the tourism industry in Jiangxi Province from 2011 to 2016 using a super-efficiency DEA model. They found that while the overall efficiency was high, there were notable spatial disparities, with regions in northern Jiangxi performing better than those in the south. In 2021, Wang [22] explored the relationship between tourism industry agglomeration and TEE in Shanxi Province from 2009 to 2018. The study showed that tourism agglomeration positively affected TEE, with spatial configuration evolving through different stages during the study period. Fang [23] examined the coordination between TEE and economic development in the Yangtze River Economic Belt from 2010 to 2018. The study revealed that while TEE generally improved, significant regional disparities persisted. The research proposed strategies to promote the coordinated development of tourism and the economy. Hong, Wang and Zhang [24] focused on the western regions of China from 2000 to 2017, analyzing the spatial and temporal evolution of TEE. They found that factors such as tourist scale, industrial structure optimization, and technological advancements contributed to increased TEE, while urbanization had a negative impact. In 2022, Li and Wu [25] investigated the spatial consistency between TEE and regional ecological security in the middle reaches of the Yangtze River from 2005 to 2018. Their study suggested that the relationship between TEE and ecological security followed an inverted U-shaped pattern, with spatial consistency improving over time. Liu [26] examined the impact of regional innovation on TEE across 30 provinces in China from 2008 to 2017, finding that innovation significantly improved tourism efficiency, particularly in the eastern regions, though there was a downward trend in overall TEE. Focusing on Shandong Province, Tian and Zhao [27] used a three-stage DEA model to evaluate the TEE of 16 cities from 2010 to 2019. Their study showed that while Shandong's TEE was relatively high, regional differences persisted, largely influenced by natural environment factors and innovation in tourism models. Finally, in 2023, Deng [28] conducted a comprehensive study on the spatiotemporal differentiation of TEE in 30 provinces across mainland China from 2000 to 2019. Using a "tourism value-added coefficient" to estimate carbon emissions, Deng applied a spatial Durbin model to investigate the factors influencing TEE. The results indicated a significant increase in carbon emissions from tourism, but also a gradual improvement in ecological efficiency, with regional disparities diminishing over time. Across these studies, the common thread is the emphasis on balancing tourism development with environmental sustainability. Table 1 shows the previous works.

Across these studies, the common thread is the emphasis on balancing tourism development with environmental sustainability. The use of various models, including DEA, SBM, and spatial analysis, has allowed researchers to assess TEE's evolution across different regions and time periods, providing valuable insights for policymakers to promote low-carbon, green, and high-quality tourism development.

Table 1. Previously works on the same issue.

Authors	Study	Contributions and results
Yao, Chen, Yin and Li [19]	They conducted an empirical analysis on the TEE of Hainan Province, using a tourism ecological efficiency model to calculate the efficiency of different tourism activities.	They concluded that optimizing the tourism model with "short distances, long stays, and high consumption" can improve TEE, while the opposite would reduce it.
In 2018, Gao [20]	They studied the spatial effects of TEE in Jiangsu Province, using tools like MaxDEA and OpenGeoda to measure efficiency across 13 cities.	The study revealed significant spatial imbalances in TEE and suggested policies for improvement
Lin and Lin [21]	They evaluated the eco-efficiency of the tourism industry in Jiangxi Province from 2011 to 2016 using a super-efficiency DEA model.	They found that while the overall efficiency was high, there were notable spatial disparities, with regions in northern Jiangxi performing better than those in the south. In 2021,
Wang [22]	They explored the relationship between tourism industry agglomeration and TEE in Shanxi Province from 2009 to 2018.	The study showed that tourism agglomeration positively affected TEE, with spatial configuration evolving through different stages during the study period.
Liu [26]	They examined the impact of regional innovation on TEE across 30 provinces in China from 2008 to 2017,	They are finding that innovation significantly improved tourism efficiency, particularly in the eastern regions, though there was a downward trend in overall TEE.
Tian and Zhao [27]	They used a three-stage DEA model to evaluate the TEE of 16 cities from 2010 to 2019.	Their study showed that while Shandong's TEE was relatively high, regional differences persisted, largely

		influenced by natural environment factors and innovation in tourism models.
Deng [28]	They conducted a comprehensive study on the spatiotemporal differentiation of TEE in 30 provinces across mainland China from 2000 to 2019. Using a "tourism value-added coefficient" to estimate carbon emissions,	Deng applied a spatial Durbin model to investigate the factors influencing TEE. The results indicated a significant increase in carbon emissions from tourism, but also a gradual improvement in ecological efficiency, with regional disparities diminishing over time.

2. Preliminaries

Wang et al. [29] put forward the SVNSs.

Definition 1 [29]. The SVNSs are put forward:

$$EA=\left\{\left(\theta, ET_A(\theta), EI_A(\theta), EF_A(\theta)\right) \middle| \theta\in\Theta\right\} \quad (1)$$

where $ET_A(\theta), EI_A(\theta), EF_A(\theta)$ is the truth-membership, indeterminacy-membership and falsity-membership, $ET_A(\theta), EI_A(\theta), EF_A(\theta)\in[0,1]$, $0\le ET_A(\theta)+EI_A(\theta)+EF_A(\theta)\le 3$.

The SVNN is put forward as $EA=(ET_A, EI_A, EF_A)$, where $ET_A, EI_A, EF_A\in[0,1]$, and $0\le ET_A+EI_A+EF_A\le 3$.

Definition 2 [30]. Let $EA=(ET_A, EI_A, EF_A)$, the score function is put forward:

$$SF(EA)=\frac{(2+ET_A-EI_A-EF_A)}{3},\ SF(EA)\in[0,1]. \quad (2)$$

Definition 3[30]. Let $EA=(ET_A, EI_A, EF_A)$, the accuracy function is put forward:

$$AF(EA)=\frac{ET_A-EF_A+1}{2},\ AF(EA)\in[0,1]. \quad (3)$$

Peng et al.[30] put forward the order relation.

Definition 4[30]. Let $EA=(ET_A, EI_A, EF_A)$, $EB=(ET_B, EI_B, EF_B)$, let $SF(EA)=\frac{(2+ET_A-EI_A-EF_A)}{3}$ and $SF(EB)=\frac{(2+ET_B-EI_B-EF_B)}{3}$, and let $AF(EA)=\frac{ET_A-EF_A+1}{2}$ and $AF(EB)=\frac{ET_B-EF_B+1}{2}$, if $SF(EA)<SF(EB)$, $PA<PB$; if $SF(EA)=SF(EB)$, (1)if $AF(EA)=AF(EB)$, $EA=EB$; (2) if $AF(EA)<AF(EB)$, $EA<EB$.

Definition 5[29]. Let $EA=(ET_A, EI_A, EF_A)$ and $EB=(ET_B, EI_B, EF_B)$, the put forward operations are:

(1) $EA \oplus EB = \left(ET_A + ET_B - ET_A ET_B, EI_A EI_B, EF_A EF_B \right);$

(2) $EA \otimes EB = \left(ET_A ET_B, EI_A + EI_B - EI_A EI_B, EF_A + EF_B - EF_A EF_B \right);$

(3) $\lambda EA = \left(1 - \left(1 - ET_A\right)^{\lambda}, \left(EI_A\right)^{\lambda}, \left(EF_A\right)^{\lambda} \right), \lambda > 0;$

(4) $\left(EA\right)^{\lambda} = \left(\left(ET_A\right)^{\lambda}, \left(EI_A\right)^{\lambda}, 1 - \left(1 - EF_A\right)^{\lambda} \right), \lambda > 0.$

Definition 6[31]. Let $EA = \left(ET_A, EI_A, EF_A \right)$ and $EB = \left(ET_B, EI_B, EF_B \right)$, then the normalized Hamming distance between $EA = \left(ET_A, EI_A, EF_A \right)$ and $EB = \left(ET_B, EI_B, EF_B \right)$ is put forward:

$$HD\left(EA, EB\right) = \frac{1}{3}\begin{pmatrix} \left|ET_A - ET_B\right| \\ + \left|EI_A - EI_B\right| + \left|EF_A - EF_B\right| \end{pmatrix} \tag{4}$$

Definition 6[32]. Let $es, et, er > 0$, $ea_i (i = 1, 2, 3, \ldots, n)$ $\left(ea_i > 0\right)$ with weight $ew = (ew_1, ew_2, \ldots, ew_n)^T$, $ew_i \in \left[0,1\right]$, $\sum_{i=1}^{n} ew_i = 1.$ The GWBM technique is put forward:

$$\begin{aligned} & \mathrm{GWBM}_{ew}^{es,et,er}(ea_1, ea_2, \ldots, ea_n) \\ & = \left(\sum_{i,j,k=1}^{n} ew_i ew_j ew_k ea_i^s ea_j^t ea_k^r \right)^{1/(es+et+er)} \end{aligned} \tag{5}$$

3. GSVNNPBM technique

Then, GSVNNWBM technique is put forward.

Definition 8. Let $es, et, er > 0$, $EA_i = (ET_i, EI_i, EF_i)$ $(i = 1, 2, 3, \ldots, n)$ be SVNNs with weight $ew_i = (ew_1, ew_2, \ldots, ew_n)^T$, $\sum_{i=1}^{n} ew_i = 1.$ If

$$\begin{aligned} & \mathrm{GSVNNWBM}_{ew}^{es,et,er}(EA_1, EA_2, \cdots, EA_n) \\ & = \left(\underset{i,j,k=1}{\overset{n}{\oplus}} ew_i ew_j ew_k \left(EA_i^{es} \otimes EA_j^{et} \otimes EA_k^{er} \right) \right)^{1/(es+et+er)} \\ & = \begin{pmatrix} (1 - \prod_{i,j,k=1}^{n} (1 - ET_i^{es} ET_j^{et} ET_k^{er})^{zw_i zw_j zw_k})^{1/(es+et+er)}, \\ 1 - (1 - \prod_{i,j,k=1}^{n} (1 - (1 - EI_i)^{es} (1 - EI_j)^{et} (1 - EI_k)^{er})^{ew_i ew_j ew_k})^{1/(es+et+er)}, \\ 1 - (1 - \prod_{i,j,k=1}^{n} (1 - (1 - EF_i)^{es} (1 - EF_j)^{et} (1 - EF_k)^{er})^{ew_i ew_j ew_k})^{1/(es+et+er)}). \end{pmatrix} \end{aligned} \tag{6}$$

then $\mathrm{GSVNNWBM}_{ew}^{s,t,r}$ is called GSVNNWBM technique.

The GSVNNWBM technique has the same properties.

Property 1 (Idempotency). If $EA_i = (ET_i, EI_i, EF_i)(i = 1, 2, \ldots, n)$ are same, that is, $EA_i = EA = (ET, EI, EF)$, then

$$\mathrm{GSVNNWBM}_{ew}^{es,et,er}(EA_1, EA_2, \cdots, EA_n) = EA \tag{7}$$

Property 2 (Monotonicity). Let $EA_i = (ET_{A_i}, EI_{A_i}, EF_{A_i})(i = 1,2,3,\ldots,n)$ and $EB_i = (ET_{B_i}, EI_{B_i}, EF_{B_i})$ $(i = 1,2,3,\ldots,n)$ be SVNNs. If $ET_{A_i} \le ET_{B_i}$, $EI_{A_i} \ge EI_{B_i}$, $EF_{A_i} \ge EF_{B_i}$ holds for all i, then

$$\begin{aligned} &\text{GSVNNWBM}_{ew}^{es,et,er}(EA_1, EA_2, \cdots, EA_n) \\ &\le \text{GSVNNWBM}_{ew}^{es,et,er}(EB_1, EB_2, \cdots, EB_n) \end{aligned} \tag{8}$$

Property 3 (Boundedness). Let $EA_i = (ET_{A_i}, EI_{A_i}, EF_{A_i})(i = 1,2,3,\ldots,n)$ be SVNNS. If $PA^+ = (\max_i(PT_i), \min_i(PI_i), \min_i(PF_i))$ and $PA^- = (\min_i(PT_i), \max_i(PI_i), \max_i(PF_i))$ then

$$EA^- \le \text{GSVNNWBM}_{ew}^{es,et,er}(EA_1, EA_2, \cdots, EA_n) \le EA^+. \tag{9}$$

Then, generalized single-valued neutrosophic number power BM (GSVNNPBM) technique is put forward in line with GSVNNWBM technique and PA technique [33].

Definition 9. Let $es, et, er > 0$, $EA_i = (ET_i, EI_i, EF_i)$ $(i = 1,2,3,\ldots,n)$ be SVNNs, The GSVNNPBM technique is put forward:

$$\begin{aligned} &GSVNNPBM^{es,et,er}((EA_1, EA_2, \cdots, EA_n) \\ &= \left(\bigoplus_{i,j,k=1}^{n} \frac{\left(1+ET\left(EA_i\right)\right)}{\sum_{i=1}^{n}\left(1+ET\left(EA_i\right)\right)} \frac{\left(1+ET\left(EA_j\right)\right)}{\sum_{j=1}^{n}\left(1+ET\left(EA_j\right)\right)} \frac{\left(1+ET\left(EA_k\right)\right)}{\sum_{k=1}^{n}\left(1+ET\left(EA_k\right)\right)} \left(EA_i^{es} \otimes EA_j^{et} \otimes EA_k^{er}\right) \right)^{1/(es+et+er)} \end{aligned} \tag{10}$$

where $ET\left(EA_a\right) = \sum_{\substack{j=1 \\ a \ne j}}^{m} Sup\left(EA_a, EA_j\right)$, $Sup\left(EA_a, EA_j\right)$ is the decision support for EA_a from EA_j , with given conditions: (1) $Sup\left(EA_a, EA_b\right) \in [0,1]$; (2) $Sup\left(EA_b, EA_a\right) = Sup\left(EA_a, EA_b\right)$; (3) $Sup\left(EA_a, EA_b\right) \ge Sup\left(EA_s, EA_t\right)$, if $HD\left(EA_a, EA_b\right) \le HD\left(EA_s, EA_t\right)$, HD is a distance.

The Theorem 1 is constructed.

Theorem 1. Let $es, et, er > 0$ and $EA_i = (ET_i, EI_i, EF_i)(i = 1,2,3,\ldots,n)$ be SVNNs. The aggregated value for SVNNGPGBM is SVNN and

$$SVNNGPGBM^{es,et,er}(EA_1, EA_2, \cdots, EA_n)$$
$$= \frac{1}{es+et+er} \mathop{\otimes}_{i,j,k=1}^{n} \left(esEA_i \oplus etEA_j \oplus erEA_k\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+E(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}$$
$$= \left(\begin{array}{l} \left(1-\left(1-\prod_{i,j,k=1}^{n}(1-(1-ET_i)^{es}(1-ET_j)^{et}(1-ET_k)^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+E(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)}, \\ \left(1-\prod_{i,j,k=1}^{n}\left(1-EI_i^{es}EI_j^{et}EI_k^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+E(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)}, \\ \left(1-\prod_{i,j,k=1}^{n}\left(1-EF_i^{es}EF_j^{et}EF_k^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+E(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)}. \end{array} \right) \tag{11}$$

Proof:

$$\begin{aligned} esEA_i &= (1-(1-PT_i)^{es}, EI_i^{es}, EF_i^{es}), \\ etEA_j &= (1-(1-ET_j)^{et}, EI_j^{et}, EF_j^{et}), \\ erEA_k &= (1-(1-ET_k)^{er}, EI_k^{er}, EF_k^{er}). \end{aligned} \tag{12}$$

$$esEA_i \oplus etEA_j \oplus erEA_k = \left(\begin{array}{l} 1-(1-ET_i)^{es}(1-ET_j)^{et}(1-ET_k)^{er}, \\ EI_i^{es}EI_j^{et}EI_k^{er}, EF_i^{es}EF_j^{et}EF_k^{er} \end{array} \right) \tag{13}$$

Thereafter,

$$\left(esEA_i \oplus etEA_j \oplus erEA_k\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}_{k}$$
$$= \left(\begin{array}{l} (1-(1-ET_i)^{es}(1-ET_j)^{et}(1-ET_k)^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}, \\ 1-(1-EI_i^{es}EI_j^{et}EI_k^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}, \\ 1-(1-EF_i^{es}EF_j^{et}EF_k^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}} \end{array} \right) \tag{14}$$

Therefore,

$$\underset{i,j,k=1}{\overset{n}{\otimes}}\left(esEA_i \oplus etEA_j \oplus erEA_k\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}$$

$$= \left(\begin{array}{l} \prod_{i,j,k=1}^{n}(1-(1-ET_i)^{es}(1-ET_j)^{et}(1-ET_k)^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}, \\ 1-\prod_{i,j,k=1}^{n}(1-EI_i^{es}EI_j^{et}EI_k^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}, \\ 1-\prod_{i,j,k=1}^{n}(1-EF_i^{es}EF_j^{et}EF_k^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}} \end{array} \right) \tag{15}$$

Thus,

$$\frac{1}{es+et+er}\underset{i,j,k=1}{\overset{n}{\otimes}}\left(esEA_i \oplus etEA_j \oplus erEA_k\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}$$

$$= \left(\begin{array}{l} \left(1-(1-\prod_{i,j,k=1}^{n}(1-(1-ET_i)^{es}(1-ET_j)^{et}(1-ET_k)^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)}, \\ \left(1-\prod_{i,j,k=1}^{n}(1-EI_i^{es}EI_j^{et}EI_k^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)}, \\ \left(1-\prod_{i,j,k=1}^{n}(1-EF_i^{es}EF_j^{et}EF_k^{er})^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))}\frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))}\frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)} \end{array} \right) \tag{16}$$

Hence, (11) is maintained.

Thereafter,

$$0 \le 1 - \left(1 - \prod_{i,j,k=1}^{n} \left(1 - (1-ET_i)^{es} (1-ET_j)^{et} (1-ET_k)^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))} \frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))} \frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}} \right)^{1/(es+et+er)} \le 1,$$

$$0 \le \left(1 - \prod_{i,j,k=1}^{n} \left(1 - EI_i^{es} EI_j^{et} EI_k^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))} \frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))} \frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}} \right)^{1/(es+et+er)} \le 1,$$

$$0 \le \left(1 - \prod_{i,j,k=1}^{n} \left(1 - EF_i^{es} EF_j^{et} EF_k^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))} \frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))} \frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}} \right)^{1/(es+et+er)} \le 1. \quad (17)$$

Therefore,

$$\begin{aligned} 0 \le\ & 1 - \left(1 - \prod_{i,j,k=1}^{n} \left(1 - (1-ET_i)^{es} (1-ET_j)^{et} (1-ET_k)^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))} \frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))} \frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)} \\ & + \left(1 - \prod_{i,j,k=1}^{n} \left(1 - EI_i^{es} EI_j^{et} EI_k^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))} \frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))} \frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)} \\ & + \left(1 - \prod_{i,j,k=1}^{n} \left(1 - EF_i^{es} EF_j^{et} EF_k^{er}\right)^{\frac{(1+ET(EA_i))}{\sum_{i=1}^{n}(1+ET(EA_i))} \frac{(1+ET(EA_j))}{\sum_{j=1}^{n}(1+ET(EA_j))} \frac{(1+ET(EA_k))}{\sum_{k=1}^{n}(1+ET(EA_k))}}\right)^{1/(es+et+er)} \le 3. \end{aligned} \quad (18)$$

Thereby finishing the proof.

The SVNNGPGBM technique has the same properties.

Property 4 (Idempotency). If $EA_i = (ET_i, EI_i, EF_i)(i = 1, 2, \ldots, n)$ are same, that is, $EA_i = EA = (ET, EI, EF)$, then

$$SVNNGPGBM^{es,et,er}(EA_1, EA_2, \cdots, EA_n) = EA \quad (19)$$

Property 5 (Monotonicity). Let $EA_i = (ET_{A_i}, EI_{A_i}, EF_{A_i})(i = 1, 2, 3, \ldots, n)$ and $EB_i = (ET_{B_i}, EI_{B_i}, EF_{B_i})$ $(i = 1, 2, 3, \ldots, n)$ be SVNNs. If $ET_{A_i} \le ET_{B_i}$, $EI_{A_i} \ge EI_{B_i}$, $EF_{A_i} \ge EF_{B_i}$ for all i, then

$$SVNNGPGBM^{es,et,er}(EA_1, EA_2, \cdots, EA_n) \le SVNNGPGBM^{es,et,er}(EB_1, EB_2, \cdots, EB_n) \quad (20)$$

Property 6 (Boundedness). Let $EA_i = (ET_{A_i}, EI_{A_i}, EF_{A_i})(i = 1, 2, 3, \ldots, n)$ be SVNNS. If $EA^+ = (\max_i(ET_i), \min_i(EI_i), \min_i(EF_i))$ and $EA^- = (\min_i(ET_i), \max_i(EI_i), \max_i(EF_i))$ then

$$EA^- \le SVNNGPGBM^{es,et,er}(EA_1, EA_2, \cdots, EA_n) \le EA^+. \quad (21)$$

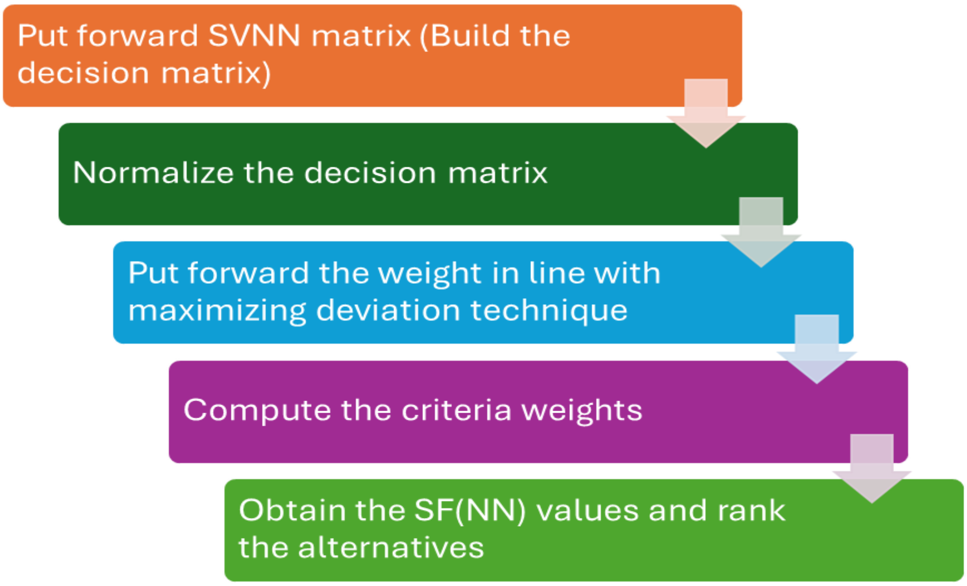

Figure 2. The steps of the MADM methodology.

4. MADM technique based on SVNNGPGBM under SVNNs

The SVNNGPGBM technique is put forward MADM with SVNSs. Let $C = C_1, C_2, \ldots, C_n$ be attributes, $ew = \{ew_1, ew_2, \ldots, ew_n\}$ be weight of C. Let $A = A_1, A_2, \ldots, A_m$ be alternatives. $EE = (EE_{ij})_{m\times n} = (ET_{ij}, EI_{ij}, EF_{ij})_{m\times n}$ is SVNN-matrix. The SVNNGPGBM technique is put forward MADM. Figure 2 shows the steps of the proposed model.

Step 1. Put forward SVNN matrix $EE = (EE_{ij})_{m\times n} = (ET_{ij}, EI_{ij}, EF_{ij})_{m\times n}$.

$$EE = \left(EE_{ij}\right)_{m\times n} = \begin{bmatrix} EE_{11} & EE_{12} & \dots & EE_{1n} \\ EE_{21} & EE_{22} & \dots & EE_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ EE_{m1} & EE_{m2} & \dots & EE_{mn} \end{bmatrix}$$

$$EE = \left(EE_{ij}\right)_{m\times n} = \left(ET_{ij}, EI_{ij}, EF_{ij}\right)_{m\times n}$$

$$= \begin{bmatrix} \left(ET_{11}, EI_{11}, PF_{11}\right) & \left(ET_{12}, EI_{1}, EF_{12}\right) & \dots & \left(ET_{1n}, EI_{1n}, EF_{1n}\right) \\ \left(ET_{21}, EI_{21}, EF_{21}\right) & \left(ET_{22}, EI_{22}, EF_{22}\right) & \dots & \left(ET_{2n}, EI_{2n}, EF_{2n}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(ET_{m1}, EI_{m1}, EF_{m1}\right) & \left(ET_{m2}, EI_{m2}, EF_{m2}\right) & \dots & \left(ET_{mn}, EI_{mn}, EF_{mn}\right) \end{bmatrix} \tag{22}$$

Step 2. Normalize $EE = \left[EE_{ij}\right]_{m\times n}$ to $NE = \left[NE_{ij}\right]_{m\times n}$.

$$NE_{ij} = \left(NET_{ij}, NEI_{ij}, NEF_{ij}\right) = \begin{cases} \left(ET_{ij}, EI_{ij}, EF_{ij}\right), & EX_j \text{ is benefit} \\ \left(EF_{ij}, 1-EI_{ij}, ET_{ij}\right), & EX_j \text{ is cost} \end{cases} \tag{23}$$

Step 3. Put forward the weight in line with maximizing deviation technique.

The maximizing deviation technique [34] under SVNSs to construct the weight values.

(1) From the $NE = \left[NE_{ij}\right]_{m\times n}$, deviation degree EA_i to other ones is constructed.

$$SVNNDD_{ij} = \sum_{t=1}^{m} ew_j \cdot \left| \frac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \frac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)} \right| \tag{24}$$

(2) Construct the total weighted deviation.

$$SVNNDD_j\left(ew\right) = \sum_{i=1}^{m} SVNNDD_{ij}\left(ew\right) = \sum_{i=1}^{m}\sum_{t=1}^{m} ew_j \left| \frac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \frac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)} \right| \tag{25}$$

(3) Construct non-linear programming techniques.

$$\left(M-1\right)\begin{cases} \max SVNNDD\left(ew\right) = \sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{m} ew_j \left| \frac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \frac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)} \right| \\ s.t.\ \ \mathrm{ew}_j \geq 0,\ \ \sum_{j=1}^{n} ew_j^2 = 1 \end{cases} \tag{26}$$

The Lagrange function is put forward this model.

$$LF\left(ew, \xi\right) = \left(\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{m} ew_j \left| \frac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \frac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)} \right| + \frac{\xi}{2}\left(\sum_{j=1}^{n} ew_j^2 - 1 \right) \right)$$

where ξ is Lagrange multiplier. The partial derivatives are constructed.

$$\begin{cases} \dfrac{\partial LF}{\partial ew_j} = \sum_{i=1}^{m}\sum_{t=1}^{m}\left|\dfrac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \dfrac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)}\right| + \xi ew_j = 0 \\ \dfrac{\partial LF}{\partial \xi} = \dfrac{1}{2}\left(\sum_{j=1}^{n} ew_j^2 - 1\right) = 0 \end{cases}$$

The weight is obtained: $ew_j^* = \dfrac{\sum_{i=1}^{m}\sum_{t=1}^{m}\left|\dfrac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \dfrac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)}\right|}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{t=1}^{m}\left|\dfrac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \dfrac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)}\right|\right)^2}}$

Finally, the normalized weights are: $ew_j = \dfrac{\sum_{i=1}^{m}\sum_{t=1}^{m}\left|\dfrac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \dfrac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)}\right|}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{m}\left|\dfrac{SF\left(NE_{ij}\right)}{AF\left(NE_{ij}\right)} - \dfrac{SF\left(NE_{tj}\right)}{AF\left(NE_{tj}\right)}\right|}$ (27)

Step 4. According to $NE = \left[NE_{ij}\right]_{m\times n}$, we can fuse all SVNNs $NE = \left[NE_{ij}\right]_{m\times n}$ through SVNNGPGBM technique to obtain the SVNNs $NE_i = \left(NET_i, NEI_i, NEF_i\right)$:

$$\begin{aligned} NE_i &= \left(NET_i, NEI_i, NEF_i\right) \\ &= SVNNGWGBM_{ew}^{es,et,er}\left(NE_{i1}, NE_{i2}, \cdots, NE_{in}\right) \\ &= \frac{1}{es+et+er}\bigotimes_{i,j,k=1}^{n}\left(esNE_{if} \oplus etNE_{ij} \oplus erNE_{ik}\right)^{\frac{\left(1+ET\left(NE_{if}\right)\right)}{\sum_{i=1}^{n}\left(1+ET\left(NE_{if}\right)\right)}\frac{\left(1+ET\left(NE_{ij}\right)\right)}{\sum_{j=1}^{n}\left(1+ET\left(NE_{ij}\right)\right)}\frac{\left(1+ET\left(NE_{ik}\right)\right)}{\sum_{k=1}^{n}\left(1+ET\left(NE_{ik}\right)\right)}} \\ &= \left(\begin{array}{c} 1-\left(1-\prod_{i,j,k=1}^{n}\left(1-(1-NET_{if})^{es}(1-NET_{ij})^{et}(1-NET_{ik})^{er}\right)^{\frac{\left(1+ET\left(NE_{if}\right)\right)}{\sum_{i=1}^{n}\left(1+ET\left(NE_{if}\right)\right)}\frac{\left(1+ET\left(NE_{ij}\right)\right)}{\sum_{j=1}^{n}\left(1+ET\left(NE_{ij}\right)\right)}\frac{\left(1+ET\left(NE_{ik}\right)\right)}{\sum_{k=1}^{n}\left(1+ET\left(NE_{ik}\right)\right)}}\right)^{1/(es+et+er)}, \\ \left(1-\prod_{i,j,k=1}^{n}\left(1-NEI_{if}^{es}NEI_{ij}^{et}NEI_{ik}^{er}\right)^{\frac{\left(1+ET\left(NE_{if}\right)\right)}{\sum_{i=1}^{n}\left(1+ET\left(NE_{if}\right)\right)}\frac{\left(1+ET\left(NE_{ij}\right)\right)}{\sum_{j=1}^{n}\left(1+ET\left(NE_{ij}\right)\right)}\frac{\left(1+ET\left(NE_{ik}\right)\right)}{\sum_{k=1}^{n}\left(1+ET\left(NE_{ik}\right)\right)}}\right)^{1/(es+et+er)}, \\ \left(1-\prod_{i,j,k=1}^{n}\left(1-NEF_{if}^{es}NEF_{ij}^{et}NEF_{ik}^{er}\right)^{\frac{\left(1+ET\left(NE_{if}\right)\right)}{\sum_{i=1}^{n}\left(1+ET\left(NE_{if}\right)\right)}\frac{\left(1+ET\left(NE_{ij}\right)\right)}{\sum_{j=1}^{n}\left(1+ET\left(NE_{ij}\right)\right)}\frac{\left(1+ET\left(NE_{ik}\right)\right)}{\sum_{k=1}^{n}\left(1+ET\left(NE_{ik}\right)\right)}}\right)^{1/(es+et+er)} \end{array}\right) \end{aligned}$$

(28)

Step 5. Obtain the $SF(NE), AF(NE_i)$ $(i=1,2,3,\cdots,m)$.

$$SF(nq_i)=\frac{(2+NET_i-NEI_i-NEF_i)}{3},$$
$$AF(NE_i)=\frac{NET_i-NEF_i+1}{2} \tag{29}$$

Step 6. Rank the alternatives

5. Empirical example and comparative analysis

5.1. Case Study

The evaluation of RTEE under high-quality development refers to measuring how the tourism industry, in the context of promoting high-quality economic growth, effectively reduces resource consumption, energy usage, and negative environmental impacts while achieving economic growth. The core idea of tourism ecological efficiency evaluation is to assess the sustainability and environmental contribution of the tourism industry by analyzing the economic output per unit of carbon emissions. Its goal is to promote the coordinated development of the regional economy and ecology by achieving environmental protection and efficient resource utilization while advancing tourism development as shown in Figure 3.

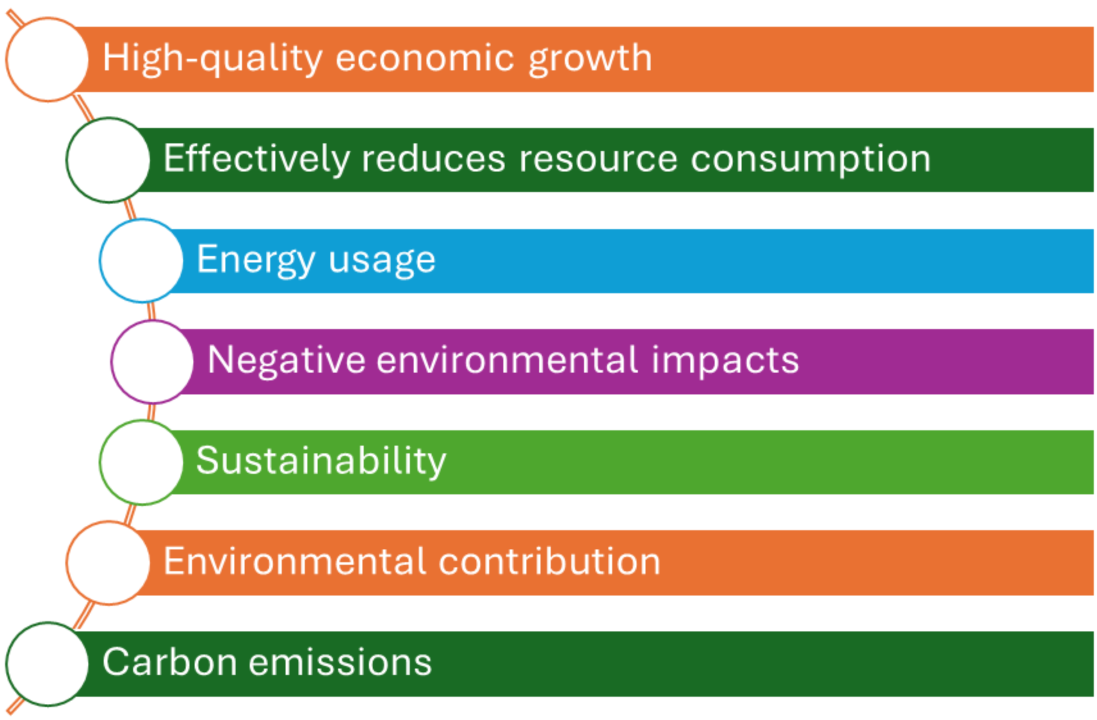

Figure 3. Evaluation of RTFE.

As the world places increasing emphasis on climate change and environmental protection, the tourism industry, being resource-intensive, needs to focus on improving ecological efficiency during its development. Through ecological efficiency evaluation, the environmental impact of the tourism industry can be quantified, helping decision-makers identify areas with high energy consumption and low output, thereby formulating more targeted policies and measures.

In summary, the evaluation of the RTEE under high-quality development is not only an important tool for assessing the sustainable development of the tourism industry but also a key pathway to achieving the green transformation of tourism and fostering a win-win situation between economic and ecological benefits. The RTEE evaluation under high-quality development is MADM. Figure 4 shows the criteria.

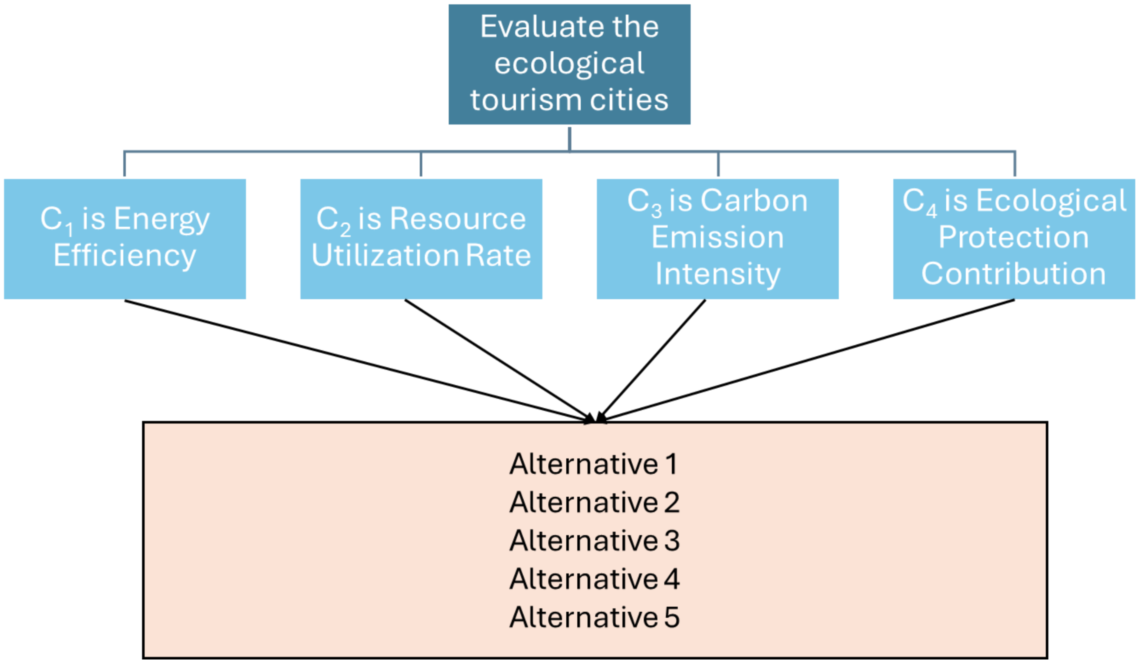

Figure 4. The list of criteria and alternatives.

Five ecological tourism cities are evaluated with four attributes:

I. Energy Efficiency (C_1)**:** This indicator measures the economic benefits generated per unit of energy consumption in the tourism industry. An improvement in energy efficiency means that tourism activities use less energy to create more economic output, helping to reduce energy waste and lower carbon emissions.;

II. Resource Utilization Rate (C_2): The resource utilization rate assesses the economic output generated per unit of resource input (such as water, land, raw materials, etc.) in the tourism industry. A high resource utilization rate indicates that resources are being used efficiently, minimizing resource waste and enhancing the sustainability of tourism.;

III. Carbon Emission Intensity (C_3): This indicator analyzes the amount of carbon emissions per unit of economic output, used to measure the tourism industry's carbon footprint. The lower the carbon emission intensity, the smaller the negative environmental impact of tourism while achieving economic growth, contributing to low-carbon development goals.

IV. Ecological Protection Contribution (C_4): This indicator examines the tourism sector's role in protecting and improving the local natural environment. It evaluates whether tourism activities help preserve ecosystems, reduce environmental pollution, and promote sustainable ecological management, ensuring that tourism development aligns with long-term ecological protection. Then, the SVNNGPGBM technique put forward MADM for solving the RTEE evaluation under high-quality development with SVNNs.

Step 1. Put forward the SVNN-matrix as shown in Table 2.
Step 2. We normalize the decision matrix as shown in Table 3.

Step 3. Constructed the weights of criteria as shown in Figure 5.

Step 4. Obtain the $NP_i\left(i=1,2,3,\cdots,5\right)$ in line with SVNNGPGBM technique
Step 5. Put forward the $SV\left(NP_i\right)\left(i=1,2,3,\cdots,5\right)$ (Table 4).
Step 6. We show the rank is $A_2 > A_5 > A_4 > A_3 > A_1$, and the best choice is A_2 as shown in Figure 6.

Table 2. The decision matrix.

	C_1	C_2	C_3	$C4_1$
A_1	(0.63, 0.55, 0.46)	(0.67, 0.46, 0.71)	(0.42, 0.13, 0.49)	(0.67, 0.47, 0.45)
A_2	(0.67, 0.34, 0.63)	(0.49, 0.53, 0.66)	(0.41, 0.16, 0.43)	(0.66, 0.39, 0.64)
A_3	(0.42, 0.63, 0.46)	(0.35, 0.42, 0.53)	(0.29, 0.17, 0.27)	(0.19, 0.17, 0.17)
A_4	(0.49, 0.44, 0.73)	(0.47, 0.53, 0.47)	(0.26, 0.14, 0.41)	(0.64, 0.36, 0.14)
A_5	(0.42, 0.38, 0.66)	(0.66, 0.37, 0.56)	(0.49, 0.26, 0.43)	(0.64, 0.37, 0.77)

Table 3. The NP matrix

	C_1	C_2	C_3	$C4_1$
A_1	(0.63, 0.55, 0.46)	(0.67, 0.46, 0.71)	(0.42, 0.13, 0.49)	(0.67, 0.47, 0.45)
A_2	(0.67, 0.34, 0.63)	(0.49, 0.53, 0.66)	(0.41, 0.16, 0.43)	(0.66, 0.39, 0.64)
A_3	(0.42, 0.63, 0.46)	(0.35, 0.42, 0.53)	(0.29, 0.17, 0.27)	(0.19, 0.17, 0.17)
A_4	(0.49, 0.44, 0.73)	(0.47, 0.53, 0.47)	(0.26, 0.14, 0.41)	(0.64, 0.36, 0.14)
A_5	(0.42, 0.38, 0.66)	(0.66, 0.37, 0.56)	(0.49, 0.26, 0.43)	(0.64, 0.37, 0.77)

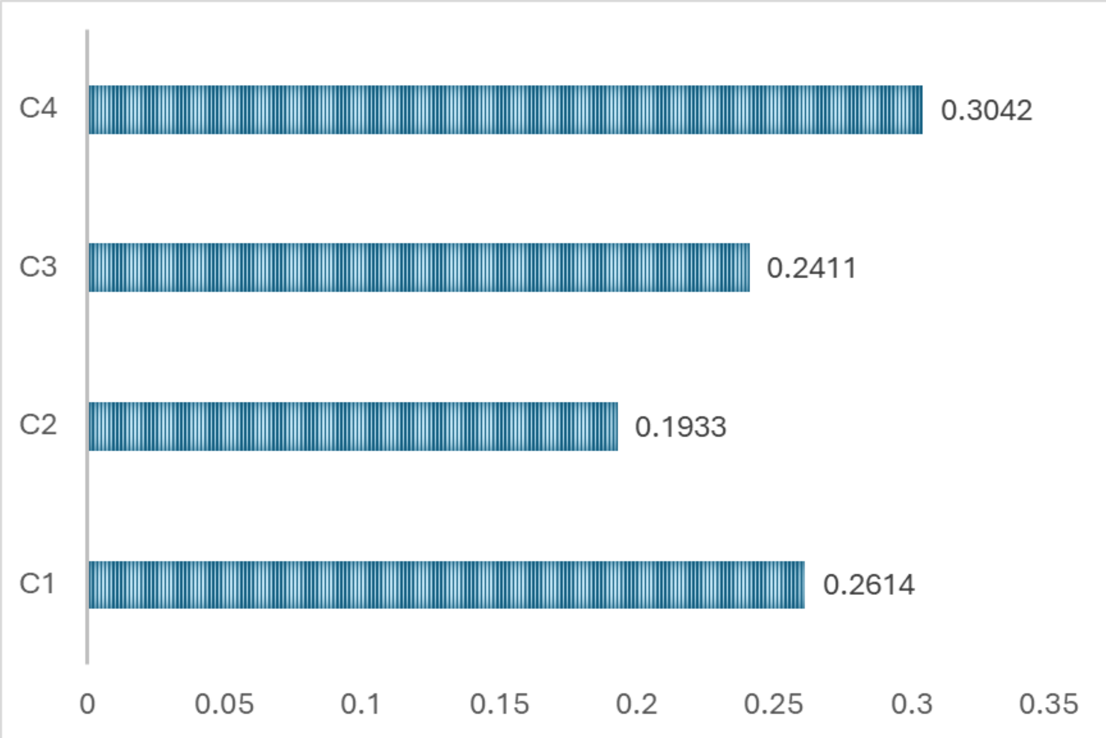

Figure 5. The criteria weights.

Table 4. The NP values

Techniques	$NP_i\left(i=1,2,\cdots,5\right)$
A_1	(0.5338,0.4831,0.3542)
A_2	(0.7035,0.3124,0.3992)
A_3	(0.5745,0.6529,0.3226)
A_4	(0.5986,0.5802,0.3416)
A_5	(0.5981,0.6547,0.1992)

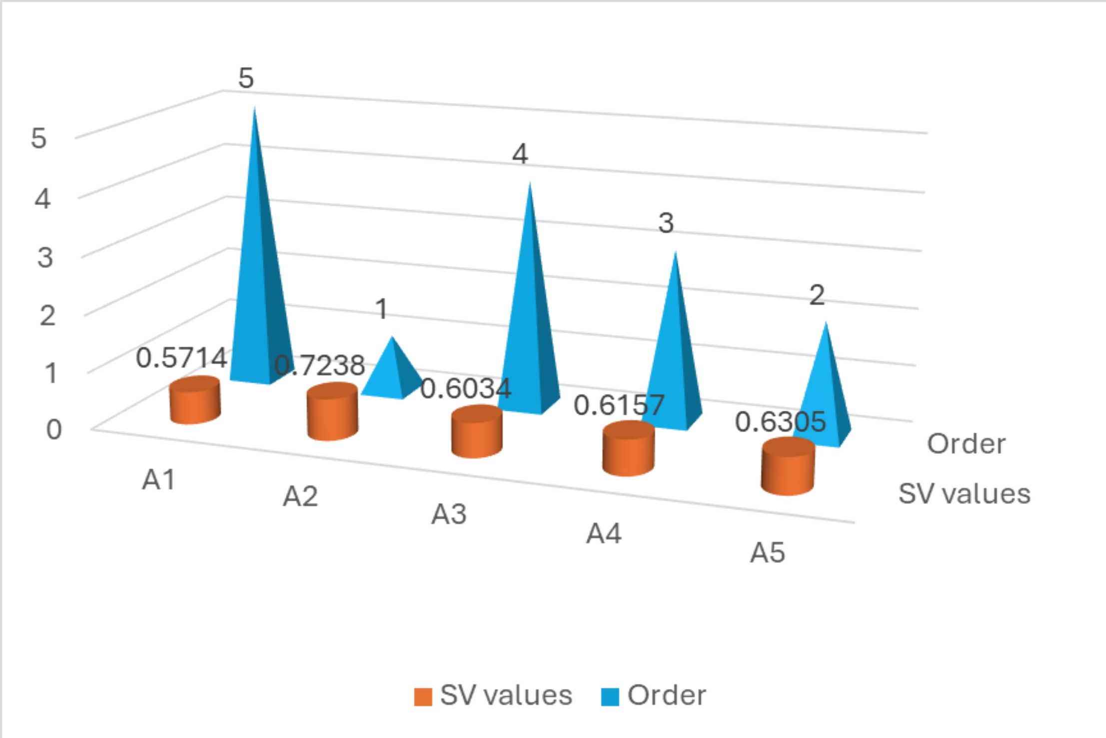

Figure 6. The order of alternatives.

5.2. Influence analysis

To illustrate the effects of different parameters on the ranking using the SVNNGPGBM method, the final results are presented in Table 6.

Table 6. Influence analysis for SVNNGPGBM technique

(es, et, er)	$SF(A_1)$	$SF(A_2)$	$SF(A_3)$	$SF(A_4)$	$SF(A_5)$	Orders
$(1,1,1)$	0.5714	0.7238	0.6034	0.6157	0.6305	$A_2 > A_5 > A_3 > A_4 > A_1$
$(2,2,2)$	0.8151	0.9184	0.8459	0.8415	0.8464	$A_2 > A_5 > A_3 > A_4 > A_1$
$(3,3,3)$	0.8821	0.9541	0.9077	0.8886	0.9109	$A_2 > A_5 > A_3 > A_4 > A_1$
$(4,4,4)$	0.9077	0.9639	0.9311	0.9029	0.9387	$A_2 > A_5 > A_3 > A_1 > A_4$
$(5,5,5)$	0.9201	0.9677	0.9431	0.9088	0.9538	$A_2 > A_5 > A_3 > A_1 > A_4$
$(6,6,6)$	0.9274	0.9697	0.9508	0.9120	0.9630	$A_2 > A_5 > A_3 > A_1 > A_4$
$(7,7,7)$	0.9321	0.9711	0.9561	0.9141	0.9692	$A_2 > A_5 > A_3 > A_1 > A_4$
$(8,8,8)$	0.9356	0.9723	0.9603	0.9159	0.9737	$A_2 > A_5 > A_3 > A_1 > A_4$
$(9,9,9)$	0.9381	0.9734	0.9637	0.9173	0.9771	$A_2 > A_5 > A_3 > A_1 > A_4$
$(10,10,10)$	0.9403	0.9745	0.9665	0.9185	0.9798	$A_2 > A_5 > A_3 > A_1 > A_4$

As seen from Table 6, the ranking changes slightly when parameter information is applied. In the MADM process, the parameter values can represent the subjective preferences of DMs. On one hand, DMs can obtain different outcomes by adjusting these parameter values; on the other hand, the parameter values also reflect the DMs' risk attitudes.

5.3. Compare analysis

The SVNNGPGBM technique is compared with SVNNWA and SVNNWG approach [30], SVNN-CODAS approach [35] and SVNN-EDAS approach [36]. The final results are shown in Figure 7.

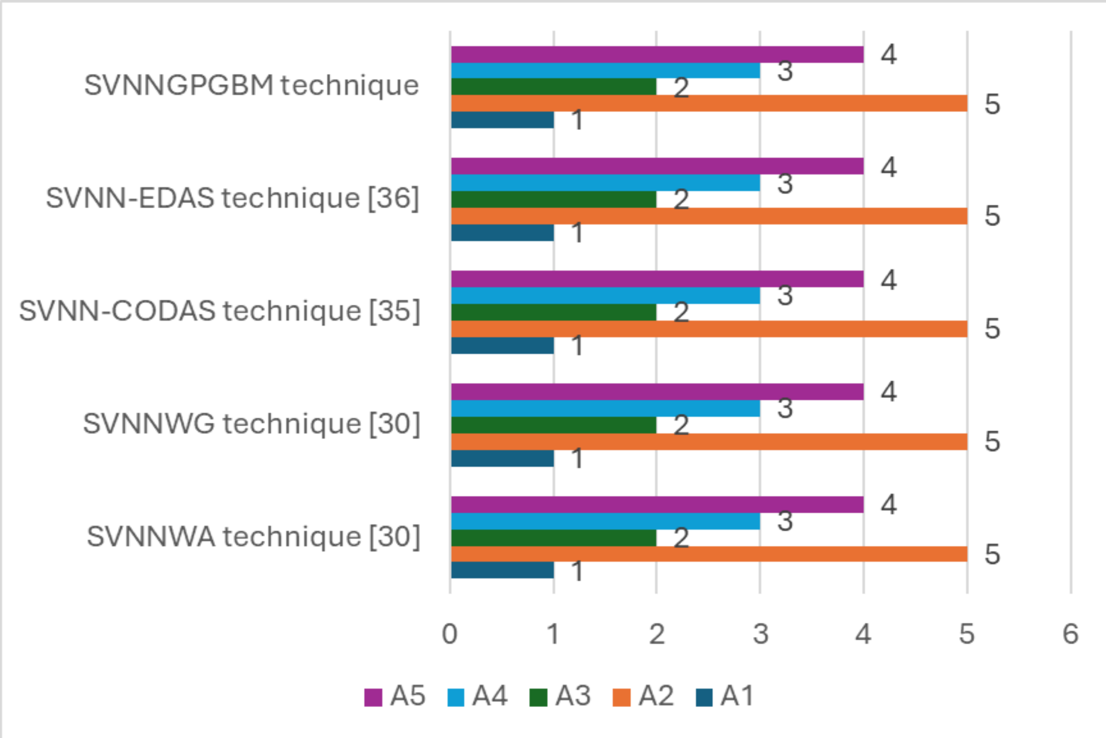

Figure 7. Comparative analysis.

From Figure 7, it is obvious that the optimal ecological tourism city is EA_2, while the worst ecological tourism city is EA_1. In other words, the order of seven approaches is slightly different. The SVNNGPGBM technique offers the following three key advantages in evaluating the satisfaction of college students' online ideological and political education in the context of big data:

- Handling Uncertainty and Ambiguity: The SVNNGPGBM technique models satisfaction using SVNSs, which can simultaneously handle truth, indeterminacy, and falsity. This allows it to accurately represent the fuzzy and uncertain factors often present in students' satisfaction evaluations. In the big data environment, where student feedback is often complex and variable, SVNNGPGBM effectively captures the inherent uncertainty and ambiguity, ensuring a more comprehensive and precise assessment.
- Integrating Multidimensional Data for Enhanced Decision-Making: SVNNGPGBM can perform comprehensive analysis across multiple evaluation dimensions. By utilizing the geometric mean and power geometric operators, this method integrates data from different sources or dimensions, avoiding bias from relying on a single evaluation criterion. This leads to a more objective and reasonable satisfaction evaluation.
- Adapting to Big Data and Improving Computational Efficiency: The SVNNGPGBM technique is well-suited to handle large-scale datasets, making it ideal for the big data context where vast amounts of student feedback need to be processed. Additionally, the algorithm is computationally efficient, enabling quick generation of comprehensive evaluation results, providing timely and effective decision-making support for educators.

6. Conclusion

The evaluation of RTEE under high-quality development aims to assess how the tourism industry promotes economic growth while minimizing resource consumption and environmental impact. The central focus is on evaluating sustainability and ecological efficiency by analyzing economic output per unit of carbon emissions. Common indicators include energy efficiency, resource utilization, carbon emission intensity, and ecological protection contribution. Energy efficiency measures economic output per unit of energy consumed, resource utilization assesses output from resource inputs, carbon emission intensity examines emissions per unit of economic output, and ecological protection contribution evaluates tourism's role in environmental preservation. By analyzing these indicators, decision-makers can optimize tourism models, achieving a balance between economic and ecological benefits, and guide the industry towards low-carbon, green, high-quality growth. RTEE evaluation under high-quality development is an MADM problem. In this study, the GWBM technique is introduced to solve MADM problems using SVNSs. The

GSVNNWBM technique is also proposed for MADM, aligned with this method. A numerical example of RTEE evaluation under high-quality development is provided, along with a comparative analysis using the GSVNNWBM technique.

7. Research limitations and further research directions

Although this study proposes an innovative method for evaluating regional tourism ecological efficiency (RTEE) and applies the single-valued neutrosophic number generalized power geometric BM (SVNNGPGBM) technique,

There are several limitations:

- High model complexity: While the SVNNGPGBM technique excels at handling uncertainty and ambiguity, its mathematical complexity is high, potentially increasing the computational cost and technical difficulty in practical applications. For non-expert users or policymakers, understanding and implementing this method may present certain challenges.
- Strong data dependency: The model relies heavily on accurate carbon emissions and resource consumption data. However, in some developing or underdeveloped regions, tourism-related environmental data may be incomplete or missing, which could limit the model's broad applicability and effectiveness.
- Lack of dynamic evaluation capability: The evaluation method in this paper is primarily based on static data, neglecting the dynamic changes in ecological efficiency as the tourism industry evolves. Over time, resource use and environmental impact may change significantly, and a static model may fail to capture these shifts.

To address these limitations, future research can explore the following three directions:

- Simplifying the model and improving operability: Future research could aim to simplify the SVNNGPGBM model to reduce its computational complexity, making it more accessible for practical use. Developing user-friendly tools or software could help policymakers and stakeholders more efficiently apply the model for decision-making.
- Enhancing data collection and processing capabilities: To address the strong data dependency, further research could focus on improving data acquisition and processing methods in areas with limited or low-quality data. Emerging technologies such as remote sensing or intelligent monitoring systems could be introduced to supplement or replace traditional data collection methods, improving the model's applicability and accuracy.

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- Building a dynamic evaluation model: To better reflect the dynamic changes in tourism ecological efficiency, future studies could incorporate time series analysis or dynamic system modeling to develop an RTEE evaluation model that evolves over time. This would allow for real-time tracking of the tourism industry's impact on the environment and resources, providing more timely and forward-looking recommendations for decision-makers.

By pursuing these research directions, the evaluation method for regional tourism ecological efficiency can be further refined, enhancing its applicability in policy formulation and practice, while providing stronger support for the green, low-carbon, and high-quality development of the tourism industry.

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