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On NeutroHyperrings and NeutroOrderedHyperrings

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Abstract. The notion of Neutrosophic sets first introduced by Smarandache in 1998 as a generalization of intuitionistic fuzzy sets. Furthermore, Al-Tahan in 2022 introduced the notion of NeutroHyperstructures. Inspired by this research, in this study, we extend NeutroAlgebra and NeutroOrderedAlgebra by introducing two new concepts: NeutroHyperring and NeutroOrderedHyperring. These new concepts enrich the existing framework by incorporating neutrosophic elements, enabling the exploration of complex relationships and uncertainties within algebraic and ordered structures. Furthermore, NeutroHyperring applications on the NeutroRing is also introduced.

Keywords: Hyperideals; Hyperstructures; NeutroHyperideal; NeutroHyperstructures; NeutroOrderedHyperring; OrderedHyperring.

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1. Introduction

In 1934, The notion of hyperstructures is introduced by Marty as generalizations of algebraic structures [1]. Subsequently, Vougiouklis further extended hyperstructures into a structure known as the H_v-structure [2]. Different types of hyperstructures, including hypergroups, hyperrings, and hypermodules [3–5]. The study of hyperstructures involves both theoretical research and practical applications in various fields, such as chemistry [6–8], physics [9, 10], and biology [11–13].

In 1998, Smarandache introduced neutrosophic sets as a generalization of intuitionistic fuzzy sets [20]. Like algebraic structures, neutrosophic sets can be defined as abstract structures referred to as NeutroAlgebraic Structures [21]. Examples of NeutroAlgebraic Structures include NeutroGroups [22], NeutroRings [23, 33], and NeutroR-modules [24]. Moreover, the neutrosophic concept can be applied to hyperstructures, resulting in the creation of NeutroHyperstructures [25, 26], which encompass structures such as NeutroH_v-semigroups [25], NeutroHypergroups [26], and Neutro-LA Semihypergroups [27]. Additionally, the application of the neutrosophic concept extends to ordered algebra and ordered hyperalgebra, leading to the development of structures like NeutroOrderedSemigroups [28, 29] and NeutroOrderedSemihypergroups [30].

Inspired by previous research on NeutroHyperstructures [25, 26], this paper focuses on applying the neutrosophic concept to hyperrings, introducing a new concept called NeutroHyperring. Furthermore, the neutrosophic concept is also extended to OrderedHyperrings, resulting in another new concept termed NeutroOrderedHyperring. This paper begins with an introduction and proceeds to present the fundamental theory required for this research in Section 2. Section 3 defines the notions of NeutroHyperring, NeutroKrasnerHyperring, and NeutroH_vRings, along with their related properties. Section 4 explores the construction of NeutroMorphisms on NeutroHyperrings (NeutroH_vRings) and analyzes properties associated with these morphisms. In Section 5, the notion of NeutroOrderedHyperring is introduced, and relevant properties are examined, In Section 6, given some application of NeutroHyperring in NeutroRing. Finally, Section 7 concludes the paper by summarizing the research findings.

2. Basic Concepts

In this section, we introduce the necessary theoretical foundations for our research. We begin with the concept of Hyperstructures and NeutroHyperstructures.

2.1. *Hyperstructures and NeutroHyperstructures*

Let $\mathfrak{H}$ be a nonempty set. The mapping $+ : \mathfrak{H} \times \mathfrak{H} \to P^*(\mathfrak{H})$ is a collection nonempty subset of $\mathfrak{H}$, is called a hyperoperation of $\mathfrak{H}$. The pair $(\mathfrak{H}, +)$ called a hypergroupoid. Here, if $\mathfrak{C}, \mathfrak{D} \subseteq \mathfrak{H}$ and $x \in \mathfrak{H}$, then we denote

$$\mathfrak{C} + \mathfrak{D} = \bigcup_{c\in\mathfrak{C}, d\in\mathfrak{D}} c + d, x + \mathfrak{C} = x + \mathfrak{C} \text{ and } \mathfrak{D} + x = \mathfrak{D} + x$$

Furthermore, a hypergroupoid $(\mathfrak{H}, +)$ is called a semihypergroup if for every $x, y, z \in \mathfrak{H}$, $x + (y + z) = (x + y) + z$. This is mean:

$$\bigcup_{c\in y+z} x + c = \bigcup_{d\in x+y} d + z$$

and $(\mathfrak{H}, +)$ is called a quasi-hypergroup if for every $x \in \mathfrak{H}$, $x + \mathfrak{H} = \mathfrak{H} + x = \mathfrak{H}$. The latter condition is called the reproduction axiom. The hypergroupoid $(\mathfrak{H}, +)$ is called a hypergroup if $(\mathfrak{H}, +)$ is a semihypergroup and quasi-hypergroup [31].

Next, we recall the definition of hyperrings. There are two types of hyperring, additive hyperrings and Krasner hyperrings, which are special case of additive hyperrings [32]. First, we recall the definition of canonical hypergroups

Definition 2.1 (3)**.** Let $(\mathfrak{R}, +)$ be a hypergroupoid. Then, R is called a canonical hypergroup if all of the following conditions are satisfied.

(1) For every $x, y, z \in \mathfrak{R}$, $x + (y + z) = (x + y) + z$.
(2) For every $x, y \in \mathfrak{R}$, $x + y = y + x$.
(3) There exists $0 \in \mathfrak{R}$ such that for every $x \in \mathfrak{R}$, $0 + x = \{x\}$.
(4) For every $x \in \mathfrak{R}$, there exists a unique element $y \in \mathfrak{R}$ such that $0 \in x + y$ (We shall write $-x$ for y and we called it the opposite of x).
(5) For every $x, y, z \in \mathfrak{R}$, If $z \in x + y$, then $y \in -x + z$ and $x \in z - y$.

Definition 2.2 (3)**.** Let $\mathfrak{R}$ be a nonempty set. The system $(\mathfrak{R}, +, \odot)$ is called a hyperring if all of the following conditions are satisfied.

(1) $(\mathfrak{R}, +)$ is a hypergroup.
(2) $(\mathfrak{R}, \odot)$ is a semihypergroup.
(3) For every $x, y, z \in \mathfrak{R}$, $x \odot (y + z) = x \odot y + x \odot z$ and $(y + z) \odot x = y \odot x + z \odot x$

Remark 2.3. Based on Definition 2.2, The system $(\mathfrak{R}, +, \odot)$ is called a Krasner hyperring if $(\mathfrak{R}, +)$ is a canonical hypergroup and $(\mathfrak{R}, \odot)$ is a semigroup with $x \odot 0 = 0 \odot x = 0$.

Definition 2.4 (3)**.** Let $(\mathfrak{R}, +, \odot)$ be a hyperring and $\mathfrak{S}$ be a nonempty subset of $\mathfrak{R}$. The set $\mathfrak{S}$ is called subhyperring of $\mathfrak{R}$ if $(\mathfrak{S}, +, \odot)$ is itself a hyperring.

Definition 2.5 (3)**.** Let $\mathfrak{S}$ be a subhyperring of a hyperring $\mathfrak{R}$. Then, $\mathfrak{S}$ is called left (right) hyperideal of $\mathfrak{R}$ if for every $r \in \mathfrak{R}$, $r \odot s \subseteq \mathfrak{S}(s \odot r \subseteq \mathfrak{S})$ for every $s \in \mathfrak{S}$. If $\mathfrak{S}$ is left and right hyperideal, then $\mathfrak{S}$ is called a hyperideal of $\mathfrak{R}$.

Next, we recall some notion of NeutroHyperstructures.

Definition 2.6 (26)**.** A nonempty set R with hyperoperation "$+ : R \times R \to P^*(R)$" is called a NeutroHypergroupoid if $(R, +)$ is a NeutroHyperoperation. $(R, +)$ is called a NeutroSemihypergroup if " $+$ " is NeutroAssociative but not an AntiHyperoperation. $(R, +)$ is called a NeutroH_v-semigroup if " $+$ " is NeutroWeakAssociative but not an AntiHyperoperation.

Definition 2.7 (25)**.** Let "$+ : R \times R \to P^*(R)$" be an NeutroOperation. Then, $(\mathfrak{R}, +)$ is called a NeutroHypergroup if $(\mathfrak{R}, +)$ is a NeutroSemihypergroup and satisfies NeutroReproduction Axiom.

For other notions of NeutroHyperstructures, it can be referred to [25] and [26].

Example 2.8. Let $\mathfrak{F} = \{\alpha, \beta, \gamma, \delta\}$ and define an hyperoperation "$\boxdot$" as follows.

$\boxdot$	α	β	γ	δ
α	α	$\{\alpha,\beta\}$	$\{\alpha,\gamma\}$	$\{\alpha,\delta\}$
β	$\{\alpha,\beta\}$	β	$\{\alpha,\delta\}$	$\{\beta,\delta\}$
γ	$\{\alpha,\gamma\}$	$\{\alpha,\delta\}$	γ	$\{\gamma,\delta\}$
δ	$\{\alpha,\delta\}$	$\{\beta,\delta\}$	$\{\gamma,\delta\}$	δ

Then, $(\mathfrak{F},\boxdot)$ is a NeutroHypergroup.

2.2. *NeutroOrderedHyperstructures*

In this section, we recall some notion of NeutroOrderedHyperstructures.

Definition 2.9. Let $(\mathfrak{R},+)$ be a NeutroSemihypergroup and define a partial order "$\leq$" on R. Then $(\mathfrak{R},+,\leq)$ is a NeutroOrderedSemihypergroup if some (or all) of following conditions are satisfied for $(T,I,F)\notin\{(1,0,0),(0,0,1)\}$.

(1) There exist $\alpha\leq\beta\in R$ with $\alpha\neq\beta$ such that $w+\alpha\leq w+\beta$ and $\alpha+w\leq\beta+w$ for every $w\in R$. (Degree of truth, "T").

(2) There exist $\alpha\leq\beta\in R$ such that $w+\alpha\nleq w+\beta$ or $\alpha+w\nleq w+\beta$ for each $w\in R$. (Degree of falsify, "F").

(3) There exist $\alpha\leq\beta\in R$ such that $w+\alpha$, $w+\beta$, $\alpha+w$, or $\beta+w$ are indeterminate, or the relation between $w+\alpha$ and $w+\beta$ or $\alpha+w$ and $\beta+w$ are indeterminate. (Degree of indeterminacy, "I").

Example 2.10 (30)**.** Let $\mathfrak{R}=\{\theta,\zeta,\omega\}$ and $+$ is a hyperoperation on R. Define $(\mathfrak{R},+)$ as follows.

$+$	θ	ζ	ω
θ	θ	θ	θ
ζ	θ	$\{\theta,\zeta\}$	ω
ω	θ	ω	ω

Based on [30], $(\mathfrak{R},+)$ is a NeutroSemihypegroup. Define an order $\leq_1$ as follows:

$$\leq_1=\{(\zeta,\theta),(\zeta,\omega),(\theta,\theta),(\zeta,\zeta),(\omega,\theta),(\omega,\omega)\}$$

It is clear that $\omega\leq_1\theta$ and for every $w\in R$, $\omega+w=w+\omega\leq_1\theta+w=w+\theta$. It is also clear that $\zeta\leq_1\omega$ but $\zeta+\zeta=\{\theta,\zeta\}\leq_1\omega+\zeta=\zeta$ does not hold.

3. **NeutroHyperring**

In this part, we introduce new notions on NeutroHyperstructures called NeutroHyperrings.

Definition 3.1. Let R be a nonempty set and $+$ and $\odot$ be hyperoperations on R. Then, R is said to be a NeutroHyperring if the following axioms is satisfied with $(T,I,F)\neq\{(1,0,0),(0,0,1)\}$.

(1) $(R, +)$ is a NeutroHypergroup

(2) $(R, \odot)$ is a NeutroSemihypergroup

(3) There exists $x, y, z, a, b, c, d, e, f \in R$ such that some (or all) following conditions are satisfied.

- $x \odot (y + z) = x \odot y + x \odot z$ and $(y + z) \odot x = y \odot x + z \odot y$
- $a \odot (b + c) \neq a \odot b + a \odot c$ or $(b + c) \odot a \neq b \odot a + c \odot a$
- $d \odot (e + f)$ is indeterminate or $d \odot e + d \odot f$ is indeterminate or $(e + f) \odot d$ is indeterminate or $e \odot d + f \odot d$ is indeterminate.

This condition is called NeutroDistributive Axiom.

If $(R, \odot)$ is a NeutroCommutative, then $(R, +, \odot)$ is said to be NeutroCommutative Hyperring.

Example 3.2. Let $R = \{f, a, k\}$. Define hyperoperations $+$ and $\odot$ as follows:

$+$	f	a	k
f	f	f	f
a	f	$\{f, a\}$	k
k	f	a	k

$\odot$	f	a	k
f	a	a	a
a	k	k	k
k	f	f	f

Then $(R, +)$ and $(R, \odot)$respectively are NeutroHypergroup and NeutroSemigroup respectively. For every $f, a, k \in R$, $f \odot (f + f) = f \odot f + f \odot f$, $(f + f) \odot f = f \odot f + f \odot f$ and $a \odot (a + k) \neq a \odot a + a \odot k$. Thus, $(R, +, \odot)$ is a NeutroHyperring.

Next, we define the Krasner NeutroHyperring.

Definition 3.3. $(R, +, \odot)$ is said to be Krasner NeutroHyperring if the following conditions are satisfied with $(T, I, F) \neq (1, 0, 0), (0, 0, 1)$.

(1) $(R, +)$ is canonical NeutroHypergroup. That is

(a) $(R, +)$ is a NeutroAssociative

(b) $(R, +)$ is a NeutroCommutative

(c) There exists $x, y, z \in R$ such that the some (or all) following condition is satisfied.

- $0 + x = \{x\}$ (Degree of truth "T")
- $0 + y \neq \{y\}$ (Degree of Falsify "F")
- $0 + z$ is indeterminate (Degree of Indetermination "I")

(d) There exists at least one condition such that $0 \in x + x'$ for each $x, x' \in R$

(e) There exists at least one condition such that if $z \in x + y$, then $y \in -x + z$ and $x \in z - y$ for each $x, y, z \in R$

(2) $(R, \odot)$ is a NeutroSemigroup

(3) $(R, +, \odot)$ is a NeutroDistributive

Example 3.4. let $R = \{0, 1, 2, 3, 4\}$. Define hyperoperations $+$ and $\odot$ as follows:

$+$	0	1	2	3	4
0	0	1	2	4	3
1	R	$\{1,2\}$	3	4	4
2	2	R	$\{2,3,4\}$	4	4
3	1	4	2	3	2
4	0	0	1	0	1

$\odot$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	2	3	4
3	0	1	2	3	4
4	0	2	3	4	1

(1) $(R,+)$ is a NeutroAssociative
(2) $(R,+)$ is a NeutroCommutative
(3) There exists $0 \in R$ such that $0+0=0$ and there exists $3 \in R$ such that $0+3=4 \neq 3$
(4) There exists $0 \in R$ such that $0 \in 0+0=0$
(5) If $0 \in 0+0$, then $0 \in 0-0$ and $0 \in 0-0$ for $0 \in R$

Then, $(R,+)$ is a canonical NeutroHypergroup. Next, for every $0 \in R$, $0 \odot (0 \odot 0) = (0 \odot 0) \odot 0 = 0$ and for every $4 \in R$, $(4 \odot 4) \odot 4 \neq 4 \odot (4 \odot 4)$. Then, $(R,\odot)$ is a NeutroSemihypergroup. Next, for every $0 \in R$, $0 \odot (0+0) = 0 \odot 0 + 0 \odot 0 = (0+0) \odot 0$ and for every $1,2 \in R$, $(1+2) \odot 1 \neq 1 \odot 1 + 2 \odot 1$. Then, $(R,+,\odot)$ is a NeutroDistibution. Thus, $(R,+,\odot)$ is a Krasner NeutroHyperring.

In this research, we only use NeutroHyperring. The definition of Krasner NeutroHyperring is just introduced.

Definition 3.5. Let R be a NeutroHyperring. Then, R is said to be a Krasner NeutroHyperfield if $(R-\{0\},\odot)$ is a NeutroGroup.

Theorem 3.6. *Let $(R,+,\odot)$ is a NeutroHyperring. Define hyperoperations "$\oplus$" and "$\boxdot$" respectively as $x \oplus y = y + x$ and $x \boxdot y = y \odot x$ for every $x, y \in R$. Then, $(R,\oplus,\boxdot)$ is a NeutroHyperring.*

Proof. The proof is straighforward. □

Definition 3.7. Let $(R,+,\odot)$ be a NeutroHyperring and S be a nonempty subset of S. Then, S is said to be a NeutroSubHyperring of R if $(S,+,\odot)$ is itself a NeutroHyperring.

Definition 3.8. Let $(R,+,\odot)$ be a NeutroHyperring and $S \subseteq R$ be a NeutroSubHyperring.

(1) If there exists $x \in S$ such that $r \odot x \subseteq S$ for every $r \in R$, then S is a NeutroLeftHyperideal of R.
(2) If there exists $x \in S$ such that $x \odot r \subseteq S$ for every $r \in R$, then S is a NeutroRightHyperideal of S.
(3) If S is NeutroLeftHyperideal and NeutroRightHyperideal, then S is a NeutroHyperideal of R.

Example 3.9. Let $A = \{1, 2, 3\}$. Define hyperoperations " + " and " $\odot$ " as follows:

$+$	1	2	3
1	1	1	3
2	$\{1,2\}$	1	3
3	1	2	3

$\odot$	1	2	3
1	1	$\{1,2\}$	3
2	2	1	3
3	1	2	3

It can be checked that $(A, +)$ is a NeutroHyperring. Then, the possible of NeutroSubHyperring are $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$. $(\{1\}, +, \odot)$, $(3, +, \odot)$, and $(\{1, 3\}, +, \odot)$ are Total Hyperrings or Classical Hyperrings, $(\{2\}, +, \odot)$ and $(\{2, 3\}, +, \odot)$ are AntiHypergroupoids, and $(\{1, 2\}, +, \odot)$ is a NeutroSubHyperring.

Example 3.10. Since $(\{1, 2\}, +, \odot)$ is a NeutroSubHyperring, then they are possible to be NeutroHyperideals. There exists $1 \in \{1, 2\}$ such that $\{1, 2, 3\} \odot 1 = \{1, 2\} \subseteq \{1, 2\}$. Then,$(\{1, 2\}, +, \odot)$ is a NeutroLeftHyperideal of R.

Furthermore, motivated by the notion of k-hyperideal which was defined by Ameri and Hedayati [33], we define k-NeutroHyperideal of NeutroHyperring.

Definition 3.11. Let $(R, +, \odot)$ be a NeutroHyperring. Then, a NeutroLeftHyperideal S is called k-NeutroLeftHyperideal if there exists $c \in S$ such that if $(c+x) \cap S \neq \emptyset$ or $(x+c) \cap S \neq \emptyset$ for every $x \in R$, then $x \in S$. For right k-NeutroRightHyperideal, it is defined similarly. If S is k-NeutroLeftHyperideal and k-NeutroRightHyperideal, then S is called k-NeutroHyperideal.

Remark 3.12. Every k-NeutroHyperideal is clearly a NeutroHyperideal. But the converse is not always true.

Example 3.13. Let $R = \{1, 2, 3\}$. Define hyperoperations " + " and " $\odot$ " as follows.

$+$	1	2	3
1	1	1	$\{1,3\}$
2	2	1	3
3	1	1	1

$\odot$	1	2	3
1	1	2	1
2	1	1	1
3	$\{1,3\}$	3	1

Then $(R, +, \odot)$ is an NeutroHyperring. It can be easily proved that $\{1, 3\}$ is a NeutroHyperideal of R. We get $(1 + 2) \cap \{1, 3\} \neq \emptyset$ but $2 \notin \{1, 3\}$ for $2 \in R$. Thus, $\{1, 3\}$ is not a k-NeutroHyperideal of R.

Proposition 3.14. *Let $\{S_k\}_{k\in K}$ be a family of k-NeutroHyperideal of a NeutroHyperring with K being a nonempty index set. Then, $\cap_{k\in K} S_k$ is a k-NeutroHyperideal of R.*

Proof. Let $r \in R$ and $s \in \cap_{k\in K} S_k$. Since $\{S_k\}_{k\in K}$ is a family of k-NeutroHyperideal, then $(r+s) \cap (\cap_{k\in K} S_k)$ and implies that $r+s \in S_k$ for every $k \in K$. Since S_k is a NeutroHyperideal of R, we can attest that $\cap_{k\in K} S_k$ is k-NeutroHyperideal of R. □

Now, we define the notion of Neutro$H_v$ring.

Definition 3.15. Let R be a non-emptyset and define hyperoperations " + " and " $\odot$ " on R. Then, a multi-value system $(R, +, \odot)$ is said to be a NeutroH_vRing if the following condition are satisfied.

(1) $(R, +)$ is a NeutroH_vGroup
(2) $(R, \odot)$ is a NeutroH_vSemigroup
(3) $(R, +, \odot)$ is a NeutroWeakDistribution, i.e. for every $x, y, z, a, b, c, d, e, f \in R$ some (or all) of the following conditions are satisfied.
   (a) $x \odot (y + z) \cap [(x \odot y) \cap (x \odot z)] \neq \emptyset$ and $(x + y) \odot z \cap [(x \odot z) + (y \odot z)] \neq \emptyset$ (Degree of Truth "T")
   (b) $a \odot (b + c) \cap [(a \odot b) \cap (a \odot c)] = \emptyset$ or $(a + b) \odot c \cap [(a \odot c) + (b \odot c)] = \emptyset$ (Degree of Falsify "F")
   (c) $(d + e) \odot f$ is indeterminate or $(d \odot f + e \odot f)$ is indeterminate or $d \odot (e + f)$ is indeterminate or $(d \odot e + d \odot f)$ is indeterminate.

If $(R, +)$ and $(R, \odot)$ are commutative, then $(R, +, \odot)$ is said to be NeutroCommutativeH_vRings. Now, we give a dual definition of NeutroH_vRings.

Definition 3.16. A NeutroH_vRing $(R, +, \odot)$ is said to be a dual NeutroH_vRing if $(R, \odot, +)$ is an NeutroH_vRing.

Example 3.17. Let $K = \{u, n, j\}$, and define hyperoperations " + " and $\odot$ as follows:

$+$	u	n	j
u	n	n	j
n	$\{u, n\}$	u	j
j	j	j	j

$\odot$	u	n	j
u	u	$\{u, n\}$	n
n	n	n	n
j	j	j	n

Now, we want to show that $(R, +, \odot)$ is a NeutroH_vRing.

(1) $(K, +)$ is a NeutroH_vGroup
(2) $(K, \odot)$ is a NeutroH_vSemigroup
(3) For every $u, n \in R$, $u \odot (n + n) \cap [(u \odot n) + (u \odot n)] = n \cap \{u, n\} \neq \emptyset$. Next, for every $u \in R$, $(u + u) \odot u \cap [(u \odot u) + u \odot u] = n \cap u = \emptyset$.

Thus, $(R, +, \odot)$ is a NeutroH_vRing.

Next we have proposition related to dual NeutroH_vRing.

Proposition 3.18. *Let $(T, \odot)$ is an NeutroH_vGroup. Then, for every NeutroHyperoperation $\boxplus$ such that $\{a, b\} \subseteq a \boxplus b$ and ($\{c, d\} \nsubseteq c \boxplus d$ or $e \boxplus f$ is indeterminate) for every $a, b, c, d, e, f \in T$, the NeutroHyperstructures $(T, \odot, \boxplus)$ is a dual NeutroH_vRing.*

Proof. First, we want to show that $(T, \odot, \boxplus)$ is a NeutroH_vRing.

(1) $(T, \odot)$ be a NeutroH_vGroup by hypothesis.

(2) For every $x, y, z, a, b, c \in T$, using Truth condition, it is clear that $\{x, y, z\} \subseteq [(x \boxplus (y \boxplus z)] \cap [(x \boxplus y) \odot z]$ and using False condition, it is clear that $\{a, b, c\} \nsubseteq [a \boxplus (b \boxplus c)] \cap [(a \boxplus b) \boxplus c] = \emptyset$. Then, $(T, \boxplus)$ is a NeutroH_vSemigroup

(3) for every $a, b, c, d, e, f \in T$, using the truth condition, we get
   - $\{a\} \cup (b \odot c) \subseteq a \boxplus (b \odot c)$
   - $(a \odot a) \cup (a \odot c) \cup (b \odot a) \cup (b \odot c) = \{a, b\} \odot \{a, c\} \subseteq (a \boxplus b) \odot (a \boxplus c)$

   Then, $b \odot c \subseteq [a \boxplus (b \odot c)] \cap [(a \boxplus b) \odot (a \boxplus c)] \neq \emptyset$. Next, using the false condition, in the same way as the truth condition, we get $e \odot f \nsubseteq [(d \odot e) \boxplus f] \cap [d \boxplus (e \boxplus f)] = \emptyset$

Then, $(T, \odot, \boxplus)$ is a NeutroH_vRing. Furthermore, we want to show that $(T, \boxplus, \odot)$ is a NeutroH_vRing.

(1) Based on the proof of $(T, \odot, \boxplus)$, we already proved that $(T, \boxplus)$ is a NeutroH_vSemigroup. Using the truth condition, for every $a \in T$, $T \subseteq \{a, T\} \subseteq a \boxplus T$ and $T \subseteq \{T, a\} \subseteq T \boxplus a$. Since $\{a, T\} = \{T, a\}$, we get $a \boxplus T = T \boxplus a = T$. Next, using the false condition, in the same way as the truth condition, for every $b \in T$, we get $b \boxplus H \neq H \boxplus b$.

(2) Since $(T, \odot)$ is a NeutroH_vGroup by hypothesis, then it is obvious that $(T, \odot)$ is a NeutroH_vSemigroup.

(3) Same with the proof of NeutroWeakDistribution for $(T, \odot, \boxplus)$.

Thus, $(T, \odot, \boxplus)$ is a dual NeutroH_vRing. □

Based on Theorem 3.4, we have Theorem 3.12 related to NeutroH_vRings.

Theorem 3.19. *Let $(R, +, \odot)$ be a NeutroH_vRing. Define hyperoperations "$\oplus$" and "$\boxdot$" respectively as $x \oplus y = y + x$ and $x \boxdot y = y \odot x$. Then, $(R, \oplus, \boxdot)$ is a NeutroH_vRing.*

Proof. We only prove for NeutroWeakDistribution axiom. The remaining axioms is easy to prove. For the truth condition, for every $x, y, z \in R$, we have $x \boxdot (y \oplus z) \cap [(x \boxdot y) \cap (x \boxdot z)]$. Using the hypothesis, we get $x \boxdot (y \oplus z) \cap [(x \boxdot y) \cap (x \boxdot z)] = (z + y) \odot x \cap [(y \odot x) \cap (z \odot x)] \neq \emptyset$ because $(R, +, \odot)$ is a NeutroWeakDistribution. For the false condition, for every $a, b, c \in R$, we get $a \boxdot (b \oplus c) \cap [(a \boxdot b) \cap (a \boxdot c)] = \emptyset$. Thus, $(R, \oplus, \boxdot)$ is a NeutroH_vRing □

Next, we have a property of NeutroHyperring related to production of NeutroHyperring.

Theorem 3.20. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ are hypergroupoids. Then $(R_1 \times R_2, +_1, \odot_1)$ is a NeutroHyperring (NeutroH_vRing) if and only if either $(R_1, +, \odot)$ is a NeutroHyperring*

(NeutroH$_v$Ring) or $(R_2, +_2, \odot_2)$ is a NeutroHyperring (NeutroH$_v$Ring) or both are NeutroHyperring (NeutroH$_v$Ring). In here, for every $(e, f), (g, h) \in R_1 \times R_2$, $(e, f) + (g, h) = \{(i, j), i \in e + g, j \in f + h\}$ and $(e, f) \odot (g, h) = \{(i, j), i \in e \odot g, j \in f \odot h\}$.

Proof. The proof is straightforward. □

4. NeutroHomomorphism of NeutroHyperrings (NeutroH_vRings)

Motivated by the notion of Neutromorphisms of NeutroSemihypergroup on Al-Tahan, et al. [26], we define NeutroMorphisms of NeutroHyperrings and NeutroH_vRings as follows.

Definition 4.1. Let $(R_1, +, \odot)$ and $(R_2, \oplus, \boxdot)$ be NeutroHyperrings (NeutroH_vRings) and $\gamma : R_1 \to R_2$ is a function. Then,

(1) If $\gamma(x + y) = \gamma(x) \oplus \gamma(y)$ and $\gamma(x \odot y) = \gamma(x) \boxdot \gamma(y)$ for some $x, y \in R_1$, then γ is called a NeutroHomomorphism.
(2) If γ is bijective NeutroHomomorphism, then γ is called a NeutroIsomorphism
(3) If for every $x, y \in R_1$, some (or all) of following conditions are satisfied.
   - $\gamma(x + y) = \gamma(x) \oplus \gamma(y)$ and $\gamma(x \odot y) = \gamma(x) \boxdot \gamma(y)$ if $x + y \subseteq R_1$ and $x \odot y \subseteq R_1$ (Degree of truth ”T”)
   - $\gamma(x + y) \neq \gamma(x) \oplus \gamma(y)$ or $\gamma(x \odot y) \neq \gamma(x) \boxdot \gamma(y)$ if $x + y \nsubseteq R_1$ or $x \odot y \nsubseteq R_1$ (Degree of falsity ”F”)
   - $\gamma(x) \oplus \gamma(y)$ is indeterminate when $x + y$ indeterminate and $\gamma(x) \boxdot \gamma(y)$ is indeterminate when $x \odot y$ indeterminate.

   then, γ is called a NeutroStrongHomomorphism
(4) if γ is a bijective NeutroStrongIsomorphism, then γ is called a NeutroStrongIsomorphism. Here, we denote $(R_1, +, \odot) \cong_{SI} (R_2, \oplus, \boxdot)$

Definition 4.2. Let $(R_1, +, \odot)$ and $(R_2, \oplus, \boxdot)$ be NeutroH_vRings and $\psi : R_1 \to R_2$ is a function. Then, ψ is called a NeutroWeakHomomorphism if $\psi(x + y) \cap (\psi(x) \oplus \psi(y)) \neq \emptyset$ for some $x, y \in R_1$.

Example 4.3. Let $(R, +, \odot)$ is defined on Example 8. We construct $(R, \oplus, \boxdot)$ using Theorem 4, we get the following table.

$\oplus$	u	n	j
u	u	$\{u, n\}$	j
n	n	u	j
j	j	j	j

$\boxdot$	u	n	j
u	u	n	j
n	$\{u, n\}$	n	j
j	n	n	n

Then, based on Theorem 3.12, $(R, \oplus, \boxdot)$ is a NeutroH_vRing. Now, define a function $\psi : (R, +, \odot) \to (R, \oplus, \boxdot)$ with $\psi(u) = u$, $\psi(n) = j$, and $\psi(j) = n$. Then, it is easy to see that ψ is a NeutroStrongIsomorphism.

Example 4.4. Based on Example 9, there exist $u, n \in R$ such that $\psi(u+n) \cap (\psi(u) \oplus \psi(n)) \neq \emptyset$. So, ψ is a NeutroWeakHomomorphism.

Furthermore, inspired by Theorem 2 on [26], we have the following theorem.

Theorem 4.5. *If R_1 and R_2 be NeutroHyperrings (NeutroH_vRings), then $R_1 \cong_{SI} R_2$ is an equivalence relation.*

Proof. The proof follows from Theorem 2 on [26]. □

Lemma 4.6. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ be NeutroHyperrings (NeutroH_vRings) and $\gamma : R_1 \to R_2$ be an injective StrongHomomorphism. If $S_1 \subseteq R_1$ is a NeutroSubhyperring (NeutroSubH_vRing) of R_1, then $\gamma(S_1)$ is a NeutroSubhyperring (NeutroSubH_vRing) of R_2*

Proof. The proof is similar to the Lemma 3 in [26]. □

Lemma 4.7. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ be NeutroHyperrings (NeutroH_vRings) and $\gamma : R_1 \to R_2$ be a NeutroStrongIsomorphism. If $S_2 \subseteq R_2$ is a NeutroSubhyperring (NeutroSubH_vRing), then $\gamma^{-1}(S_2)$ is a NeutroSubhyperring (NeutroSubH_vRing).*

Proof. The proof is similar to the Lemma 4 in [26]. □

Theorem 4.8. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ be NeutroHyperrings (NeutroH_vRings) and $\gamma : R_1 \to R_2$ be a NeutroStrongIsomorphism. Then, $T \subseteq R_1$ is a NeutroSubhyperring (NeutroH_vRing) of R_1 if and only if $\gamma(T)$ is a NeutroSubhyperring (NeutroH_vRing).*

Proof. Here, we only prove for cases where R_1 and R_2 are NeutroHyperrings. The proof of cases where R_1 and R_2 are NeutroH_vRings is done similarly.
($\Rightarrow$) It follows from Lemma 1
($\Leftarrow$) Let $\gamma(T)$ be a NeutroSubhyperring. We want to show that T is NeutroSubhyperring. Since $\gamma(T)$ is a NeutroSubhyperring, then $(\gamma(T), +_2)$ is a NeutroHypergroup, $(\gamma(T), \odot_2)$ is a NeutroSemigroup, and $(\gamma(T), +_2, \odot_2)$ is a NeutroHyperring. Next, since γ is a NeutroStrongIsomorphism and $\cong_{SI}$ is an equivalence relation, then using property of symmetry, we have $\gamma : R_2 \to R_1$ is a NeutroStrongIsomorphism i.e. $(T = \gamma^{-1}(W), +_1, \odot_1)$ is a NeutroSubhyperring with $W \subseteq R_2$. Thus, $T \subseteq R_1$ is a NeutroSubhyperring. □

Next, we present lemmas and theorem of NeutroMorphisms related to NeutroIdeals. These lemmas and theorem are inspired from [19].

Lemma 4.9. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ be NeutroHyperrings (NeutroH$_v$Rings) and $\gamma : R_1 \to R_2$ be a NeutroStrongIsomorphism. If $S_1 \subseteq R_1$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of R_1, then $\gamma(S_1)$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroLeftHyperideal) of R_2.*

Proof. The proof is similar to the Lemma 5 in [26]. □

Lemma 4.10. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ be NeutroHyperrings (NeutroH$_v$Rings) and $\gamma : R_1 \to R_2$ be a NeutroStrongIsomorphism. If $S_2 \subseteq R_2$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of R_2, then $\gamma^{-1}(S_2)$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of H.*

Proof. The proof is similar to the Lemma 6 in [26]. □

Theorem 4.11. *Let $(R_1, +_1, \odot_1)$ and $(R_2, +_2, \odot_2)$ be NeutroHyperrings (NeutroH$_v$Rings) and $\gamma : R_1 \to R_2$ be a NeutroStrongIsomorphism. Then, $S_1 \subseteq R_1$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of R_1 if and only if $\gamma(M)$ is a NeutroLeftHyperideal (NeutroRightHyperideal or NeutroHyperideal) of R_2.*

Proof. The proof is similar to the Theorem 4 in [26]. □

5. **NeutroOrderedHyperring**

First, we define the notion of NeutroOrderedHyperring.

Definition 5.1. Let $(R, +, \odot)$ be a NeutroHyperring and " $\leq$ " be a partial order on R. Then, the system $(R, +, \odot, \leq)$ is a NeutroOrderedHyperring if some(or all) of the following conditions are satisfied with $(T, I, F) \neq \{(1, 0, 0), (0, 0, 1)\}$.

(1) There exists $x \leq y \in R$ and for every $z \in R$ such that $z + x \leq z + y$, $x + z \leq y + z$, $z \odot x \leq z \odot y$, and $x \odot z \leq y \odot z$ (Degree of Truth "T").

(2) There exists $x \leq y \in R$ and for some $z \in R$ such that $z + x \nleq z + y$ or $x + z \nleq y + z$ or $z \odot x \nleq z \odot y$ or $x \odot z \nleq y \odot z$ (Degree of Falsify "F").

(3) There exists $x \leq y \in R$ with $z + x$ or $z + y$ or $x + z$ or $y + z$ or $z \odot x$ or $z \odot y$ or $x \odot z$ or $y \odot z$ are indeterminate or the relation between $z + x$ and $z + y$ or the relation between $x + z$ and $y + z$ or the relation between $z \odot x$ and $z \odot y$ or the relation between $x \odot z$ and $y \odot z$ are indeterminate (Degree of Indeterminacy "I")

Definition 5.2. Let $(R, +, \odot, \leq)$ be a NeutroHyperring. If the relation $\leq$ is a total order on R, then R is called Neutro Total OrderedHypering

Example 5.3. Let $R = \{a, b, c, d\}$. Define hyperoperations " $+$ " and " $\odot$ " as follows.

$+$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	a
b	$\{a,b\}$	$\{a,b,c\}$	$\{a,c\}$	d
c	$\{a,c\}$	$\{a,c\}$	c	d
d	a	d	$\{c,d\}$	d

$\odot$	a	b	c	d
a	a	$\{a,b\}$	$\{a,c\}$	a
b	$\{a,b\}$	$\{a,b,c\}$	$\{a,c\}$	d
c	$\{a,c\}$	$\{a,c\}$	c	d
d	a	d	$\{c,d\}$	c

Then, $(R, +, \odot)$ is a NeutroHyperring. Now, define a partial order as follows.

$$\leq_1 = \{(a,a),(b,b),(c,c),(d,d),(a,b),(a,c),(a,d),(b,a),(b,c),(b,d),(c,a),(c,d)\}$$

For some $a, b \in R$, we get $a + x \leq_1 b + x$, $x + a \leq_1 x + b$, $x \odot a \leq_1 x \odot b$, and $a \odot x \leq_1 b \odot x$ for every $a, b \in R$. Next, for some $b, c \in R$, we get $c + b \not\leq_1 b + b$ for some $b \in R$. Thus, $(R, +, \odot, \leq_1)$ is a NeutroOrderedHyperring.

Next, we define the notion of Neutro OrderedSubhyperring.

Definition 5.4. Let $(R, +, \odot, \leq)$ be a NeutroOrderedHyperring and $S \subseteq R$ is a nonempty set. S is said to be a NeutroOrderedSubhyperring if $(S, +, \odot, \leq)$ is a NeutroOrderedHyperring and there exists $s \in S$ such that $(s] = \{r \in R, r \leq s\} \subseteq S$.

Definition 5.5. Let $(R, +, \odot, \leq)$ be an NeutroOrderedHyperring, and $S \subseteq R$ be a nonempty set. If S is a Neutro OrderedSubhyperring of R, then S is a:

(1) Neutro OrderedLeftHyperideal of R if there exists $s \in S$ such that $r \odot s \subseteq S$ for all $r \in R$.

(2) Neutro OrderedRightHyperideal of R if there exists $s \in S$ such that $s \odot r \subseteq S$ for all $r \in R$.

(3) Neutro OrderedHyperideal of R if S are Neutro OrderedLeftHyperideal and Neutro OrderedRightHyperideal.

Example 5.6. Let $R = \{1, 2, 3\}$. Define hyperoperations $+$ and $\odot$ as follows.

$+$	1	2	3
1	$\{1,2\}$	2	1
2	2	$\{2,3\}$	1
3	1	1	1

$\odot$	1	2	3
1	$\{1,2\}$	2	1
2	2	$\{2,3\}$	1
3	1	2	2

It can be shown that $(R, +, \odot)$ is a NeutroHyperring. Now, define a partial order as follows.

$$\leq_1 = \{(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)\}.$$

Then, $(R, +, \odot, \leq_1)$ is a NeutroOrderedHyperring. Now, suppose that $S = \{1, 2\} \subseteq R$. Then, $\{1, 2\}$ is a Neutro OrderedSubhyperring.

Example 5.7. Based on Example 12, there exist $1 \in \{1, 2\}$ such that $1 \odot \{1, 2\} \subseteq R$ and $\{1, 2\} \odot 1 \subseteq R$. Thus, $\{1, 2\}$ is a Neutro OrderedHyperideal of R.

Proposition 5.8. *Let $(R, +, \odot, \leq)$ be a NeutroOrderedHyperring. Then, S is a NeutroLeftHyperideal of $(R, +, \odot, \leq)$ if and only if $(R.+, \odot, \leq)$ is a NeutroRightHyperideal of $(R, +, \boxdot, \leq)$ with $x \boxdot y = y \odot x$ for some $x, y \in R$.*

Proof. The proof is similar to the Proposition 3.8 in [19]. $\square$

Next, motivated by [28], we define a NeutroMorphism of NeutroOrderedHyperring.

Definition 5.9. Let $(R_1, +_1, \odot_1, \leq_1)$ and $(R_2, +_2, \odot_2, \leq_2)$ be NeutroOrderedHyperrings and define a function $\psi : R_1 \to R_2$. Then,

(1) ψ is a NeutroOrderedHomomorphism if there exist $x, y \in R$ such that $\psi(x +_1 y) = \psi(x) +_2 \psi(y)$, $\psi(x \odot_1 y) = \psi(x) \odot_2 \psi(y)$, and $\psi(x) \leq_2 \psi(y)$ if $x \leq_1 y$.
(2) ψ is a NeutroOrderedIsomorphism if ψ is a bijective NeutroHomomorphism.
(3) ψ is called a NeutroOrderedStrongHomomorphism if $\psi(x +_1 y) = \psi(x) +_2 \psi(y)$ for every $x, y \in R_1$ and $x \leq_1 y \in R_1$ is equivalent to $\psi(x) \leq_2 \psi(y) \in R_2$.
(4) ψ is called a NeutroOrderedStrongIsomorphism if ψ is a bijective NeutroOrderedStrongHomomorphism.

Example 5.10. Let $(R, +, \odot, \leq)$ be a NeutroOrderedHyperring and define an identity mapping $\psi : R_1 \to R_1$. Then, ψ is a NeutroOrderedStrongIsomorphism.

Theorem 5.11. *Let $(R_1, +_1, \odot_1, \leq_1)$ and $(R_2, +_2, \odot_2, \leq_2)$ be NeutroOrderedHyperrings and $\psi : R_1 \to R_2$ be a NeutroOrderedStrongIsomorphism. If $S_1 \subseteq R_1$ is a NeutroOrderedSubhyperring of S_1, then $\psi(S_1)$ is a NeutroOrderedSubhyperring of R_2.*

Proof. First, we prove that $\{\psi(S_1), +_2\}$ is a NeutroHypergroup. Since $(S_1, +_1)$ is a NeutroHypergroup, then $(S_1, +_1)$ is either NeutroOperation or NeutroAssociative and $(S, +_1)$ is satisfies NeutroReproduction Axiom.

(1) If $(S_1, +_1)$ is a NeutroOperation, there exist $x, y, a, b, c, d \in S_1$ such that $x +_1 y \in S_1$ and $a +_1 b \notin S_1$ or $a +_1 b$ is indeterminate. Then, we get for some $\psi(x), \psi(y), \psi(a), \psi(b), \psi(c), \psi(d) \in \psi(S_1)$, $\psi(x) +_2 \psi(y) = \psi(x +_1 y) \in \psi(S_1)$ and $\psi(a) +_2 \psi(b) = \psi(a +_1 b) \notin \psi(S_1)$ or $\psi(x) +_2 \psi(y) = \psi(x +_1 y)$ is indeterminate.
(2) If $(S_1, +_1)$ is a NeutroAssociative, there exist $x, y, z, a, b, c \in S_1$ such that $(x +_1 y) +_1 z = x +_1 (y +_1 z)$ and $(x +_1 y) +_1 z \neq x +_1 (y +_1 z)$. Since ψ is one-to-one, we get for some $\psi(x), \psi(y), \psi(z), \psi(a), \psi(b), \psi(c) \in \psi(S_1)$, $(\psi(x) +_2 \psi(y)) +_2 \psi(z) = \psi(x) +_2 (\psi(y) +_2 \psi(z))$ and $(\psi(a) +_2 \psi(b)) +_2 \psi(c) \neq \psi(a) +_2 (\psi(b) +_2 \psi(c))$.
(3) If $(S_1, +_1)$ satisfies the NeutroReproduction Axiom, there exist $a, b \in S_1$ such that $a +_1 S_1 = S_1 +_a = S_1$ and $b + S_1 \neq S_1 + b \neq S_1$. Since ψ is one-to-one, we get

for some $\psi(a), \psi(b), \psi(S_1) \in \psi(S_1)$, $\psi(a) +_2 \psi(S_1) = \psi(S_1) + \psi(a) = \psi(S_1)$ and $\psi(b) +_2 \psi(S_1) = \psi(S_1) + \psi(b) = \psi(S_1)$.

Then, $(\psi(S_1), +_2)$ is a NeutroHypergroup. To prove that $(\psi, \odot_2)$ is a NeutroSemihypergroup, the proof is similar to that proof of NeutroOperation and NeutroAssociative of $(\psi(S_1), +_2)$. Now, we prove that $(\psi(S_1), +_2, \odot_2)$ is a NeutroDistributive. Since $(R_1, +, \odot_1)$ is a NeutroDistributive, then there exists $x, y, z, a, b, c \in R_1$ such that $x \odot_1 (y +_1 z) = x \odot_1 y +_1 x \odot_1 z$, $(y +_1 z) \odot_1 x = y \odot_1 x +_1 z \odot_1 x$, and $a \odot_1 (b +_1 c) \neq a \odot_1 b +_1 c \odot_1 a$. We get for some $\psi(x), \psi(y), \psi(z), \psi(a), \psi(b), \psi(c) \in \psi(S_1)$, $\psi(x) \odot_2 (\psi(y) +_2 \psi(z)) = \psi(x) \odot_2 \psi(y) +_2 \psi(x) \odot_2 \psi(z)$, $(\psi(y) +_2 \psi(z)) \odot_2 \psi(x) = \psi(y) \odot_2 \psi(x) +_2 \psi(z) \odot_2 \psi(x)$, and $\psi(a) \odot_2 (\psi(b) +_2 \psi(c)) \neq \psi(a) \odot_2 \psi(b) +_2 \psi(a) \odot_2 \psi(c)$ Thus, $(\psi(S_1), +_1, \odot)$ is a NeutroSubhyperring of R_2.

Since S_1 is a NeutroOrderedSubhyperring of R_1, then there exists $a \in S_1$ such that $(a] \subseteq S_1$. It is clear that $[\psi(a)] \subseteq \psi(S_1)$ and for every $j \in R_2$, there exists $k \in R_1$ such that $j = \psi(k)$. For $\psi(k) \leq_2 \psi(a)$, we have $k \leq_1 a$ and it implies that $k \in S_1$ and hence $j \in \psi(S_1)$. The proof of the NeutroOrdered relation is similar to the Lemma 3.28 in [28]. Therefore, $\psi(S_1)$ is a NeutroOrderedSubhyperring of R_2. □

Theorem 5.12. *Let $(R_1, +_1, \odot_1, \leq_1)$ and $(R_2, +_2, \odot_2, \leq_2)$ be NeutroOrderedHyperrings and $\psi : R_1 \to R_2$ be a NeutroOrderedStrongIsomorphism. If $S_1 \subseteq R_1$ is a NeutroOrderedLeftHyperideal (NeutroOrderedRightHyperideal or NeutroOrderedHyperideal) of S_1, then $\psi(S_1)$ is a NeutroOrderedLeftHyperideal (NeutroOrderedRightHyperideal or NeutroOrderdedHyperideal) of R_2.*

Proof. The proof is similar to Lemma 3.29 in [28]. □

Example 5.13. Based on Example 12, define hyperoperations on R_2 as $+_1 = +_2$, $\odot_1 = \odot_2$ and define a partial order $\leq_2$ is same as $\leq_1$. Let $\psi : R_1 \to R_2$ be a mapping defined by $\psi(1) = 1$, $\psi(2) = 3$ and $\psi(3) = 2$. Then, $(R_2, +_2, \odot_2, \leq_2)$ is a NeutroOrderedIsomorphism.

Next, based on Theorem 4.9, we have a theorem related to the productional on NeutroOrderedHyperring.

Theorem 5.14. *Let $(R_1, +_1, \odot_1, \leq_2)$ and $(R_2, +_2, \odot_2, \leq_2)$ be NeutroOrderedHyperrings. Then, $(R_1 \times R_2, \odot, +, \leq)$ is an NeutroOrderedHyperring. Here, $(a, b) \leq (c, d)$ is equivalent to $a \leq_1 c$ and $b \leq_2 d$.*

Proof. Based on Theorem 4.9, $(R_1 \times R_2, +, \odot)$ is a NeutroHyperring. Now, we only show that it satisfies the NeutroOrdered axiom. Having $(R_1, +_1, \odot_1, \leq_1)$ and $(R_2, +_2, \odot_2, \leq_2)$ NeutroOrderedHyperrings. Then, we get

(1) There exists $a \leq b \in R_1$ with $a \neq b$ such that $c +_1 a \leq c +_1 b$ and $a + c_1 \leq b +_1 c$ for every $c \in R_1$. (Degree of truth, "T").

(2) There exists $a \leq b \in R_1$ such that $c +_1 a \nleq c +_1 b$ and $a +_1 c \nleq b +_1 c$. (Degree of Falsify, "F").

(3) There exists $a \leq b \in R_1$ such that $c +_1 a$ or $c +_1 b$ or $a +_1 c$ or $b +_1 c$ are indeterminate, or the relation between $c +_1 a$ and $c +_1 b$ or $a +_1 c$ and $b +_1 c$ are indeterminate. (Degree of indeterminacy, "I").

(4) There exists $a' \leq b' \in R_2$ with $a' \leq b'$ such that $c' +_2 a' \leq c' +_2 b'$ and $a + c_2 \leq b +_2 c$ for every $c \in R_1$. (Degree of truth, "T").

(5) There exists $a' \leq b' \in R_2$ with $a' \nleq b'$ such that $c' +_2 a' \leq c' +_2 b'$ and $a + c_2 \leq b +_2 c$ for every $c \in R_1$. (Degree of truth, "T").

(6) There exists $a' \leq b' \in R_2$ such that $c' +_1 a'$ or $c' +_1 b'$ or $a' +_1 c'$ or $b' +_1 c'$ are indeterminate, or the relation between $c' +_1 a'$ and $c' +_1 b'$ or $a' +_1 c'$ and $b' +_1 c'$ are indeterminate. (Degree of indeterminacy, "I").

For $(R, \odot_1)$ and $(R, \odot_2)$, we just replace $+_k$ with $\odot_k$. If $(R_1 \times R_2, +, \odot, \leq)$ is satisfied Condition 3 or Condition 6, then $(R_1 \times R_2, +, \odot, \leq)$ is a NeutroOrderedHyperring. Now, suppose that Conditions 1,2,4, and 5 are satisfied. Then, without loss of generality, we get

(1) There exist $(a, a') \leq (b, b') \in R_1 \times R_2$ with $(a, a') \neq (b, b')$ such that $(c, c') + (a, a') \leq (c, c') + (b, b')$, $(a, a') + (c, c') \leq (b, b') + (c, c')$, $(c, c') \odot (a, a') \leq (c, c') \odot (b, b')$, and $(a, a') \odot (c, c') \leq (b, b') \odot (c, c')$. (Degree of truth "T").

(2) There exist $(a, a') \leq (b, b') \in R_1 \times R_2$ such that $(c, c') + (a, a') \nleq (c, c') + (b, b')$ or $(a, a') + (c, c') \nleq (b, b') + (c, c')$ or $(c, c') \odot (a, a') \nleq (c, c') \odot (b, b')$ or $(a, a') \odot (c, c') \nleq (b, b') \odot (c, c')$. (Degree of falsify "F").

Then, $(R_1 \times R_2, +, \odot, \leq)$ is a NeutroOrderedHyperring. □

Theorem 5.15. *Let $(R_1, +_1, \odot_1, \leq_1)$ and $(R_2, +_2, \odot_2, \leq_2)$ be NeutroOrderedHyperring and S_1 and S_2 are NeutroOrderedSubhyperring respectively for R_1 and R_2. Then $S_1 \times S_2$ is a NeutroOrderedHyperring of $R_1 \times R_2$.*

Proof. Based on Theorem 4.9, we have $(S_1 \times S_2, +, \odot, \leq)$ is a NeutroOrderedHyperring. Based on hypothesis, S_1 and S_2 are NeutroOrderedHyperrings of R_1 and R_2, then there exist $s_1 \in S_1$, $s_2 \in S_2$ such that $(s_1] \subseteq S_1$ and $(s_2] \subseteq S_2$. We get, $((s_1 \times s_2)] = (s_1] \times (s_2] \subseteq S_1 \times S_2$. Thus, $S_1 \times S_2$ is a NeutroOrderedSubhyperring of $R_1 \times R_2$. □

6. Application of NeutroHyperring On NeutroRing

The notion of NeutroRing first introduced by Agboola in 2020 [23]. We have following relation between NeutroHyperring and NeutroRing.

Theorem 6.1. *Every NeutroRing is a NeutroHyperring.*

Proof. Since every ring is a hyperring, then it is obvious that every NeutroRing is a NeutroHyperring. □

Theorem 6.2. *Let $R = \bigcap_{i=1}^{n} R_i$ be a NeutroRing. Then, it is also a NeutroHyperring.*

Proof. Since every NeutroRing is a NeutroHyperring, then it is clear that if $R = \bigcap_{i=1}^{n} R_i$ is a NeutroRing, then it is also a NeutroHyperring. □

Theorem 6.3. *Let $\prod_{i=1}^{n} R_i$ be a NeutroRing. Then, it is also a NeutroHyperring.*

Proof. The proof is straightforward. □

Theorem 6.4. *Let $(R, +, \odot)$ be a NeutroRing and $\{T_i\}$ with $i = 1, 2, ..., n$ be a family of NeutroSubring of R. $T = \bigcap_{i=1}^{n} S_i$ and $T = \prod_{i=1}^{n} R_i$ is NeutroSubrings. Then, it also NeutroSubHyperrings.*

Proof. The proof is straightforward. □

Theorem 6.5. *Let $(R, +, \odot)$ be a NeutroRing and $\{I_i\}$ with $i = 1, 2, ..., n$ be a family of NeutroIdeal of R. $I = \bigcap_{i=1}^{n} I_i$ and $I = \sum_{i=1}^{n} I_i$ is NeutroIdeals. Then, it also NeutroHyperideals.*

Proof. The proof is straightforward. □

7. Conclusion

Based on explanation above, we already define the new notions on NeutroHyperstructures and NeutroOrderedHyperstructures, that is NeutroHyperrings and NeutroOrderedHyperrings. Besides that, we also find properties related to NeutroHyperrings and NeutroOrderedHyperrings also application of NeutroHyperring in NeutroRing. For future research, it is interesting to define NeutroHypermodules, NeutroPolygroups, NeutroOrderedHypermodules, and NeutroOrderedPolygroups. We may find unique mathematical links and get a deeper knowledge of these algebraic systems by examining the characteristics and structures of these new concepts.

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