Neutrosophic Approaches for Analyzing the Interaction Between the Sports Industry and Community Health: A Multi-Attribute Decision-Making Perspective

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Abstract: In the context of integrating sports and medicine, the evaluation of collaborative development models in community healthcare and the sports health industry can be analyzed through four aspects: resource integration efficiency, health service coverage, improvement in residents' health levels, and policy support and implementation effectiveness. These indicators help assess the model's effectiveness and promote residents' well-being and industry collaboration. The collaborative development patterns evaluation of community medical and sports health industry based on sports medicine integration is multiple-attribute decision-making (MADM). In such a study, the generalized weighted geometric Bonferroni mean (GWGBM) approach is conducted for MADM with single-valued neutrosophic sets (SVNSs). Then, the single-valued neutrosophic number generalized power weighted geometric BM (SVNNGPWGBM) approach is conducted and then the MADM decision approaches are put forward based on the GWGBM approach and power geometric (PG) approach we used the Maximizing Deviation Approach (MDA) to obtain the weights of criteria and rank the alternatives. Finally, an example of collaborative development patterns evaluation of community medical and sports health industry based on sports medicine integration

Keywords: MADM; SVNSs; GWGBM approach; collaborative development patterns evaluation

1. Introduction

As society and the economy develop and people's living standards rise, health issues have gradually become a topic of widespread concern [1]. In this context, the concept of sports-medical integration has emerged and is gradually becoming an important approach to promoting public health. Simply put, sports-medical integration combines healthcare with physical exercise, using scientifically-based exercise interventions to help improve people's health, prevent diseases, and promote rehabilitation. Particularly today, with the increasing prevalence of chronic diseases and the acceleration of aging, traditional medical methods alone are insufficient to cope with the increasingly complex health challenges [2]. Therefore, sports-medical integration provides a new approach to solving these problems.

Community healthcare is an important component of the primary healthcare system and, due to its wide service coverage and proximity to residents, has become an ideal platform for implementing sports-medical integration. Community healthcare can not only provide basic medical services to residents but also play a role in disease prevention and health management [3]. The sports and health industry, on the other hand, encompasses various forms of services such as fitness and sports rehabilitation, and boasts abundant resources and professional talent. The combination of community healthcare and the sports health industry creates a complementary relationship, providing residents with more comprehensive and scientifically sound health management services. This collaborative development model can be reflected in several ways. First, community healthcare institutions can collaborate with local gyms, sports clubs, and other organizations to offer residents personalized exercise plans and health management services. Community doctors assess the health status of residents, issue exercise prescriptions, and monitor their exercise outcomes and health changes in real time. Meanwhile, professionals from the sports health industry are responsible for specific exercise guidance and rehabilitation services, ensuring the safety and effectiveness of residents' exercise programs. With the support of information technology, the collaborative development between community healthcare and the sports health industry has gained even more momentum. By establishing a health management platform, residents' health data and exercise data can be shared in real time, allowing doctors and fitness coaches to dynamically adjust health intervention plans based on this data [4]. This information-based approach not only enhances the precision of health management but also makes health interventions more personalized and intelligent for residents. Moreover, the collaboration between community healthcare institutions and the sports health industry is also reflected in health education and promotion [5]. By organizing health lectures, community activities, and other forms of outreach, they can promote the concept of sports-medical integration to residents, encouraging more people to actively participate in exercise and take control of their health [6]. This health education not only improves residents' knowledge of health but also strengthens their health awareness, fostering the adoption of healthier lifestyles. In terms of chronic disease management, the collaborative model of sports-medical integration also demonstrates unique advantages. For residents with chronic diseases, community healthcare can leverage the expertise of the sports health industry to provide personalized exercise rehabilitation services [7]. Through scientifically-guided exercise interventions, patients can improve their condition and enhance their physical fitness, thereby reducing their reliance on medications and improving their quality of life [8]. In this process, community healthcare institutions and the sports health industry work closely together to form a complete health management cycle. This collaborative development model offers multiple benefits[9]. On the one hand, it can effectively alleviate the pressure on healthcare resources and reduce medical expenses; on the other hand, it promotes the development of the sports health industry, bringing new opportunities for diversification and professionalization within the sector. At the same time, residents gain access to more comprehensive health management services through this model, allowing them to better prevent diseases, maintain their health, and improve their overall quality of life [10]. In summary, the collaborative development model between community healthcare and the sports health industry under the framework of sports-medical integration is an effective way to address modern health challenges. By organically combining healthcare and sports, community healthcare and the sports health industry can play a greater role in disease prevention and chronic disease management, while also promoting a shift from a "disease-centered" approach to a "health-centered" one. The promotion of this model not only contributes to improving public health levels but also points the way forward for the future development of healthcare systems and the health industry.

In real life, decision-making is an essential part of our daily activities, ranging from personal choices such as clothing, food, housing, and transportation, to large-scale decisions like national policy formulation. To handle inconsistencies in decision-making information, Smarandache [11] introduced neutrosophic sets (NSs), which provide a way to represent uncertainty and inconsistency. Building on this, Wang et al. [12] proposed SVNSs to enhance this framework. In evaluating the collaborative development patterns of community medical and sports health industries based on sports medicine integration, the problem can be framed as a MADM issue. However, real-world MADM problems often involve input variables that are interrelated and dependent on each other, which presents a challenge not adequately addressed by traditional integration methods. To address this issue, the Bonferroni Mean (BM) approach [13] was introduced, which has the unique ability to reflect the interdependence and interactions between data. Yager [14] later extended this idea by proposing the Generalized Bonferroni Mean (GBM), which was applied to MADM problems. Following this, Zhu et al. [15] developed the Geometric BM (GBM) by combining the Bonferroni Mean with the geometric mean, providing a more robust method for handling decision-making scenarios with interrelated attributes. Xia, Xu, and Zhu [16] further advanced this work by proposing the GWGBM, which was designed specifically to address MADM problems under the framework of SVNSs. A subsequent development in this area is the SVNNGPWGBM approach. This method integrates the GWGBM and the power geometric (PG) [17] approaches, further refining the process of addressing complex decision-making scenarios with interacting attributes. The SVNNGPWGBM approach serves as a foundational method for MADM problems, particularly in cases where there is a need to account for the interdependence of attributes. In this study, the focus is on applying the SVNNGPWGBM approach to evaluate the collaborative development patterns of community medical and sports health industries, under the framework of sports medicine integration.

The key motivations and objectives of this study are constructed:

Maximizing Deviation Approach for Attribute Weighting: The first step involves employing the maximizing deviation method to determine the weight of each attribute in the decision-making process. This ensures that the most significant factors are given appropriate emphasis in the evaluation.

Implementing the SVNNGPWGBM Approach: Using the SVNNGPWGBM method, the study aims to effectively handle the interrelationships between different attributes, providing a more accurate and comprehensive evaluation of the collaborative development patterns in the medical and sports health sectors.

Case Study: Finally, the study applies the SVNNGPWGBM approach to a real-world example evaluating the collaborative development patterns of community medical and sports health industries based on sports medicine integration.

In summary, this research addresses a significant challenge in MADM problems—namely, the interdependence of input variables—by utilizing advanced methods like the Bonferroni Mean and its extensions. The study not only introduces the SVNNGPWGBM method as a powerful tool for decision-making but also applies it to the evaluation of collaborative development patterns in the community medical and sports health industries. The results demonstrate the effectiveness of this approach in reflecting the complex interrelationships between attributes, offering a valuable contribution to decision-making in fields that require the integration of medical and sports health services.

To achieve this, the study framework is planned: Section 2 introduces SVNSs. In Section 3, the SVNNGPWGBM approach is detailed. Section 4 discusses the MADM process based on the

SVNNGPWGBM approach. An example application evaluating collaborative development patterns in the community medical and sports health industry through sports medicine integration is provided in Section 5. Finally, the conclusion is presented in Section 6.

2. Preliminaries

Wang et al. [12] planned the SVNSs

Definition 1 [12]. The SVNSs PA in Θ is planned:

$$UA=\left\{\left(\theta,UT_A\left(\theta\right),UI_A\left(\theta\right),UF_A\left(\theta\right)\right)\middle|\theta\in\Theta\right\} \quad (1)$$

where $UT_A\left(\theta\right),UI_A\left(\theta\right),UF_A\left(\theta\right)$ is truth-membership, indeterminacy-membership and falsity-membership, $UT_A\left(\theta\right),UI_A\left(\theta\right),UF_A\left(\theta\right)\in\left[0,1\right]$, $0\le UT_A\left(\theta\right)+UI_A\left(\theta\right)+UF_A\left(\theta\right)\le 3$.

The SVNN is planned as $UA=\left(UT_A,UI_A,UF_A\right)$, where $UT_A,UI_A,UF_A\in\left[0,1\right]$, and $0\le UT_A+UI_A+UF_A\le 3$.

Definition 2 [18]. Let $UA=\left(UT_A,UI_A,UF_A\right)$, the score function is planned:

$$SF\left(UA\right)=\frac{\left(2+UT_A-UI_A-UF_A\right)}{3},\ SF\left(UA\right)\in\left[0,1\right]. \quad (2)$$

Definition 3[18]. Let $UA=\left(UT_A,UI_A,UF_A\right)$, the accuracy function be planned:

$$AF\left(UA\right)=\frac{UT_A-UF_A+1}{2},\ AF\left(UA\right)\in\left[0,1\right]\ . \quad (3)$$

Peng et al.[18] planned the order relation.

Definition 4[18]. Let $UA=\left(UT_A,UI_A,UF_A\right)$, $UB=\left(UT_B,UI_B,UF_B\right)$, let $SF\left(UA\right)=\frac{\left(2+UT_A-UI_A-UF_A\right)}{3}$ and $SF\left(UB\right)=\frac{\left(2+UT_B-UI_B-UF_B\right)}{3}$, and let $AF\left(UA\right)=\frac{UT_A-UF_A+1}{2}$ and $AF\left(UB\right)=\frac{UT_B-UF_B+1}{2}$, if $SF\left(UA\right)<SF\left(UB\right)$, $UA<UB$; if $SF\left(UA\right)=SF\left(UB\right)$, (1)if $AF\left(UA\right)=AF\left(UB\right)$, $UA=UB$; (2) if $AF\left(UA\right)<AF\left(UB\right)$, $UA<UB$.

Definition 5[12]. Let $UA=\left(UT_A,UI_A,UF_A\right)$ and $UB=\left(UT_B,UI_B,UF_B\right)$, the planned operations are:

(1) $UA\oplus UB=\left(UT_A+UT_B-UT_AUT_B,UI_AUI_B,UF_AUF_B\right)$;

(2) $UA\otimes UB=\left(UT_AUT_B,UI_A+UI_B-UI_AUI_B,UF_A+UF_B-UF_AUF_B\right)$;

(3) $\lambda UA=\left(1-\left(1-UT_A\right)^{\lambda},\left(UI_A\right)^{\lambda},\left(UF_A\right)^{\lambda}\right),\lambda>0$;

(4) $\left(UA\right)^{\lambda}=\left(\left(UT_A\right)^{\lambda},\left(UI_A\right)^{\lambda},1-\left(1-UF_A\right)^{\lambda}\right),\lambda>0.$

Definition 6[19]. Let $UA=\left(UT_A,UI_A,UF_A\right)$ and $UB=\left(UT_B,UI_B,UF_B\right)$, the SVNN Hamming distance (SVNNHD) between $UA=\left(UT_A,UI_A,UF_A\right)$ and $UB=\left(UT_B,UI_B,UF_B\right)$ is planned:

$$SVNNHD\left(UA,UB\right)=\frac{1}{3}\left(\left|UT_A-UT_B\right|+\left|UI_A-UI_B\right|+\left|UF_A-UF_B\right|\right) \tag{4}$$

Definition 7[16]. Let $s,t,r>0$, $\beta_i(i=1,2,3,\ldots,n)$ be positive numbers along with weight $uw=(uw_1,uw_2,\ldots,uw_n)^T$, $uw_i\in\left[0,1\right],\sum_{i=1}^{n}uw_i=1.$ The GWGBM is planned:

$$\mathrm{GWGBM}^{s,t,r>0}(\beta_1,\beta_2,\ldots,\beta_n)=\frac{1}{s+t+r}\prod_{i,j,k=1}^{n}(s\beta_i+t\beta_j+r\beta_k)^{pw_ipw_jpw_k} \tag{5}$$

3. SVNNGPWGBM approach

Then, the SVNNGWGBM approach [20] is planned on the GWGBM approach.

Definition 8[20]. Let $s,t,r>0$, $UA_i=(UT_i,UI_i,UF_i)(i=1,2,\ldots,n)$ be SVNNs with weight $uw_i=(uw_1,uw_2,\ldots,uw_n)^T$, $\sum_{i=1}^{n}uw_i=1.$ If

$$\begin{aligned}
&SVNNGWGBM_{uw}^{s,t,r}(UA_1,UA_2,\cdots,UA_n)\\
&=\frac{1}{s+t+r}\underset{i,j,k=1}{\overset{n}{\otimes}}(sUA_i\oplus tUA_j\oplus rUA_k)^{uw_iuw_juw_k}\\
&=\left(\begin{array}{l}
1-(1-\prod_{i,j,k=1}^{n}(1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{uw_iuw_juw_k})^{1/(s+t+r)},\\
(1-\prod_{i,j,k=1}^{n}(1-UI_i^sUI_j^tUI_k^r)^{uw_iuw_juw_k})^{1/(s+t+r)},\\
(1-\prod_{i,j,k=1}^{n}(1-UF_i^sUF_j^tUF_k^r)^{uw_iuw_juw_k})^{1/(s+t+r)}.
\end{array}\right)
\end{aligned} \tag{6}$$

then $\mathrm{SVNNGWGBM}_{uw}^{s,t,r}$ is called SVNNGWGBM approach.

The SVNNGWGBM approach [20] has the same properties.

Property 1 (Idempotency). If $UA_i = (UT_i, UI_i, UF_i)(i = 1, 2, \ldots, n)$ are equal, that is, $UA_i = UA = (UT, UI, UF)$, then

$$SVNNGWGBM_{uw}^{s,t,r}(UA_1, UA_2, \cdots, UA_n) = UA \quad (7)$$

Property 2 (Monotonicity). Let $UA_i = (UT_{A_i}, UI_{A_i}, UF_{A_i})(i = 1, 2, 3, \ldots, n)$ and $UB_i = (UT_{B_i}, UI_{B_i}, UF_{B_i})\ (i = 1, 2, 3, \ldots, n)$ be two sets of SVNNs. If $UT_{A_i} \le UT_{B_i}$, $UI_{A_i} \ge UI_{B_i}$, $UF_{A_i} \ge UF_{B_i}$ holds for all i, then

$$\begin{aligned} & SVNNGWGBM_{uw}^{s,t,r}(UA_1, UA_2, \cdots, UA_n) \\ & \le SVNNGWGBM_{uw}^{s,t,r}(UB_1, UB_2, \cdots, UB_n) \end{aligned} \quad (8)$$

Property 3 (Boundedness). Let $UA_i = (UT_{A_i}, UI_{A_i}, UF_{A_i})(i = 1, 2, 3, \ldots, n)$ be SVNNS. If

$UA^+ = (\max_i(UT_i), \min_i(UI_i), \min_i(UF_i))$ and $UA^- = (\min_i(UT_i), \max_i(UI_i), \max_i(UF_i))$

then

$$UA^- \le SVNNGWGBM_{uw}^{s,t,r}(UA_1, UA_2, \cdots, UA_n) \le UA^+. \quad (9)$$

The SVNNGPWGBM approach is planned on the SVNNGWGBM approach [20] and the PG approach [17].

Definition 9. Let $s, t, r > 0$, $UA_i = (UT_i, UI_i, UF_i)(i = 1, 2, \ldots, n)$ be SVNNs with their weight $uw_i = (uw_1, uw_2, \ldots, uw_n)^T$, $\sum_{i=1}^{n} uw_i = 1$. The SVNNGPWGBM approach is planned:

$$\begin{aligned} & SVNNGPWGBM_{uw}^{s,t,r}(UA_1, UA_2, \cdots, UA_n) \\ & = \frac{1}{s+t+r} \mathop{\otimes}_{i,j,k=1}^{n} (sUA_i \oplus tUA_j \oplus rUA_k)^{uw_i uw_j uw_k} \end{aligned} \quad (10)$$

where $T(UA_a) = \sum_{\substack{j=1 \\ a \ne j}}^{m} uw_j Sup(UA_a, UA_j)$, $Sup(UA_a, UA_j)$ is decision support for UA_a from UA_j, with conditions: (1) $Sup(UA_a, UA_b) \in [0,1]$; (2) $Sup(UA_b, UA_a) = Sup(UA_a, UA_b)$; (3) $Sup(UA_a, UA_b) \ge Sup(UA_s, UA_t)$, if $SVNNHD(UA_a, UA_b) \le SVNNHD(UA_s, UA_t)$.

Theorem 1 is obtained.

Theorem 1. Let $s,t,r>0$ and $UA_i=(UT_i,UI_i,UF_i)(i=1,2,\ldots,n)$ be SVNNs. The aggregated result of SVNNGPWGBM is a SVNN and

$$
\begin{aligned}
&SVNNGPWGBM_{uw}^{s,t,r}(UA_1,UA_2,\cdots,UA_n)\\
&=\frac{1}{s+t+r}\underset{i,j,k=1}{\overset{n}{\otimes}}(sUA_i\oplus tUA_j\oplus rUA_k)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}}\\
&=\left(\begin{array}{l}
1-(1-\prod_{i,j,k=1}^{n}(1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)},\\
(1-\prod_{i,j,k=1}^{n}(1-UI_i^sUI_j^tUI_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)},\\
(1-\prod_{i,j,k=1}^{n}(1-UF_i^sUF_j^tUF_k^r)^{uw\,\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}\,uw_juw_k})^{1/(s+t+r)}
\end{array}\right)
\end{aligned}
\tag{11}
$$

Proof:

$$
\begin{aligned}
sUA_i&=(1-(1-UT_i)^s,UI_i^s,UF_i^s),\\
tUA_j&=(1-(1-UT_j)^t,UI_j^t,UF_j^t),\\
rUA_k&=(1-(1-UT_k)^r,UI_k^r,UF_k^r).
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&sUA_i\oplus tUA_j\oplus rUA_k\\
&=\left(\begin{array}{l}1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r,\\ UI_i^sUI_j^tUI_k^r,UF_i^sUF_j^tUF_k^r\end{array}\right)
\end{aligned}
\tag{13}
$$

Thereafter,

$$\left(sUA_i \oplus tUA_j \oplus rUA_k\right)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}}$$

$$= \left(\begin{array}{l} (1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}}, \\ 1-(1-UI_i^s UI_j^t UI_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}}, \\ 1-(1-UF_i^s UF_j^t UF_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}} \end{array} \right) \quad (14)$$

Therefore,

$$\underset{i,j,k=1}{\overset{n}{\otimes}}\left(sUA_i \oplus tUA_j \oplus rUA_k\right)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}}$$

$$= \left(\begin{array}{l} \prod_{i,j,k=1}^{n}(1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}}, \\ 1-\prod_{i,j,k=1}^{n}(1-UI_i^s UI_j^t UI_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}}, \\ 1-\prod_{i,j,k=1}^{n}(1-UF_i^s UF_j^t UF_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n} uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n} uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n} uw_k(1+T(UA_k))}} \end{array} \right) \quad (15)$$

Thus,

$$\frac{1}{s+t+r}\bigotimes_{i,j,k=1}^{n}\left(sUA_i\oplus tUA_j\oplus rUA_k\right)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}}$$

$$=\left(\begin{array}{l}1-(1-\prod_{i,j,k=1}^{n}(1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)},\\ (1-\prod_{i,j,k=1}^{n}(1-UI_i^sUI_j^tUI_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)},\\ (1-\prod_{i,j,k=1}^{n}(1-UF_i^sUF_j^tUF_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)}.\end{array}\right)$$

(16)

Hence, (11) is maintained.

Thereafter,

$$0\le 1-(1-\prod_{i,j,k=1}^{n}(1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)}\le 1,$$

$$0\le(1-\prod_{i,j,k=1}^{n}(1-UI_i^sUI_j^tUI_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)}\le 1,$$

$$0\le(1-\prod_{i,j,k=1}^{n}(1-UF_i^sUF_j^tUF_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)}\le 1.$$

(17)

Therefore,

$$0 \le 1-(1-\prod_{i,j,k=1}^{n}(1-(1-UT_i)^s(1-UT_j)^t(1-UT_k)^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)}$$
$$+(1-\prod_{i,j,k=1}^{n}(1-UI_i^sUI_j^tUI_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)}$$
$$+(1-\prod_{i,j,k=1}^{n}(1-UF_i^sUF_j^tUF_k^r)^{\frac{uw_i(1+T(UA_i))}{\sum_{i=1}^{n}uw_i(1+T(UA_i))}\frac{uw_j(1+T(UA_j))}{\sum_{j=1}^{n}uw_j(1+T(UA_j))}\frac{uw_k(1+T(UA_k))}{\sum_{k=1}^{n}uw_k(1+T(UA_k))}})^{1/(s+t+r)} \le 3. \tag{18}$$

Thereby completing the proof.

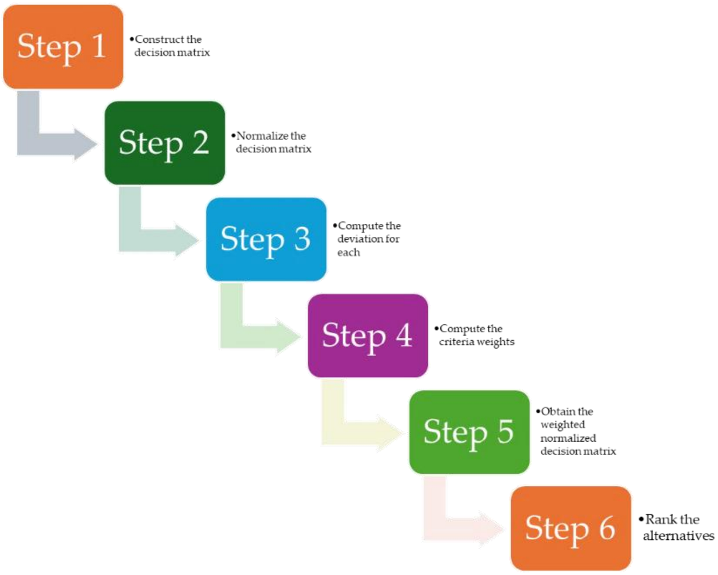

Figure 1. Phases of the MCDM method.

4. The MADM method based on SVNNGPWGBM with SVNNs

The SVNNGPWGBM method is planned for MADM with SVNSs. Let $UX=\{UX_1,UX_2,\ldots,UX_n\}$ be attributes, $uw=\{uw_1,uw_2,\ldots,uw_n\}$ be the weight of UX_j. Let $UA=\{UA_1,UA_2,\ldots,UA_m\}$ be alternatives. $UU=(uu_{ij})_{m\times n}=(UT_{ij},UI_{ij},UF_{ij})_{m\times n}$ is SVNN-matrix. The SVNNGPWGBM method for MADM will be planned. Figure 1 shows the steps of the MCDM method.

Step 1. Plan the SVNN matrix $UU=(uu_{ij})_{m\times n}=(UT_{ij},UI_{ij},UF_{ij})_{m\times n}$.

$$UU=\left(UU_{ij}\right)_{m\times n}=\begin{bmatrix} UU_{11} & UU_{12} & \cdots & UU_{1n} \\ UU_{21} & UU_{22} & \cdots & UU_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ UU_{m1} & UU_{m2} & \cdots & UU_{mn} \end{bmatrix}$$

$$UU=\left(UU_{ij}\right)_{m\times n}=\left(UT_{ij},UI_{ij},UF_{ij}\right)_{m\times n} \tag{19}$$

$$=\begin{bmatrix} \left(UT_{11},UI_{11},UF_{11}\right) & \left(UT_{12},UI_{1},UF_{12}\right) & \cdots & \left(UT_{1n},UI_{1n},UF_{1n}\right) \\ \left(UT_{21},UI_{21},UF_{21}\right) & \left(UT_{22},UI_{22},UF_{22}\right) & \cdots & \left(UT_{2n},UI_{2n},UF_{2n}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(UT_{m1},UI_{m1},UF_{m1}\right) & \left(UT_{m2},UI_{m2},UF_{m2}\right) & \cdots & \left(UT_{mn},UI_{mn},UF_{mn}\right) \end{bmatrix}$$

Step 2. Normalize the $UU=\left[UU_{ij}\right]_{m\times n}$ to $NP=\left[NP_{ij}\right]_{m\times n}$.

$$NU_{ij}=\left(NUT_{ij},NUI_{ij},NUF_{ij}\right)$$
$$=\begin{cases}\left(UT_{ij},UI_{ij},UF_{ij}\right), & UX_j \text{ is a benefit criterion} \\ \left(UF_{ij},1-UI_{ij},UT_{ij}\right), & UX_j \text{ is a cost criterion}\end{cases} \tag{20}$$

Step 3. Plan the weight with a maximizing deviation approach.

The maximizing deviation approach [21] is utilized to derive the weight.

(1) Depending on $NU=\left[NU_{ij}\right]_{m\times n}$, the deviation degree PA_i to other choices is obtained.

$$SVNNDD_{ij}=\sum_{t=1}^{m}uw_j\cdot\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right| \tag{21}$$

(2) Obtain the total weighted deviation.

$$SVNNDD_j\left(uw\right)=\sum_{i=1}^{m}SVNNDD_{ij}\left(uw\right)$$
$$=\sum_{i=1}^{m}\sum_{t=1}^{m}uw_j\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right| \tag{22}$$

(3) Obtain a non-linear programming approach.

$$\left(M-1\right)\begin{cases}\max SVNNDD\left(uw\right)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{m}pw_j\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right| \\ s.t.\ \ \mathrm{uw}_j\geq 0,\ \ \sum_{j=1}^{n}uw_j^2=1\end{cases} \tag{23}$$

The Lagrange function is utilized to address this proposed approach.

$$LF\left(uw,\xi\right)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{m}uw_j\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right|+\frac{\xi}{2}\left(\sum_{j=1}^{n}uw_j^2-1\right)$$

Where ξ is the planned Lagrange multiplier. The partial derivatives are calculated.

$$\begin{cases}\dfrac{\partial LF}{\partial uw_j}=\sum_{i=1}^{m}\sum_{t=1}^{m}\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right|+\xi uw_j=0\\ \dfrac{\partial LF}{\partial \xi}=\dfrac{1}{2}\left(\sum_{j=1}^{n}uw_j^2-1\right)=0\end{cases}$$

The weight information is obtained: $uw_j^*=\dfrac{\sum_{i=1}^{m}\sum_{t=1}^{m}\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right|}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{t=1}^{m}\left|AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)\right|\right)^2}}$

The weights are obtained:

$$uw_j=\frac{\sum_{i=1}^{m}\sum_{t=1}^{m}AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{t=1}^{m}AF\left(NU_{ij}\right)-AF\left(NU_{tj}\right)}\tag{24}$$

Step 4. From the $NU=\left[NU_{ij}\right]_{m\times n}$, the SVNNs $NU=\left[NU_{ij}\right]_{m\times n}$ are planned through the SVNNGPWGBM approach to obtain the SVNNs $NU_i=\left(NUT_i, NUI_i, NUF_i\right)$:

$$\begin{aligned}&NU_i=\left(NUT_i, NUI_i, NUF_i\right)\\&=SVNNGWGBM_{uw}^{s,t,r}(NU_{i1},NU_{i2},\cdots,NU_{in})\\&=\frac{1}{s+t+r}\underset{i,j,k=1}{\overset{n}{\otimes}}(sNU_{if}\oplus tNU_{ij}\oplus rNU_{ik})^{\frac{uw_i(1+T(NU_i))}{\sum_{i=1}^{n}uw_i(1+T(NU_i))}\frac{uw_j(1+T(NU_j))}{\sum_{j=1}^{n}uw_j(1+T(NU_j))}\frac{uw_k(1+T(NU_k))}{\sum_{k=1}^{n}uw_k(1+T(NU_k))}}\\&=\left(\begin{array}{l}1-(1-\prod_{i,j,k=1}^{n}(1-(1-NUT_{if})^s(1-NUT_{ij})^t(1-NUT_{ik})^r)^{\frac{uw_i(1+T(NU_i))}{\sum_{i=1}^{n}uw_i(1+T(NU_i))}\frac{uw_j(1+T(NU_j))}{\sum_{j=1}^{n}uw_j(1+T(NU_j))}\frac{uw_k(1+T(NU_k))}{\sum_{k=1}^{n}uw_k(1+T(NU_k))}})^{1/(s+t+r)},\\(1-\prod_{i,j,k=1}^{n}(1-NUI_{if}^sNUI_{ij}^tNUI_{ik}^r)^{\frac{uw_i(1+T(NU_i))}{\sum_{i=1}^{n}uw_i(1+T(NU_i))}\frac{uw_j(1+T(NU_j))}{\sum_{j=1}^{n}uw_j(1+T(NU_j))}\frac{uw_k(1+T(NU_k))}{\sum_{k=1}^{n}uw_k(1+T(NU_k))}})^{1/(s+t+r)},\\(1-\prod_{i,j,k=1}^{n}(1-NUF_{if}^sNUF_{ij}^tNUF_{ik}^r)^{\frac{uw_i(1+T(NU_i))}{\sum_{i=1}^{n}uw_i(1+T(NU_i))}\frac{uw_j(1+T(NU_j))}{\sum_{j=1}^{n}uw_j(1+T(NU_j))}\frac{uw_k(1+T(NU_k))}{\sum_{k=1}^{n}uw_k(1+T(NU_k))}})^{1/(s+t+r)}\end{array}\right)\end{aligned}\tag{25}$$

Step 5. Obtain the weighted decision matrix.

Step 6. Rank the alternatives based on the sum of each row of the weighted normalized decision matrix.

Figure 2. The 14 criteria.

5. Numerical Example

In this section, we will implement the proposed method to illustrate its effectiveness through a numerical example. This demonstration aims to provide clarity on how the method operates in practice and the results it yields. By analyzing specific data and applying the outlined techniques, we will showcase the practical applications and outcomes of our approach, offering insights into its potential benefits in real-world scenarios.

5.1 Case study

With the improvement of people's material living standards and the rapid development of urbanization, the challenges faced by urban living environments are mainly manifested in the increasing demand for improving the quality of living space and public services. The discipline of urban and rural planning has gradually shifted its research perspective from material space to social space. In this process, the connotation of "health" continues to enrich, and its construction framework is further expanded, involving multiple dimensions such as society, economy, and technology, and including various requirements on the material and spiritual levels. As the basic management unit of a city, communities are the frontline of epidemic prevention and control, as well as the most effective line of defense against external input and internal spread. With the normalization of the epidemic, public health, joint prevention, self-protection, and lifestyle changes centered around individuals and communities have become the key to epidemic prevention and control. Community medical service facilities are the gatekeepers of people's health. Setting up public health units based on community living circles, strengthening the construction of community medical service facilities, and the allocation of health resources, are of great significance for improving the urban disaster prevention space system, enhancing community resilience, and ensuring people's safety. Since the 1990s, healthy communities have gradually become a hot topic of international research, undergoing development from material to spiritual and then to interdisciplinary integration. In recent years, the academic community has returned to the community itself, focusing on research on topics such as community development, community resilience, and green spaces. Domestic research started relatively late and is still in its early stages. It mainly focuses on the creation of material and spiritual spaces in healthy communities and the construction of evaluation standards, such as community planning techniques combined with urban planning and design, health impact factors, and health evaluation indicators. The report of the 19th National Congress of the Communist Party of China in 2017 officially incorporated the "Healthy China Strategy" into the basic national development strategy, and the construction of a healthy China has become an important part of China's national governance goals. The "2030 Plan Outline for Healthy China" proposes the strategic task of developing new formats of health services, actively promoting the integration of health and sports-related industries, and giving birth to new industries, formats, and models of health. As a pillar industry of national health, the interaction, correlation, and integration development between the sports industry and the health industry play a positive role in continuously promoting the construction of a healthy China and meeting the growing health needs of the people. As a core quantitative indicator for judging the development quality of the sports and health industries, industrial added value has diagnostic significance for evaluating the development level and radiative driving ability of the two industries. The collaborative development patterns evaluation of the community medical and sports health industry based on sports medicine integration is a classical MADM issue. In this study, an empirical application of collaborative development patterns evaluation of the community medical and sports health industry based on sports medicine integration is conducted through the SVNNGPGBM technique. There are five collaborative development patterns of the community medical and sports health industry based on sports medicine integration and evaluating their embedded computers' performance. To assess five collaborative development patterns of community medical and sports health industry based on sports medicine integration fairly, the experts give their information with the 14 decision attributes as shown in Figure 2. Then, the SVNNGPGBM technique is applied to MADM for solving the collaborative development patterns evaluation of the community medical and sports health industry based on sports medicine integration with SVNNs. The SVNNGPGBM technique involves the decision steps below:

Step 1. The decision matrix is built between the criteria and alternatives as shown in Table 1.

Step 2. Then we normalize the decision matrix as shown in Table 2.

Step 3. Then we compute the deviation for each criterion.

Step 4. Then we compute the weights of the criteria as shown in Figure 3.

Step 5. Then we compute the weighted normalized decision matrix as shown in Table 3.

Step 6. Then we rank the alternatives as shown in Figure 4.

Table 1. The assessment matrix.

	C_1	**C_2**	**C_3**	**C_4**	**C_5**	**C_6**	**C_7**	**C_8**	**C_9**	**C_{10}**	**C_{11}**	**C_{12}**	**C_{13}**	**C_{14}**
A_1	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)
A_2	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.5,0.5,0.5)
A_3	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.7,0.4,0.3)
A_4	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.8,0.3,0.2)
A_5	(0.7,0.4,0.3)	(0.9,0.2,0.1)	(0.5,0.5,0.5)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.2,0.8,0.7)	(0.8,0.3,0.2)	(0.9,0.2,0.1)
A_6	(0.5,0.5,0.5)	(0.8,0.3,0.2)	(0.3,0.7,0.6)	(0.7,0.4,0.3)	(0.1,0.9,0.8)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.2,0.8,0.7)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.3,0.7,0.6)	(0.9,0.2,0.1)	(0.1,0.9,0.8)
A_7	(0.3,0.7,0.6)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.5,0.5,0.5)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.3,0.7,0.6)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.5,0.5,0.5)	(0.1,0.9,0.8)	(0.2,0.8,0.7)
A_8	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.3,0.7,0.6)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.2,0.8,0.7)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.3,0.7,0.6)
A_9	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.3,0.7,0.6)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.3,0.7,0.6)	(0.8,0.3,0.2)	(0.3,0.7,0.6)	(0.5,0.5,0.5)
A_{10}	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.5,0.5,0.5)	(0.7,0.4,0.3)
A_{11}	(0.2,0.8,0.7)	(0.8,0.3,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.3,0.7,0.6)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.7,0.4,0.3)	(0.8,0.3,0.2)
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
A_1	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.3,0.2)	(0.5,0.5,0.5)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.5,0.5,0.5)	(0.5,0.5,0.5)
A_2	(0.8,0.3,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)
A_3	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.7,0.4,0.3)
A_4	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.5,0.5,0.5)
A_5	(0.5,0.5,0.5)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.5,0.5,0.5)
A_6	(0.5,0.5,0.5)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.7,0.4,0.3)	(0.7,0.4,0.3)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.7,0.4,0.3)
A_7	(0.7,0.4,0.3)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.8,0.3,0.2)
A_8	(0.8,0.3,0.2)	(0.3,0.7,0.6)	(0.3,0.7,0.6)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.9,0.2,0.1)
A_9	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.3,0.7,0.6)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.3,0.7,0.6)	(0.8,0.3,0.2)	(0.3,0.7,0.6)	(0.1,0.9,0.8)
A_{10}	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.5,0.5,0.5)	(0.7,0.4,0.3)
A_{11}	(0.2,0.8,0.7)	(0.8,0.3,0.2)	(0.5,0.5,0.5)	(0.5,0.5,0.5)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.7,0.4,0.3)	(0.5,0.5,0.5)	(0.3,0.7,0.6)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.7,0.4,0.3)	(0.8,0.3,0.2)
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}

A_1	(0.3,0.7,0.6)	(0.3,0.7,0.6)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.5,0.5,0.5)
A_2	(0.7,0.4,0.3)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.3,0.7,0.6)	(0.7,0.4,0.3)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.3,0.7,0.6)	(0.3,0.7,0.6)	(0.5,0.5,0.5)	(0.7,0.4,0.3)	(0.3,0.7,0.6)
A_3	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.3,0.7,0.6)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.3,0.7,0.6)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.3,0.7,0.6)
A_4	(0.3,0.7,0.6)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.3,0.7,0.6)	(0.9,0.2,0.1)	(0.7,0.4,0.3)
A_5	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.7,0.4,0.3)	(0.8,0.3,0.2)	(0.9,0.2,0.1)
A_6	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.9,0.2,0.1)
A_7	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.8,0.3,0.2)
A_8	(0.8,0.3,0.2)	(0.2,0.8,0.7)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.2,0.8,0.7)	(0.9,0.2,0.1)
A_9	(0.9,0.2,0.1)	(0.9,0.2,0.1)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.3,0.7,0.6)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.9,0.2,0.1)	(0.3,0.7,0.6)	(0.1,0.9,0.8)
A_{10}	(0.1,0.9,0.8)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.7,0.4,0.3)	(0.2,0.8,0.7)	(0.9,0.2,0.1)	(0.8,0.3,0.2)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.2,0.8,0.7)	(0.2,0.8,0.7)	(0.1,0.9,0.8)	(0.5,0.5,0.5)	(0.2,0.8,0.7)

Table 2. The normalized assessment matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
A_1	-0.08846	-0.02179	0.044872	0.211538	0.32265	0.333761	0.455983	0.578205	0.478205	0.478205	-0.15513	-0.05513	0.100427	0.211538
A_2	0.444872	0.389316	0.033761	0.044872	0.32265	0.267094	0.244872	0.478205	-0.15513	-0.02179	0.044872	0.211538	0.32265	0.155983
A_3	0.455983	0.389316	0.244872	0.355983	0.278205	0.455983	0.200427	-0.02179	0.100427	0.267094	0.211538	0.389316	0.511538	0.267094
A_4	0.244872	0.511538	0.378205	0.42265	0.389316	0.511538	0.300427	0.178205	0.100427	0.333761	0.32265	0.089316	0.544872	0.355983
A_5	0.32265	0.511538	0.42265	0.478205	0.267094	0.300427	0.400427	0.333761	0.32265	0.42265	0.478205	0.267094	0.511538	0.455983
A_6	0.333761	0.544872	0.400427	0.444872	0.300427	0.544872	0.300427	0.511538	0.300427	0.478205	0.544872	0.400427	0.333761	0.267094
A_7	0.333761	0.02265	0.089316	0.455983	0.544872	0.055983	0.478205	0.389316	0.367094	0.300427	0.300427	0.211538	-0.15513	0.300427
A_8	0.300427	0.011538	0.22265	-0.12179	0.089316	0.367094	0.333761	0.544872	0.478205	0.12265	0.12265	0.411538	-0.05513	0.400427
A_9	0.333761	0.578205	0.267094	0.233761	0.089316	0.044872	-0.08846	-0.15513	0.333761	0.12265	-0.02179	0.511538	0.044872	-0.03291
A_{10}	-0.12179	-0.15513	0.300427	0.378205	0.367094	0.578205	0.478205	0.233761	0.089316	-0.05513	-0.12179	0.333761	0.211538	0.233761
A_{11}	-0.05513	0.478205	0.211538	0.211538	0.578205	0.578205	0.478205	0.378205	0.12265	0.044872	-0.05513	-0.12179	0.378205	0.478205

Table 3. The weighted normalized assessment matrix.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}
A_1	0.007634	0.009954	0.001833	0.000352	4.57E-22	7.65E-05	0.001037	0.005953	0.004297	0.004437	0.008109	0.006333	0.001931	0.000254
A_2	0.003602	0.000849	0.00205	0.00389	4.57E-22	0.000689	0.000399	0.002304	0.010446	0.004331	0.000986	6.2E-05	0.000457	0.000821
A_3	0.00398	0.000849	2.46E-06	0.000362	0.000109	0.000544	0.000958	0.009559	0.001191	0.000115	0.000305	0.001594	0.005914	1.05E-05
A_4	2.25E-05	0.004551	0.000971	0.00133	0.000246	0.001437	3.89E-05	0.001557	0.001191	0.000804	0.002506	0.00166	0.007517	0.000293
A_5	0.000689	0.004551	0.001683	0.002602	0.000171	0.000306	0.000341	3.47E-05	0.000593	0.002694	0.009155	4.99E-05	0.005914	0.001599
A_6	0.000859	0.006071	0.001302	0.001788	2.73E-05	0.002177	3.89E-05	0.003332	0.000341	0.004437	0.013278	0.001842	0.000607	1.05E-05
A_7	0.000859	0.007367	0.001086	0.002042	0.00273	0.006666	0.00142	0.000488	0.001305	0.000381	0.001896	6.2E-05	0.014177	1.93E-05
A_8	0.000404	0.007978	1.13E-05	0.011232	0.003009	6.32E-23	3.99E-06	0.004548	0.004297	0.000759	7.38E-05	0.002107	0.008041	0.000744
A_9	0.000859	0.007809	4.23E-05	0.000168	0.003009	0.007151	0.010473	0.018672	0.000745	0.000759	0.002596	0.005299	0.003633	0.005167
A_{10}	0.009329	0.020044	0.000193	0.000617	0.000109	0.003069	0.00142	0.000541	0.001403	0.005571	0.006444	0.000624	0.000127	0.000118
A_{11}	0.00611	0.00325	3.4E-05	0.000352	0.00361	0.003069	0.00142	0.000355	0.00082	0.002319	0.003688	0.009508	0.001422	0.002032

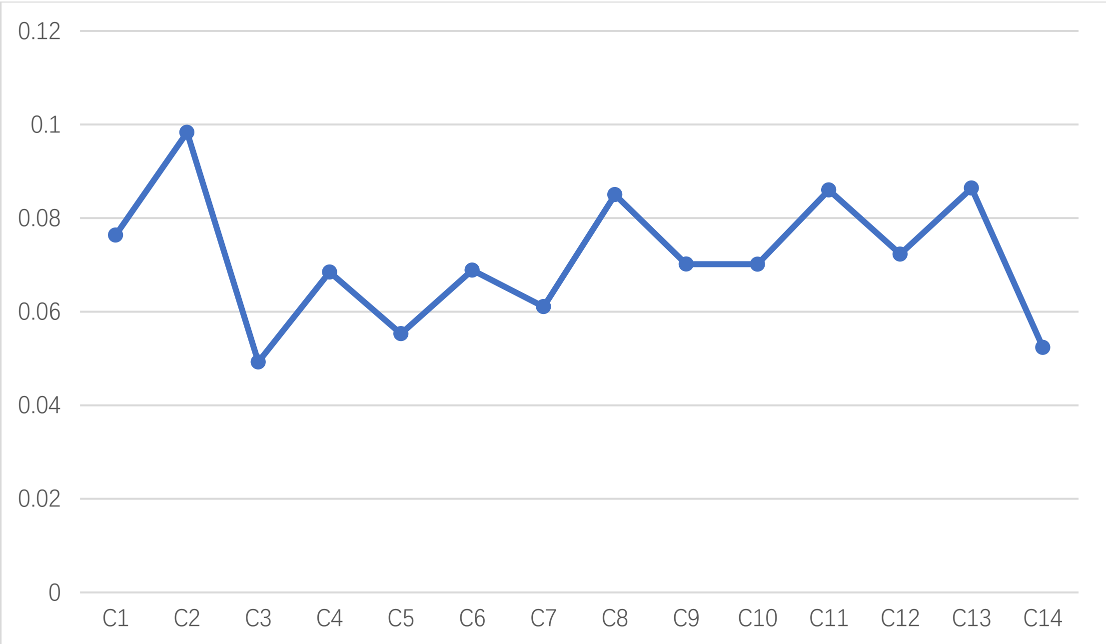

Figure 3. The criteria weights.

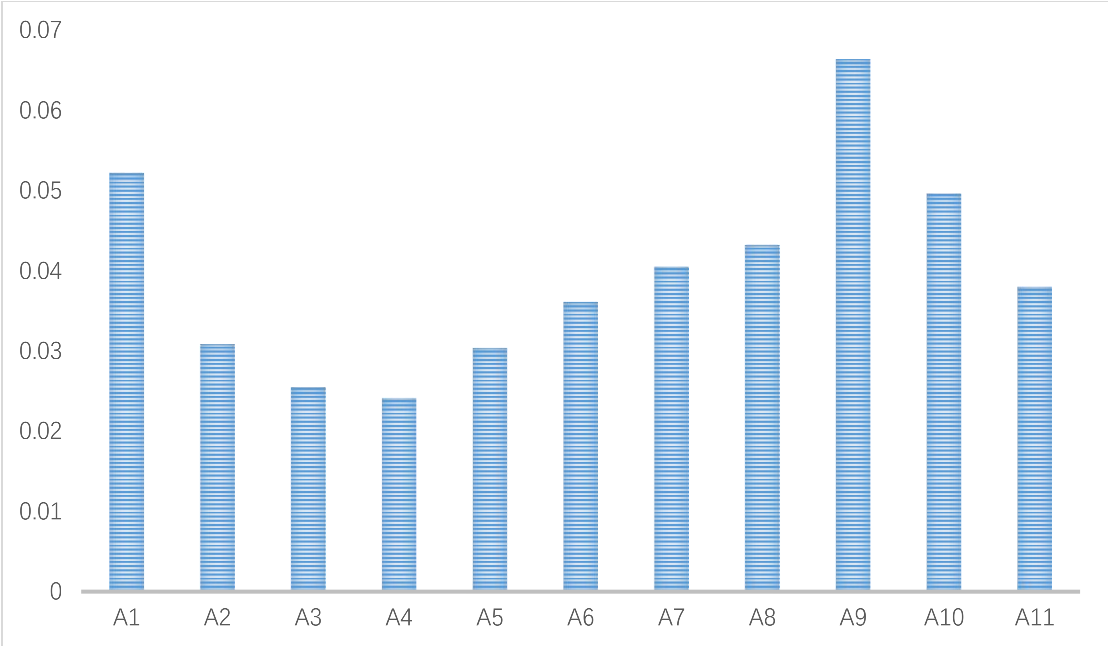

Figure 4. The rank of alternatives.

5.4. Managerial Implications

The constructed SVNNGPWGBM approach, based on the GGBM and PG methods, offers valuable insights into evaluating collaborative development patterns in the community medical and sports health industry, particularly through sports medicine integration. This approach provides a practical framework for MADM. Firstly, the SVNNGPWGBM approach incorporates SVNSs, enabling experts to express uncertainty effectively. This inclusion allows for a nuanced representation of expert opinions, which is crucial when dealing with complex decision-making scenarios. Secondly, the approach objectively determines the importance of attributes using the PG approach. By doing so, it rationally derives attribute weight information based on existing data. This enhances the practicality of the SVNNGPWGBM approach in evaluating collaborative development patterns within the community medical and sports health industry, ensuring decisions are grounded in reliable information. Thirdly, leveraging SVNSs, the GWBM approach, and the PG method, the SVNNGPWGBM approach effectively addresses the influence of unreasonable variables. It adeptly portrays the interrelationships among multiple variables, providing a more accurate ranking of collaborative development patterns. This flexibility makes it a practical tool for assessing the integration of sports medicine into community health initiatives. As a result, the SVNNGPWGBM approach offers an efficient method for evaluating collaborative development patterns in this industry. It serves as a valuable reference for decision-makers, guiding them towards more informed and reasonable decisions. By utilizing this approach, stakeholders can better navigate the complexities of integrating sports medicine with community health, ultimately enhancing collaborative efforts and outcomes.

6. Conclusion

Health is a timeless pursuit for humanity, providing the essential foundation for engaging in all productive activities and striving for a better life. The seamless integration and development of the sports and health industries play a crucial role in advancing China's national fitness and health strategies. They are vital not only for adjusting the country's economic structure in the context of the New Normal but also for fostering high-quality economic growth. Moreover, these industries are fundamental to building a prosperous, strong, democratic, civilized, and harmonious socialist modern state. To achieve these goals, it is imperative to align with contemporary trends, seize emerging opportunities, and address the diverse sports and health needs of the public. This involves catering to personalized, differentiated, and high-end consumption demands through strategic government policies, establishment of industrial clusters, application of advanced technologies, expansion of business opportunities, promotion of a healthy social atmosphere, and cultivation of versatile talents. By working together, we can drive the high-quality integrated development of the sports and health industries. Evaluating collaborative development patterns in the community medical and sports health industry, particularly through sports medicine integration, presents a classic MADM challenge. In this context, the SVNNGPWGBM approach is employed, building on the GSVNNWBM method. This approach is exemplified through a case study that evaluates collaborative development patterns, demonstrating its effectiveness.

The primary contributions of this study are highlighted as follows:

(1) Introduction of the SVNNGPWGBM Approach: This method is developed by integrating the WGBM approach with the power geometric method. This innovative approach offers a robust framework for addressing complex decision-making scenarios in the sports and health sectors.

(2) Use of the Maximizing Deviation Approach: To derive weight information, the maximizing deviation method is utilized. This ensures that the decision-making process is grounded in objective and reliable data, enhancing the accuracy and credibility of the evaluations.

(3) Application to Solve MADM Problems: The SVNNGPWGBM approach is effectively applied to solve MADM issues, demonstrating its versatility and applicability in real-world scenarios. This method provides decision-makers with a powerful tool to navigate the complexities of integrating sports medicine into community health initiatives.

(4) Case Study: An illustrative example is provided, showcasing the evaluation of collaborative development patterns in the community medical and sports health industry.

By implementing these strategies, the study offers valuable insights and practical solutions for promoting the integrated development of the sports and health industries. The findings serve as a useful reference for policymakers and stakeholders, guiding them in making informed decisions that enhance collaboration and drive progress in this vital sector. Through these efforts, we can contribute to a healthier, more prosperous society, aligned with the goals of modern economic and social development.

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