Neutrosophic Sets and Systems, Vol. 77, 2025
University of New Mexico

Neutrosophic SuperHyperSoft Sets

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Abstract:

Neutrosophic SuperHyperSoft Sets (NSHSS) represent an innovative extension of traditional soft set theory, integrating the principles of Neutrosophy to address uncertainties in decision-making processes. In this study, we recall the NSHSS and establish its foundational principles, including the relationships between various attributes and their corresponding values. The framework facilitates a structured evaluation process, allowing decision-makers to assess multiple proposals based on criteria such as environmental impact, feasibility, community engagement, cost-effectiveness, and innovation. Each criterion is categorized into distinct levels, ranging from low to transformative, thereby enabling a nuanced analysis of project proposals. Through illustrative examples and tabular representations, we demonstrate the practical applications of NSHSS in selecting projects with the highest potential for positive environmental impact. The findings indicate that NSHSS can significantly enhance the decision-making process, ensuring that resources are allocated efficiently to initiatives that align with sustainability goals.

Keywords: Neutrosophic SuperHyperSoft Sets; Extension of Neutrosophic HyperSoft Sets; Extension of SuperHyperSoft Sets; Environmental Sustainability.

1. Introduction and Preliminaries

The concept of soft set was introduced by Molodstov [5], a mathematical tool to deal with uncertainty associated with truth and false membership function. Soft set is a generalization of fuzzy set theory and rough set theory, aimed at providing a flexible approach for modeling problems where data is not always precisely defined. Further, Maji [3, 4] developed several basic operations within soft set theory and explored applications in decision making. HyperSoft Set was introduced by Smarandache [9], generalized a hypersoft set from the idea of soft set by replacing the argument function into multi argument function. In a hypersoft set, each parameter is not directly associated with a set of elements in the universal set. Instead, it is associated with a family of sub-parameters. These sub-parameters refine the information provided by the original parameters, offering more detailed granularity.

Neutrosophic set [8] was designed to handle uncertainty, incomplete information, inconsistency and indeterminacy data. Maji [2] later combined these ideas to present the Neutrosophic soft set, a novel model that integrates the concepts of soft sets and Neutrosophic sets. Truth, falsity membership and indeterminacy are Neutrosophic soft set (NSS) attributes that are independent in nature functions. Various sets have been proposed as hypothesis of the Neutrosophic soft set, with their properties and applications being exactly studied. Smarandache [1,7,11,12] also introduced the Neutrosophic hypersoft set, merging the principles of Neutrosophic and hypersoft sets. This new model aims to tackle uncertainty more accurately, which helps us better understand and apply these mathematical concepts.

Definition 1.1: [3] Let ɥ be the universal set and let € denote the set of attributes related to ɥ. Define P(ɥ) as the set of Neutrosophic values associated with ɥ, where A is a subset of 𝕀. The combination (F, A) is known as a Neutrosophic soft set over ɥ, and the relationship is expressed as F: A → (ɥ).

Definition 1.2: [9] Let ɥ represent the universal set and (ɥ) denotes its power set. Consider well defined attributes $\zeta 1\zeta, 2\zeta, 3\zeta \ldots, \mathbf{n}$ for $\mathbf{n} \geq 1$. The corresponding attributes values are the sets $L_1, L_2, L_3 \ldots L_n$ with $Li \cap Lj = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3 \ldots n\}$. The pair (F , $L_1 \times L_2 \times L_3 \times \ldots \times L_n$) is referred to as a Hypersoft set over ɥ with F being a function that maps where $F : L_1 \times L_2 \times L_3 \times \ldots \times L_n \rightarrow$ (ɥ).

Definition 1.3: [7] Let ɥ represent the universal set and (ɥ) be the power set of ɥ. Consider $\zeta 1\zeta, 2\zeta, 3, \ldots \zeta n$ for $n \geq 1$, along with the corresponding values represented by the set $L_1, L_2, L_3 \ldots L_n$ with $Li \cap Lj = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3 \ldots n\}$. The relationship among these sets is expressed as $L_1 \times L_2 \times L_3 \times rF : L_1 \times L_2 \times L_3 \times \ldots \times L_n \rightarrow$ (ɥ) and F($L_1 \times L_2 \times L_3 \times \ldots \times L_n$) = {< x, T(F(𝕀)), I(F(𝕀)), F(F(𝕀)) >, $x \in$ ɥ }. In this context T represents the truth membership value, I indicates indeterminate membership value and F denotes the falsity membership value such that T, I, F: ɥ → [0,1] also $0 \leq T$(F(𝕀)) + I(F(𝕀)) + F(F(𝕀)) ≤ 3.

2. Neutrosophic SuperHyperSoft Set

Definition 2.1 [10] Let Δ be the universe of discourse, $\mathcal{P}(\Delta)$ be the power set of Δ, Let $r_1, r_2, \ldots \ldots, r_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the set $R_1, R_2, \ldots \ldots, R_n$ with $R_i \cap R_j = \emptyset$ for $i \neq j$. Let $Þ(R_1), Þ(R_2), \ldots\ldots Þ(R_n)$ be the power sets of $R_1, R_2, \ldots \ldots, R_n$ respectively. A Neutrosophic SuperHyperSoft Set (NSHSS) over Δ as the pair $(\mathcal{S}, Þ(R_1) \times Þ(R_2) \times \ldots\ldots \times Þ(R_n))$ where $\mathcal{S}$: $\mathcal{P}(R_1) \times \mathcal{P}(R_2) \times \ldots\ldots \times \mathcal{P}(R_n) \rightarrow \mathcal{P}(\Delta)$ *and* $\mathcal{S}, (\mathcal{P}(R_1) \times \mathcal{P}(R_2) \times \ldots\ldots \times \mathcal{P}(R_n)) =$
$\{\alpha, \langle x, T_{\mathcal{S}(\alpha)}(x), I_{\mathcal{S}(\alpha)}(x), F_{\mathcal{S}(\alpha)}(x) \rangle : x \in \Delta, \alpha \in (Þ(R_1) \times Þ(R_2) \times \ldots\ldots \times Þ(R_n)) \subseteq R_1, R_2, \ldots \ldots, R_n\}$.

For example, in the context of student internship selection, decision-making based on multiple criteria is often challenging due to the imprecision and uncertainty associated with evaluating students. Traditional methods may not capture the complexity involved when different aspects such as academic performance, skills, and extracurricular activities need to be considered simultaneously. To address this, the Neutrosophic HyperSoft Set model can be applied, which allows for a multi-parameter and multi-attribute decision framework, providing more granular and flexible assessment.

The goal of this example is to assign internships to students based on the following main criteria:

1. Academic Performance
2. Skills
3. Extracurricular Activities.

For each of these criteria, evaluations are represented in neutrosophic terms, where the triplet (T, I, F) captures the truth, indeterminacy, and falsity degrees respectively. Two students, represented by F1 and F2 are considered for selection.

Table 1. Values for Students Selection for Internship using Neutrosophic Hypersoft Set

criteria	F1	F2
Academic performance	(.8, .1, .5)	(.9, .1, .1)
Skills	(.7, .15, .1)	(.8, .1, .05)
Extracurricular Activities	(.6, .2, .1)	(.7, .2, .1)

In the context of Neutrosophic SuperHypersoft Set, we assign internships to students based on the following main and sub criteria

1. Academic Performance (Cumulative grade point average and Continuous internal assessments)

2. Skills (Technical skills and soft skills)
3. Extracurricular Activities (Leadership roles and Volunteer work)

Table 2. Values for Students Selection for Internship using Neutrosophic Super Hypersoft Set

Main Criteria	Sub Criteria	F_1	F_2
Academic Performance	Cumulative grade point average	(.3, .5, .1)	(.1, .2, .3)
	Continuous internal assessments	(.6, .7, .4)	(.3, .4, .5)
Skills	Technical skills	(.4, .3, .3)	(.7, .2, .1)
	Soft skills	(.3, .1, .2)	(.5, .15, .1)
Extracurricular Activities	Leadership roles	(.7, .1, .05)	(.8, .2, .4)
	Volunteer work	(.6, .7, .4)	(.6, .4, .3)

The above two tables clearly explain the concepts of Neutrosophic Hypersoft Set and Neutrosophic Super Hypersoft Set.

2.3. Evaluating Proposals for an Environmental Sustainability Initiative

An organization is seeking to fund innovative projects that promote environmental sustainability within the community. Various teams have submitted proposals and the organization needs to evaluate these proposals based on specific criteria to determine which projects will receive funding.

The outcome of this evaluation process will be the selection of projects that demonstrate the greatest potential for positive environmental impact, community engagement and long-term sustainability. By funding these projects, the organization aims to contribute to the development of a more sustainable community and serve as a catalyst for further environmental initiatives. The evaluation will result in a prioritized list of projects that meet the established criteria, along with a clear justification for the selection. This outcome will guide the organization's decision-making process and ensure that the allocated funds are directed towards initiatives that have the highest likelihood of success and impact.

2.4 Selection Criteria for Proposal Evaluation

1. Impact on Environmental Sustainability:

   - ✓ Low: Minimal effect on environmental sustainability; does not address key issues.
   - ✓ Moderate: Some positive effects, such as reducing waste or energy consumption, but limited in scope.
   - ✓ High: Significant contributions to environmental sustainability, such as large-scale waste reduction, habitat restoration, or carbon footprint reduction.
   - ✓ Transformative: Projects that fundamentally change community practices or policies towards sustainability, leading to long-term environmental benefits.

2. Feasibility:

   - ✓ Low: High risk of failure due to lack of resources, unclear implementation plans, or significant barriers.
   - ✓ Moderate: Some challenges anticipated but manageable with available resources and planning.
   - ✓ High: Well-defined implementation plan with clear timelines, resources, and stakeholder engagement.
   - ✓ Exceptional: Strong evidence of previous success in similar projects, with robust support from the community and stakeholders.

3. Community Engagement:

- ✓ Low: Minimal involvement of community members or stakeholders in project planning or execution.
- ✓ Moderate: Some community involvement, but limited to feedback rather than active participation.
- ✓ High: Active participation from community members in all phases of the project, including planning, implementation, and evaluation.
- ✓ Exceptional: Strong partnerships with local organizations, schools, and residents, fostering a sense of ownership and commitment to the project.

4. Cost-Effectiveness:

- ✓ Low: High costs relative to the expected benefits; poor allocation of resources.
- ✓ Moderate: Reasonable costs with some benefits, but potential for better resource management.
- ✓ High: Good balance of costs and benefits, demonstrating efficient use of resources.
- ✓ Exceptional: Outstanding value for money, with innovative funding strategies and potential for scalability.

5. Innovation:

- ✓ Low: Standard approaches with little creativity or uniqueness.
- ✓ Moderate: Some novel ideas or methods, but largely based on existing practices.
- ✓ High: Creative and original approaches that introduce new concepts or technologies to address sustainability challenges.
- ✓ Transformative: Groundbreaking ideas that could redefine practices in sustainability and inspire similar projects elsewhere.

2.5 Decision-Making under Uncertain Conditions

The evaluation committee must review each proposal against these criteria while considering uncertainties such as varying levels of community support, potential regulatory challenges, and the availability of funding. For instance, a project that demonstrates high impact and exceptional feasibility may be prioritized over a moderately innovative project that has low community engagement. This structured evaluation process allows the organization to objectively assess proposals, ensuring that funding is allocated to projects that align with its sustainability goals and have the potential for meaningful environmental impact. By doing so, the organization can effectively contribute to fostering a more sustainable community.

Let ξ be the set of decision making in selecting suitable proposal $\xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ also consider the set of attributes as α^1 = Impact on Environmental Sustainability, α^2 = Feasibility, α^3 = Community Engagement, α^4 = Cost-Effectiveness, α^5 = Innovation and their respective attributes are given as

1. α^1= Impact on Environmental Sustainability = {Low Impact, Moderate Impact, High Impact, Transformative Impact}
2. α^2 = Feasibility = {Low Feasibility, Moderate Feasibility, High Feasibility, Transformative Feasibility}
3. α^3 = Community Engagement = {Low Community Engagement, Moderate Community Engagement, High Community Engagement, Transformative Community Engagement}

4. α^4 = Cost-Effectiveness = {Low Cost-Effectiveness, Moderate Cost-Effectiveness, High Cost-Effectiveness, Transformative Cost-Effectiveness}
5. α^5 = Innovation = {Low Innovation, Moderate Innovation, High Innovation, Transformative Innovation}

Let $W: \alpha^1 \times \alpha^2 \times \alpha^3 \times \alpha^4 \times \alpha^5 \to \alpha^n$. Below are the tables of their NSHS values

Table 3: Decision maker NSHS values for impact on environmental sustainability

Impact on Environmental Sustainability	ξ_1	ξ_2	ξ_3	ξ_4
Low Impact	(.2, .4, .6)	(.4, .5, .5)	(.7, .6, .3)	(.6, .3, .7)
Moderate Impact	(.6, .4, .7)	(.4, .3, .1)	(.7, .6, .2)	(.6, .3, .5)
High Impact	(.7, .3, .4)	(.6, .3, .5)	(.6, .5, .4)	(.4, .6, .3)
Transformative Impact	(.4, .6, .3)	(.6, .4, .2)	(.5, .3, .4)	(.5, .3, .2)

Table 4: Decision maker NSHS values for feasibility

Feasibility	ξ_1	ξ_2	ξ_3	ξ_4
Low Feasibility	(.5, .3, .4)	(.6, .4, .3)	(.7, .5, .6)	(.5, .4, .7)
Moderate Feasibility	(.3, .6, .4)	(.8, .4, .7)	(.5, .6, .7)	(.3, .6, .7)
High Feasibility	(.6, .1, .4)	(.7, .5, .3)	(.4, .5, .6)	(.4, .3, .5)
Transformative Feasibility	(.4, .6, .3)	(.6, .4, .2)	(.5, .3, .4)	(.5, .3, .2)

Table 5: Decision maker NSHS values for community engagement

Community Engagement	ξ_1	ξ_2	ξ_3	ξ_4
Low Community Engagement	(.5, .1, .2)	(.5, .2, .6)	(.2, .6, .5)	(.7, .6, .3)
Moderate Community Engagement	(.3, .1, .7)	(.4, .6, .3)	(.1, .4, .7)	(.1, .7, .4)
High Community Engagement	(.3, .4, .5)	(.6, .7, .3)	(.6, .3, .5)	(.2, .4, .5)
Transformative Community Engagement	(.6, .5, .6)	(.2, .4, .3)	(.8, .2, .1)	(.6, .4, .6)

Table 6: Decision maker NSHS values for cost-effectiveness

Cost-Effectiveness	ξ_1	ξ_2	ξ_3	ξ_4
Low Cost-Effectiveness	(.4, .6, .7)	(.4, .2, .5)	(.6, .2, .4)	(.5, .4, .4)
Moderate Cost-Effectiveness	(.8, .4, .5)	(.5, .3, .7)	(.7, .4, .7)	(.4, .6, .2)
High Cost-Effectiveness	(.5, .3, .2)	(.1, .4, .6)	(.5, .6, .4)	(.5, .4, .6)
Transformative Cost-Effectiveness	(.4, .3, .5)	(.4, .6, .3)	(.6, .1, .3)	(.6, .2, .3)

Table 7: Decision maker NSHS values for innovation

Innovation	ξ_1	ξ_2	ξ_3	ξ_4
Low Innovation	(.4, .6, .3)	(.3, .2, .5)	(.2, .7, .6)	(.5, .2, .1)
Moderate Innovation	(.2, .3, .4)	(.2, .4, .5)	(.4, .3, .2)	(.9, .4, .3)
High Innovation	(.6, .5, .4)	(.6, .5, .3)	(.3, .5, .4)	(.7, .3, .5)
Transformative Innovation	(.3, .2, .6)	(.7, .4, .6)	(.6, .4, .1)	(.4, .7, .6)

NSHSS is define as, $\boldsymbol{W}: \ \alpha^{\mathbf{1}} \times \alpha^{\mathbf{2}} \times \alpha^{\mathbf{3}} \times \alpha^{\mathbf{4}} \times \alpha^{\mathbf{5}} \to \alpha^{\boldsymbol{n}}(\boldsymbol{X})$.

Let us assume $W(V) = W$ (High Impact, High Feasibility, High Community Engagement) = {ξ_1, ξ_3}

Then NSHSS of above assumed relation is

$W(V) = W$ (High Impact, High Feasibility, High Community Engagement) = {< ξ_1, (High Impact (.7, .3, .4), High Feasibility (.6, .1, .4), High Community Engagement (.3, .4, .5)>, < ξ_3, (High Impact (.6, .5, .4), High Feasibility (.4, .5, .6), High Community Engagement (.6, .3, .5) >}

Table 8: Tabular Representation of NSHS Set

$W(V)$ = W (High Impact, High Feasibility, High Community Engagement)	ξ1	ξ3
High Impact	(.7, .3, .4)	(.6, .5, .4)
High Feasibility	(.6, .1, .4)	(.4, .5, .6)
High Community Engagement	(.3, .4, .5)	(.6, .3, .5)

Definition 2.6: Let $W(V^1)$ and $W(V^2)$ be two NSHS set over ξ. We have attributes $f^{1}, f^{2}, \dots f^{n}$ for $n \geq 1$, which are clearly defined attributes. The corresponding attributive values are the set $F^{1}, F^{2}, \dots F^{n}$ with $F^{i} \cap F^{j} = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3 \dots n\}$. Their relationship is described by $F^{1} \times F^{1} \times \dots \times F^{n} = V$. $W(V^1)$ is the NSHS subset of $W(V^2)$, if it meets the following conditions:

$$<T(W(V^1)) \leq T(W(V^2)), I(W(V^1)) \leq I(W(V^2)), F(W(V^1)) \geq F(W(V^2))>$$

Numerical Example of Subset

Consider the NSHSS W (V^1) and NSHSS $W(V^2)$ over the universe ξ = {ξ1, ξ2, ξ3, ξ4}. The NSHSS $W(V^1)$ = W (High Impact, High Feasibility, High Community Engagement) = {ξ1, ξ3} is the subset of NSHSS $W(V^2)$=W (High Impact, High Feasibility) = {ξ1, ξ3}.

Table 9: Tabular Representation of NSHSS $W(V^1)$

$W(V^1)$ = W (High Impact, High Feasibility, High Community Engagement)	ξ1	ξ3
High Impact	(.7, .3, .4)	(.6, .5, .4)
High Feasibility	(.6, .1, .4)	(.4, .5, .6)
High Community Engagement	(.3, .4, .5)	(.6, .3, .5)

Table 10: Tabular Representation of NSHSS $W(V^2)$

$W(V^2)$ = W (High Impact, High Feasibility)	ξ1
High Impact	(.8, .5, .3)
High Feasibility	(.7, .3, .2)

This can also be written as $W(V^1) \subset W(V^2) = W$ (High Impact, High Feasibility, High Community Engagement) ⊂ W (High Impact, High Feasibility) , {< ξ1, High Impact, (.7, .3, .4), High Feasibility (.6, .1, .4), High Community Engagement (.3, .4, .5) >, < ξ3, High Impact (.6, .5, .4), High Feasibility (.4, .5, .6), High Community Engagement (.6, .3, .5) >} ⊂ {< ξ1, High Impact (.8, .5, .3), High Feasibility (.7, .3, .2)> }

Definition 2.7: Let $W(V^1)$ and $W(V^2)$ be two NSHS set over ξ. For $n \geq 1$, consider the well-defined attributes $f^{1}, f^{2}, \dots f^{n}$. The corresponding values of these attributes forms the sets $F^{1}, F^{2}, \dots F^{n}$ with $F^{i} \cap F^{j} = \emptyset$, for $i \neq j$, $i, j \in \{\{1, 2, 3 \dots n\}$. The relation among these sets is expressed as $F^{1} \times F^{1} \times \dots \times F^{n} = V$. The NSHS set $W(V^1)$ is deemed equal to the NSHS $W(V^2)$ if the following conditions are met:

$$< T(W(V^1)) = T(W(V^2)), I(W(V^1)) = I(W(V^2)), F(W(V^1)) = F(W(V^2))>$$

Consider the NSHSS $W(V^1)$ and $W(V^2)$ over the same universe ξ = {ξ1, ξ2, ξ3, ξ4}. The NSHSS W(V^1) = W (High Impact, High Feasibility, High Community Engagement) = {ξ1, ξ3} is the equal to NSHSS $W(V^2)$ = W (High Impact, High Feasibility) = {ξ1 } if $T(W(V^1)) = T(W(V^2))$, $I(W(V^1))$ =

$I(W(V^2))$, $F(W(V^1))$ = $F(W(V^2))$

10: Tabular Representation of NSHSS $W(V^1)$

$W(V^1)$ = W (High Impact, High Feasibility, High Community Engagement)	ξ1	ξ3
High Impact	(.7, .3, .4)	(.6, .5, .4)
High Feasibility	(.6, .1, .4)	(.4, .5, .6)
High Community Engagement	(.3, .4, .5)	(.6, .3, .5)

Table 11: Tabular Representation of $W(V^2)$

$W(V^2)$ = W (High Impact, High Feasibility)	ξ1
High Impact	(.7, .3, .4)
High Feasibility	(.6, .1, .4)

This can also be written as $W(V^1)$ = W(V2) = W (High Impact, High Feasibility, High Community Engagement)= W (High Impact, High Feasibility), { < ξ1, (High Impact (.7, .3, .4), High Feasibility (.6, .1, .4), High Community Engagement (.3, .4, .5) >, < ξ3, (High Impact (.6, .5, .4), High Feasibility (.4, .5, .6), High Community Engagement (.6, .3, .5) >} = {< ξ1, (High Impact (.7, .3, .4), High Feasibility (.6, .1, .4)>}

Definition 2.8: Let $W(V^1)$ and $W(V^2)$ stand for two NSHS set over ξ. Consider the attributes $f^1, f^2, \ldots f^n$ for $n \geq 1$. These attributes have corresponding values in the sets $F^1, F^2, \ldots F^n$ with $F^i \cap F^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3 \ldots n\}$. Their relationship can be expressed as $F^1 \times F^1 \times \ldots \times F^n = V$. Futhermore, $W(V^1)$ is classified as null NSHSS i f < $T(W(V^1))$ = 0, $I(W(V^1))$ = 0, $F(W(V^1))$ = 0.

Numerical Example of Null NSHS Set

Consider the NSHSS $W(V^1)$ over the ξ = {ξ1, ξ2, ξ3, ξ4}. The NSHSS $W(V^1)$ = W (High Impact, High Feasibility, High Community Engagement) = {ξ1, ξ3} is said to be null NSHSS if its Neutrosophic values are 0.

Table 12: Tabular Representation of NSHSS $W(V^1)$

$W(V^1)$ = W (High Impact, High Feasibility, High Community Engagement)	ξ1	ξ3
High Impact	(0, 0, 0)	(0, 0, 0)
High Feasibility	(0, 0, 0)	(0, 0, 0)
High Community Engagement	(0, 0, 0)	(0, 0, 0)

This can be written as $W(V^1)$= W (High Impact, High Feasibility, High Community Engagement) = W (High Impact, High Feasibility) {< ξ1, (High Impact (0, 0, 0), High Feasibility (0, 0, 0), High Community Engagement (0, 0, 0) >, < ξ3, (High Impact (0, 0, 0), High Feasibility (0, 0, 0), High Community Engagement (0, 0, 0) >}

Definition 2.9: Let $W(V^1)$ and $W(V^2)$ represent two NSHS set over ξ. Let $f^1, f^2, \ldots f^n$ for $n \geq 1$, be well defined attributes, with corresponding values as sets $F^1, F^2, \ldots F^n$ with $F^i \cap F^j = \emptyset$, for $i \neq j$ and $i, j \in \{\{1, 2, 3 \ldots n\}$. The relationship among these sets is given by $F^1 \times F^1 \times \ldots \times F^n = V$. In this context, the complement $W^c(V^1)$ is the complement of NSHSS of $W(V^1)$. It can be expressed as $W^c(V^1)$: $(\neg F^1 \times \neg F^1 \times \ldots \neg F^n) \rightarrow P^n(X)$ ensuring that < $T^C(W(V^1))$ = $F(W(V^1))$, $I^C(W(V^1))$ = $I(W(V^1))$, $F^C(W(V^1))$ = $T(W(V^1))$>

Numerical Example of Compliment of NHSS

Consider the NSHSS $W(V^1)$ over the universe ξ = {ξ1, ξ2, ξ3, ξ4}. The compliment of NSHSS $W(V^1)$ = W (Transformative Impact, Transformative Feasibility, Transformative Innovation) = {ξ1, ξ3} is given as $T(W(V^1)) = F(W(V^1)), I^C(W(V^1)) = I(W(V^1)), F^C(W(V^1)) = T(W(V^1))$.

Table 13: Tabular Representation of NSHSS $W(V^1)$

$W(V^1)$ = W **(Transformative Impact, Transformative Feasibility, Transformative Innovation)**	ξ1	ξ3
Transformative Impact	(.4, .6, .3)	(.5, .3, .4)
Transformative Feasibility	(.4, .6, .3)	(.5, .3, .4)
Transformative Innovation	(.3, .2, .6)	(.6, .4, .1)

Table 14: Tabular Representation of NSHSS $W(V^1)$

$W^c(V^1)$ = W **(Not Transformative Impact, Not Transformative Feasibility, Not Transformative Innovation)**	ξ1	ξ3
Not Transformative Impact	(.3, .6, .4)	(.4, .3, .5)
Not Transformative Feasibility	(.3, .6, .4)	(.4, .3, .5)
Not Transformative Innovation	(.6, .2, .3)	(.1, .4, .6)

This might also be written as $W^c(V^1)$ = W (Not Transformative Impact, Not Transformative Feasibility, Not Transformative Innovation) = {< ξ1, (Not Transformative Impact (.3, .6, .4), Not Transformative Feasibility (.3, .6, .4), Not Transformative Innovation (.6, .2, .3)) >, < ξ3, (Not Transformative Impact (.4, .3, .5), Not Transformative Feasibility (.4, .3, .5), Not Transformative Innovation (.1, .4, .6) >}

Definition 2.10: Let $\boldsymbol{W(V^1)}$ and $\boldsymbol{W(V^2)}$ represents two NSHS set over ξ. Consider $\boldsymbol{f^1, f^2, \dots f^n}$ for $\boldsymbol{n \geq 1}$, be well defined attributes, whose attributes values are respectively the set $\boldsymbol{F^1, F^2, \dots F^n}$ with $\boldsymbol{F^i \cap F^j = \emptyset}$, for $i \neq j$ and $\boldsymbol{i, j \in \{\{1, 2, 3 \dots n\}}$ and their relation $\boldsymbol{F^1 \times F^1 \times \dots \times F^n = V}$. The union of $\boldsymbol{W(V^1)}$ and $\boldsymbol{W(V^2)}$ is represented as $\boldsymbol{W(V^1) \cup W(V^2)}$ is given as

$$T(W(V^1) \cup W(V^2)) = \begin{cases} T\left(W\left(V^1\right)\right) if\ x \in V^1 \\ T\left(W\left(V^2\right)\right) if\ x \in V^2 \\ \max\left(T\left(W\left(V^1\right)\right), T\left(W\left(V^2\right)\right)\right) if\ x \in V^1 \cap V^2 \end{cases}$$

$$I(W(V^1) \cup W(V^2)) = \begin{cases} I\left(W\left(V^1\right)\right) if\ x \in V^1 \\ I\left(W\left(V^2\right)\right) if\ x \in V^2 \\ \dfrac{I\left(W\left(V^1\right)\right) + I\left(W\left(V^2\right)\right)}{2} if\ x \in V^1 \cap V^2 \end{cases}$$

$$F(W(V^1) \cup W(V^2) = \begin{cases} F\left(W\left(V^1\right)\right) if\ x \in V^1 \\ F\left(W\left(V^2\right)\right) if\ x \in V^2 \\ \min\left(F\left(W\left(V^1\right)\right), F\left(W\left(V^2\right)\right)\right) if\ x \in V^1 \cap V^2 \end{cases}$$

Numerical Example of Union

Consider two NSHSS $W(V^1)$ and NSHSS $W(V^2)$ over ξ = {ξ1, ξ2, ξ3, ξ4}. The NSHSS $W(V^1)$ = W (Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness, Moderate Innovation) = {ξ2, ξ4} and NSHSS $W(V^2)$ = W (Moderate Feasibility, Moderate Cost-Effectiveness, Moderate Innovation) = {ξ2}

Table 15: Tabular Representation of NSHSS $W(V^1)$

$W(V^1)$ = W **(Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness, Moderate Innovation)**	ξ2	ξ4
Moderate Feasibility	(.8, .4, .7)	(.3, .6, .7)
Moderate Community Engagement	(.4, .6, .3)	(.1, .7, .4)
Moderate Cost-Effectiveness	(.5, .3, .7)	(.4, .6, .2)
Moderate Innovation	(.2, .4, .5)	(.9, .4, .3)

Table 16: Tabular Representation of NSHSS $W(V^2)$

$W(V^2)$ = W **(Moderate Feasibility, Moderate Cost-Effectiveness, Moderate Innovation)**	ξ2
Moderate Feasibility	(.8, .6, .5)
Moderate Cost-Effectiveness	(.6, .7, .2)
Moderate Innovation	(.3, .6, .3)

The union of above NSHSS is given as

Table 17: Union of NSHSS $W(V^1)$ and NSHSS $W(V^2)$

$W(V^1) \cup W(V^2)$	ξ2	ξ4
Moderate Feasibility	(.8, .5, .5)	(.3, .6, .7)
Moderate Community Engagement	(.4, .3, .0)	(.1, .7, .4)
Moderate Cost-Effectiveness	(.6, .5, .2)	(.4, .6, .2)
Moderate Innovation	(.3, .5, .3)	(.9, .4, .3)

This can also be written as $W(V^1) \cup W(V^2)$ = W (Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness, Moderate Innovation) ∪ W (Moderate Feasibility, Moderate Cost-Effectiveness, Moderate Innovation)= {< ξ2, (Moderate Feasibility (.8, .5, .5), Moderate Community Engagement (.4, .3, .0), Moderate Cost-Effectiveness (.6, .5, .2), Moderate Innovation (.3, .5, .3)) >, < ξ4, (Moderate Feasibility (.3, .6, .7), Moderate Community Engagement (.1, .7, .4), Moderate Cost-Effectiveness (.4, .6, .2), Moderate Innovation (.9, .4, .3)) >}

Definition 2.11: Let $W(V^1)$ and $W(V^2)$ represents two NSHS set over ξ. Consider $f^1, f^2, \ldots f^n$ for $n \geq 1$. The attributes have corresponding values that form the sets $F^1, F^2, \ldots F^n$ with $F^i \cap F^j = \emptyset$, for $i \neq j$ and $i, j \in \{\{1, 2, 3 \ldots n\}$. The relationship among these sets is expressed $F^1 \times F^1 \times \ldots \times F^n =$

$\boldsymbol{V}$. Consequently the intersection of $\boldsymbol{W}(\boldsymbol{V}^{1})$ and $\boldsymbol{W}(\boldsymbol{V}^{2})$ is represented as $\boldsymbol{W}(\boldsymbol{V}^{1}) \cap \boldsymbol{W}(\boldsymbol{V}^{2})$ is given as

$$T\left(W(V^1) \cap W(V^2)\right) = \begin{cases} T\left(W\left(V^1\right)\right) if\ x \in V^1 \\ T\left(W\left(V^2\right)\right) if\ x \in V^2 \\ \min\left(T\left(W\left(V^1\right)\right), T\left(W\left(V^2\right)\right)\right) if\ x \in V^1 \cap V^2 \end{cases}$$

$$I(W(V^1) \cap W(V^2)) = \begin{cases} I\left(W\left(V^1\right)\right) if\ x \in V^1 \\ I\left(W\left(V^2\right)\right) if\ x \in V^2 \\ \dfrac{I\left(W\left(V^1\right)\right) + I\left(W\left(V^2\right)\right)}{2} if\ x \in V^1 \cap V^2 \end{cases}$$

$$F(W(V^1) \cap W(V^2)) = \begin{cases} F\left(W\left(V^1\right)\right) if\ x \in V^1 \\ F\left(W\left(V^2\right)\right) if\ x \in V^2 \\ \max\left(F\left(W\left(V^1\right)\right), F\left(W\left(V^2\right)\right)\right) if\ x \in V^1 \cap V^2 \end{cases}$$

Numerical Example of Intersection

Consider the two NSHSS $\boldsymbol{W}(\boldsymbol{V}^{1})$ and NSHSS $\boldsymbol{W}(\boldsymbol{V}^{2})$ over the same universe $\xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$. The NSHSS $\boldsymbol{W}(\boldsymbol{V}^{1}) = \boldsymbol{W}$ (Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness, Moderate Innovation) = $\{\xi_2, \xi_4\}$ and NSHSS $\boldsymbol{W}(\boldsymbol{V}^{2}) = \boldsymbol{W}$ (Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness) = $\{\xi_2\}$.

Table 18: Tabular Representation of NSHSS $W(V^1)$

$\boldsymbol{W}(\boldsymbol{V}^{1}) = \boldsymbol{W}$ **(Moderate Feasibility, Moderate Engagement, Moderate Possibility, Moderate Innovation)**	ξ_2	ξ_4
Moderate Feasibility	(.8, .4, .7)	(.3, .6, .7)
Moderate Community Engagement	(.4, .6, .3)	(.1, .7, .4)
Moderate Cost-Effectiveness	(.5, .3, .7)	(.4, .6, .2)
Moderate Innovation	(.2, .4, .5)	(.9, .4, .3)

Table 19: Tabular Representation of NSHSS $W(V^2)$

$\boldsymbol{W}(\boldsymbol{V}^{2}) = \boldsymbol{W}$ **(Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness)**	ξ_2
Moderate Feasibility	(.8, .6, .5)
Moderate Community Engagement	(.6, .7, .2)
Moderate Cost-Effectiveness	(.5, .7, .2)

The intersection of above NSHSS is given as:

Table 20: Intersection of NSHSS $W(V^1)$ and NSHSS $W(V^2)$

$\boldsymbol{W}(\boldsymbol{V}^{1}) \cap \boldsymbol{W}(\boldsymbol{V}^{2})$	ξ_2
Moderate Feasibility	(.8, .5, .7)
Moderate Community Engagement	(.4, .7, .3)
Moderate Cost-Effectiveness	(.5, .5, .7)

Moderate Innovation	(0, .4, .5)

This may also be written as $W(V^1) \cap W(V^2)$ = W (Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness, Moderate Innovation) ∩ W (Moderate Feasibility, Moderate Community Engagement, Moderate Cost-Effectiveness) = {< ξ_2, (Moderate Feasibility (.8, .5, .7), Moderate Community Engagement (.4, .7, .3), Moderate Cost-Effectiveness (.5, .5, .7), Moderate Innovation (0, .4, .5) >}

Definition 2.12: Let $\boldsymbol{W(V^1)}$ and $\boldsymbol{W(V^2)}$ represents two NSHS set over ξ. Consider the attributes $\boldsymbol{f^1, f^2, \dots f^n}$ for $\boldsymbol{n \geq 1}$, denoting n distinct attributes. The related attributes values are indicated by the sets $\boldsymbol{F^1, F^2, \dots F^n}$ with $\boldsymbol{F^i \cap F^j = \emptyset}$, for $i \neq j$ and $\boldsymbol{i, j \epsilon \{1, 2, 3 \dots n\}}$. Their connection can be represented as $\boldsymbol{F^1 \times F^1 \times \dots \times F^n = V}$. The AND operation is defined as $\boldsymbol{W(V^1) \wedge W(V^2) = W(V^1 \times V^2)}$ and is expressed by the following conditions:

$$T(V^1 \times V^2) = min(T(W(V^1)), T(W(V^2)))$$
$$I(V^1 \times V^2) = \frac{(I(W(V^1)), I(W(V^2)))}{2}$$
$$F(V^1 \times V^2) = max(F(W(V^1)), F(FW(V^2)))$$

Numerical example of AND operation

Consider the two NSHSS $W(V^1)$ and NSHSS $W(V^2)$ over the same universe ξ = {ξ_1, ξ_2, ξ_3, ξ_4}. The NSHSS $W(V^1)$ = W (Low Impact, Low Cost-Effectiveness, Low Innovation) = {ξ_1, ξ_4} and NSHSS $W(V^2)$ = W (Low Impact, Low Innovation) = {ξ_1}.

Table 21: Tabular Representation of NSHSS $\boldsymbol{W(V^1)}$

$\boldsymbol{W(V^1)}$ = $\boldsymbol{W}$ **(Low Impact, Low Cost-Effectiveness, Low Innovation)**	ξ_1	ξ_4
Low Impact	(.2, .4, .6)	(.6, .3, .7)
Low Cost-Effectiveness	(.4, .6, .7)	(.5, .4, .4)
Low Innovation	(.4, .6, .3)	(.5, .2, .1)

Table 22: Tabular Representation of NSHSS $\boldsymbol{W(V^2)}$

$\boldsymbol{W(V^2)}$ = $\boldsymbol{W}$ **(Low Impact, Low Innovation)**	ξ_1
Low Impact	(.3, .6, .3)
Low Innovation	(.5, .8, .2)

The AND operation of above NSHSS is given as:

Table 23: AND operation of NSHSS $\boldsymbol{W(V^1)}$ and NSHSS $\boldsymbol{W(V^2)}$

$\boldsymbol{W(V^1) \wedge W(V^2)}$	ξ_1	ξ_4
Low Impact × Low Impact	(.2, .5, .6)	(0, .15, .7)
Low Impact × Low Innovation	(.2, .6, .6)	(0, .15, .7)
Low Cost-Effectiveness × Low Impact	(.3, .6, .7)	(0, .2, .4)
Low Cost-Effectiveness × Low Innovation	(.4, .7, .7)	(0, .2, .4)
Low Innovation ×High Impact	(.3, .6, .3)	(0, .1, .1)
Low Innovation × Low Innovation	(.4, .7, .3)	(0, .1, .1)

Definition 2.13: Let $\boldsymbol{W(V^1)}$ and $\boldsymbol{W(V^2)}$ be two NSHSS over ξ. Consider $\boldsymbol{f^1, f^2, \dots f^n}$ for $\boldsymbol{n \geq 1}$ Denoting n distinct attributes. The related attributes values are indicated by the sets $\boldsymbol{F^1, F^2, \dots F^n}$ with $\boldsymbol{F^i \cap F^j = \emptyset}$, for $\boldsymbol{i \neq j}$ and $\boldsymbol{i, j \epsilon \{1, 2, 3 \dots n\}}$. Their connection can be represented as $\boldsymbol{F^1 \times F^1 \times \dots \times F^n = V}$. The OR operation is defined as $\boldsymbol{W(V^1) \vee W(V^2) = W(V^1 \times V^2)}$ and is expressed

by the following conditions:

$$T(V^1 \times V^2) = max(\,T(W(V^1)), T(W(V^2)))$$
$$I(V^1 \times V^2) = \frac{(I(W(V^1)), I(W(V^2)))}{2}$$
$$F(V^1 \times V^2) = min(\,F(W(V^1)), F(W(V^2)))$$

Numerical example of OR operation

Consider the two NSHSS $W(V^1)$ and NSHSS $W(V^2)$ over the same universe $\xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$. The NSHSS $W(V^1)$ = W (Low Impact, Low Cost-Effectiveness, Low Innovation) = $\{\xi_1, \xi_4\}$ and NSHSS $W(V^2)$ = W (Low Impact, Low Innovation) = $\{\xi_1\}$.

Table 24: Tabular Representation of NSHSS $W(V^1)$

$W(V^1)$ = W (Low Impact, Low Cost-Effectiveness, Low Innovation)	ξ_1	ξ_4
Low Impact	(.2, .4, .6)	(.6, .3, .7)
Low Cost-Effectiveness	(.4, .6, .7)	(.5, .4, .4)
Low Innovation	(.4, .6, .3)	(.5, .2, .1)

Table 25: Tabular Representation of NSHSS $W(V^2)$

$W(V^2)$ = W (Low Impact, Low Innovation)	ξ_1
Low Impact	(.3, .6, .3)
Low Innovation	(.5, .8, .2)

Then the OR Operation of above NSHSS is given as :

Table 26: OR operation of NSHSS $W(V^1)$ and NSHSS $W(V^2)$

$W(V^1) \wedge W(V^2)$	ξ_1	ξ_4
Low Impact × Low Impact	(.3, .5, .3)	(.6, .15, 0)
Low Impact × Low Innovation	(.5, .6, .2)	(.6, .15, 0)
Low Cost-Effectiveness × Low Impact	(.4, .6, .3)	(.5, .2, 0)
Low Cost-Effectiveness × Low Innovation	(.5, .7, .2)	(.5, .2, 0)
Low Innovation ×High Impact	(.4, .6, .3)	(.5, .1, 0)
Low Innovation × Low Innovation	(.5, .7, .2)	(.5, .1, 0)

5. Conclusions

This paper discusses the operations of Neutrosophic SuperHyperSoft Set, including union, intersection, compliment, AND and OR operations are presented. It demonstrates the validity and application of these operations through appropriate examples. The Neutrosophic SuperHyperSoft Set (NSHSS) is recalled as a new tool that can be useful for decision-making problems, particularly in suitable selections.

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Received: Aug 14, 2024. Accepted: Nov 5, 2024