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NEUTROSOPHIC SOFT MULTI-ATTRIBUTE DECISION MAKING BASED ON GREY RELATIONAL PROJECTION METHOD

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Abstract. The present paper proposes neutrosophic soft multi-attribute decision making based on grey relational projection method. Neutrosophic soft sets is a combination of neutrosophic sets and soft sets and it is a new mathematical apparatus to deal with realistic problems in the fields of medical sciences, economics, engineering, etc. The rating of alternatives with respect to choice parameters is represented in terms of neutrosophic soft sets. The weights of the choice parameters are completely unknown to the decision maker and information entropy method is used to determine unknown weights. Then, grey relational projection method is applied in order to obtain the ranking order of all alternatives. Finally, an illustrative numerical example is solved to demonstrate the practicality and effectiveness of the proposed approach.

Keywords: Neutrosophic sets; Neutrosophic soft sets; Grey relational projection method; Multi-attribute decision making.

1 Introduction

In real life, we often encounter many multi-attribute decision making (MADM) problems that cannot be described in terms of crisp numbers due to inderminacy and inconsistency of the problems. Zadeh [1] incorporated the degree of membership and proposed the notion of fuzzy set to handle uncertainty. Atanassov [2] introduced the degree of non-membership and defined intuitionistic fuzzy set to deal with imprecise or uncertain decision information. Smarandache [3, 4, 5, 6] initiated the idea of neutrosophic sets (NSs) by using the degree of indeterminacy as independent component to deal with problems involving imprecise, indeterminate and inconsistent information which usually exist in real situations. In NSs, indeterminacy is quantified and the truth-membership, indeterminacy-membership, falsity-membership functions are independent and they assume the value from $]^{-}0, 1^{+}[$. However, from scientific and realistic point of view Wang et al. [7] proposed single valued NSs (SVNSs) and then presented the set theoretic operators and various properties of SVNSs.

Molodtsov [8] introduced the soft set theory for dealing with uncertain, fuzzy, not clearly described objects in 1999. Maji et al. [9] applied the soft set theory for solving decision making problem. Maji et al. [10] also defined the operations AND, OR, union, intersection of two soft sets and also proved several propositions on soft set operations. However, Ali et al. [11] and Yang [12] pointed out that some assertions of Maji et al. [10] are not true in general, by counterexamples. The soft set theory have received a great deal of attention from the researchers and many researchers have combined soft sets with other sets to make different hybrid structures like fuzzy soft sets [13], intuitionistic fuzzy soft sets [14], vague soft sets [15] generalized fuzzy soft sets [16], generalized intuitionistic fuzzy soft [17], possibility vague soft set [18], etc. The different hybrid systems have had quite impact on solving different practical decision making problems such as medical diagnosis [16, 18], plot selection, object recognition [19], etc where data set are imprecise and uncertain. Maji et al. [13, 14] incorporated fuzzy soft sets and intuitionistic soft sets based on the nature of the parameters involved in the soft sets. Cağman et al. [20] redefined fuzzy soft sets and their properties and then developed fuzzy soft aggregation operator for decision problems. Recently, Maji [21] introduced the concept of neutrosophic soft sets (NSSs) which is a combination of neutrosophic sets [3, 4, 5, 6] and soft sets [8], where the parameters are neutrosophic sets. He also introduced several definitions and operations on NSSs and presented an application of NSSs in house selection problem. Maji

[22] further studied weighted NSSs by imposing some weights on the parameters. Based on the concept of weighted NSSs, Maji [23] solved a multi-criteria decision making problem.

MADM problem generally comprises of selecting the most suitable alternative from a set of alternatives with respect to their attributes and it has received much attention to the researchers in the field of decision science, management, economics, investment [24, 25], school choice [26], etc. Grey relational analysis (GRA) [27] is an effective tool for modeling MADM problems with complicated interrelationships between numerous factors and variables. GRA is applied in a range of MADM problems such as agriculture, economics, hiring distribution [28], marketing, power distribution systems [29], personal selection, teacher selection [30], etc. Biswas et al. [24] investigated entropy based GRA method for solving MADM problems under single valued neutrosophic assessments. Biswas et al. [25] also studied GRA based single valued neutrosophic MADM problems with incomplete weight information. Mondal and Pramanik [26] presented a methodological approach to select the best elementary school for children using neutrosophic MADM with interval weight information based on GRA. Mondal and Pramanik [31] also developed rough neutrosophic MADM based on modified GRA.

Zhang et al. [32] developed a new grey relational projection (GRP) method for solving MADM problems in which the attribute value takes the form of intuitionistic trapezoidal fuzzy number, and the attribute weights are unknown. In this paper, we have extended the concept of Zhang et al. [32] to develop a methodology for solving neutrosophic soft MADM problems based on grey relational projection method with unknown weight information.

Rest of the paper is organized as follows. Section 2 presents some definitions concerning NS, SVNS, soft sets, and neutrosophic soft sets. A neutrosophic soft MADM based on GRP method is discussed in Section 3. In Section 4, we have solved a numerical example in order to demonstrate the proposed procedure. Finally, Section 5 concludes the paper.

2 Preliminaries

In this section we briefly present some basic definitions regarding NSs, SVNSs, soft sets, and NSSs.

2.1 Neutrosophic set

Definition 1 [3, 4, 5, 6] Consider X be a universal space of objects (points) with generic element in X denoted by x. Then a NS is defined as follows:

$$A = \{x, \langle T_A(x), I_A(x), F_A(x) \rangle \mid x \in X\}.$$

where, $T_A(x)$, $I_A(x)$, $F_A(x): X \rightarrow\ ]^-0, 1^+[$ are the truth-membership, indeterminacy-membership, and falsity-membership functions, respectively and $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$. We consider the NS which assmes the value from the subset of [0, 1] because] $^-0$, 1^+[will be hard to apply in real world science and engineering problems.

Definition 2 [7] Let X be a universal space of points with generic element in X represented by x. Then a SVNS $\tilde{N} \subset X$ is characterized by a truth-membership function $T_{\tilde{N}}(x)$, a indeterminacy-membership function $I_{\tilde{N}}(x)$, and a falsity-membership function $F_{\tilde{N}}(x)$ with $T_{\tilde{N}}(x)$, $I_{\tilde{N}}(x)$, $F_{\tilde{N}}(x): X \rightarrow [0, 1]$ for each point $x \in X$ and we have, $0 \le \sup T_{\tilde{N}}(x) + \sup I_{\tilde{N}}(x) + \sup F_{\tilde{N}}(x) \le 3$.

Definition 3 [7] The complement of a SVNS $\tilde{N}$ is represented by $\tilde{N}^C$ and is defined by

$T_{\tilde{N}^C}(x) = F_{\tilde{N}}(x)$; $I_{\tilde{N}^C}(x) = 1 - I_{\tilde{N}}(x)$; $F_{\tilde{N}^C}(x) = T_{\tilde{N}}(x)$

Definition 4 [7] For two SVNSs $\tilde{N}_A$ and $\tilde{N}_B$

$$\tilde{N}_A = \{x, \langle T_{\tilde{N}_A}(x), I_{\tilde{N}_A}(x), F_{\tilde{N}_A}(x) \rangle \mid x \in X\}$$

and

$$\tilde{N}_B = \{x, \langle T_{\tilde{N}_B}(x), I_{\tilde{N}_B}(x), F_{\tilde{N}_B}(x) \rangle \mid x \in X\}$$

1. $\tilde{N}_A \subseteq \tilde{N}_B$ if and only if

$T_{\tilde{N}_A}(x) \le T_{\tilde{N}_B}(x)$; $I_{\tilde{N}_A}(x) \ge I_{\tilde{N}_B}(x)$; $F_{\tilde{N}_A}(x) \ge F_{\tilde{N}_B}(x)$

2. $\tilde{N}_A = \tilde{N}_B$ if and only if

$T_{\tilde{N}_A}(x) = T_{\tilde{N}_B}(x)$; $I_{\tilde{N}_A}(x) = I_{\tilde{N}_B}(x)$; $F_{\tilde{N}_A}(x) = F_{\tilde{N}_B}(x)$; $\forall\, x \in X$.

3. $\tilde{N}_A \cup \tilde{N}_B$

$$= \{\left\langle \begin{array}{l} x, \max\{T_{\tilde{N}_A}(x), T_{\tilde{N}_B}(x)\}, \min\{I_{\tilde{N}_A}(x), I_{\tilde{N}_B}(x)\}, \\ \min\{F_{\tilde{N}_A}(x), F_{\tilde{N}_B}(x)\} \mid x \in X \end{array} \right\rangle\}$$

4. $\tilde{N}_A \cap \tilde{N}_B$

$$= \{\left\langle \begin{array}{l} x, \min\{T_{\tilde{N}_A}(x), T_{\tilde{N}_B}(x)\}, \max\{I_{\tilde{N}_A}(x), I_{\tilde{N}_B}(x)\}, \\ \max\{F_{\tilde{N}_A}(x), F_{\tilde{N}_B}(x)\} \mid x \in X \end{array} \right\rangle\}$$

Definition 5 [7] The Hamming distance between $\tilde{N}_A$ = $\{x_i, \left\langle T_{\tilde{N}_A}(x_i), I_{\tilde{N}_A}(x_i), F_{\tilde{N}_A}(x_i) \right\rangle \mid x_i \in X\}$ and $\tilde{N}_B$ = $\{x_i, \left\langle T_{\tilde{N}_B}(x_i), I_{\tilde{N}_B}(x_i), F_{\tilde{N}_B}(x_i) \right\rangle \mid x_i \in X\}$ is defined as given below.

$$H(\tilde{N}_A, \tilde{N}_B) = \frac{1}{3}\sum_{i=1}^{n} \left| T_{\tilde{N}_A}(x_i) - T_{\tilde{N}_B}(x_i) \right| + \left| I_{\tilde{N}_A}(x_i) - I_{\tilde{N}_B}(x_i) \right| + \left| F_{\tilde{N}_A}(x_i) - F_{\tilde{N}_B}(x_i) \right| \quad (1)$$

with the property: $0 \le H(\tilde{N}_A, \tilde{N}_B) \le 1$.

2.2 Soft sets and Neutrosophic soft sets

Definition 6 [8] Suppose U is a universal set, F is a set of parameters and P (U) is a power set of U. Consider a non-empty set A, where $A \subset F$. A pair (M, A) is called a soft set over U, where M is a mapping given by M: $A \rightarrow P(U)$.

Definition 7 [21] Let U be an initial universal set. Let F be a set of parameters and A be a non-empty set such that $A \subset F$. P(U) represents the set of all neutrosophic subsets of U. A pair (M, A) is called a NSS over U, where M is a mapping given by M: $A \rightarrow P(U)$.

In other words, (M, A) over U is a parameterized family f of all neutrosophic sets over U.

Example: Let U be the universal set of objects or points. F = {very large, large, medium large, medium low, low, very low, attractive, cheap, expensive} is the set of parameters and each parameter is a neutrosophic word or sentence concerning neutrosophic word. To define neutrosophic soft set means to find out very large objects, large objects, medium large objects, attractive objects, and so on. Let U = (u_1, u_2, u_3, u_4, u_5, u_6) be the universal set consisting of six objects and F = {f_1, f_2, f_3, f_4} be a set of parameters. Here, f_1, f_2, f_3, f_4 stand for the parameters 'very large', 'large', 'attractive', 'expensive' respectively. Suppose that,

M (very large) = {< u_1, 0.8, 0.3, 0.4>, < u_2, 0.7, 0.3, 0.5>, < u_3, 0.8, 0.2, 0.3>, < u_4, 0.6, 0.4, 0.5>, < u_5, 0.9, 0.3, 0.3>, < u_6, 0.8, 0.4, 0.5>},

M (large) = {< u_1, 0.7, 0.3, 0.2>, < u_2, 0.6, 0.3, 0.4>, < u_3, 0.6, 0.4, 0.4>, < u_4, 0.6, 0.3, 0.2>, < u_5, 0.7, 0.5, 0.4>, < u_6, 0.6, 0.5, 0.6>},

M (attractive) = {< u_1, 0.9, 0.2, 0.2>, < u_2, 0.8, 0.3, 0.2>, < u_3, 0.8, 0.2, 0.3>, < u_4, 0.9, 0.4, 0.2>, < u_5, 0.8, 0.5, 0.4>, < u_6, 0.7, 0.4, 0.6>},

M (expensive) = {< u_1, 0.8, 0.2, 0.3>, < u_2, 0.9, 0.1, 0.2>, < u_3, 0.8, 0.3, 0.5>, < u_4, 0.9, 0.3, 0.3>, < u_5, 0.8, 0.4, 0.5>, < u_6, 0.8, 0.2, 0.5>}

Therefore, M (very large) means very large objects, M (attractive) means attractive objects, etc. Now we can represent the above NSS (M, A) over U in the form of a table (See the Table 1).

Table 1. Tabular form of the NSSs (M, A)

U	f_1 = very large	f_2 = large	f_3 = attractive	f_4 = expensive
u_1	(0.8, 0.3, 0.4)	(0.7, 0.3, 0.2)	(0.9, 0.2, 0.2)	(0.8, 0.2, 0.3)
u_2	(0.7, 0.3, 0.5)	(0.6, 0.3, 0.4)	(0.8, 0.3, 0.2)	(0.9, 0.1, 0.2)
u_3	(0.8, 0.2, 0.3)	(0.6, 0.4, 0.4)	(0.8, 0.2, 0.3)	(0.8, 0.3, 0.5)
u_4	(0.6, 0.4, 0.5)	(0.6, 0.3, 0.2)	(0.9, 0.4, 0.2)	(0.9, 0.3, 0.3)
u_5	(0.9, 0.3, 0.3)	(0.7, 0.5, 0.4)	(0.8, 0.5, 0.4)	(0.8, 0.4, 0.5)
u_6	(0.8, 0.4, 0.5)	(0.6, 0.5, 0.6)	(0.7, 0.4, 0.6)	(0.8, 0.2, 0.5)

Definition 8 [21]: Consider two NSSs (M_1, A) and (M_2, B) over a common universe U. (M_1, A) is said to be neutrosophic soft subset of (M_2, B) if $M_1 \subset M_2$, and $T_{M_1(f)}(x) \le T_{M_2(f)}(x)$, $I_{M_1(f)}(x) \le I_{M_2(f)}(x)$, $F_{M_1(f)}(x) \le F_{M_2(f)}(x)$, $\forall f \in A$, $x \in U$. We represent it by $(M_1, A) \subseteq (M_2, B)$.

Definition 9 [21]: Let (M_1, A) and (M_2, B) be two NSSs over a common universe U. They are said to be equal i.e. $(M_1, A) = (M_2, B)$ if $(M_1, A) \subseteq (M_2, B)$ and $(M_2, B) \subseteq (M_1, A)$.

Definition 10 [21]: Consider $F = \{f_1, f_2, \ldots, f_q\}$ be a set of parameters. Then, the *NOT* of F is defined by *NOT* F = $\{$*not* f_1, *not* f_2, …, *not* $f_q\}$, where it is to be noted that *NOT* and *not* are different operators.

Definition 11 [21]: The complement of a neutrosophic soft set (M, A) is denoted by $(M, A)^C$ and is represented as $(M, A)^C = (M^C,$ *NOT* A) with $T_{M^C(f)}(x) = F_{M(f)}(x)$; $I_{M^C(f)}(x) = I_{M(f)}(x)$; $F_{M^C(f)}(x) = T_{M(f)}(x)$, where M^C: *NOT* $A \rightarrow P(U)$.

Definition 12 [21]: A NSS (M, A) over a universe U is called a null NSS with respect to the parameter A if $T_{M(f)}(m) = I_{M(f)}(m) = F_{M(f)}(m) = 0$, $\forall f \in A$, $\forall m \in U$.

Definition 13 [21]: Let (M_1, A) and (M_2, B) be two NSSs over a common universe U. The union (M_1, A) and (M_2, B) is defined by $(M_1, A) \cup (M_2, B) = (M, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership functions are defined as follows:

$$\begin{aligned} T_{M(f)}(m) &= T_{M_1(f)}(m), \text{ if } f \in M_1 - M_2, \\ &= T_{M_2(f)}(m), \text{ if } f \in M_2 - M_1, \\ &= \max(T_{M_1(f)}(m), T_{M_2(f)}(m)), \text{ if } f \in M_1 \cap M_2. \\ I_{M(f)}(m) &= I_{M_1(f)}(m), \text{ if } f \in M_1 - M_2, \\ &= I_{M_2(f)}(m), \text{ if } f \in M_2 - M_1, \\ &= \frac{I_{M_1(f)}(m) + I_{M_2(f)}(m)}{2} \text{ if } f \in M_1 \cap M_2. \\ F_{M(f)}(m) &= F_{M_1(f)}(m), \text{ if } f \in M_1 - M_2, \\ &= F_{M_2(f)}(m), \text{ if } f \in M_2 - M_1, \\ &= \min(F_{M_1(f)}(m), F_{M_2(f)}(m)), \text{ if } f \in M_1 \cap M_2. \end{aligned}$$

Definition 14 [21]: Suppose (M_1, A) and (M_2, B) are two NSSs over a common universe U. The intersection (M_1, A) and (M_2, B) is defined by $(M_1, A) \cap (M_2, B) = (N, D)$, where $D = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership functions of (N, D) are as follows:

$T_{N(f)}(m) = \min(T_{M_1(f)}(m), T_{M_2(f)}(m))$; $I_{N(f)}(m) = \dfrac{I_{M_1(f)}(m) + I_{M_2(f)}(m)}{2}$; $F_{N(f)}(m) = \max(F_{M_1(f)}(m), F_{M_2(f)}(m))$.

3 A neutrosophic soft MADM based on grey relational projection method

Assume $G = \{g_1, g_2, \ldots, g_p\}$, $(p \geq 2)$ be a discrete set of alternatives and $A = \{a_1, a_2, \ldots, a_q\}$, $(q \geq 2)$ be a set of choice parameters under consideration in a MADM problem. The rating of performance value of alternative g_i, i = 1, 2, …, p with respect to the choice parameter a_j, j = 1, 2, …, q is represented by a tuple $t_{ij} = (T_{M(a_j)}(g_i), I_{M(a_j)}(g_i), F_{M(a_j)}(g_i))$, where for a fixed i the value t_{ij} (i = 1, 2, …, p; j = 1, 2, …, q) denotes NSS of all the p objects. Let $w = \{w_1, w_2, \ldots, w_q\}$ be the weight vector assigned for the choice parameters, where $0 \leq w_j \leq 1$ with $\sum_{j=1}^{q} w_j = 1$, but specific value of w_j is unknown. Now the steps of decision making based on neutrosophic soft information are described as given below.

Step 1. Construction of criterion matrix with SVNSs

GRA method is appropriate for dealing with quantitative attributes. However, in the case of qualitative attribute, the performance values are taken as SVNSs. The performance values $t_{\tilde{N}_{ij}}$ (i = 1, 2, …, p; j = 1, 2, …, q) could be arranged in the matrix called criterion matrix and whose rows are labeled by the alternatives and columns are labeled by the choice parameters. The criterion matrix is presented as follows:

$$D_{\tilde{N}} = \left\langle t_{\tilde{N}_{ij}} \right\rangle_{p \times q} = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1q} \\ t_{21} & t_{22} & \cdots & t_{2q} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ t_{p1} & t_{p2} & \cdots & t_{pq} \end{bmatrix}$$

where $t_{ij} = (T_{ij}, I_{ij}, F_{ij})$ where $T_{ij}, I_{ij}, F_{ij} \in [0, 1]$ and $0 \leq T_{ij} + I_{ij} + F_{ij} \leq 3$, i = 1, 2, …, p; j = 1, 2, …, q.

Step 2. Determination of weights of the attributes

In the decision making situation, the decision maker encounters problem of identifying the unknown attributes weights, where it may happen that the weights of attributes are different. In this paper, we use information entropy method in order to obtain unknown attribute weight. The entropy measure can be used when weights of attributes are dissimilar and completely unknown to the decision maker. The entropy measure [33] of a SVNS $\tilde{N}$ = $\{x, \langle T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x) \rangle$ is defined as given below.

$$E_i(\tilde{N}) = 1 - \frac{1}{n}\sum_{i=1}^{p}(T_{\tilde{N}}(x_i) + F_{\tilde{N}}(x_i))\left|I_{\tilde{N}}(x_i) - I^{C}_{\tilde{N}}(x_i)\right| \quad (2)$$

which has the following properties:

(i). $E_i(\tilde{N}) = 0$ if $\tilde{N}$ is a crisp set and $I_{\tilde{N}}(x_i) = 0, \forall\, x \in X$.

(ii). $E_i(\tilde{N}) = 0$ if $\langle T_{\tilde{N}}(x_i), I_{\tilde{N}}(x_i), F_{\tilde{N}}(x_i) \rangle$ = <0.5, 0.5, 0.5>, $\forall\, x \in X$.

(iii). $E_i(\tilde{N}_1) \le E_i(\tilde{N}_2)$ if $\tilde{N}_1$ is more uncertain than $\tilde{N}_2$ i.e.

$T_{\tilde{N}_1}(x_i) + F_{\tilde{N}_1}(x_i) \le T_{\tilde{N}_2}(x_i) + F_{\tilde{N}_2}(x_i)$ and $\left|I_{\tilde{N}_1}(x_i) - I^{C}_{\tilde{N}_1}(x_i)\right| \le \left|I_{\tilde{N}_2}(x_i) - I^{C}_{\tilde{N}_2}(x_i)\right|$.

(iv). $E_i(\tilde{N}) = E_i(\tilde{N}^{C}), \forall\, x \in X$.

Therefore, the entropy value E_j of the j-th attribute can be obtained as follows:

$$E_j = 1 - \frac{1}{q}\sum_{i=1}^{p}(T_{ij}(x_i) + F_{ij}(x_i))\left|I_{ij}(x_i) - I^{C}_{ij}(x_i)\right|,$$

(j = 1, 2,…, q). (3)

Here, $0 \le E_j \le 1$ and according to Hwang and Yoon [34] and Wang and Zhang [35] the entropy weight of the j-th attribute is defined as follows:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{q} 1 - E_j}, \text{ with } 0 \le w_j \le 1 \text{ and } \sum_{j-1}^{q} w_j = 1 \quad (4)$$

Step 3. Determination of ideal neutrosophic estimates reliability solution (INERS) and ideal neutrosophic estimates un-reliability solution (INEURS)

Dezart [36] proposed the idea of single valued neutrosophic cube. From this cube one can easily obtain ideal neutrosophic estimates reliability solution (INERS) and ideal neutrosophic estimates un-reliability solution (INEURS). An INERS $P^{+}_{\tilde{N}}$ = [$p^{+}_{\tilde{N}_1}$, $p^{+}_{\tilde{N}_2}$, …, $p^{+}_{\tilde{N}_q}$] is a solution in which every element $p^{+}_{\tilde{N}_j}$ = < T^{+}_j, I^{+}_j, F^{+}_j >, where $T^{+}_j = \max_i$ {T_{ij}}, $I^{+}_j = \min_i$ {I_{ij}}, $F^{+}_j = \min_i$ {F_{ij}} in the criteria matrix $D_{\tilde{N}}$ = < T_{ij}, I_{ij}, F_{ij} >$_{p \times q}$ for i = 1, 2, …, p; j = 1, 2, …, q. Also, an INEURS $P^{-}_{\tilde{N}}$ = [$p^{-}_{\tilde{N}_1}$, $p^{-}_{\tilde{N}_2}$, …, $p^{-}_{\tilde{N}_q}$] is a solution in which every element $p^{-}_{\tilde{N}_j}$ = < T^{-}_j, I^{-}_j, F^{-}_j >$_{p \times q}$, where $T^{-}_j = \min_i$ {T_{ij}}, $I^{-}_j = \max_i$ {I_{ij}}, $F^{-}_j = \max_i$ {F_{ij}} in the criterion matrix $D_{\tilde{N}}$ = < T_{ij}, I_{ij}, F_{ij} >$_{p \times q}$ for i = 1, 2, …, p; j = 1, 2, …, q.

Step 4. Grey relational projection method

3.1 Projection method

Definition 15 [37, 38]: Consider $a = (a_1, a_2, \dots, a_q)$ and $b = (b_1, b_2, \dots, b_q)$ are two vectors, then cosine of included angle between vectors a and b is defined as follows:

$$\text{Cos}\,(a, b) = \frac{\sum_{j=1}^{q}(a_j b_j)}{\sqrt{\sum_{j=1}^{q} a_j^2} \times \sqrt{\sum_{j=1}^{q} b_j^2}} \quad (5)$$

Obviously, 0 < Cos $(a, b) \le 1$, and the direction of a and b is more accordant according to the bigger value of Cos (a, b).

Definition 16 [37, 38]: Let $a = (a_1, a_2, \dots, a_q)$ be a vector, then norm of a is given by

$$\| a \| = \sqrt{\sum_{j=1}^{q} a_j^2} \quad (6)$$

The direction and norm are two important parts of a vector. However, Cos (a, b) can only compute whether their directions are accordant, but cannot determine the magnitude of norm. Therefore, the closeness degree of two vectors can be defined by the projection value in order to take the norm magnitude and cosine of included angle together.

Definition 17 [37, 38]: Let $a = (a_1, a_2, \dots, a_q)$ and $b = (b_1, b_2, \dots, b_q)$ be two vectors, then the projection of vector a onto vector b is defined as follows:

Pr (a) = || a || Cos (a, b) =

$$\sqrt{\sum_{j=1}^{q} a_j^2} \times \frac{\sum_{j=1}^{q}(a_j b_j)}{\sqrt{\sum_{j=1}^{q} a_j^2} \times \sqrt{\sum_{j=1}^{q} b_j^2}} = \frac{\sum_{j=1}^{q}(a_j b_j)}{\sqrt{\sum_{j=1}^{q} b_j^2}} \quad (7)$$

The bigger the value of Pr (a) is, the more close the vector b to the vector a is.

3.2 Grey correlation projection method

The grey correlation projection method is a combination of grey correlation method and projection method. The method is presented in the following steps.

Step-1. The grey relational coefficient of each alternative from INERS is obtained from the following formula:

$$\zeta_{ij}^{+} = \frac{\min_i \min_j \Omega_{ij}^{+} + \sigma \max_i \max_j \Omega_{ij}^{+}}{\Omega_{ij}^{+} + \sigma \max_i \max_j \Omega_{ij}^{+}} \quad (8)$$

where Ω_{ij}^{+} = d($t_{\tilde{N}_j}$, $p_{\tilde{N}_j}^{+}$) = Hamming distance between $t_{\tilde{N}_j}$ and $p_{\tilde{N}_j}^{+}$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Also, the grey relational coefficient of each alternative from INEURS is obtained from the formula given below:

$$\zeta_{ij}^{-} = \frac{\min_i \min_j \Omega_{ij}^{-} + \sigma \max_i \max_j \Omega_{ij}^{-}}{\Omega_{ij}^{-} + \sigma \max_i \max_j \Omega_{ij}^{-}} \quad (9)$$

where Ω_{ij}^{-} = d($t_{\tilde{N}_j}$, $p_{\tilde{N}_j}^{-}$) = Hamming distance between $t_{\tilde{N}_j}$ and $p_{\tilde{N}_j}^{-}$, (i = 1, 2, ..., p; j = 1, 2, ..., q).

Here, $\sigma \in [0, 1]$ represents the environmental or resolution coefficient and it is used to adjust the difference of the relation coefficient. Generally, we set $\sigma = 0.5$.

Step-2. Grey correlation coefficient matrix ζ^{+} between every alternative and INERS is formulated as given below.

$$\zeta^{+} = \begin{bmatrix} \zeta_{11}^{+} & \zeta_{12}^{+} & \cdots & \zeta_{1q}^{+} \\ \zeta_{21}^{+} & \zeta_{12}^{+} & \cdots & \zeta_{2q}^{+} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \zeta_{p1}^{+} & \zeta_{12}^{+} & \cdots & \zeta_{pq}^{+} \end{bmatrix}$$

and correlation coefficient between INERS and INERS is:

$$\zeta_0^{+} = (\zeta_{01}^{+}, \zeta_{02}^{+}, \ldots, \zeta_{0q}^{+}) = (1, 1, \ldots, 1)$$

Grey correlation coefficient matrix ζ^{-} between every alternative and INEURS is constructed as follows.

$$\zeta^{-} = \begin{bmatrix} \zeta_{11}^{-} & \zeta_{12}^{-} & \cdots & \zeta_{1q}^{-} \\ \zeta_{21}^{-} & \zeta_{12}^{-} & \cdots & \zeta_{2q}^{-} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \zeta_{p1}^{-} & \zeta_{12}^{-} & \cdots & \zeta_{pq}^{-} \end{bmatrix}$$

Similarly, the correlation coefficient between INEURS and INEURS is:

$$\zeta_0^{-} = (\zeta_{01}^{-}, \zeta_{02}^{-}, \ldots, \zeta_{0q}^{-}) = (1, 1, \ldots, 1)$$

Step-3. Weighted neutrosophic grey correlation coefficient matrix G between every alternative and INERS is formulated as given below.

$$G^{+} = \begin{bmatrix} w_1\zeta_{11}^{+} & w_2\zeta_{12}^{+} & \cdots & w_q\zeta_{1q}^{+} \\ w_1\zeta_{21}^{+} & w_2\zeta_{12}^{+} & \cdots & w_q\zeta_{2q}^{+} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ w_1\zeta_{p1}^{+} & w_2\zeta_{12}^{+} & \cdots & w_q\zeta_{pq}^{+} \end{bmatrix}$$

The weighted correlation coefficient between INERS and INERS is:

$$G_0^{+} = (w_1\zeta_{01}^{+}, w_2\zeta_{02}^{+}, \ldots, w_q\zeta_{0q}^{+}) = (w_1, w_2, \ldots, w_q)$$

Weighted neutrosophic grey correlation coefficient matrix G^{-} between every alternative and INEURS is presented as follows:

$$G^{-} = \begin{bmatrix} w_1\zeta_{11}^{-} & w_2\zeta_{12}^{-} & \cdots & w_q\zeta_{1q}^{-} \\ w_1\zeta_{21}^{-} & w_2\zeta_{12}^{-} & \cdots & w_q\zeta_{2q}^{-} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ w_1\zeta_{p1}^{-} & w_2\zeta_{12}^{-} & \cdots & w_q\zeta_{pq}^{-} \end{bmatrix}$$

and similarly, weighted correlation coefficient between INEURS and INEURS is presented as follows:

$$G_0^{-} = (w_1\zeta_{01}^{-}, w_2\zeta_{02}^{-}, \ldots, w_q\zeta_{0q}^{-}) = (w_1, w_2, \ldots, w_q)$$

Step-4. Calculation of the weighted grey correlation of alternative g_i onto the INERS can be obtained as: $Pr_i^+ = \| G_i \| \operatorname{Cos}(G_i, G_0^+) = \sqrt{\sum_{j=1}^{q}(w_j\zeta_{ij}^+)^2} \times \frac{\sum_{j=1}^{q}((w_j\zeta_{ij}^+)\times w_j)}{\sqrt{\sum_{j=1}^{q}(w_j\zeta_{ij}^+)^2}\times\sqrt{\sum_{j=1}^{q}w_j^2}} = \frac{\sum_{j=1}^{q}((w_j\zeta_{ij}^+)\times w_j)}{\sqrt{\sum_{j=1}^{q}w_j^2}}$

$$= \frac{\sum_{j=1}^{q}(w_j^2\zeta_{ij}^+)}{\sqrt{\sum_{j=1}^{q}w_j^2}} \quad (10)$$

Similarly, the weighted grey correlation of alternative g_i onto the INEURS can be obtained as follows:

$Pr_i^- = \| G_i \| \operatorname{Cos}(G_i, G_0^-) = \sqrt{\sum_{j=1}^{q}(w_j\zeta_{ij}^-)^2} \times \frac{\sum_{j=1}^{q}((w_j\zeta_{ij}^-)\times w_j)}{\sqrt{\sum_{j=1}^{q}(w_j\zeta_{ij}^-)^2}\times\sqrt{\sum_{j=1}^{q}w_j^2}} = \frac{\sum_{j=1}^{q}((w_j\zeta_{ij}^-)\times w_j)}{\sqrt{\sum_{j=1}^{q}w_j^2}}$

$$\frac{\sum_{j=1}^{q}(w_j^2\zeta_{ij}^-)}{\sqrt{\sum_{j=1}^{q}w_j^2}} \quad (11)$$

Step-5. Calculation of the neutrosophic relative relational degree

The ranking order of all alternatives can be obtained according to the value of the neutrosophic relative relational degree. We calculate the neutrosophic relative relational degree by using the following equation

$$C_i = \frac{Pr_i^+}{Pr_i^+ + Pr_i^-}, \ i = 1, 2, \ldots, p. \quad (12)$$

Rank the alternatives according to the values of C_i, i = 1, 2, …, p in descending order and choose the alternative with biggest C_i.

4 A numerical example

We consider the decision making problem for selecting the most suitable house for Mr. X [21]. Let Mr. X desires to select the most suitable house out of *p* houses on the basis of *q* parameters. Also let, the rating of or performance value of the house g_i, i = 1, 2, ..., p with respect to parameter a_j, j = 1, 2, …, q is represented by $t_{\tilde{N}_{ij}} = (T_{G(f_j)}(g_i), I_{G(f_j)}(g_i), F_{G(f_j)}(g_i))$ such that for a fixed i, $t_{\tilde{N}_{ij}}$ denotes neutrosophic soft set of all the *q* objects. Let, A = {beautiful, cheap, in good repairing, moderate, wooden} be the set of choice parameters. The criterion decision matrix (see Table 2) is presented as follows:

Table 2. Tabular form of criterion decision matrix

U	beautiful	cheap	in good repairing	moderate	wooden
g_1	(0.6,0.3, 0.8)	(0.5, 0.2, 0.6)	(0.7, 0.3, .4)	(0.8, 0.5, 0.6)	(0.6, 0.7, 0.2)
g_2	(0.7, 0.2, 0.6)	(0.6, 0.3, 0.7)	(0.7, 0.5, .6)	(0.6, 0.8, 0.3)	(0.8, 0.1, 0.8)
g_3	(0.8, 0.3, 0.4)	(0.8, 0.5, 0.1)	(0.3, 0.5, 0.6)	(0.7, 0.2, 0.1)	(0.7, 0.2, 0.6)
g_4	(0.7, 0.5, 0.6)	(0.6, 0.8, 0.7)	(0.7, 0.6, 0.8)	(0.8, 0.3, 0.6)	(0.8, 0.3, 0.8)
g_5	(0.8, 0.6, 0.7)	(0.5, 0.6, 0.8)	(0.8, 0.7,0.6)	(0.7, 0.8, 0.3)	(0.7, 0.2, 0.6)

The proposed procedure is presented in the following steps.

Step 1. Calculation of the weights of the attribute

Entropy value E_j (j = 1, 2, …, 5) of the j-th attribute can be obtained from the equation (3) as follows:
$E_1 = 0.576$, $E_2 = 0.556$, $E_3 = 0.74$, $E_4 = 0.564$, $E_5 = 0.24$.
Then the corresponding normalized entropy weights are obtained as given below.
$w_1 = 0.2155$, $w_2 = 0.2076$, $w_3 = 0.2763$, $w_4 = 0.2111$, $w_5 = 0.0895$, where $\sum_{j=1}^{5} w_j = 1$.

Step 2. Calculation of INERS and INEURS

The INERS ($P_{\tilde{N}}^+$) and INEURS ($P_{\tilde{N}}^-$) of the decision matrix are shown as follows:

$P_{\tilde{N}}^+ = <$ (0.8, 0.2, 0.4); (0.8, 0.2, 0.1); (0.8, 0.3, 0.4); (0.8, 0.2, 0.1); (0.8, 0.1, 0.2) $>$

$P_{\tilde{N}}^- = <$ (0.6, 0. 6, 0.8); (0.5, 0.8, 0.8); (0.3, 0.7, 0.8); (0.6, 0.8, 0.6); (0.6, 0.7, 0.8) $>$

Step 3. Determine the grey relational coefficient of each alternative from INERS and INEURS

The grey relational coefficient of each alternative from INERS can be determined as follows:

$$\zeta_{ij}^+ = \begin{bmatrix} 0.571 & 0.532 & 1.000 & 0.532 & 0.532 \\ 0.799 & 0.500 & 0.665 & 0.470 & 0.615 \\ 1.000 & 0.799 & 0.500 & 1.000 & 0.615 \\ 0.615 & 0.380 & 0.532 & 0.615 & 0.532 \\ 0.571 & 0.380 & 0.615 & 0.500 & 0.615 \end{bmatrix}$$

Similarly, the neutrosophic grey relational coefficient of each alternative from INEURS can be obtained as given below.

$$\zeta_{ij}^{-}=\begin{bmatrix}1.000 & 0.516 & 0.405 & 0.650 & 0.599\\ 0.555 & 0.555 & 0.516 & 0.788 & 0.516\\ 0.483 & 0.384 & 0.714 & 0.405 & 0.516\\ 0.714 & 0.880 & 0.650 & 0.555 & 0.599\\ 1.000 & 0.880 & 0.555 & 0.714 & 0.516\end{bmatrix}$$

Step 4. Calculation of the weighted grey correlation projection

Calculation of the weighted grey correlation projection of alternative g_i onto the INERS and INEURS can be obtained from the equations (10) and (11) respectively as follows:

$\Pr_1^+ = 0.1538$, $\Pr_2^+ = 0.1353$, $\Pr_3^+ = 0.1686$, $\Pr_4^+ = 0.1172$, $\Pr_5^+ = 0.117$;

$\Pr_1^- = 0.1333$, $\Pr_2^- = 0.1283$, $\Pr_3^- = 0.1157$, $\Pr_4^- = 0.1502$, $\Pr_5^- = 0.1627$.

Step 5. Calculate the grey relative relational degree

We compute the grey relative relational degree by using equation (12) as follows:

$C_1 = 0.5357$, $C_2 = 0.5133$, $C_3 = 0.5930$, $C_4 = 0.4188$, $C_5 = 0.4183$.

Step 6. The ranking order of the houses can be obtained according to the value of grey relative relational degree. It is observed that $C_3 > C_1 > C_2 > C_4 > C_5$ and so the highest value of grey relative relational degree is C_3. Therefore, the house g_3 is the best alternative for Mr. X.

Note: We now compare our proposed method with the method discussed by Maji [21]. Maji [21] first constructed the comparison matrix and then computed the score S_i of g_i, $\forall$ i. The preferable alternative is selected based on the maximum score of S_i. The ranking order of the houses is given by $g_5 > g_3 > g_4 > g_1 > g_2$. In the present paper, a neutrosophic soft MADM problem through grey correlation projection method is proposed with unknown weights information. The ranking of alternatives are determined by the relative closeness to INERS which combines grey relational projection values from INERS and INEURS to each alternative. The ranking order of the houses is presented as $g_3 > g_1 > g_2 > g_4 > g_5$. However, if he rejects the house h_3 for any reason, his next preference will be g_1.

5 Conclusion

In this paper, we have presented a new approach for solving neutrosophic soft MADM problem based on GRP method with unknown weight information of the choice parameters. The proposed approach is a hybrid model of neutrosophic soft sets and GRP method where the choice parameters are represented in terms of single valued neutrosophic information. The weights of the parameters are determined by using information entropy method. In the proposed approach, grey relative relational degrees of all alternatives are calculated in order to rank the alternatives and then the most suitable option is selected. An illustrative example for house selection is provided in order to verify the practicality and effectiveness of the proposed approach. We hope that that the proposed approach can be effective in dealing with different MADM problems such as cluster analysis, image processing, medical diagnosis, pattern recognition, object selection.

In the future, we shall investigate generalized neutrosophic soft GRP, interval neutrosophic soft GRP, intuitionistic soft GRP methods for practical MADM problems.

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