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Neutrosophic fuzzy metric spaces and fixed points for contractions of nonlinear type

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Abstract. In this research, we present fixed point theorems pertaining to nonlinear contractions, situated within the sophisticated framework of neutrosophic fuzzy metric spaces. Furthermore, we establish several fixed point results that are pertinent to this specific context.

Keywords: Fixed point; Neutrosophic fuzzy metric; Geraghty; nonlinear contractions.

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1. **Introduction**

The concept of Fuzzy Sets (FSs), initially proposed by Zadeh [1], has had a profound impact across numerous scientific disciplines since its inception. Although this framework is highly relevant for practical applications, it has not always provided effective solutions to various challenges over time. Consequently, there has been a resurgence of interest in research aimed at addressing these issues. In this regard, Atanassov [2] introduced Intuitionistic Fuzzy Sets (IFSs) as a means to confront such challenges. Additionally, the Neutrosophic Set (NS), developed by Smarandache [3], represents a complex extension of conventional set theory. Other significant generalizations include interval-valued FS [4], interval-valued IFS [5], as well as paraconsistent, dialetheist, paradoxist, and tautological sets [6], along with Pythagorean fuzzy sets [7]. Neutrosophic sets exhibit a diverse range of applications across multiple domains. For instance, Barbosa and Smarandache [8] presented the Neutrosophic One-Round Zero-Knowledge Proof protocol (N-1-R) ZKP, which enhances the One-Round (1-R) ZKP framework by integrating

Neutrosophic numbers. Furthermore, the authors in [10] provide a comprehensive characterization of efficient and optimally suitable solutions related to scalar optimization problems, as well as outlining the Kuhn-Tucker conditions relevant to both efficiency and proper efficiency. For a more thorough exploration of the applications of neutrosophic sets and their extensive uses, it is recommended to consult the literature referenced in [9–20, 23, 24, 48]. Our objective is to integrate our research with novel concepts, as demonstrated in the applications of [30, 31, 37, 38, 49, 53].

The Banach fixed-point theorem [21], often referred to as the Banach contraction principle, is a fundamental theorem in mathematics, particularly in the study of metric spaces. It ensures the existence and uniqueness of fixed points for specific self-maps within these spaces, thus providing a systematic method for identifying such points. The theorem serves as a foundational element for Picard's method of successive approximations. Introduced by Stefan Banach in 1922, this theorem has motivated numerous mathematicians to investigate various extensions and generalizations across a wide array of mathematical disciplines, as evidenced by the citations in [**?**, **?**, 22, 25–29, 34–36, 39–42, 44]. A prominent example of its application is found in the concept of neutrosophic metric space (NMS), which was initially proposed by Kirisci and Simsek [**?**]. This framework has been utilized to analyze a range of fixed point theorems.

2. Preliminary

In this framework, the interval $]0-, 1+[$ is characterized as a non-standard unit interval. Within this context, non-standard finite numbers are articulated as $(1+) = 1 + \epsilon$, where "1" represents the standard component and ϵ denotes the non-standard element. In a similar manner, $(0-) = 0 - \epsilon$, with "0" indicating the standard component and ϵ as the non-standard element. The numbers 0 and 1 can be interpreted as non-standard values that are infinitesimally small yet less than 0 or infinitesimally small yet greater than 1, respectively, and these values are encompassed within the non-standard unit interval $]0-, 1+[$.

Definition 2.1. [1]In relation to a universal set U, a fuzzy set F is defined by the notation $F = \{< a, \mu_F(\xi) >: 0 \leq \mu_F(\xi) \leq 1, \xi \in U\}$. In this context, $\mu_F(\xi)$ represents the degree of membership of the element ξ within the fuzzy set F.

Definition 2.2. [3] A neutrosophic set V relative to a universal set U is defined as $V = \{< \xi, (T_N(\xi), I_N(\xi), F_N(\xi)) >: \xi \in U, T_N(\xi), I_N(\xi), F_N(\xi) \in]0-, 1+[\}$. In this context, $T_N(\xi)$, $I_N(\xi)$, and $F_N(\xi)$ represent the membership degrees of truth, indeterminacy, and falsity for an element ξ within the set V, respectively, while $]0-, 1+[$ signifies a non-standard unit interval.

Definition 2.3. [50] A neutrosophic fuzzy set B within a universal set U is characterized as follows: $B = \{< x, (\mu_B(\xi), T_B(\xi, \mu), I_B(\xi, \mu), F(\xi, \mu)) >: \xi \in U, \mu_B(\xi) \in [0, 1],$

$T_B(\xi,\mu), I_B(\xi,\mu), F(\xi,\mu) \in]0-, 1+[$ } In this framework, the membership degree $\mu_B(\xi)$ is represented by three distinct components: the truth membership grade $T_B(\xi,\mu)$, the indeterminacy membership grade $I_B(\xi,\mu)$, and the falsity membership grade $F(\xi,\mu)$. The notation $]0-, 1+[$ signifies a nonstandard unit interval.

Triangular norms (shortly TN), initially introduced by Menger [51], are a fundamental concept in mathematical analysis. Menger's innovative approach involved using probability distributions to assess the distance between two elements within a specific space, moving beyond the traditional reliance on numerical values. This technique facilitates the extension of the triangle inequality in metric spaces through the application of triangular norms. In contrast, triangular conorms (shortly CN) serve as the dual counterparts to t-norms. Both TN and CN are essential in fuzzy operations, especially concerning intersections and unions.

In this manuscript, we denote $\mathbb{R}^+$ as the interval $(0,\infty)$ and I as the interval $[0,1]$.

Definition 2.4. Consider an operation $\diamond : I \times I \to I$. This operation is classified as continuous TN (CTN) if it meets the following criteria: for any elements $\sigma, \sigma', t, t' \in I$.

(1) $\sigma \diamond 1 = \sigma$,
(2) If $\sigma \leq \sigma'$ and $t \leq t'$, than $\sigma \diamond t \leq \sigma' \diamond t'$,
(3) $\diamond$ is continuous,
(4) $\diamond$ is commutative and associate.

Definition 2.5. Consider an operation $\bullet : I \times I \to I$. This operation is classified as continuous TN (CTN) if it meets the following criteria: for all elements $\sigma, \sigma', t, t' \in I$.

(1) $\sigma \bullet 0 = \sigma$,
(2) If $\sigma \leq \sigma'$ and $t \leq t'$, than $\sigma \bullet t \leq \sigma' \bullet t'$,
(3) $\bullet$ is continuous,
(4) $\bullet$ is commutative and associate.

Definition 2.6. [**?**] A 6-tuple $(\mathcal{X}, \mathcal{A}, \mathcal{C}, \mathcal{D}, \diamond, \bullet)$ is referred to as a Neutrophic Metric Space (NMS) if the set $\mathcal{X}$ is a non-empty arbitrary collection, $\diamond$ signifies a continuous t-norm, $\bullet$ indicates a continuous t-conorm, and the elements $\mathcal{A}, \mathcal{C}$, and $\mathcal{D}$ are three fuzzy sets established on the Cartesian product $\mathcal{X}^2 \times (0,\infty)$. These components must satisfy the following specific conditions for all elements $\xi, \omega, c \in \mathcal{X}$ and for all positive real numbers λ, ρ.

(1) $0 \leq \mathcal{A}(\xi,\omega,\lambda) \leq 1$, $0 \leq \mathcal{C}(\xi,\omega,\lambda) \leq 1$, $0 \leq \mathcal{D}(\xi,\omega,\lambda) \leq 1$,
(2) $0 \leq \mathcal{A}(\xi,\omega,\lambda) + \mathcal{C}(\xi,\omega,\lambda) + \mathcal{D}(\xi,\omega,\lambda) \leq 3$,

(3) $\mathcal{A}(\xi,\omega,\lambda) = 1$, for $\lambda > 0$ iff $\xi = \omega$
(4) $\mathcal{A}(\xi,\omega,\lambda) = H(\omega,\xi,\lambda)$, for $\lambda > 0$

(5) $\mathcal{A}(\xi,\omega,\lambda)\diamond\mathcal{A}(\omega,c,\rho)\le\mathcal{A}(\xi,c,\lambda+\rho)$
(6) $\mathcal{A}(\xi,\omega,\cdot):\mathbb{R}^+\to I$ is continuous
(7) $\lim_{\lambda\to\infty}\mathcal{A}(\xi,\omega,\lambda)=1$
(8) $\mathcal{C}(\xi,\omega,\lambda)=0$ iff $\xi=\omega$
(9) $\mathcal{C}(\xi,\omega,\lambda)=\mathcal{C}(\omega,\xi,\lambda)$,
(10) $\mathcal{C}(\xi,\omega,\lambda)\bullet\mathcal{C}(\omega,c,\rho)\ge\mathcal{C}(\xi,c,\lambda+\rho)$,
(11) $\mathcal{C}(\xi,\omega,\cdot):\mathbb{R}^+\to I$ is continuous
(12) $\lim_{\lambda\to\infty}\mathcal{C}(\xi,\omega,\lambda)=0$
(13) $\mathcal{D}(\xi,\omega,\lambda)=0$, for $\lambda>0$ iff $\xi=\omega$
(14) $\mathcal{D}(\xi,\omega,\lambda)=\mathcal{D}(\omega,\xi,\lambda)$,
(15) $\mathcal{D}(\xi,\omega,\lambda)\bullet\mathcal{D}(\omega,c,\rho)\ge S(\xi,c,\lambda+\rho)$,
(16) $\mathcal{D}(\xi,\omega,\cdot):\mathbb{R}^+\to I$ is continuous
(17) $\lim_{\lambda\to\infty}\mathcal{D}(\xi,\omega,\lambda)=0$
(18) If $\lambda\le 0$, then $\mathcal{A}(\xi,\omega,\lambda)=0$, $\mathcal{C}(\xi,\omega,\lambda)=\mathcal{D}(\xi,\omega,\lambda)=1$

The functions $\mathcal{A}(\xi,\omega,\lambda)$, $\mathcal{C}(\xi,\omega,\lambda)$, and $\mathcal{D}(\xi,\omega,\lambda)$ represent the degrees of nearness, neutralness, and non-nearness between the elements ξ and ω in relation to the parameter λ, respectively.

Recently, Ghosh et al. [52] presented the notion of neutrosophic fuzzy metric spaces and examined various topological characteristics associated with this concept.

Definition 2.7. [52] A 7-tuple $(\mathcal{X},\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\diamond,\bullet)$ is known as a Neutrophic Fuzzy Metric Space (NFMS) if $\mathcal{X}$ is an arbitary set, $\diamond$ is a continuous t-norm, $\bullet$ is a continuous t-conorm, and $\mathcal{A},\mathcal{B},\mathcal{C}$, and $\mathcal{D}$ are fuzzy sets on $\mathcal{X}^2\times(0,\infty)$ satisfying the following conditions for all $\xi,\omega,c,\in\mathcal{X}$ and $\lambda,\rho>0$.

(1) $0\le\mathcal{A}(\xi,\omega,\lambda)\le1$, $0\le\mathcal{B}(\xi,\omega,\lambda)\le1$, $0\le\mathcal{C}(\xi,\omega,\lambda)\le1$, $0\le\mathcal{D}(\xi,\omega,\lambda)\le1$,
(2) $0\le\mathcal{A}(\xi,\omega,\lambda)+\mathcal{B}(\xi,\omega,\lambda)+\mathcal{C}(\xi,\omega,\lambda)+\mathcal{D}(\xi,\omega,\lambda)\le4$,

(3) $\mathcal{A}(\xi,\omega,\lambda)=1$, iff $\xi=\omega$
(4) $\mathcal{A}(\xi,\omega,\lambda)=H(\omega,\xi,\lambda)$,
(5) $\mathcal{A}(\xi,\omega,\lambda)\diamond\mathcal{A}(\omega,c,\rho)\le\mathcal{A}(\xi,c,\lambda+\rho)$, for $\rho,\lambda>0$
(6) $\mathcal{A}(\xi,\omega,\cdot):\mathbb{R}^+\to I$ is continuous
(7) $\lim_{\lambda\to\infty}\mathcal{A}(\xi,\omega,\lambda)=1$
(8) $\mathcal{B}(\xi,\omega,\lambda)=1$, iff $\xi=\omega$
(9) $\mathcal{B}(\xi,\omega,\lambda)=\mathcal{B}(\omega,\xi,\lambda)$, for $\lambda>0$
(10) $\mathcal{B}(\xi,\omega,\lambda)\diamond\mathcal{B}(\omega,c,\rho)\le\mathcal{B}(\xi,c,\lambda+\rho)$,
(11) $\mathcal{B}(\xi,\omega,\cdot):\mathbb{R}^+\to I$ is continuous

(12) $\lim_{\lambda\to\infty} \mathcal{B}(\xi,\omega,\lambda) = 1$
(13) $\mathcal{C}(\xi,\omega,\lambda) = 0$, iff $\xi = \omega$
(14) $\mathcal{C}(\xi,\omega,\lambda) = \mathcal{C}(\omega,\xi,\lambda)$,
(15) $\mathcal{C}(\xi,\omega,\lambda) \bullet \mathcal{C}(\omega,c,\rho) \geq \mathcal{C}(\xi,c,\lambda+\rho)$,
(16) $\mathcal{C}(\xi,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
(17) $\lim_{\lambda\to\infty} \mathcal{C}(\xi,\omega,\lambda) = 0$
(18) $\mathcal{D}(\xi,\omega,\lambda) = 0$, iff $\xi = \omega$
(19) $\mathcal{D}(\xi,\omega,\lambda) = \mathcal{D}(\omega,\xi,\lambda)$,
(20) $\mathcal{D}(\xi,\omega,\lambda) \bullet \mathcal{D}(\omega,c,\rho) \geq S(\xi,c,\lambda+\rho)$,
(21) $\mathcal{D}(\xi,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
(22) $\lim_{\lambda\to\infty} \mathcal{D}(\xi,\omega,\lambda) = 0$
(23) If $\lambda \leq 0$, then $\mathcal{A}(\xi,\omega,\lambda) = \mathcal{B}(\xi,\omega,\lambda) = 0$, $\mathcal{C}(\xi,\omega,\lambda) = \mathcal{D}(\xi,\omega,\lambda) = 1$

In this context, $\mathcal{A}(\xi,\omega,\lambda)$ represents the certainity that distance between ξ and ω is less than λ, $\mathcal{B}(\xi,\omega,\lambda)$ represents the degree of nearness, $\mathcal{C}(\xi,\omega,\lambda)$ stadns for the degree of neutralness, and $\mathcal{D}(\xi,\omega,\lambda)$ denotes the degree of non-nearness between ξ and ω with respect to λ, respectively.

The convergence, Cauchyness, completeness are given as follows.

Definition 2.8. [52] Let (ξ_n) be a sequence in a NFMS $(\mathcal{X},\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\diamond,\bullet)$. Then

(1) (ξ_n) converges to $\xi \in \mathcal{X}$ iff for a given $\epsilon \in (0,1)$, $\lambda > 0$ there is $n_0 \in \mathbb{N}$ such that for each $n \geq n_0$

$$\mathcal{A}(\xi_n,\xi,\lambda) > 1-\epsilon,\ \mathcal{C}(\xi_n,\xi,\lambda) < \epsilon,\ \mathcal{D}(\xi_n,\xi,\lambda) < \epsilon$$

i.e.,

$$\lim_{n\to\infty} \mathcal{A}(\xi_n,\xi,\lambda) = 1,\ \lim_{n\to\infty} \mathcal{B}(\xi_n,\xi,\lambda) = 1,\ \lim_{n\to\infty} \mathcal{C}(\xi_n,\xi,\lambda) = 0,\ \lim_{n\to\infty} \mathcal{D}(\xi_n,\xi,\lambda) = 0$$

(2) (ξ_n) is called Cauchy iff for a given $\epsilon \in (0,1)$, $\lambda > 0$ there is $n_0 \in \mathbb{N}$ such that for each $n,m \geq n_0$

$$\mathcal{A}(\xi_n,\xi_m,\lambda) > 1-\epsilon,\ \mathcal{B}(\xi_n,\xi_m,\lambda) > 1-\epsilon,\ \mathcal{C}(\xi_n,\xi_m,\lambda) < \epsilon,\ \mathcal{D}(\xi_n,\xi_m,\lambda) < \epsilon$$

i.e.,

$$\lim_{n.m\to\infty} \mathcal{A}(\xi_n,\xi_m,\lambda) = 1,\ \lim_{n.m\to\infty} \mathcal{B}(\xi_n,\xi_m,\lambda) = 1,\ \lim_{n,m\to\infty} \mathcal{C}(\xi_n,\xi_m,\lambda) = 0,\ \lim_{n,m\to\infty} \mathcal{D}(\xi_n,\xi_m,\lambda) = 0$$

(3) $(\mathcal{X},\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\diamond,\bullet)$ is called complete if each Cauchy sequence is convergent to an element in $\mathcal{X}$.

3. Main Result

Definition 3.1. In this context, we define a real-valued function of three variables on $\mathcal{X}^2 \times (0,\infty)$ where $\mathcal{X}$ is any non-empty set, denoted as $\mathcal{G}$ to possess the property (UC) if for any sequences (ξ_n) and (ω_n) in $\mathcal{X}$, the following equality holds:

$$\lim_{\lambda\to\lambda_0}\lim_{n\to\infty}\mathcal{G}(\xi_n,\omega_n,\lambda)=\lim_{n\to\infty}\lim_{\lambda\to\lambda_0}\mathcal{G}(\xi_n,\omega_n,\lambda).$$

whenever the two limits are exist.

Throughout the remainder of this study, we will assume that each of the fuzzy sets $\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}$ exhibits the UC property.

We will commence with several pertinent lemmas.

Lemma 3.2. *Let $(\mathcal{X},\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\diamond,\bullet)$ be a NFMS. Then*

(1) $\mathcal{A}(\xi,\omega,\cdot):\mathbb{R}^+\to\mathbb{R}^+$ *is non-decreasing*

(2) $\mathcal{B}(\xi,\omega,\cdot):\mathbb{R}^+\to\mathbb{R}^+$ *is non-decreasing*

(3) $\mathcal{C}(\xi,\omega,\cdot):\mathbb{R}^+\to\mathbb{R}^+$ *is non-increasing*

(4) $\mathcal{D}(\xi,\omega,\cdot):\mathbb{R}^+\to\mathbb{R}^+$ *is non-increasing*

Proof. (1) Let $\lambda_1,\lambda_2>0$, with $\lambda_1>\lambda_2$. Then, there is $\delta>0$ such that $\lambda_1=\lambda_2+\delta$. From (5), we get

$$\begin{aligned}\mathcal{A}(\xi,\omega,\lambda_1)&=\mathcal{A}(\xi,\omega,\lambda_2+\delta)\\&\geq\mathcal{A}(\xi,\omega,\lambda_2)\diamond\mathcal{A}(\omega,\omega,\delta)\\&=\mathcal{A}(\xi,\omega,\lambda_2).\end{aligned}$$

The proofs for (2),(3) and (4) are identical to that of (1). □

Lemma 3.3. *Let $(\mathcal{X},\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\diamond,\bullet)$ be a NFMS, and let (ξ_n) be a sequence such that for* $\lambda>0$

$$\begin{aligned}\mathcal{A}(\xi_p,\xi_q,\lambda)&\geq\mathcal{A}(\xi_{p-1},\xi_{q-1},\lambda)\\\mathcal{B}(\xi_p,\xi_q,\lambda)&\geq\mathcal{B}(\xi_{p-1},\xi_{q-1},\lambda)\\\mathcal{C}(\xi_p,\xi_q,\lambda)&\leq\mathcal{C}(\xi_{p-1},\xi_{q-1},\lambda)\\\mathcal{D}(\xi_p,\xi_q,\lambda)&\leq\mathcal{D}(\xi_{p-1},\xi_{q-1},\lambda)\end{aligned}\tag{1}$$

and

$$\begin{aligned}\lim_{n\to\infty}\mathcal{A}(\xi_n,\xi_{n+1},\lambda)&=1,\\\lim_{n\to\infty}\mathcal{B}(\xi_n,\xi_{n+1},\lambda)&=1,\\\lim_{n\to\infty}\mathcal{C}(\xi_n,\xi_{n+1},\lambda)&=0,\\\lim_{n\to\infty}\mathcal{D}(\xi_n,\xi_{n+1},\lambda)&=0.\end{aligned}\tag{2}$$

If (ξ_n) is not Cauchy, then there exist an $1 > \epsilon > 0$ and $\lambda > 0$ along with two subsequences (ξ_{n_k}) and (ξ_{m_k}) derived from (ξ_n), where (m_k) such that one at least of the following holds.

$$\lim_{k\to\infty} \mathcal{A}(\xi_{n_k}, \xi_{m_k}, \lambda) = 1 - \epsilon,$$

$$\lim_{k\to\infty} \mathcal{B}(\xi_{n_k}, \xi_{m_k}, \lambda) = 1 - \epsilon,$$

$$\lim_{k\to\infty} \mathcal{C}(\xi_{n_k}, \xi_{m_k}, \lambda) = \epsilon,$$

$$\lim_{k\to\infty} \mathcal{D}(\xi_{n_k}, \xi_{m_k}, \lambda) = \epsilon.$$

Proof. If (ξ_n) is not Cauchy, then for each $\lambda > 0$

$$\lim_{n,m\to\infty} \mathcal{A}(\xi_n, \xi_m, \lambda) \neq 1,$$

$$\lim_{n,m\to\infty} \mathcal{B}(\xi_n, \xi_m, \lambda) \neq 1,$$

$$\lim_{n,m\to\infty} \mathcal{C}(\xi_n, \xi_m, \lambda) \neq 0,$$

or

$$\lim_{n,m\to\infty} \mathcal{D}(\xi_n, \xi_m, \lambda) \neq 0.$$

Case 1: If $\lim_{n,m\to\infty} \mathcal{A}(\xi_n, \xi_m, \lambda) \neq 1$, then there are $\lambda > 0$, and $\epsilon > 0$ along with two subsequences (ξ_{n_k}) and (ξ_{m_k}) derived from (ξ_n), where (m_k) is selected as the smallest index satisfying the condition.

$$\mathcal{A}(\xi_{n_k}, \xi_{m_k}, \lambda) \leq 1 - \epsilon, \quad m_k > n_k > k. \tag{3}$$

This implies that

$$\mathcal{A}(\xi_{n_k}, \xi_{m_k - 1}, \lambda) > 1 - \epsilon. \tag{4}$$

chose $\delta > 0$. Then

$$\begin{aligned} \mathcal{A}(\xi_{n_k}, \xi_{m_k}, \lambda + \delta) &\geq \mathcal{A}(\xi_{n_k}, \xi_{m_k - 1}, \lambda) \diamond \mathcal{A}(\xi_{m_k - 1}, \xi_{m_k}, \delta) \\ &> (1 - \epsilon) \diamond \mathcal{A}(\xi_{m_k - 1}, \xi_{m_k}, \delta) \end{aligned}$$

Using Equation 2, we get

$$\liminf_{k\to\infty} \mathcal{A}(\xi_{n_k}, \xi_{m_k}, \lambda + \delta) \geq (1 - \epsilon).$$

Also,

$$\begin{aligned}(1-\epsilon) &\leq \lim_{\delta\to 0^+}\liminf_{k\to\infty}\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda+\delta)\\ &=\liminf_{k\to\infty}\lim_{\delta\to 0^+}\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda+\delta)\\ &=\liminf_{k\to\infty}\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda).\end{aligned}$$

Also, from 3, it follows

$$\limsup_{k\to\infty}\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda)\leq(1-\epsilon).$$

So, we get

$$\lim_{k\to\infty}\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda)=(1-\epsilon).$$

Again, we have

$$\begin{aligned}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda+\delta) &\geq \mathcal{A}(\xi_{n_k-1},\xi_{n_k},\delta)\diamond\mathcal{A}(\xi_{n_k},\xi_{m_k-1},\lambda)\\ &>\mathcal{A}(\xi_{n_k-1},\xi_{n_k},\delta)\diamond(1-\epsilon).\end{aligned}$$

Using Equation 2, we get $\liminf_{k\to\infty}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda+\delta)\geq(1-\epsilon)$.
Also,

$$\begin{aligned}(1-\epsilon) &\leq \lim_{\delta\to 0^+}\liminf_{k\to\infty}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda+\delta)\\ &=\liminf_{k\to\infty}\lim_{\delta\to 0^+}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda+\delta)\\ &=\liminf_{k\to\infty}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda).\end{aligned}$$

From Eq 3, we get

$$\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)\leq\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda)\leq(1-\epsilon).$$

So,

$$\limsup_{k\to\infty}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)\leq(1-\epsilon).$$

Hence,

$$\lim_{k\to\infty}\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)=(1-\epsilon).$$

The demonstration for the remaining cases is Similar to that of Case (1). □

In order to support our primary conclusion, we require the subsequent category of functions as delineated by Geraghty in [56].

Definition 3.4. [56] Let S represent the set of all functions $\alpha : \mathbb{R}^+ \to [0,1)$ that fulfill the following condition:

$$\alpha(t_n) \to 1 \implies t_n \to 0.$$

Definition 3.5. Let $(\mathcal{X}, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \diamond, \bullet)$ be a NFMS, $\alpha \in S$. A mapping $f : \mathcal{X} \to \mathcal{X}$ is called α-neutrosophic fuzzy contraction if for each $\xi, \omega \in \mathcal{X}$ and each $\lambda > 0$, we have

$$\frac{1}{\mathcal{A}(f\xi, f\omega, \lambda)} - 1 \leq \alpha\left(\frac{1}{\mathcal{A}(\xi, \omega, \lambda)} - 1\right)\left(\frac{1}{\mathcal{A}(\xi, \omega, \lambda)} - 1\right),$$
$$\frac{1}{\mathcal{B}(f\xi, f\omega, \lambda)} - 1 \leq \alpha\left(\frac{1}{\mathcal{B}(\xi, \omega, \lambda)} - 1\right)\left(\frac{1}{\mathcal{B}(\xi, \omega, \lambda)} - 1\right),$$
$$\mathcal{C}(f\xi, f\omega, \lambda) \leq \alpha(\mathcal{C}(\xi, \omega, \lambda))(\mathcal{C}(\xi, \omega, \lambda)),$$

and

$$\mathcal{D}(f\xi, f\omega, \lambda) \leq \alpha(\mathcal{D}(\xi, \omega, \lambda))(\mathcal{D}(\xi, \omega, \lambda)).$$

Theorem 3.6. *Let $(\mathcal{X}, \mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \diamond, \bullet)$ be a complete NFMS, Suppose that there is $\alpha \in S$ such that $f : \mathcal{X} \to \mathcal{X}$ is α-neutrosophic fuzzy contraction. Consequently, the function f possesses a unique fixed point.*

Proof. Let $\xi_0 \in \mathcal{X}$ represents an arbitrary point. We examine the Picard sequence (ξ_n) characterized by the relation $\xi_{n+1} = f(\xi_n)$ for all $n \geq 0$. By Definition 3.5 we have

$$\frac{1}{\mathcal{A}(\xi_n, \xi_{n+1}, \lambda)} - 1 \leq \alpha\left(\frac{1}{\mathcal{A}(\xi_{n-1}, \xi_n, \lambda)} - 1\right)\left(\frac{1}{\mathcal{A}(\xi_{n-1}, \xi_n, \lambda)} - 1\right),$$
$$\frac{1}{\mathcal{B}(\xi_n, \xi_{n+1}, \lambda)} - 1 \leq \alpha\left(\frac{1}{\mathcal{B}(\xi_{n-1}, \xi_n, \lambda)} - 1\right)\left(\frac{1}{\mathcal{B}(\xi_{n-1}, \xi_n, \lambda)} - 1\right),$$

$$\mathcal{C}(\xi_n, \xi_{n+1}, \lambda) \leq \alpha(\mathcal{C}(\xi_{n-1}, \xi_n, \lambda))(\mathcal{C}(\xi_{n-1}, \xi_n, \lambda)),$$

and

$$\mathcal{D}(\xi_n, \xi_{n+1}, \lambda) \leq \alpha(\mathcal{D}(\xi_{n-1}, \xi_n, \lambda))(\mathcal{D}(\xi_{n-1}, \xi_n, \lambda)).$$

Thus,

$$\frac{\frac{1}{\mathcal{A}(\xi_n, \xi_{n+1}, \lambda)} - 1}{\left(\frac{1}{\mathcal{A}(\xi_{n-1}, \xi_n, \lambda)} - 1\right)} \leq \alpha\left(\frac{1}{\mathcal{A}(\xi_{n-1}, \xi_n, \lambda)} - 1\right), \tag{5}$$

$$\frac{\frac{1}{\mathcal{B}(\xi_n, \xi_{n+1}, \lambda)} - 1}{\left(\frac{1}{\mathcal{B}(\xi_{n-1}, \xi_n, \lambda)} - 1\right)} \leq \alpha\left(\frac{1}{\mathcal{B}(\xi_{n-1}, \xi_n, \lambda)} - 1\right), \tag{6}$$

$$\frac{\mathcal{C}(\xi_n, \xi_{n+1}, \lambda)}{(\mathcal{C}(\xi_{n-1}, \xi_n, \lambda))} \leq \alpha(\mathcal{C}(\xi_{n-1}, \xi_n, \lambda)), \tag{7}$$

$$\frac{\mathcal{D}(\xi_n, \xi_{n+1}, \lambda)}{(\mathcal{D}(\xi_{n-1}, \xi_n, \lambda))} \leq \alpha(\mathcal{D}(\xi_{n-1}, \xi_n, \lambda)). \tag{8}$$

And also, we get

$$\frac{1}{\mathcal{A}(\xi_n,\xi_{n+1},\lambda)}-1<\frac{1}{\mathcal{A}(\xi_{n-1},\xi_n,\lambda)}-1,$$
$$\frac{1}{\mathcal{B}(\xi_n,\xi_{n+1},\lambda)}-1<\frac{1}{\mathcal{B}(\xi_{n-1},\xi_n,\lambda)}-1,$$

$$\mathcal{C}(\xi_n,\xi_{n+1},\lambda)<(\mathcal{C}(\xi_{n-1},\xi_n,\lambda)),$$

and

$$\mathcal{D}(\xi_n,\xi_{n+1},\lambda)<(\mathcal{D}(\xi_{n-1},\xi_n,\lambda)).$$

So, we have

(1) the sequence $(\mathcal{A}(\xi_n,\xi_{n+1},\lambda):n\in\mathbb{N})$ is nondecreasing in [0,1], and hence, there is $r_{\mathcal{A}}\leq 1$ such that $r_{\mathcal{A}}$ is the limit of this sequence.

(2) the sequence $(\mathcal{B}(\xi_n,\xi_{n+1},\lambda):n\in\mathbb{N})$ is nondecreasing in [0,1], and hence, there is $r_{\mathcal{B}}\leq 1$ such that $r_{\mathcal{B}}$ is the limit of this sequence.

(3) the sequence $(\mathcal{C}(\xi_n,\xi_{n+1},\lambda):n\in\mathbb{N})$ is nonincreasing in [0,1], and hence, there is $r_{\mathcal{C}}\geq 0$ such that $r_{\mathcal{C}}$ is the limit of this sequence.
and

(4) the sequence $(\mathcal{D}(\xi_n,\xi_{n+1},\lambda):n\in\mathbb{N})$ is nonincreasing in [0,1], and hence, there is $r_{\mathcal{D}}\geq 0$ such that $r_{\mathcal{D}}$ is the limit of this sequence..

Case 1: If $r_{\mathcal{A}}<1$, by taking the limit in Eq 5, we get

$$\lim_{n\to\infty}\alpha\left(\frac{1}{\mathcal{A}(\xi_{n-1},\xi_n,\lambda)}-1\right)=1$$

which implies that

$$\lim_{n\to\infty}\frac{1}{\mathcal{A}(\xi_n,\xi_{n+1},\lambda)}-1=0,$$

a contradiction. So $r_{\mathcal{A}}=1$. By the same way we conclude that $r_{\mathcal{B}}=1$, $r_{\mathcal{C}}=0$ and $r_{\mathcal{D}}=0$.

Now, we claim that (ξ_n) i Cauchy. If not then by Lemma 3.3, then there exist an $\epsilon>0$ and $\lambda>0$ along with two subsequences (ξ_{n_k}) and (ξ_{m_k}) derived from (ξ_n), where (m_k) such that one of the following holds

$$\lim_{k\to\infty}\mathcal{A}(\xi_n,\xi_m,\lambda)=1-\epsilon,$$
$$\lim_{k\to\infty}\mathcal{B}(\xi_n,\xi_m,\lambda)=1-\epsilon,$$
$$\lim_{k\to\infty}\mathcal{C}(\xi_n,\xi_m,\lambda)=\epsilon,$$
$$\lim_{k\to\infty}\mathcal{D}(\xi_n,\xi_m,\lambda)=\epsilon.$$

Using Definition 3.5, we deduce that one of the following holds

$$\frac{1}{\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda)}-1\leq\alpha\left(\frac{1}{\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)\left(\frac{1}{\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right),$$
$$\frac{1}{\mathcal{B}(\xi_{n_k},\xi_{m_k},\lambda)}-1\leq\alpha\left(\frac{1}{\mathcal{B}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)\left(\frac{1}{\mathcal{B}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right),$$
$$\mathcal{C}(\xi_{n_k},\xi_{m_k},\lambda)\leq\alpha(\mathcal{C}(\xi_{n_k-1},\xi_{m_k-1},\lambda))(\mathcal{C}(\xi_{n_k-1},\xi_{m_k-1},\lambda)),$$

or

$$\mathcal{D}(\xi_{n_k},\xi_{m_k},\lambda)\leq\alpha(\mathcal{D}(\xi_{n_k-1},\xi_{m_k-1},\lambda))(\mathcal{D}(\xi_{n_k-1},\xi_{m_k-1},\lambda))$$

So,

$$\frac{\frac{1}{\mathcal{A}(\xi_{n_k},\xi_{m_k},\lambda)}-1}{\left(\frac{1}{\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)}\leq\alpha\left(\frac{1}{\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right),$$
$$\frac{\frac{1}{\mathcal{B}(\xi_{n_k},\xi_{m_k},\lambda)}-1}{\left(\frac{1}{\mathcal{B}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)}\leq\alpha\left(\frac{1}{\mathcal{B}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right),$$
$$\frac{\mathcal{C}(\xi_{n_k},\xi_{m_k},\lambda)}{(\mathcal{C}(\xi_{n_k-1},\xi_{m_k-1},\lambda))}\leq\alpha(\mathcal{C}(\xi_{n_k-1},\xi_{m_k-1},\lambda)),$$

or

$$\frac{\mathcal{D}(\xi_{n_k},\xi_{m_k},\lambda)}{(\mathcal{D}(\xi_{n_k-1},\xi_{m_k-1},\lambda))}\leq\alpha(\mathcal{D}(\xi_{n_k-1},\xi_{m_k-1},\lambda)).$$

Hence, by taking the limit on $k\to\infty$, we get

$$\lim_{k\to\infty}\alpha\left(\frac{1}{\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)=1,$$

$$\lim_{k\to\infty}\alpha\left(\frac{1}{\mathcal{B}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)=1,$$

$$\lim_{k\to\infty}\alpha\left(\mathcal{C}(\xi_{n_k-1},\xi_{m_k-1},\lambda)\right)=1,$$

or

$$\lim_{k\to\infty}\alpha\left(\mathcal{D}(\xi_{n_k-1},\xi_{m_k-1},\lambda)\right)=1.$$

which implies that

$$\lim_{k\to\infty}\left(\frac{1}{\mathcal{A}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)=0,$$

$$\lim_{k\to\infty}\left(\frac{1}{\mathcal{B}(\xi_{n_k-1},\xi_{m_k-1},\lambda)}-1\right)=0,$$

$$\lim_{k\to\infty}\left(\mathcal{C}(\xi_{n_k-1},\xi_{m_k-1},\lambda)\right)=0,$$

or

$$\lim_{k\to\infty}(\mathcal{D}(\xi_{n_k-1},\xi_{m_k-1},\lambda))=0.$$

which leads to a contradiction in each single case.

Hence (ξ_n) is a Cauchy sequence, thus, there is $u\in\mathcal{X}$ such that $\xi_n\to u$.

Definition 3.5 gives that

$$\frac{1}{\mathcal{A}(fu,\xi_{n+1},\lambda)}-1\leq\alpha\left(\frac{1}{\mathcal{A}(u,\xi_n,\lambda)}-1\right)\left(\frac{1}{\mathcal{A}(u,\xi_n,\lambda)}-1\right)\to 0 \text{ as } n\to\infty,$$
$$\frac{1}{\mathcal{B}(fu,\xi_{n+1},\lambda)}-1\leq\alpha\left(\frac{1}{\mathcal{B}(u,\xi_n,\lambda)}-1\right)\left(\frac{1}{\mathcal{B}(u,\xi_n,\lambda)}-1\right)\to 0 \text{ as } n\to\infty,$$
$$\mathcal{C}(fu,\xi_{n+1},\lambda)\leq\alpha(\mathcal{C}(u,\xi_n,\lambda))(\mathcal{C}(u,\xi_n,\lambda))\to 0 \text{ as } n\to\infty,$$

and

$$\mathcal{D}(fu,\xi_{n+1},\lambda)\leq\alpha(\mathcal{D}(u,\xi_n,\lambda))(\mathcal{D}(u,\xi_n,\lambda))\to 0 \text{ as } n\to\infty.$$

Which implies that ξ_{n+1} converges to fu, hence $u=fu$.

Let $v\in\mathcal{X}$ with $v=fv$. If $u\neq v$, then from Definition 3.5, it follows that

$$\frac{1}{\mathcal{A}(u,v,\lambda)}-1=\frac{1}{\mathcal{A}(fu,fv,\lambda)}-1\leq\alpha\left(\frac{1}{\mathcal{A}(u,v,\lambda)}-1\right)\left(\frac{1}{\mathcal{A}(u,v,\lambda)}-1\right)<\frac{1}{\mathcal{A}(u,v,\lambda)}-1,$$
$$\frac{1}{\mathcal{B}(u,v,\lambda)}-1=\frac{1}{\mathcal{B}(fu,fv,\lambda)}-1\leq\alpha\left(\frac{1}{\mathcal{B}(u,v,\lambda)}-1\right)\left(\frac{1}{\mathcal{B}(u,v,\lambda)}-1\right)<\frac{1}{\mathcal{B}(u,v,\lambda)}-1,$$
$$\mathcal{C}(u,v,\lambda)=\mathcal{C}(fu,fv,\lambda)\leq\alpha(\mathcal{C}(u,v,\lambda))(\mathcal{C}(u,v,\lambda))<\mathcal{C}(u,v,\lambda),$$

and

$$\mathcal{D}(u,v,\lambda)=\mathcal{D}(fu,fv,\lambda)\leq\alpha(\mathcal{D}(u,v,\lambda))(\mathcal{D}(u,v,\lambda))<\mathcal{D}(u,v,\lambda).$$

which is a contradiction. So $u=v$. □

By defining the function $\alpha(s)=q$, with the constant q restricted to the interval $[0,1)$, we can draw the following conclusion.

Corollary 3.7. *Let $(\mathcal{X},\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D},\diamond,\bullet)$ be a complete NFMS, Suppose that $f:\mathcal{X}\to\mathcal{X}$ satisfies the following for each $\xi,\omega\in\mathcal{X}$ and each $\lambda>0$, we have:*

$$\frac{1}{\mathcal{A}(f\xi,f\omega,\lambda)}-1\leq q\left(\frac{1}{\mathcal{A}(\xi,\omega,\lambda)}-1\right),$$
$$\frac{1}{\mathcal{B}(f\xi,f\omega,\lambda)}-1\leq q\left(\frac{1}{\mathcal{B}(\xi,\omega,\lambda)}-1\right),$$
$$\mathcal{C}(f\xi,f\omega,\lambda)\leq q\mathcal{C}(\xi,\omega,\lambda),$$

and

$$\mathcal{D}(f\xi,f\omega,\lambda)\leq q\mathcal{D}(\xi,\omega,\lambda).$$

Consequently, the function f possesses a unique fixed point.

4. Conclusion

In this research, we presented fixed point theorems related to nonlinear contractions within the advanced framework of neutrosophic fuzzy metric spaces. Additionally, we established several fixed point results relevant to this specific context. For future studies, these mathematical tools can be used effectively with other tools and techniques that can be observed through [57]– [77]

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