Neutrosophic Sets and Systems, Vol. 77, 2025

University of New Mexico

Modified 2-tuple Linguistic Neutrosophic Hybrid GRA Framework for Comprehensive Strength Evaluation for High-Quality Development in the Cultural Industry

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Abstract: The evaluation of comprehensive strength for the high-quality development of the cultural industry is a systematic assessment of the overall performance of the cultural sector across multiple dimensions. This evaluation covers aspects such as economic benefits, innovation capacity, social impact, cultural influence, and sustainability. By using both quantitative indicators and qualitative analysis, it measures the industry's overall competitiveness and growth potential, providing a scientific basis for policymaking, resource allocation, and industry optimization. This evaluation system helps promote high-quality and sustainable growth in the cultural industry. The comprehensive strength evaluation for high-quality development in the cultural industry is a multiple-attribute group decision-making (MAGDM) approach. In this study, entropy is put forward weight numbers along with 2-tuple linguistic neutrosophic sets (2TLNSs), and the 2-tuple linguistic neutrosophic number hybrid grey relational analysis (2TLNN-HGRA) approach is put forward based on grey relational analysis (GRA) and 2TLNSs. Finally, a numerical example of comprehensive strength evaluation for high-quality development in the cultural industry was put forward and some comparisons are put forward to illustrate the 2TLNN-HGRA approach.

Keywords: MAGDM problem; 2TLNSs; HGRA approach; Comprehensive strength evaluation for high-quality development in the cultural industry

1. Introduction

The comprehensive evaluation of the high-quality development of the cultural industry encompasses various dimensions, each contributing to the overall strength and sustainability of the sector. Key indicators include economic performance, such as the industry's contribution to GDP, employment generation, and revenue growth. The integration of innovation and technology plays a crucial role, with digital transformation and the use of creative technologies boosting productivity and expanding market reach. Cultural influence is another vital aspect, measuring the industry's ability to preserve and promote cultural heritage, foster creativity, and enhance global cultural exchanges. Sustainability is assessed through the industry's environmental impact, resource efficiency, and adaptability to changing social and economic conditions. Additionally, government policies and financial support are critical, as they provide the necessary infrastructure and incentives for growth. The robustness of the value chain—from content creation to distribution—along with the industry's capacity for international collaboration, further reflects its comprehensive strength. A well-rounded evaluation ensures that the cultural industry not only thrives economically but also enriches society and culture, both locally and globally.

Multiple-attribute group decision-making (MAGDM) is a process that involves evaluating and selecting the best option from a set of alternatives based on multiple attributes, with input from a group of decision-makers [1]. Each attribute represents a criterion that needs to be considered, and the preferences or judgments of the decision-makers are aggregated to arrive at a collective decision. In MAGDM, different methods can be applied to handle the complexity of multiple criteria and the varying opinions within the group, such as the weighted sum model, analytic hierarchy process (AHP), or fuzzy approaches. The decision-making process typically involves several steps, including defining the problem, identifying relevant attributes, collecting individual judgments, and applying a decision rule or algorithm to combine the group's preferences [2]. Challenges in MAGDM include managing conflicting opinions, weighting different attributes appropriately, and ensuring a fair and transparent decision process. This approach is widely used in areas such as business strategy, public policy, and resource allocation, where decisions must account for a range of factors and stakeholders' inputs [3]. The comprehensive strength evaluation for high-quality development in the cultural industry is approached through MAGDM. The 2TLNNSs, referenced in sources [4], serve as an effective tool for managing uncertain information during this evaluation. Deng [5] introduced several grey relational laws and developed the GRA approach. Compared to other MADM methods, the GRA approach [6] is adept at handling the shape similarity among choices from the PIA and NIA. In this study, the 2TLNN-HGRA method is introduced, integrating both GRA principles and 2TLNNSs. A numerical example assessing the ecological tourism environmental carrying capacity is demonstrated, and several comparisons are conducted to validate the 2TLNN-HGRA approach.

The primary research objectives and motivations of this study include:(1)Incorporation of entropy to assign weight numbers within 2TLNNSs; (2) Implementation of the 2TLNN-HGRA approach, leveraging both 2TLNNHD and 2TLNNLD under 2TLNNSs; (3) Application of 2TLNN-HGRA method to address MAGDM challenges under 2TLNNSs; (4) Presentation of practical numerical example to showcase the effectiveness of the 2TLNN-HGRA approach; (5) Conducting a thorough comparative analysis with several existing decision-making approaches to demonstrate efficiency.

The structure of this study is presented as follows: Section 2 introduces the literature review. Section 3 introduces the 2TLNNSs. The 2TLNN-HGRA method for addressing the MAGDM is detailed in Section 4. A numerical example evaluating the comprehensive strength for high-quality development in the cultural industry is provided in Section 5. Section 6 presents the conclusions. Section 6 presents the future research directions.

2. Literature review

In recent years, scholars have increasingly focused on the high-quality development of the cultural industry, with research covering various aspects such as policy, technological empowerment, and the restructuring of industrial chains. This review provides a chronological summary of 17 relevant studies, outlining the research progress in the field of high-quality development of the cultural industry. The earliest study dates back to 2018 when Zhou [7] pointed out that Sichuan's cultural industry should seize historical opportunities and develop into a pillar industry of the national economy. This study proposed development strategies centered on building spatial and industrial layouts. Following this, He and Ren [8] in 2019 analyzed the impact of different types of cultural industry policies on high-quality development in China, finding that indirect guidance policies were more effective in the eastern regions, while all three types of policies were effective in the central and western regions. That same year, Yao [9] examined the high-quality development of the cultural industry in Haining, emphasizing the need to accelerate the construction of a modern cultural industry based on local resources in the context of the new era. Li [10] explored the connotation, driving forces, and pathways of high-quality development in the cultural industry, highlighting that the transformation to high-quality development is key to meeting people’s demand for a better life. Huang and Zhao [11] studied financing innovations for promoting the high-quality development of Tibet's cultural industry, emphasizing the leveraging role of national financial investments. Fan [12] focused on the dual-effect unity of the cultural industry in the context of the digital economy, stressing the importance of digital transformation for high-quality development. Meanwhile, Yang [13] pointed out that the cultural industry should maintain cultural confidence and expand internationally to enhance its global competitiveness during its transformation and upgrading. By 2021, as the digital economy developed rapidly, Luo and Zhou [14] examined the high-quality transformation of small and micro cultural enterprises, identifying delays in digital platform development as a major issue. Feng [15] emphasized the new requirements of the times for the cultural industry, noting that optimizing the industrial structure is crucial for building a cultural power. Li and Huang [16] further discussed the "Culture+" model, arguing that empowering the cultural industry with digital technology is the core pathway to achieving high-quality development. In 2022, research delved deeper into specific mechanisms. Zhou and Tan [17] analyzed how big data empowers the high-quality development of the digital cultural industry by optimizing operational structures and infrastructure. Xiang [18] proposed a fusion mechanism for the digital cultural industry, emphasizing the collaborative roles of social drivers, cultural management, technological accumulation, and other factors. In 2023, Ban and Sun [19] pointed out in their news report that the high-quality development of the cultural industry requires continued policy support, especially in developing and utilizing unique cultural resources. By 2024, research increasingly focused on the paths of digital empowerment in the cultural industry. Hui and Zhang [20] conducted an empirical study analyzing the impact of the digital economy on high-quality development, finding that the digital economy significantly enhances the level of the cultural industry, especially in pilot areas of the "Broadband China" initiative. Jiang and Ye [21] proposed suggestions for enhancing high-quality development by restructuring the cultural industry chain through extension and optimization. Zhang, Li and Yang [22] explored the challenges faced by the cultural industry under the wave of digitalization and proposed constructing an innovation-driven industrial ecosystem to promote digital transformation. Finally, Li and Fu [23] pointed out that the high-quality development of the cultural industry depends on improving market competitiveness and innovation capabilities, as well as strengthening international cooperation. In summary, research on the high-quality development of the cultural industry in recent years has gradually expanded across multiple dimensions, including policy, technological empowerment, and industrial chain development. As the digital economy and big data technologies continue to advance, the cultural industry is evolving toward greater intelligence and innovation.

2.1 Importance of Neutrosophic Sets in the Comprehensive Strength Evaluation for High-Quality Development in the Cultural Industry

This section explores the role of neutrosophic sets, particularly the 2-tuple linguistic neutrosophic sets (2TLNSs), in evaluating the comprehensive strength of the cultural industry. By integrating entropy-based weight numbers and hybrid grey relational analysis (2TLNN-HGRA), this approach provides a robust framework for assessing multiple attributes of the cultural sector, including economic benefits, innovation capacity, social impact, and sustainability, contributing to informed decision-making and sustainable industry growth.

3. Preliminaries

Wang et al. [24] put forward the 2TLNSs.

Definition 1 [24]. Let $u\vartheta$ be linguistic term sets (LTSs), and

$$u\vartheta = \{u\vartheta_0 = exceedingly\ terrible, u\vartheta_1 = very\ terrible,$$
$$u\vartheta_2 = terrible, u\vartheta_3 = medium, u\vartheta_4 = well,$$
$$u\vartheta_5 = very\ well, u\vartheta_6 = exceedingly\ well\},$$

Then the 2TLNSs are put forward:

$$u\xi = \langle (u\vartheta_t, ux,), (u\vartheta_i, uy), (u\vartheta_f, uz) \rangle \quad (1)$$

with 2-tuple linguistic values $(u\vartheta_t, ux,), (u\vartheta_i, uy), (u\vartheta_f, uz)$ which is membership, indeterminacy, and non-membership and $0 \le \Delta^{-1}(u\vartheta_t, ux,) + \Delta^{-1}(u\vartheta_i, uy) + \Delta^{-1}(u\vartheta_f, uz) \le 3u\pi$.

Definition 2 [24]. Let $u\xi_1 = \langle (u\vartheta_{t_1}, ux_1,), (u\vartheta_{i_1}, ry_1), (u\vartheta_{f_1}, uz_1) \rangle$, $u\xi_2 = \langle (u\vartheta_{t_2}, ux_2,), (u\vartheta_{i_2}, uy_2), (u\vartheta_{f_2}, uz_2) \rangle$ be 2TLNN (2-tuple linguistic neutrosophic number), $\lambda > 0$, the operational laws are put forward:

$$(1)\ u\xi_1 \oplus u\xi_2 = \left\{ \begin{array}{l} \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{t_1}, ux_1,)}{u\pi} + \frac{\Delta^{-1}(u\vartheta_{t_2}, ux_2,)}{u\pi} - \frac{\Delta^{-1}(u\vartheta_{t_1}, ux_1,)}{u\pi} \cdot \frac{\Delta^{-1}(u\vartheta_{t_2}, ux_2,)}{u\pi}\right)\right), \\ \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{i_1}, uy_1)}{u\pi} \cdot \frac{\Delta^{-1}(u\vartheta_{i_2}, uy_2)}{u\pi}\right)\right), \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{f_1}, uz_1)}{u\pi} \cdot \frac{\Delta^{-1}(u\vartheta_{f_2}, uz_2)}{u\pi}\right)\right) \end{array} \right\}; \quad (2)$$

$$(2)\ \ u\xi_1 \otimes u\xi_2 = \begin{Bmatrix} \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{t_1},ux_{1,})}{u\pi}+\frac{\Delta^{-1}(u\vartheta_{t_2},ux_{2,})}{u\pi}\right)\right), \\ \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)}{\pi}+\frac{\Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi}-\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)}{u\pi}\cdot\frac{\Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi}\right)\right), \\ \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)}{u\pi}+\frac{\Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi}-\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)}{u\pi}\cdot\frac{\Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi}\right)\right) \end{Bmatrix};\quad (3)$$

$$(3)\ \ \lambda u\xi_1 = \begin{Bmatrix} \Delta\left(u\pi\left(1-\left(1-\frac{\Delta^{-1}(u\vartheta_{t_1},ux_{1,})}{u\pi}\right)^{\lambda}\right)\right), \\ \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)}{u\pi}\right)^{\lambda}\right),\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)}{u\pi}\right)^{\lambda}\right) \end{Bmatrix};\quad (4)$$

$$(4)\ \ (u\xi_1)^{\lambda} = \begin{Bmatrix} \Delta\left(u\pi\left(\frac{\Delta^{-1}(u\vartheta_{t_1},ux_{1,})}{u\pi}\right)^{\lambda}\right), \\ \Delta\left(u\pi\left(1-\left(1-\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)}{u\pi}\right)^{\lambda}\right)\right),\Delta\left(u\pi\left(1-\left(1-\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)}{u\pi}\right)^{\lambda}\right)\right) \end{Bmatrix}.\quad (5)$$

Wang et al. [24] put forward the score function (SF) and accuracy function (AF) under 2TLNNs.

Definition 3[24]. Let $u\xi_1 = \langle(u\vartheta_{t_1}, ux_{1,}),(u\vartheta_{i_1}, ry_1),(u\vartheta_{f_1}, uz_1)\rangle$, $u\xi_2 = \langle(u\vartheta_{t_2}, ux_{2,}),(u\vartheta_{i_2}, uy_2),(u\vartheta_{f_2}, uz_2)\rangle$, the SF and AF are put forward:

$$SF(u\xi_1) = \frac{\begin{pmatrix}2u\pi+\Delta^{-1}(u\vartheta_{t_1},ux_{1,}) \\ -\Delta^{-1}(u\vartheta_{i_1},uy_1)-\Delta^{-1}(u\vartheta_{f_1},uz_1)\end{pmatrix}}{3u\pi},\qquad SF(u\xi_1)\in[0,1]\quad (6)$$

$$SF(u\xi_2) = \frac{\begin{pmatrix}2u\pi+\Delta^{-1}(u\vartheta_{t_2},ux_2) \\ -\Delta^{-1}(u\vartheta_{i_2},uy_2)-\Delta^{-1}(u\vartheta_{f_2},uz_2)\end{pmatrix}}{3u\pi},\ SF(u\xi_2)\in[0,1]\quad (7)$$

$$AF(u\xi_1) = \frac{1}{2u\pi}\begin{pmatrix}u\pi+\Delta^{-1}(u\vartheta_{t_1},ux_{1,}) \\ -\Delta^{-1}(u\vartheta_{f_1},uz_1)\end{pmatrix},\qquad AF(u\xi_1)\in[0,1]\quad (8)$$

$$AF(u\xi_2) = \frac{1}{2u\pi}\begin{pmatrix}u\pi+\Delta^{-1}(u\vartheta_{t_2},ux_2) \\ -\Delta^{-1}(u\vartheta_{f_2},uz_2)\end{pmatrix},\qquad AF(u\xi_2)\in[0,1]\quad (9)$$

For two $u\xi_1 = \langle(u\vartheta_{t_1}, ux_{1,}),(u\vartheta_{i_1}, ry_1),(u\vartheta_{f_1}, uz_1)\rangle$, $u\xi_2 = \langle(u\vartheta_{t_2}, ux_{2,}),(u\vartheta_{i_2}, uy_2),(u\vartheta_{f_2}, uz_2)\rangle$, the order is put forward: (1) $if\ SF(u\xi_1) \prec SF(u\xi_2),\ u\xi_1 \prec u\xi_2$; (2) $if\ SF(u\xi_1) = SF(u\xi_2), AF(u\xi_1) \prec AF(u\xi_2), u\xi_1 \prec u\xi_2$; $if\ SF(u\xi_1) = SF(u\xi_2), AF(u\xi_1) = AF(u\xi_2),\ u\xi_1 = u\xi_2$.

Then, the 2TLNNWA approach [24] is put forward.

Definition 4[24]. Let $u\xi_j = \left\langle\left(u\vartheta_{t_j}, ux_{j,}\right),\left(u\vartheta_{i_j}, uy_j\right),\left(u\vartheta_{f_j}, uz_j\right)\right\rangle$ be 2TLNNs, the 2TLNNWA approach is put forward:

$$\begin{aligned}&\text{2TLNNWA}(u\xi_1, u\xi_2, \ldots, u\xi_n) \\ &= uw_1u\xi_1 \oplus uw_2u\xi_2 \ldots \oplus uw_nu\xi_n = \overset{j=1}{\underset{n}{\oplus}}\ uw_ju\xi_j\end{aligned}$$

$$= \begin{pmatrix} \Delta\left(u\pi\left(1-\prod_{j=1}^{n}\left(1-\frac{\Delta^{-1}(u\vartheta_{t_j},ux_{j,})}{u\pi}\right)^{uw_j}\right)\right), \\ \Delta\left(u\pi\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(u\vartheta_{i_j},uy_j)}{u\pi}\right)^{uw_j}\right),\Delta\left(u\pi\prod_{j=1}^{n}\left(\frac{\Delta^{-1}(u\vartheta_{i_j},uy_j)}{u\pi}\right)^{uw_j}\right) \end{pmatrix}\quad (10)$$

with$uw_j$is weight value of $r\xi_j$,$\sum_{j=1}^{n} uw_j = 1$.

Then the 2TLNNHD (2TLNN Hamming distance) [4] and 2TLNNLD (2TLNN Logarithmic distance) is put forward.

Definition 5[4]. Let $u\xi_1 = \langle (u\vartheta_{t_1}, ux_1,), (u\vartheta_{i_1}, ry_1), (u\vartheta_{f_1}, uz_1) \rangle$, $u\xi_2 = \langle (u\vartheta_{t_2}, ux_2,), (u\vartheta_{i_2}, uy_2), (u\vartheta_{f_2}, uz_2) \rangle$, then the 2TLNNHD are put forward:

$$2TLNNHD(u\xi_1, u\xi_2) = \frac{1}{3}\left(\left|\frac{\Delta^{-1}(u\vartheta_{t_1},ux_1) - \Delta^{-1}(u\vartheta_{t_2},ux_2)}{u\pi}\right| + \left|\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1) - \Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi}\right| + \left|\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1) - \Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi}\right| \right) \quad (11)$$

Definition 5. Let $u\xi_1 = \langle (u\vartheta_{t_1}, ux_1,), (u\vartheta_{i_1}, ry_1), (u\vartheta_{f_1}, uz_1) \rangle$, $u\xi_2 = \langle (u\vartheta_{t_2}, ux_2,), (u\vartheta_{i_2}, uy_2), (u\vartheta_{f_2}, uz_2) \rangle$, then the 2TLNNLD are put forward:

$$2TLNNLD(u\xi_1, u\xi_2) = \frac{1}{3}\left(\begin{array}{l} \frac{\Delta^{-1}(u\vartheta_{t_1},ux_1)}{u\pi} log \frac{\frac{2\Delta^{-1}(u\vartheta_{t_1},ux_1)}{u\pi}}{\frac{\Delta^{-1}(u\vartheta_{t_1},ux_1)+\Delta^{-1}(u\vartheta_{t_2},ux_2)}{u\pi}} \\ + \frac{\Delta^{-1}(u\vartheta_{t_2},ux_2)}{u\pi} log \frac{\frac{2\Delta^{-1}(u\vartheta_{t_2},ux_2)}{u\pi}}{\frac{\Delta^{-1}(u\vartheta_{t_1},ux_1)+\Delta^{-1}(u\vartheta_{t_2},ux_2)}{u\pi}} \\ + \frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)}{u\pi} log \frac{\frac{2\Delta^{-1}(u\vartheta_{i_1},uy_1)}{u\pi}}{\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)+\Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi}} \\ + \frac{\Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi} log \frac{\frac{2\Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi}}{\frac{\Delta^{-1}(u\vartheta_{i_1},uy_1)+\Delta^{-1}(u\vartheta_{i_2},uy_2)}{u\pi}} \\ + \frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)}{u\pi} log \frac{\frac{2\Delta^{-1}(u\vartheta_{f_1},uz_1)}{u\pi}}{\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)+\Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi}} \\ + \frac{\Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi} log \frac{\frac{2\Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi}}{\frac{\Delta^{-1}(u\vartheta_{f_1},uz_1)+\Delta^{-1}(u\vartheta_{f_2},uz_2)}{u\pi}} \end{array} \right) \quad (12)$$

4. 2TLNN-HGRA approach for MAGDM with 2TLNSs

The 2TLNN-HGRA is put forward by the MAGDM. Let $UX = (UX_1, UX_2, \dots, UX_m)$ be alternatives and $UY = (UY_1, UY_2, \dots, UY_m)$ be attributes with weight values$uw = (uw_1, uw_2, \cdots, uw_n)$and experts$UZ = (UZ_1, UZ_2, \dots, UZ_m)$ with weight $u\omega = (u\omega_1, u\omega_2, \cdots, u\omega_q)$, the 2TLNN-HGRA approach are put forward MAGDM problem.

Step 1. Put forward the 2TLNN-matrix $UR = \left[UR_{ij}^{(t)}\right]_{m\times n}$:

$UY_1 \quad UY_2 \quad \dots \quad UY_n$

$$UR^{(t)} = \left[UR_{ij}^{(t)}\right]_{m\times n} = \begin{matrix} UX_1 \\ UX_2 \\ \vdots \\ UX_m \end{matrix} \begin{bmatrix} UR_{11}^{(t)} & UR_{12}^{(t)} & \dots & UR_{1n}^{(t)} \\ UR_{21}^{(t)} & UR_{22}^{(t)} & \dots & UR_{2n}^{(t)} \\ \vdots & \vdots & \vdots & \vdots \\ UR_{m1}^{(t)} & UR_{m2}^{(t)} & \dots & UR_{mn}^{(t)} \end{bmatrix} \quad (13)$$

where$UR_{ij}^{(t)} = \left(\left(u\vartheta_{t_{ij}}^{(t)}, ux_{ij}^{(t)},\right), \left(u\vartheta_{i_{ij}}^{(t)}, uy_{ij}^{(t)},\right), \left(u\vartheta_{f_{ij}}^{(t)}, uz_{ij}^{(t)},\right)\right)$ is 2TLNNs.

Step 2. Put forward the 2TLNN overall matrix $UR = \left[UR_{ij}\right]_{m\times n}$:

$$UR = \left[UR_{ij}\right]_{m\times n} = \begin{matrix} \\ UX_1 \\ UX_2 \\ \vdots \\ UX_m \end{matrix} \begin{matrix} UY_1 & UY_2 & \dots & UY_n \\ UR_{11} & UR_{12} & \dots & UR_{1n} \\ UR_{21} & UR_{22} & \dots & UR_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ UR_{m1} & UR_{m2} & \dots & UR_{mn} \end{matrix} \quad (14)$$

In line with the 2TLNNWA, the $UR = \left[UR_{ij}\right]_{m\times n}$ is put forward:

$$\begin{aligned} UR_{ij} &= \left(\left(u\vartheta_{t_{ij}}, ux_{ij}\right), \left(u\vartheta_{i_{ij}}, uy_{ij}\right), \left(u\vartheta_{f_{ij}}, uz_{ij}\right)\right) \\ &= u\omega_1 UR_{ij}^{(1)} \oplus u\omega_2 UR_{ij}^{(2)} \oplus \cdots \oplus u\omega_q UR_{ij}^{(q)} \\ &= \left\{ \begin{array}{l} \Delta\left(u\pi\left(1 - \prod_{t=1}^{q}\left(1 - \frac{\Delta^{-1}\left(u\vartheta_{t_{ij}}^{(t)}, ux_{ij}^{(t)}\right)}{u\pi}\right)^{u\omega_t}\right)\right), \\ \Delta\left(u\pi \prod_{t=1}^{q}\left(\frac{\Delta^{-1}\left(u\vartheta_{i_{ij}}^{(t)}, uy_{ij}^{(t)}\right)}{u\pi}\right)^{u\omega_t}\right), \Delta\left(u\pi \prod_{t=1}^{q}\left(\frac{\Delta^{-1}\left(u\vartheta_{f_{ij}}^{(t)}, uz_{ij}^{(t)}\right)}{u\pi}\right)^{u\omega_t}\right) \end{array} \right\} \end{aligned} \quad (15)$$

Step 3. Put forward the 2TLNN normalized matrix$NUR = \left[NUR_{ij}\right]_{m\times n}$[25].

For benefit attributes:

$$\begin{aligned} NUR_{ij} = UR_{ij} &= \left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}^{n}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) \\ &= \left(\left(u\vartheta_{t_{ij}}, ux_{ij}\right), \left(u\vartheta_{i_{ij}}, uy_{ij}\right), \left(u\vartheta_{f_{ij}}, uz_{ij}\right)\right) \end{aligned} \quad (16)$$

For cost attributes:

$$\begin{aligned} NUR_{ij} &= \left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}^{n}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) \\ &= \begin{pmatrix} \Delta\left(u\pi - \Delta^{-1}\left(u\vartheta_{t_{ij}}, ux_{ij}\right)\right), \Delta\left(u\pi - \Delta^{-1}\left(u\vartheta_{i_{ij}}, uy_{ij}\right)\right), \\ \Delta\left(u\pi - \Delta^{-1}\left(u\vartheta_{f_{ij}}, uz_{ij}\right)\right) \end{pmatrix} \end{aligned} \quad (17)$$

Step 4. Cultivate the weight numbers through entropy.

Entropy [26] is cultivated for weight numbers. The normalized decision matrix (NDM) is cultivated:

$$NDM_{ij} = \frac{\begin{pmatrix} SF\left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}^{n}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) \\ +AF\left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}^{n}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) + 1 \end{pmatrix}}{\sum_{i=1}^{m} \begin{pmatrix} SF\left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}^{n}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) \\ +AF\left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}^{n}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) + 1 \end{pmatrix}}, \quad (18)$$

The 2TLNN Shannon decision entropy (2TLNNSDE) is cultivated:

$$2TLNNSDE_j = -\frac{1}{\ln m}\sum_{i=1}^{m} NDM_{ij} \ln NDM_{ij} \tag{19}$$

and $NDM_{ij} \ln NDM_{ij} = 0$ if $NDM_{ij} = 0$.

Then, the weight numbers are cultivated:

$$uw_j = \frac{1-2TLNNSDE_j}{\sum_{j=1}^{n}(1-2TLNNSDE_j)} \tag{20}$$

Step 5. Put forward the 2TLNN positive ideal sorting alternative (2TLNNPISA) and 2TLNN negative ideal sorting alternative (2TLNNNISA)[25]:

$$2TLNNPISA = \{2TLNNPISA_j\} \tag{21}$$

$$2TLNNNISA = \{2TLNNNISA_j\} \tag{22}$$

$$2TLNNPISA_j = \left(\left(u\vartheta_{t_j}^{n+}, ux_j^{n+}\right), \left(u\vartheta_{i_j}^{n+}, uy_j^{n+}\right), \left(u\vartheta_{f_j}^{n+}, uz_j^{n+}\right)\right) \tag{23}$$

$$2TLNNNISA_j = \left(\left(u\vartheta_{t_j}^{n-}, ux_j^{n-}\right), \left(u\vartheta_{i_j}^{n-}, uy_j^{n-}\right), \left(u\vartheta_{f_j}^{n-}, uz_j^{n-}\right)\right) \tag{24}$$

$$SF\left(\left(u\vartheta_{t_j}^{n+}, ux_j^{n+}\right), \left(u\vartheta_{i_j}^{n+}, uy_j^{n+}\right), \left(u\vartheta_{f_j}^{n+}, uz_j^{n+}\right)\right) = \max_i SF\left(\left(u\vartheta_{t_{ij}}^{n}, ux_{ij}\right), \left(u\vartheta_{i_{ij}}^{n}, uy_{ij}^{n}\right), \left(u\vartheta_{f_{ij}}^{n}, uz_{ij}^{n}\right)\right) \tag{25}$$

$$SF\left(\left(u\vartheta_{t_j}^{n-}, ux_j^{n-}\right), \left(u\vartheta_{i_j}^{n-}, uy_j^{n-}\right), \left(u\vartheta_{f_j}^{n-}, uz_j^{n-}\right)\right) = \min_i SF\left(\left(u\vartheta_{t_j}^{n+}, ux_j^{n+}\right), \left(u\vartheta_{i_j}^{n+}, uy_j^{n+}\right), \left(u\vartheta_{f_j}^{n+}, uz_j^{n+}\right)\right) \tag{26}$$

Step 6. Put forward the hybrid grey rational coefficients (HGRC) from 2TLNNPISA and 2TLNNNISA:

$$2TLNNPISA(HGRC_{ij}) = \frac{1}{2}\left(\frac{\begin{pmatrix}\min\limits_{1\le i\le m} 2TLNNHD(NUR_{ij}, 2TLNNPISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNHD(NUR_{ij}, 2TLNNPISA_j)\end{pmatrix}}{\begin{pmatrix}2TLNNHD(NUR_{ij}, 2TLNNPISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNHD(NUR_{ij}, 2TLNNPISA_j)\end{pmatrix}} + \frac{\begin{pmatrix}\min\limits_{1\le i\le m} 2TLNNLD(NUR_{ij}, 2TLNNPISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNLD(NUR_{ij}, 2TLNNPISA_j)\end{pmatrix}}{\begin{pmatrix}2TLNNLD(NUR_{ij}, 2TLNNPISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNLD(NUR_{ij}, 2TLNNPISA_j)\end{pmatrix}}\right) \tag{27}$$

$$2TLNNNISA(HGRC_{ij}) = \frac{1}{2}\left(\frac{\begin{pmatrix}\min\limits_{1\le i\le m} 2TLNNHD(NUR_{ij}, 2TLNNNISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNHD(NUR_{ij}, 2TLNNNISA_j)\end{pmatrix}}{\begin{pmatrix}2TLNNHD(NUR_{ij}, 2TLNNNISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNHD(NUR_{ij}, 2TLNNNISA_j)\end{pmatrix}} + \frac{\begin{pmatrix}\min\limits_{1\le i\le m} 2TLNNLD(NUR_{ij}, 2TLNNNISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNLD(NUR_{ij}, 2TLNNNISA_j)\end{pmatrix}}{\begin{pmatrix}2TLNNLD(NUR_{ij}, 2TLNNNISA_j)\\ +\rho \max\limits_{1\le i\le m} 2TLNNLD(NUR_{ij}, 2TLNNNISA_j)\end{pmatrix}}\right) \tag{28}$$

Step 7. Put forward the hybrid grey relation degree (HGRD) from 2TLNNPISA and 2TLNNNISA:

$$2TLNNPIDA(HGRD_i) = \sum_{j=1}^{n} uw_j 2TLNNPIDA(HGRC_{ij}) \tag{29}$$

$$2TLNNNIDA(HGRD_i) = \sum_{j=1}^{n} uw_j 2TLNNNIDA(HGRC_{ij}) \tag{30}$$

Step 8. Put forward the 2TLNN hybrid relative relational degree (2TLNNHRRD):

$$2TLNNHRRD_i = \frac{2TLNNPISA(HGRD_i)}{2TLNNNISA(HGRD_i) + 2TLNNPISA(HGRD_i)} \tag{31}$$

Step 9. Put forward the optimal choice in line with the largest 2TLNNHRRD.

5. Numerical examples and comparisons

This section presents numerical examples and comparisons to illustrate the effectiveness and accuracy of the proposed methods.

5.1 Numerical example of comprehensive strength evaluation for high-quality development in the cultural industry

The comprehensive strength evaluation for high-quality development in the cultural industry is an essential process that assesses the overall performance and potential of the cultural sector in contributing to economic growth, social progress, and cultural innovation. This evaluation framework typically spans multiple dimensions, including economic benefits, innovation capacity, social impact, cultural influence, and sustainability. By systematically analyzing these areas, the evaluation aims to provide a holistic understanding of the industry's strengths and areas for improvement, facilitating informed decision-making for stakeholders. One of the primary focuses of this evaluation is economic contribution, where factors such as revenue generation, job creation, and market growth are considered. A thriving cultural industry not only boosts local economies but also fosters entrepreneurship and creative industries. In parallel, innovation capacity is assessed by measuring the industry's ability to adapt to new technologies, trends, and creative processes. This includes the adoption of digital platforms, the development of new content formats, and the integration of cutting-edge technologies like virtual reality and artificial intelligence into cultural products and services. Another critical dimension is the social impact of the cultural industry. The evaluation examines how cultural activities contribute to social well-being, community building, and the preservation of cultural heritage. It also considers the industry's role in promoting social inclusivity and cultural diversity. This aspect is crucial as it highlights the cultural sector's potential to foster cross-cultural understanding and cohesion in increasingly diverse societies. Furthermore, the evaluation assesses the cultural influence of the industry, focusing on its ability to shape cultural values, trends, and identities both locally and globally. This dimension looks at how cultural products, such as films, music, literature, and art, resonate with audiences and contribute to shaping public discourse and societal norms. Global cultural exports, soft power, and international collaborations are key indicators in this area. Finally, sustainability is a growing concern in the context of the cultural industry’s development. The evaluation considers how well the industry integrates sustainable practices, including environmental responsibility, ethical production, and long-term viability. This is particularly important as the cultural industry, like many others, faces increasing pressure to operate in ways that minimize environmental impact while maximizing social and economic benefits. To conduct a comprehensive strength evaluation, both quantitative and qualitative methods are employed. Quantitative metrics might include financial performance, audience reach, and innovation indices, while qualitative assessments could involve expert evaluations, stakeholder interviews, and cultural impact studies. This combination enables a more nuanced understanding of the industry's performance, offering a balance between measurable outcomes and the more subjective aspects of cultural value. In conclusion, the comprehensive strength evaluation for high-quality development in the cultural industry is an indispensable tool for understanding the complex interplay between economic, social, and cultural factors. It provides policymakers, industry leaders, and other stakeholders with valuable insights that can guide strategic planning, resource allocation, and policy formulation. By fostering a balanced approach to growth, this evaluation supports the cultural industry's ability to thrive in a rapidly changing global landscape while maintaining its core mission of enriching society through creativity and cultural expression. The comprehensive strength evaluation for high-quality development in the cultural industry is MAGDM. Five cultural industry demonstration bases $UX_i(i = 1,2,3,\cdots,5)$ are evaluated in light with different attributes: ①UY_1 is Economic Benefits: The economic benefits indicator measures the cultural industry's contribution to the overall economy, including revenue generation, profit growth, job creation, and market share. This indicator reflects the market performance and profitability of the cultural industry, as well as its role in driving regional and national economic growth. ②UY_2 is Innovation Capacity: The innovation capacity indicator evaluates the cultural industry's performance in terms of innovation in technology, content, and business models. This includes the development of new products, the application of digital technologies, and the exploration of emerging cultural forms. Innovation is crucial for maintaining the competitiveness of the cultural industry and adapting to rapidly changing market demands. ③UY_3 is Social Impact: The social impact indicator assesses the cultural industry's contribution to society, particularly in enhancing social cohesion, promoting cultural identity, education, and increasing public participation. The cultural industry plays an important role in disseminating cultural values, promoting diversity, and enhancing public welfare. ④UY_4 is Sustainability Capacity: The sustainability capacity indicator measures the cultural industry's performance in areas such as environmental protection, resource efficiency, and cultural heritage preservation, ensuring that economic and social benefits are achieved while minimizing negative environmental impacts and promoting long-term ecological and cultural sustainability. Five cultural industry demonstration bases are evaluated through 2TLNNs with expert weights $u\omega = \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$. The 2TLNN-HGRA is put forward for comprehensive strength evaluation for high-quality development in the cultural industry.

Step 1. Put forward the 2TLNN-matrix $RR = \left[RR_{ij}^{(t)}\right]_{5\times 4}$ $(t = 1,2,3)$ (Table 1).

Table 1. 2TLNNs for three experts.

	C_1	C_2	C_3	C_4
A_1	$((u\vartheta_4, 0), (u\vartheta_5, 0), (u\vartheta_2, 0))$	$((u\vartheta_2, 0), (u\vartheta_4, 0), (u\vartheta_1, 0))$	$((u\vartheta_4, 0), (u\vartheta_3, 0), (u\vartheta_1, 0))$	$((u\vartheta_2, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$
A_2	$((u\vartheta_2, 0), (u\vartheta_2, 0), (u\vartheta_4, 0))$	$((u\vartheta_3, 0), (u\vartheta_1, 0), (u\vartheta_5, 0))$	$((u\vartheta_5, 0), (u\vartheta_2, 0), (u\vartheta_1, 0))$	$((u\vartheta_5, 0), (u\vartheta_1, 0), (u\vartheta_1, 0))$
A_3	$((u\vartheta_1, 0), (u\vartheta_3, 0), (u\vartheta_2, 0))$	$((u\vartheta_2,0), (u\vartheta_4, 0), (u\vartheta_3, 0))$	$((u\vartheta_4, 0), (u\vartheta_2, 0), (u\vartheta_3, 0))$	$((u\vartheta_2, 0), (u\vartheta_3, 0), (u\vartheta_1, 0))$
A_4	$((u\vartheta_5, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$	$((u\vartheta_4, 0), (u\vartheta_1, 0), (u\vartheta_2, 0))$	$((u\vartheta_3, 0), (u\vartheta_4, 0), (u\vartheta_3, 0))$	$((u\vartheta_4, 0), (u\vartheta_5, 0), (u\vartheta_4, 0))$
A_5	$((u\vartheta_3, 0), (u\vartheta_2, 0), (u\vartheta_1, 0))$	$((u\vartheta_4, 0), (u\vartheta_2, 0), (u\vartheta_5, 0))$	$((u\vartheta_2, 0), (u\vartheta_1, 0), (u\vartheta_5, 0))$	$((u\vartheta_1, 0), (u\vartheta_1, 0), (u\vartheta_4, 0))$
	C_1	C_2	C_3	C_4
A_1	$((u\vartheta_2, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$	$((u\vartheta_2, 0), (u\vartheta_1, 0), (u\vartheta_5, 0))$	$((u\vartheta_3,0), (u\vartheta_2,0), (u\vartheta_5,0))$	$((u\vartheta_2,0), (u\vartheta_4,0), (u\vartheta_5,0))$
A_2	$((u\vartheta_2, 0), (u\vartheta_4, 0), (u\vartheta_3, 0))$	$((u\vartheta_5, 0), (u\vartheta_2, 0), (u\vartheta_1, 0))$	$((u\vartheta_5,0), (u\vartheta_2,0), (u\vartheta_3,0))$	$((u\vartheta_5,0), (u\vartheta_2,0), (u\vartheta_3,0))$
A_3	$((u\vartheta_4,0), (u\vartheta_2,0), (u\vartheta_1,0))$	$((u\vartheta_4, 0), (u\vartheta_3, 0), (u\vartheta_1, 0))$	$((u\vartheta_4, 0), (u\vartheta_3, 0), (u\vartheta_2, 0))$	$((u\vartheta_5, 0), (u\vartheta_1, 0), (u\vartheta_2, 0))$
A_4	$((u\vartheta_2, 0), (u\vartheta_4, 0), (u\vartheta_5, 0))$	$((u\vartheta_1, 0), (u\vartheta_2, 0), (u\vartheta_4, 0))$	$((u\vartheta_3, 0), (u\vartheta_2, 0), (u\vartheta_5, 0))$	$((u\vartheta_1, 0), (u\vartheta_5, 0), (u\vartheta_2, 0))$
A_5	$((u\vartheta_2, 0), (u\vartheta_4, 0), (u\vartheta_3, 0))$	$((u\vartheta_5, 0), (u\vartheta_2, 0), (u\vartheta_3, 0))$	$((u\vartheta_2,0), (u\vartheta_2,0), (u\vartheta_3,0))$	$((u\vartheta_2, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$

	C_1	C_2	C_3	C_4
A_1	$((u\vartheta_2, 0), (u\vartheta_1, 0), (u\vartheta_4, 0))$	$((u\vartheta_1, 0), (u\vartheta_3, 0), (u\vartheta_5, 0))$	$((u\vartheta_3, 0), (u\vartheta_4, 0), (u\vartheta_2, 0))$	$((u\vartheta_3, 0), (u\vartheta_5, 0), (u\vartheta_2, 0))$
A_2	$((u\vartheta_4, 0), (u\vartheta_1, 0), (u\vartheta_2, 0))$	$((u\vartheta_5, 0), (u\vartheta_2, 0), (u\vartheta_1, 0))$	$((u\vartheta_4, 0), (u\vartheta_1, 0), (u\vartheta_2, 0))$	$((u\vartheta_5, 0), (u\vartheta_3, 0), (u\vartheta_2, 0))$
A_3	$((u\vartheta_5, 0), (u\vartheta_4, 0), (u\vartheta_4, 0))$	$((u\vartheta_2 ,0), (u\vartheta_5, 0), (u\vartheta_4, 0))$	$((u\vartheta_3, 0), (u\vartheta_2, 0), (u\vartheta_5, 0))$	$((u\vartheta_1, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$
A_4	$((u\vartheta_5, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$	$((u\vartheta_1, 0), (u\vartheta_4, 0), (u\vartheta_3, 0))$	$((u\vartheta_2, 0), (u\vartheta_4, 0), (u\vartheta_1, 0))$	$((u\vartheta_3, 0), (u\vartheta_5, 0), (u\vartheta_2, 0))$
A_5	$((u\vartheta_1, 0), (u\vartheta_2, 0), (u\vartheta_4, 0))$	$((u\vartheta_4, 0), (u\vartheta_2, 0), (u\vartheta_3, 0))$	$((u\vartheta_2, 0), (u\vartheta_3, 0), (u\vartheta_4, 0))$	$((u\vartheta_4, 0), (u\vartheta_2, 0), (u\vartheta_1, 0))$

Step 2. The experts weight values are$u\omega = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$, the $UR = [UR_{ij}]_{5\times4}$is put forward in Table 2.

Table 2. The$UR = [UR_{ij}]_{5\times4}$

	C_1	C_2	C_3	C_4
A_1	$\{(u\vartheta_2, 0.2692), (u\vartheta_3, 0.1873), (u\vartheta_1, 0.1421)\}$	$\{(u\vartheta_2, 0.3282), (u\vartheta_3, -0.3214), (u\vartheta_4, 0.3356)\}$	$\{(u\vartheta_2, 0.3342), (u\vartheta_3, 0.1529), (u\vartheta_2, 0.3625)\}$	$\{(u\vartheta_3, 0.1487), (u\vartheta_4, 0.4075), (u\vartheta_2, 0.4123)\}$
A_2	$\{(u\vartheta_5, -0.1452), (u\vartheta_2, 0.2287), (u\vartheta_3, -0.2493)$	$\{(u\vartheta_2, 0.2209), (u\vartheta_4, 0.3843), (u\vartheta_1, -0.2523)\}$	$\{(u\vartheta_5, 0.1507), (u\vartheta_2, 0.1629), (u\vartheta_1, 0.3845)\}$	$\{(u\vartheta_5, -0.4272), (u\vartheta_4, 0.1433), (u\vartheta_4, -0.1292)\}$
A_3	$\{(u\vartheta_3, -0.1943), (u\vartheta_2, 0.1562), (u\vartheta_2, -0.1475)$	$\{(u\vartheta_3, -0.1642), (u\vartheta_2, 0.1485), (u\vartheta_5, -0.3256)\}$	$\{(u\vartheta_2, -0.2732), (u\vartheta_3, 0.2214), (u\vartheta_4, -0.3328)\}$	$\{(u\vartheta_3, 0.4296), (u\vartheta_2, 0.3147), (u\vartheta_3, 0.1785)\}$
A_4	$\{(u\vartheta_2, 0.1793), (u\vartheta_5, -0.1872), (u\vartheta_1, 0.1128)\}$	$\{(u\vartheta_2, 0.1334), (u\vartheta_4, -0.2645), (u\vartheta_1, 0.1856)\}$	$\{(u\vartheta_3, 0.1933), (u\vartheta_2, -0.3372), (u\vartheta_3, -0.3527)$	$\{(u\vartheta_2, 0.1223), (u\vartheta_2, -0.3465), (u\vartheta_3, -0.1256)$
A_5	$\{(u\vartheta_2, 0.1294), (u\vartheta_2, 0.1178), (u\vartheta_3, 0.2394)\}$	$\{(u\vartheta_3, 0.3246), (u\vartheta_3, -0.2126), (u\vartheta_5, -0.4345)$	$\{(u\vartheta_1, 0.2803), (u\vartheta_2, 0.2124), (u\vartheta_2, 0.3643)\}$	$\{(u\vartheta_2, 0.2106), (u\vartheta_3, -0.3425), (u\vartheta_3, -0.1379)$

Step 3. All attributes are benefit-oriented, the$NUR = [NUR_{ij}]_{5\times4}$ (Table 3) is same to$UR = [UR_{ij}]_{5\times4}$.

Table 3. The$NUR = [NUR_{ij}]_{5\times4}$

	C_1	C_2	C_3	C_4
UX_1	$\{(u\vartheta_2, 0.2692), (u\vartheta_3, 0.1873), (u\vartheta_1, 0.1421)\}$	$\{(u\vartheta_2, 0.3282), (u\vartheta_3, -0.3214), (u\vartheta_4, 0.3356)\}$	$\{(u\vartheta_2, 0.3342), (u\vartheta_3, 0.1529), (u\vartheta_2, 0.3625)\}$	$\{(u\vartheta_3, 0.1487), (u\vartheta_4, 0.4075), (u\vartheta_2, 0.4123)\}$
UX_2	$\{(u\vartheta_5, -0.1452), (u\vartheta_2, 0.2287), (u\vartheta_3, -0.2493)$	$\{(u\vartheta_2, 0.2209), (u\vartheta_4, 0.3843), (u\vartheta_1, -0.2523)\}$	$\{(u\vartheta_5, 0.1507), (u\vartheta_2, 0.1629), (u\vartheta_1, 0.3845)\}$	$\{(u\vartheta_5, -0.4272), (u\vartheta_4, 0.1433), (u\vartheta_4, -0.1292)\}$
UX_3	$\{(u\vartheta_3, -0.1943), (u\vartheta_2, 0.1562), (u\vartheta_2, -0.1475)$	$\{(u\vartheta_3, -0.1642), (u\vartheta_2, 0.1485), (u\vartheta_5, -0.3256)\}$	$\{(u\vartheta_2, -0.2732), (u\vartheta_3, 0.2214), (u\vartheta_4, -0.3328)\}$	$\{(u\vartheta_3, 0.4296), (u\vartheta_2, 0.3147), (u\vartheta_3, 0.1785)\}$
UX_4	$\{(u\vartheta_2, 0.1793), (u\vartheta_5, -0.1872), (u\vartheta_1, 0.1128)\}$	$\{(u\vartheta_2, 0.1334), (u\vartheta_4, -0.2645), (u\vartheta_1, 0.1856)\}$	$\{(u\vartheta_3, 0.1933), (u\vartheta_2, -0.3372), (u\vartheta_3, -0.3527)$	$\{(u\vartheta_2, 0.1223), (u\vartheta_2, -0.3465), (u\vartheta_3, -0.1256)$
UX_5	$\{(u\vartheta_2, 0.1294), (u\vartheta_2, 0.1178), (u\vartheta_3, 0.2394)\}$	$\{(u\vartheta_3, 0.3246), (u\vartheta_3, -0.2126), (u\vartheta_5, -0.4345)$	$\{(u\vartheta_1, 0.2803), (u\vartheta_2, 0.2124), (u\vartheta_2, 0.3643)\}$	$\{(u\vartheta_2, 0.2106), (u\vartheta_3, -0.3425), (u\vartheta_3, -0.1379)$

Step 4. Cultivate the weight numbers: $uw_1 = 0.2654, uw_2 = 0.3475, uw_3 = 0.2057, uw_4 = 0.1814$

Step 5. Put forward 2TLNNPISA and 2TLNNNISA (Table 4).

Table 4. 2TLNNPISA and 2TLNNNISA

	C_1	C_2	C_3	C_4
2TLNNPISA	$\{(u\vartheta_5, -0.1452), (u\vartheta_2, 0.2287), (u\vartheta_3, -0.2493)\}$	$\{(u\vartheta_3, 0.3246), (u\vartheta_3, -0.2126), (u\vartheta_5, -0.4345)\}$	$\{(u\vartheta_5, 0.1507), (u\vartheta_2, 0.1629), (u\vartheta_1, 0.3845)\}$	$\{(u\vartheta_5, -0.4272), (u\vartheta_4, 0.1433), (u\vartheta_4, -0.1292)\}$
2TLNNNISA	$\{(u\vartheta_2, 0.1294), (u\vartheta_2, 0.1178), (u\vartheta_3, 0.2394)\}$	$\{(u\vartheta_2, 0.2209), (u\vartheta_4, 0.3843), (u\vartheta_1, -0.2523)\}$	$\{(u\vartheta_1, 0.2803), (u\vartheta_2, 0.2124), (u\vartheta_2, 0.3643)\}$	$\{(u\vartheta_2, 0.1223), (u\vartheta_2, -0.3465), (u\vartheta_3, -0.1256)\}$

Step 6. Put forward $2TLNNPISA(HGRC_{ij})$, $2TLNNNISA(HGRC_{ij})$ (Table 5).

Table 5. The$2TLNNPISA(HGRC_{ij})$ and $2TLNNNISA(HGRC_{ij})$

Alternatives	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
A_1	0.7975	0.5343	0.3092	0.4158	0.7168	0.6981	0.6827	0.8582
A_2	1.0000	0.6396	1.0000	1.0000	0.8263	1.0000	0.8639	0.7744
A_3	0.8288	0.7975	0.4269	0.5139	0.8380	0.8582	0.6747	0.7676
A_4	0.6043	0.5821	0.4504	0.5186	1.0000	0.9531	1.0000	1.0000
A_5	0.5676	1.0000	0.4269	0.5343	0.8190	0.7240	0.9027	0.7739

Step 7. Put forward the $2TLNNPISA(HGRD_i)$,$2TLNNNISA(HGRD_i)$ (Table 6):

Table 6. The $2TLNNPISA(HGRD_i)$,$2TLNNNISA(HGRD_i)$

	$2TLNNPISA(HGRD_i)$	$2TLNNNISA(HGRD_i)$	$2TLNNHRRD_i$
A_1	0.5363	0.7289	0.4239
A_2	0.8748	0.8850	0.4971
A_3	0.6781	0.7986	0.4592
A_4	0.5494	0.9837	0.3584
A_5	0.6829	0.7950	0.4621

Step 8. Put forward the$2TLNNHRRD_i$

Step 9. In light with $2TLNNHRRD_i$, the order is:$A_2 > A_5 > A_3 > A_1 > A_4$ andA_2 is the optimal cultural industry demonstration base as shown in Figure 1.

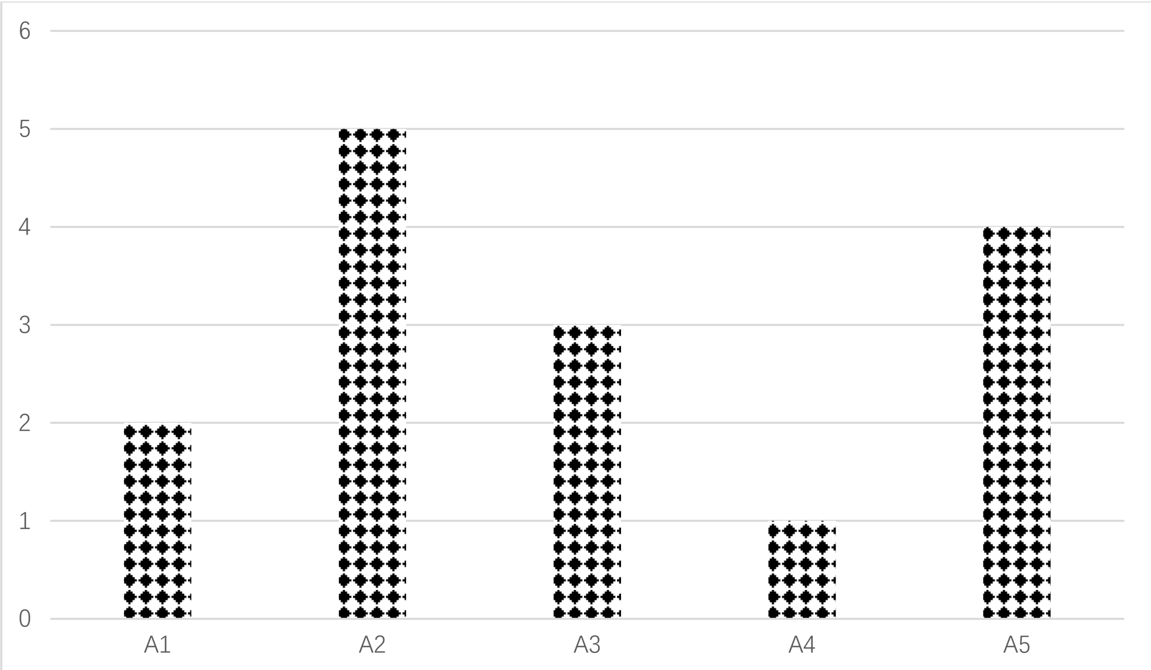

Figure 1. The rank of alternatives.

5.2. Comparative analysis

The 2TLNN-HGRA approach is compared with 2TLNNWA approach [24], 2TLNNWG approach [24], 2TLNN-CLVA approach [27], 2TLNN-TODIM approach [4], 2TLNN-GLDS approach[28] and 2TLNN-EDAS approach[29]. The comparative results are put forward in Figure 2.

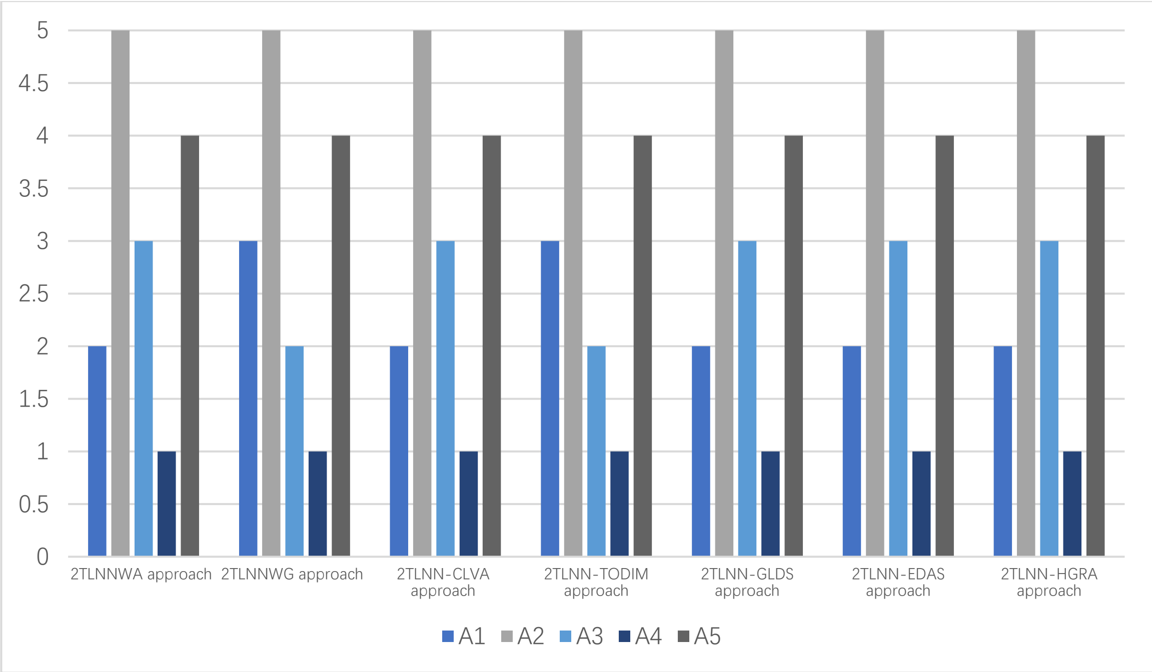

Figure 2. The comparative analysis ranks.

In a comparative analysis, the ranking produced by the 2TLNN-HGRA approach aligns with those generated by 2TLNNWA approach [24], 2TLNN-CLVA approach [27], 2TLNN-GLDS approach [28], and 2TLNN-EDAS approach [29]; however, it slightly differs from the rankings by 2TLNNWG approach [24] and 2TLNN-TODIM approach [4]. Additionally, all these methods identified the same best cultural industry demonstration base and worst cultural industry demonstration base. This consistency across different approaches confirms the effectiveness of the 2TLNN-HGRA method.

The 2TLNN-HGRA method proposed in this paper demonstrates the following three key advantages in handling multi-attribute group decision-making (MAGDM) problems: (1) Effectively handles uncertainty and fuzziness: By incorporating the principles of 2TLNNS, this method can manage the inherent fuzziness and uncertainty in the decision-making process. The use of 2TLNNS allows decision-makers to express their fuzzy preferences for different options more precisely, enhancing the accuracy and robustness of the decision outcomes. (2) Incorporates entropy weighting for objective weight allocation: The method introduces the entropy weighting method to automatically assign weights to various attributes, reducing the influence of subjective human factors on the decision outcome. The entropy method adjusts the weights based on the information divergence between attributes, ensuring that each attribute's importance is objectively reflected in the decision process, thus improving the fairness and scientific rigor of the results. (3) Utilizes multiple distance measures to enhance decision precision: This approach combines 2TLNNHD and 2TLNNLD, assessing the distance of each option from the ideal solution from multiple perspectives. This multi-metric evaluation provides a more comprehensive assessment of the alternatives, leading to more precise and reliable decision results.

These three advantages make the 2TLNN-HGRA method highly adaptable, objective, and precise in addressing complex decision-making problems, particularly in high-quality evaluations.

6. Managerial Implications

The evaluation of high-quality development in the cultural industry has numerous managerial implications:

i. Informed Strategic Planning and Investment

Guiding Investment Priorities: Evaluations can reveal high-potential areas within the cultural sector, helping managers allocate resources efficiently, such as by investing in digital transformation, sustainable practices, or new cultural experiences.

Targeting Emerging Markets and Audiences: Analysis can identify demographic trends and audience preferences, allowing managers to tailor offerings to emerging markets and engage audiences through targeted marketing efforts.

ii. Enhanced Resource Allocation and Efficiency

Optimizing Budget and Resource Allocation: Insights from evaluations allow managers to allocate budgets to the most impactful areas, maximizing return on investment while reducing waste in less productive projects or initiatives.

Efficient Use of Cultural Resources: Managers can optimize the use of tangible and intangible cultural assets, ensuring that resources are not overused or underutilized, thus preserving cultural value for future generations.

iii. Boosting Cultural Innovation and Adaptation

Encouraging Innovative Practices: High-quality development emphasizes innovation; managers can prioritize investments in new technologies, business models, or art forms that keep cultural organizations relevant and engaging.

Adapting to Changing Consumer Preferences: Through regular evaluation, managers can stay informed on consumer trends, adapting offerings to match audience interests and keeping cultural products and services attractive.

iv. Strengthening Cultural Identity and Social Value

Preserving and Promoting Cultural Heritage: Evaluations focused on quality development enable managers to prioritize initiatives that reinforce cultural identity and heritage, contributing to societal cohesion and pride.

Enhancing Social Responsibility: High-quality cultural projects can enhance social well-being. Managers can design programs that emphasize accessibility, inclusivity, and educational value, ensuring that cultural projects serve diverse community needs.

7. Conclusion

The evaluation of comprehensive strength for the high-quality development of the cultural industry is a multi-dimensional assessment that seeks to measure the overall performance and competitiveness of the sector. This evaluation typically examines key aspects such as economic contributions, innovation capacity, social impact, cultural influence, and sustainability. By integrating both quantitative metrics (e.g., revenue growth, market share, and employment rates) and qualitative assessments (e.g., cultural value, creative output, and social benefits), it provides a holistic view of how well the industry is positioned for long-term sustainable development. The evaluation aims to identify strengths and weaknesses within the cultural sector, serving as a scientific basis for informed decision-making regarding resource allocation, policy formulation, and strategic planning. Ultimately, the goal is to enhance the competitiveness of the cultural industry while ensuring it contributes positively to social and economic development, cultural preservation, and innovation. The comprehensive strength evaluation for high-quality development in the cultural industry utilizes a MAGDM framework. In this context, the 2TLNN-HGRA method has been developed by integrating GRA with 2TLNNS principles. This study presents a numerical example to assess the comprehensive strength of high-quality development in the cultural industry, using various comparative analyses to verify the effectiveness of the 2TLNN-HGRA approach.

The key contributions of this research are as follows: (1) Introduction of entropy to assign weight values within 2TLNNSs; (2) Implementation of the 2TLNN-HGRA approach, leveraging both 2TLNNHD and 2TLNNLD under 2TLNNSs; (3) Application of 2TLNN-HGRA method to solve MAGDM problems using 2TLNNSs; (4) Demonstration of the 2TLNN-HGRA method through a practical numerical example in the comprehensive strength evaluation for high-quality development in the cultural industry; and (5) Execution of an efficient comparative analysis against several existing decision-making approaches.

8. Potential research directions

Building on the current research, future studies can extend and deepen the application of the 2TLNN-HGRA method from multiple perspectives, enhancing its utility and decision-making efficiency. Below are three potential research directions with detailed descriptions:

(1) Expanding the Application of the 2TLNN-HGRA Method: The current study focuses on evaluating the high-quality development of the cultural industry, demonstrating the effectiveness of the 2TLNN-HGRA method in addressing complex multi-attribute group decision-making (MAGDM) problems. However, future research could explore its applicability in other fields. For instance, areas such as environmental management, smart city planning, innovation technology assessment, and financial investment also face complex MAGDM challenges with uncertainty and fuzziness. Applying the 2TLNN-HGRA method in these new domains can further validate its generalizability and robustness, while also expanding its application prospects in various complex decision-making scenarios.

(2) Introducing a Dynamic Weight Allocation Mechanism: The existing research uses the entropy method to assign static weights to different attributes. However, this approach assumes that the weights remain constant, which may not reflect the dynamic nature of real-world decision-making environments. Especially in rapidly evolving sectors like the cultural industry, decision contexts often change over time or due to external conditions. Future studies could introduce a dynamic weight allocation mechanism that adjusts attribute weights in real-time according to changing environmental conditions or decision needs. This approach would better capture real-time shifts in the decision-making process, improving the flexibility and accuracy of the model. In particular, when evaluating objects at different stages of development, a dynamic weighting mechanism will be more applicable.

(3) Integrating Machine Learning with the 2TLNN-HGRA Method: With the rapid advancement of big data and artificial intelligence, combining machine learning (ML) techniques with the 2TLNN-HGRA method could become a significant trend in future decision-making research. ML models can automatically identify and

process large volumes of decision-related data, which would not only improve the precision of weight allocation but also predict trends in decision attributes. This integration would markedly enhance the level of intelligence in the decision-making process, especially when dealing with high-dimensional data and complex uncertainties. ML can assist decision-makers by extracting key information from the data more efficiently, thereby optimizing decision outcomes. Additionally, ML could accelerate the computational process of the 2TLNN-HGRA method, significantly improving decision efficiency and enhancing its ability to handle fuzzy information and uncertainty.

By pursuing these research directions, the 2TLNN-HGRA method could not only be applied more widely across various fields but also better handle dynamic changes and data processing demands in complex decision-making problems. This would provide more powerful tools for evaluating high-quality development in the cultural industry and other related sectors.

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Received: Aug 10, 2024. Accepted: Nov 4, 2024