NSS

Neutrosophic Sets and Systems, Vol. 77, 2025

University of New Mexico

Inventory Control of Single Perishable Item in Neutrosophic Environment: with Stock-based Demand, Preservation technology, Shortage and Two-stage price discount for Imperfect items

Geremew Assefa Gudeta[1,*],Natesan Thillaigovindan [2] and Getinet Alemayehu Wole[3]

[1]Mathematics Department, ArbaMinch University, Ethiopia; gerem2009@gmail.com

[2]Mathematics Department, ArbaMinch University, Ethiopia; thillaigovindan.natesan@gmail.com

[3]Mathematics Department, Haramaya University, Ethiopia; getalem2014@gmail.com

[*]Correspondence: gerem2009@gmail.com; Tel.: +251913287543/961394234; P.O.Box:21

Abstract. In this article, we study an inventory model for a single perishable item which can contribute to economic growth, organizational benefit as well as individual life. Both supplier and retailer consider price discount for defective items up to a specified percentage, in a situation where the parameters involved are vague and imprecise. Such consideration has not been reported in the literature. By recognizing the power of neutrosophic environment to handle uncertainty and impreciseness inherited or existing in the problem, we use single-valued triangular neutrosophic numbers (SVNs) for representing stock-based demand, retail price, ordering cost, holding cost, wastage cost, imperfect item price, and preservation cost with de-neutrosophication. The Taylor series approximation is used to transform the time variables in the profit function into the order quantity and maximum stock variables. We determine the theoretical optimality condition and verify the concavity and uniqueness of the global optimal solution using Hessian matrix. Our aim is to determine the maximum value of the retailer's profit by making decisions on the order quantity and maximum stock level, as well as to estimate shortages, stock-out time, cycle length, and identify the model's inherent impreciseness. We develop neutrosophic fuzzy optimization technique (NeFOT), an extension of the intuitionistic fuzzy optimization technique (IFOT) to evaluate the compromise acceptance, indeterminacy, and rejection levels inherited in the model and optimal solution. We support the study by illustrating a numerical example and graph with Mathematica 11.3 and LINGO 18.0.

Keywords: Single item perishable inventory, Backlogged shortage, Preservation, Two-stage price discount of imperfect items, Stock based demand, Neutrosophic environment.

1. Introduction

The inventory problem is the technique of managing items in stock for the purpose of daily operations. Inventory deals with how much of a given item to be kept in hand (stock level), how much to order (reorder quantity), when to place an order again (reorder point), and how frequently to check the stock (continuous/periodic review) to meet demand satisfactorily. The strategy to maintain a trade-off between minimizing total cost and maximizing profit as well as customer satisfaction, is known as optimal inventory control. Economic order quantity (EOQ) is one of the strategy initially considered under the infinite life time assumption Harris [1], but this is unrealistic because many products have a limited lifetime–perishables that decay or deteriorate and lose quality during loading/unloading, transportation and storage.

Nahmias [2] categorized deteriorating items as: *fixed lifetime*(best before the day-BBD) means that utility of the item decreases on its lifetime, *deteriorate totally*–no value, and *random lifetime*–no specified lifetime means probabilistic or imprecise life time. Perishability of baked goods, milk, yogurt, fresh fruits, meat products, vegetables, pharmaceuticals, healthcare products, and other items cannot be ignored. The deterioration or decay is a natural process by which things lose their quality and freshness and have negative effect on both environment and human health, even after they have been used. Giri *et al.* [3] considered an inventory model for deteriorating items with stock-dependent demand.

A part of the demand may not be satisfied with available inventory, due to stock-out. Thus, shortage or stock-out is a natural phenomenon in any supplier-retailer model due to factors such as uncertain demand, selling price discount, deterioration rate, lead time, and so on. There are three possibilities, (a) all customers are waiting for the next order arrival (full back-order case) (b) no one is willing to wait (lost sales case) (c) some customers are willing to wait (partially backlogged) and the rest lost. As perishable items are more sensitive, defects may occur during stocking at the supplier level or before arrival to stock due to loading/unloading, transportation, deterioration and other causes. Removing or returning to supplier is one of the ways to treat imperfect items, while others recommend sale of imperfect items to customers at discounted price.

To minimize loss of items, and to maximize profit, today science and technology play an important role to preserve freshness of perishable items and also to attract customers, which cannot be ignored during model formulation. The current ways of preservation are fermentation, refrigeration, irradiation, pasteurization, pre-cooling, refrigerated storage and transportation, drying and dry storage, controlled atmosphere storage, improved packaging and utilization, regulated ripening, growth-regulating chemicals. These technologies provide high quality fruits and vegetables, and other perishable items for customers and reduce post-harvest losses.

In present day competitive market, there are different ways of attracting costumers to improve the demand level of business, techniques like price discount, quantity discount, stock based demand, advertising, stock-based pricing and others are used. Among several scholars, Mahata and Debnath [4] considered defective items due to transportation and deterioration with a price discount on selling at retailer level rather than returning to supplier with price-based demand under preservation technology, neglecting stock-out and price discount at supplier level for imperfect items. Defective items are an issue in profit cases due to imperfect products, which increases loss and decreases popularity Dey *et al.* [5].

In our daily life activities, we need a systematic and scientific way to control inventory of perishable items. However, in addition to the mentioned issues, due to factors like uncertain demand, selling price discount, deterioration rate, lead time, transportation, quality and others, perishable items lose their utility or marginal value, and it is impossible to consistently satisfy customer or retailer demand. Hence, to regulate the business, there is need to develop a model, and make the right decision on how to control inventories of perishable items. In scenarios requiring measurement, observation, experimentation, quantification, and data analysis, there is always some degree of uncertainty, inconsistency, and ambiguity when it comes to computational process or decision-making, which require novel mathematical frame work Momena *et al.* [6]. Impreciseness,vagueness, and inconsistency can occur in parameters of the model as well as estimated solution. Contract between supplier and retailer and cost of preservation technology are also issues in inventory modeling of perishable items.

Many studies focused on perishable items to maximize retailer's profit and reduce loss from deterioration. Perishable items were the subject of several research that sought to optimize and reduce decay loss. Asma and Amirtharaj [7] developed an inventory model of deteriorating items and employed fuzzy programming. Poswal *et al.* [8] discussed perishable items with stock-dependent demand and backlogged shortages under fuzzy conditions. Imperfect products have been studied by Mondal and Khara [9]. Shunmugam and Karuppuchamy [10] presented an EOQ inventory model with stock dependent demand, preservation technology, and exponential holding cost in fuzzy environment.A fuzzy EOQ inventory system for perishable items with a constant perish rate was developed by Mahapatra *et al.* [11]. A fuzzy environment EOQ model of deteriorating items with partial backlog and time-varying demand has been studied by Barman *et al.* [12]. Chaudhary *et al.* [13] discussed a sustainable inventory model for defective items under fuzzy condition. Kar *et al.* [14] discussed three inventory models of deteriorating items where vagueness and impreciseness arise due to objective goals and different parameters. Karthick and Uthayakumar [15] studied optimization of an imperfect production model with varying setup cost, price discount, and lead time under fuzzy demand. Asma and Amirtharaj [16] studied an IFOT to solve an inventory model of deteriorating item without and with budget constraints. Pawar *et al.* [17] have emphasized multi-objective inventory problem in an intuitionistic fuzzy environment. Tsegaye *et al.* [18] utilized an intuitionistic fuzzy multi-objective optimization problems and studied an efficient method to solve it. Panda and Sahoo [19] utilized fuzzy NLP and intuitionistic fuzzy techniques for solving multi-objective problems of inventory models for deteriorating items under constraints (budget and space).

In 1995, Smarandache introduced a generalization of fuzzy and intuitionistic fuzzy sets (IFs) known as neutrosophic sets (NSs). As an extension of neutrosophic set, single-valued neutrosophic sets (SVNSs) was introduced by Wang *et al.* [20]. To minimize the incorrectness due to unclear, erroneous, missing, and wrong information, neutrosophic set is an appropriate tool. Mohamed *et al.* [21] employed SVNSs to deal with vague information in sustainable supplier selection. Currently, Christianto and Smarandache [22] recognizing the power of neutrosophic environment used SVNSs to overcome uncertainty information in the evaluation process. Again, Mohamed *et al.* [23] emphasized the importance of neutrosophic set as one of the most effective tools for dealing with ambiguity since it addresses the problem of uncertainty and utilized SVNSs in decision-making process.

Furthermore, deciding and making comparisons between any two neutrosophic numbers is not as easy as in crisp case in application areas. An approximate crisp value of neutrosophic numbers is required. Abdel-Basset *et al.* [24] introduced a novel ranking method for de-neutrosophication of SVNSs and applied it to production planning LPP. Smarandache *et al.* [25] defined a score function for intuitionistic fuzzy numbers and used an accuracy function for de-neutrosophication in optimization problems.

Neutrosophic numbers are one of the recent tools introduced to handle inventory problems Mullai and Broumi [26], Mullai and Surya [27], Mondal *et al.* [28] and Mullai and Surya [29], under different scenarios. Pal *et al.* [30] studied an EOQ model for deteriorating products under shortage in a neutrosophic environment. Mondal *et al.* [31] discussed the inventory control of seasonal items with logistic-growth demand rate under fully permissible delay in payment in neutrosophic environment. Kar *et al.* [32] discussed price, marketing, service, and green-dependent neutrosophic demand under uncertain resource constraints of the EOQ model via GP. Bhavani *et al.* [33] considered deterioration and discount on defective items and other cases in neutrosophic condition for fixed life perishable products using PSO method. Bhavani and Mahapatra [34] analyzed maximum life-time-based deterioration using generalized triangular neutrosophic cost pattern with novel demand via PSO. Mohanta *et al.* [35] examined the EOQ inventory model as demand based on retail price and promotional effort with preservation technology in neutrosophic environment. Again, Mohanta *et al.* [36] studied a model with deterioration rate, demand rate, and other inventory costs as trapezoidal SVN and employed mean interval ranking method. Product deteriorating continuously but has a maximum lifespan, linear demand without shortage, neutrosophic environment via goal programming was discussed in Deb and Islam [37]. Surya *et al.* [38] studied a neutrosophic inventory system for decaying items with demand that is influenced by price to find the neutrosophic optimal total cost and neutrosophic optimal interval of time for the inventory system via defuzzification process with the aid of signed distance method.

We aim to design and analyses an inventory model for a single perishable item to maximize total profit per cycle and determine the EOQ, maximum stock level, stock-out time, cycle length, shortages, and defective goods in neutrosophic environment as the key targets. By extending Gudeta *et al.* [39] work to neutrosophic environment, we take into account single perishable item with stock-based demand, a specified percentage of defect items, backlogged shortages, wastage costs, preservation technology and its investment, and price discounts at both supplier and retailer levels for imperfect items. The time variant model is converted to a quantity and maximum stock level variables model using the Taylor series approximation up to the second degree, allowing for establishment of theoretical optimality condition. With the neutrosophic set addressing truth membership grade, indeterminacy grade, and falsity membership grade for every parameter, exact solution approaches are applied using a single-valued triangular neutrosophic number with de-neutrosophication. Further, by extending the IFOT approach of Panda and Sahoo [19], a new technique, NeFOT technique (*Neutrosophic Fuzzy Optimization Technique*) is developed to solve the EOQ inventory model. This method converts the model to an equivalent multi-objective deterministic model corresponding to the original problem. To determine the maximum profit of the retailer, Mathematica 18.0 and LINGO 11.3 software are utilized.

1.1. *Motivation and Novelties*

In reality, from individual life to complex organizations, utilize their resources for the intended purposes such as development of economic growth, organizational benefit, as well as individual life. Perishable resources are one of the backbones for such economic growth that needs systematic study and scientific techniques to use effectively and efficiently. Thus, perishable inventory control is required to minimize the related costs, minimize the loss of items due to nature of items and other causes, and preserve freshness to attract customers and gain profit. Stock-based demand is one of the costumer attraction approaches for the business centers to be more profitable. Shortages naturally occur due to inflated demand, deterioration, price discount for imperfect items, and others. Moreover, the expiration dates can lead to shortages of fresh products, such as fruits and vegetables, and other perishable items. These items have a limited shelf life, and their quality deteriorates with time. For perishable items, due to loading/unloading, post-harvest losses, deterioration and other causes at the supplier stage, retailer may receive maximum stock with a specified defect percentage of imperfect items. Thus, a contract between supplier and retailer to exchange imperfect items with price discount up to a specified defect percentage is required at two levels, for retailer and downstream to consumers. Some items get spoiled due to their nature, even if preservation technology exists. Wastage cost of deteriorated items at retailer stock also need consideration. Hence, we appreciate the power of neutrosophic environment that incorporates all the ways of handling uncertainty and inconsistency that occur during the process as well as those inherent in the problem. One of the main issue with the earlier study by Gudeta *et al.* [39] was that they ignored the effect of neutrosophic environment. Thus, we are motivated to extend this work utilizing SVNNs for representing the parameters: stock-based demand, specified percentage of defect items, backlogged shortages, wastage costs, preservation technology and its investment, and price discounts at both supplier and retailer levels for imperfect items to maximize total profit per cycle.

The novelty of the proposed model:

- The retailer aims to maximize his profit in a neutrosophic environment.
- Significant reduction in operating costs and minimizes losses.
- At the retail level, demand is based on actual requirement, stock level factors for consumer attraction, and imperfect items with specified price discount percentages at both supplier-retailer levels are considered.
- Preservation technology, with its investment is used to maintain freshness and minimize deterioration.
- Shortage of items is considered with backlogging.
- Taylor series approximation is used to transform the time variables to order quantity and maximum stock level variables.
- Single valued triangular neutrosophic numbers with de-neutrosophication is considered.
- A new method, the NeFOT (*Neutrosophic Fuzzy Optimization Technique*), is developed to solve the EOQ inventory model.
- Inherent uncertainty can be evaluated using the developed NeFOT.

A few scholars have made contributions towards inventory control for perishable items in a neutrosophic environment. However, to the best of our knowledge, no research is carried out combining all the scenarios mentioned under motivation for a single perishable item in a neutrosophic environment.

Need, limitations and impacts of study:

Need: Perishable items/goods are the most usable resources in every human activity, organization, and government for the purpose of economic growth and business activities. However, these resources are scarce, and have short lifetime due to their nature. Further, uncertainty or vagueness and inconsistency in information inherited in the problem, need appropriate handling techniques. Hence, neutrosophic environment is used to solve such problems for better utilization of resources, and single-value triangular neutrosophic numbers are considered for representing the parameters.

Limitation: The trend of using systematic and scientific inventory systems may not be adopted and the companies may not have fully documented resources or records about their inventory systems. Due to the complexity of inventory models, the study limited its scope by focusing on specific aspects such as single perishable item with stock-based demand, a specified percentage of defective items, backlogged shortages, wastage costs, preservation technology and its investment, and price discounts at both supplier and retailer levels for imperfect items to maximize total profit per cycle in neutrosophic environment. In addition, the study is limited to model development, theoretical optimality conditions, and numerical example due to lack of availability of data, especially concerning neutrosophic inputs and historical inventory records for perishable items, focusing on continuous review policy with planned shortages.

Impact: Studying inventory models for perishable items in a neutrosophic environment has impact on academics, countries, business centers, organizations across various sectors, and society as a whole, making it a valuable area of research and applications.

- Studying these models contributes to advancing knowledge in fields such as operations research, supply chain management, and decision sciences.
- It provides opportunities for theoretical exploration, development of new methodologies, and validation of existing theories in complex and uncertain environments.
- Help interdisciplinary collaboration, encourage exchange of ideas, and contribute to academic community's understanding of inventory control in dynamic settings.
- In agricultural sectors optimizing perishable inventory control of crops, reducing post-harvest losses, and improving food security can lead to economic benefits.
- Efficient inventory control can reduce wastage of perishable item and enhance the competitiveness of businesses.
- Optimizing perishable inventory control can lead to cost savings, increased revenue, enhanced customer satisfaction, protect human health as well as environment and better utilization of resources.

2. Methodology

2.1. *Basic Concepts and Definitions*

Definition 2.1. [25] Single Valued Neutrosophic Sets (SVNSs): Let X be an universe of discourse. A single valued neutrosophic set $\tilde{N}$ over X is an object having the form $\tilde{N} = \left\{\left\langle x, \mu_{\tilde{N}}(x), \varphi_{\tilde{N}}(x), v_{\tilde{N}}(x)\right\rangle : x \in X\right\}$,where $\mu_{\tilde{N}}(x), \varphi_{\tilde{N}}(x), v_{\tilde{N}}(x) : X \to [0,1]$, with $0 \leq \mu_{\tilde{N}}(x) + \varphi_{\tilde{N}}(x) + v_{\tilde{N}}(x) \leq 3, \forall x \in X$. The values $\mu_{\tilde{N}}(x), \varphi_{\tilde{N}}(x)$ and $v_{\tilde{N}}(x)$ denote truth-membership degree, indeterminacy degree and falsity membership degree of x to $\tilde{N}$, respectively

Definition 2.2. [6] A SVN set $\tilde{N}$ of X is said to be an Neutrosophic fuzzy number (NFN) if

i. $\tilde{N}$ is Neut-normal if $\exists\ x_o \in X$, such that $\mu_{\tilde{N}}(x_o) = 1, \varphi_{\tilde{N}}(x_o) = 0 = v_{\tilde{N}}(x_o)$ (x_o– core/mean value of $\tilde{N}$)

ii. $\mu_{\tilde{N}}(x)$ is convex, $\forall x_1, x_2 \in X, \lambda \in [0,1]$ if

$$\mu_{\tilde{N}}(\lambda x_1 + (1-\lambda)x_2) \geq min\left\{\mu_{\tilde{N}}(x_1), \mu_A(x_2)\right\}$$

iii. $\varphi_{\tilde{N}}(x), v_{\tilde{N}}(x)$ are concave $\forall x_1, x_2 \in X, \lambda \in [0,1]$, if

$$\varphi_{\tilde{N}}(\lambda x_1 + (1-\lambda)x_2) \leq max\left\{\varphi_{\tilde{N}}(x_1), \varphi_{\tilde{N}}(x_2)\right\}$$

$$v_{\tilde{N}}(\lambda x_1 + (1-\lambda)x_2) \leq max\left\{v_{\tilde{N}}(x_1), v_{\tilde{N}}(x_2)\right\}$$

iv. $\mu_{\tilde{N}}(x), \varphi_{\tilde{N}}(x), v_{\tilde{N}}(x) : X \to [0,1]$ are upper,lower and lower semi-continuous functions, respectively.

Definition 2.3. [25] A neutrosophic fuzzy number $\tilde{N}$ is characterized by upper and lower semi-continuous functions $\mu_{\tilde{N}}, \varphi_{\tilde{N}}, \nu_{\tilde{N}} : X \longrightarrow [0,1]$ in the space of neutrosophic fuzzy numbers with the following properties:

1. There exists $a, b \in X, a \leq b$ such that $\mu_{\tilde{A}}(x) = 0$, $\varphi_{\tilde{N}}(x) = 1 = \nu_{\tilde{N}}(x)$ outside $[a, b]$.
2. There exists $c, d \in X, c \leq d$ such that:
   i. $\mu_{\tilde{N}}$ is nondecreasing, and $\varphi_{\tilde{N}}$ & $\nu_{\tilde{N}}$ are nonincreasing on $[a, c]$;
   ii. $\mu_{\tilde{N}}(x) = 1$ and $\varphi_{\tilde{N}}(x) = 0 = \nu_{\tilde{N}}(x)$, for all $x \in [c, d]$;
   iii $\mu_{\tilde{N}}$ is nonincreasing and $\varphi_{\tilde{N}}$ & $\nu_{\tilde{N}}$ are nondecreasing on $[d, b]$.

The set $\left\{x \in X : \mu_{\tilde{N}}(x) = 1, \varphi_{\tilde{N}}(x) = 0 = \mu_{\tilde{N}}(x)\right\}$ is called the core of $\tilde{N}$, denoted by $core(\tilde{N})$. The closure of the set $\left\{x \in X : \mu_{\tilde{N}}(x) \geq 0; \varphi_{\tilde{N}}(x) \leq 1; \nu_{\tilde{N}}(x) \leq 1\right\}$ is called the support of $\tilde{N}$, denoted by $supp(\tilde{N})$. From the above definition it is immediate that $supp(\tilde{N})$ is a bounded interval. If $\mu_{\tilde{N}}$, $\varphi_{\tilde{N}}$ and $\nu_{\tilde{N}}$ are continuous and $supp(\tilde{N}) = [a, b]$ $core(\tilde{N}) = [c, d]$, then we have necessarily $a < c \leq d < b$.

Definition 2.4. [6] Let $\tilde{N} = \{\langle h_l, h, h_u; m_l, m, m_u; k_l, k, k_u\rangle : h, m, k \in X\}$ be a neutrosophic number. The truth membership,indeterminacy and falsity membership functions are defined as :

$$\mu_{\tilde{N}}(x) = \begin{cases} \frac{x-h_l}{h-h_l} & , h_l \le x \le h \\ 1 & , x = h \\ \frac{h_u-x}{h_u-h} & , h \le x \le h_u \\ 0 & , otherwise \end{cases}, \quad \varphi_{\tilde{N}}(x) = \begin{cases} \frac{m-x}{m-m_l} & , m_l \le x \le m \\ 0 & , x = m \\ \frac{x-m}{m_u-m} & , m \le x \le m_u \\ 1 & , otherwise \end{cases},$$

$$v_{\tilde{N}}(x) = \begin{cases} \frac{k-x}{k-k_l} & , k_l \le x \le k \\ 0 & , x = k \\ \frac{x-k}{k_u-k} & , k \le x \le k_u \\ 1 & , otherwise \end{cases} \text{ where } \begin{cases} 0 \le \mu_{\tilde{N}_l}(x) + \varphi_{\tilde{N}_l}(x) + v_{\tilde{N}_l}(x) \le 3, \forall x \in \tilde{N}. \\ 0 \le \mu_{\tilde{N}_u}(x) + \varphi_{\tilde{N}_u}(x) + v_{\tilde{N}_u}(x) \le 3, \forall x \in \tilde{N}. \end{cases}$$

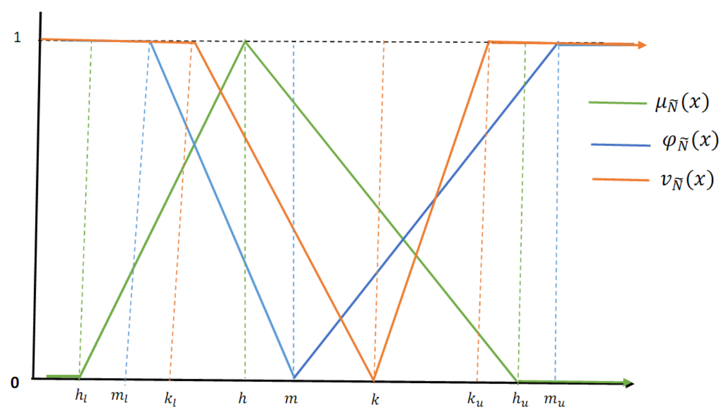

FIGURE 1. Independent Single Valued Triangular Neutrosophic Number Representation

Definition 2.5. [6]. Arithmetic operation on single-valued triangular neutrosophic numbers. Let $\tilde{N}_1 = \{\langle h_{1l}, h_1, h_{1u}; m_{1l}, m_1, m_{1u}; k_{1l}, k_1, k_{1u}\rangle : h_1, m_1, k_1 \in X\}$
$\tilde{N}_2 = \{\langle h_{2l}, h_2, h_{2u}; m_{2l}, m_2, m_{2u}; k_{2l}, k_2, k_{2u}\rangle : h_2, m_2, k_2 \in X\}$ be two SVTNNs. Addition, subtraction, multiplication,inverse and scalar multiplication are defined as:

i. $\tilde{N}_1 \oplus \tilde{N}_2$=
$(h_{1l} + h_{2l}, h_1 + h_2, h_{1u} + h_{2u}; m_{1l} + m_{2l}, m_1 + m_2, m_{1u} + m_{2u}; k_{1l} + k_{2l}, k_1 + k_2, k_{1u} + k_{2u})$

ii. $\tilde{N}_1 \ominus \tilde{N}_2$=
$(h_{1l} - h_{2u}, h_1 - h_2, h_{1u} - h_{2l}; m_{1l} - m_{2u}, m_1 - m_2, m_{1u} - m_{2l}; k_{1l} - k_{2u}, k_1 - k_2, k_{1u} - k_{2l})$

iii. $\tilde{N}_1 \otimes \tilde{N}_2$= $(h_{1l}h_{2l}, h_1h_2, h_{1u}h_{2u}; m_{1l}m_{2l}, m_1m_2, m_{1u}m_{2u}; k_{1l}k_{2l}, k_1k_2, k_{1u}k_{2u})$

iv. $\tilde{N}^{-1} = \begin{cases} \left\langle \frac{1}{h_u}, \frac{1}{h}, \frac{1}{h_l}; \frac{1}{m_u}, \frac{1}{m}, \frac{1}{m_l}; \frac{1}{k_u}, \frac{1}{k}, \frac{1}{k_l} \right\rangle, h, m, k > 0 \\ \left\langle \frac{1}{h_l}, \frac{1}{h}, \frac{1}{h_u}; \frac{1}{m_l}, \frac{1}{m}, \frac{1}{m_u}; \frac{1}{k_l}, \frac{1}{k}, \frac{1}{k_u} \right\rangle, h, m, k < 0 \end{cases}$

iv. $\lambda\tilde{N} = \begin{cases} \tilde{N} = \langle \lambda h_l, \lambda h, \lambda h_u; \lambda m_l, \lambda m, \lambda m_u; \lambda k_l, \lambda k, \lambda k_u \rangle; \lambda > 0 \\ \tilde{N} = \langle \lambda h_u, \lambda h, \lambda h_l; \lambda m_u, \lambda m, \lambda m_l; \lambda k_u, \lambda k, \lambda k_l \rangle; \lambda < 0 \end{cases}$

Definition 2.6. [35] Single-Valued Neutrosophic Score, Accuracy and De-neutrosophication are defined as:

$$\begin{aligned} \text{Score function } S(\tilde{N}) &= \frac{S_\mu(\tilde{N}) + (1 - S_\varphi(\tilde{N})) + (1 - S_v(\tilde{N}))}{3} \\ &= \frac{(8 + (h_l + 2h + h_u) - (m_l + 2m + m_u) - (k_l + 2k + k_u))}{12} \\ \text{Accuracy function } a(\tilde{N}) &= S_\mu(\tilde{N}) - S_v(\tilde{N}) \\ &= \frac{(h_l + 2h + h_u) - (k_l + 2k + k_u)}{4} \\ \text{De-neutrosophication } De(\tilde{N}) &= \frac{S_\mu(\tilde{N}) + S_\varphi(\tilde{N}) + S_v(\tilde{N})}{3} \\ &= \frac{(h_l + 2h + h_u) + (m_l + 2m + m_u) + (k_l + 2k + k_u)}{12} \end{aligned}$$

where, $S_\mu(\tilde{N}) = \frac{h_l+2h+h_u}{4}$, $S_\varphi(\tilde{N}) = \frac{m_l+2m+m_u}{4}$ and $S_v(\tilde{N}) = \frac{k_l+2k+k_u}{4}$ are score functions of truth, indeterminacy and falsity degrees, respectively [25].

Theorem 2.7. *[35] If $\tilde{N}_1 = \langle h_{1l}, h_1, h_{1u}; m_{1l}, m_1, m_{1u}; k_{1l}, k_1, k_{1u} \rangle$ and $\tilde{N}_2 = \langle h_{2l}, h_2, h_{2u}; m_{2l}, m_2, m_{2u}; k_{2l}, k_2, k_{2u} \rangle$ are two neutrosophic numbers, then the following properties hold*

i. $De(\lambda_1\tilde{N}_1 \oplus \lambda_2\tilde{N}_2) = \lambda_1 De(\tilde{N}_1) + \lambda_2 De(\tilde{N}_2)$, $\forall, \lambda_1, \lambda_2 > 0$.

ii. $De(\lambda_1\tilde{N}_1 \ominus \lambda_2\tilde{N}_2) = \lambda_1 De(\tilde{N}_1) - \lambda_2 De(\tilde{N}_2)$, $\forall, \lambda_1, \lambda_2 > 0$ *, De denotes de-neutrosophication.*

Multi-objective optimization problems: Optimization problems involving objective functions that are conflicting each other are known as multi-objective optimization problems, defined as:

$$\begin{aligned} &Optimize\ G_k(x), k = 1, 2, ..., K \\ &\text{S.t } g_j(x)(\leq, =, \geq) b_j, j = 1, 2, ..., m \\ &x = (x_i) \geq 0, i = 1, 2, ..., n \end{aligned} \tag{1}$$

where $G_k(x)$ are objective functions for all $k = 1, 2, ..., K$ and $g_j(x)$ are constraints for all $j = 1, 2, ..., m$ which are non linear functions for all decision variables $x \in R^n$. Each objective function $G_k(x), k = 1, 2, ..., K$ can be either maximization or minimization under constraints $g_j(x), j = 1, 2, ..., m$.

Definition 2.8. [40] *Unique optimal solution*: A solution x^* is said to be unique optimal solution (1) of

i. maximization optimization problems, if $\exists x^* \in R^n$, which satisfies the constraints, such that $G_k(x^*) \geq G_k(x)$, $\forall x \in R^n$.

ii. minimization optimization problems , if $\exists x^* \in R^n$, which satisfies the constraints, such that $G_k(x^*) \leq G_k(x)$, $\forall x \in R^n$.

Definition 2.9. [40] *Pareto optimal solution*: Let $x^*, \bar{x} \in R^n$ be points that satisfy the constraints of (1) and x^* is said to be Pareto optimal solution for (1)

i. maximization problem, if there does not exist another $\bar{x}$ such that $G_k(x^*) \leq G_k(\bar{x})$, for all k = 1,2,...,K and $G_k(x^*) < G_k(\bar{x})$ for at least one k = 1,2,...,K.

ii. minimization problem, if there is no another $\bar{x}$ such that $G_k(x^*) \geq G_k(\bar{x})$, for all k = 1,2,...,K and $G_k(x^*) > G_k(\bar{x})$ for at least one k = 1,2,...,K.

For multi-objective problems there is no unique optimal solution, so we look for Pareto optimal solutions. A Pareto optimal solution that satisfies all the aspiration levels of the decision-maker is called a compromised. Thus, an efficient or Pareto optimal solution (POS) is said to be optimal for multi-objective problem if there is no other solution that is good according to all criteria and strictly better according to at least one criterion. If POS is in the interior of a non-convex, feasible objective region, it is impossible to improve the optimal solution. However, globally, POS are always located on its boundary, and it is possible to improve them using appropriate tools. Obviously, any globally Pareto-optimal solution is locally Pareto-optimal. The converse is valid for convex problems [40].

2.2. *Proposed Method: Neutrosophic Fuzzy Optimization Technique (NeFOT)*

Neutrosophic fuzzy numbers allow for representation and handling of impreciseness, ambiguity, and indeterminacy of information inherent in real-world problems. Solving a neutrosophic optimization problem using de-neutrosophication method involves converting neutrosophic parameters into crisp values and then solving the resulting crisp optimization problem. However, this approach may not sufficiently address the impreciseness, inconsistency, and ambiguity of information inherent in original neutrosophic model, potentially leading to sub-optimal solutions or neglecting important decision-making considerations. Hence, we recognize that the optimal value of the de-neutrosophied model is an approximate value of the optimal solution, so that it can be treated with other techniques to obtain a more appropriate solution. NeFOT is considered with some tolerance on the optimal value for defining the truth membership, indeterminacy, and falsity membership functions to find the compromise Pareto optimal solution.

Thus, NeFOT approach ensures and allows for more comprehensive treatment of impreciseness, ambiguity and indeterminacy in that the resulting solutions are robust, taking into account the tolerance for deviations from optimal values and capturing the degrees of truth, falsity, and indeterminacy inherent in the neutrosophic parameters.

Definition 2.10. In real-life multi-objective problems, objectives are conflicting in nature. So, to get a compromise solution to multi-objective problems, the FOT and IFO are techniques that yield better results and can give a strong Pareto optimal solution [16] and [19] suggested it as a novel method for solving MOOP.

Definition 2.11. In real-life problems due to ambiguous, inadequate, and inconsistent information, the optimal solution is insufficient to draw a decision, so that for MOOP in neutrosophic environment, using NeFOT yields better result and can give a strong Pareto optimal solution.

Crisp number to Neutrosophic number:

For a given crisp number x_0, the truth, indeterminacy, and falsity intervals can be defined using the tolerance for each around x_0 to define a neutrosophic number. The tolerance intervals around the crisp value capture the uncertainty more comprehensively and give a nuanced understanding of the degree to which the crisp value may deviate. Neutrosophic numbers provide flexible modeling of imprecise, uncertain, and vague or inconsistent information by adjusting the tolerance interval level to different scenarios to ensure consistent handling of such information within Neutrosophic framework Smarandache [41].

Definition 2.12. For a given crisp number x_o the corresponding neutrosophic number $\tilde{N}$ with truth, indeterminacy, and falsity intervals around x_o using lower and upper tolerances to capture the uncertainty or vagueness and inconsistency can be expressed as
$\tilde{N} = \{(x_0 - T_l, x_0, x_0 + T_u; x_0 - I_l, x_0, x_0 + I_u; x_0 - F_l, x_0, x_0 + F_u) : T, I, F \in [0, 1]\}$.

For instance,

i. $\tilde{N}_1 = \langle 0.7, 0.8, 0.9; 0.75, 0.8, 0.85; 0.77, 0.8, 0.83\rangle$
ii. $\tilde{N}_2 = \langle 0.4, 0.5, 0.6; 0.3, 0.5, 0.7; 0.2, 0.5, 0.8\rangle$.
iii. $\tilde{N}_3 = \langle 4, 5, 6; 3, 5, 7; 2, 5, 8\rangle$.

Definition 2.13. For any three real numbers (l, m, y) the corresponding interval numbers can be represented as $\{(l - \triangle_1, l, l + \triangle_2), (m - \eta_1, m, m + \eta_2), (y - \varepsilon_1, y, y + \varepsilon_2)\}$, where $\triangle_1, \eta_1, \varepsilon_1 > 0$ and $\triangle_2, \eta_2, \varepsilon_2 > 0$ are the left and right spans of (l, m, y) respectively.

Extending this definition to the neutrosophic triangular numbers we have

Definition 2.14. Single valued triangular neutrosophic fuzzy numbers:
$\tilde{S}_i = \{(l_i - \triangle_{l_{i1}}, l_i, l_i + \triangle_{l_{i2}}), (m_i - \eta_{m_{i1}}, m_i, m_i + \eta_{m_{i2}}), (y_i - \varepsilon_{y_{i1}}, y_i, y_i + \varepsilon_{y_{i2}})\}$, with cores

(l_i, m_i, y_i) are single valued triangular neutrosophic fuzzy numbers with membership degrees $\mu_{\tilde{S}_i}(x)$, indeterminacy degrees $\varphi_{\tilde{S}_i}(x)$ and non-membership degrees $v_{\tilde{S}_i}(x)$ respectively.

Definition 2.15. In Definition 2.14 , $\tilde{S}_i$ are also single valued triangular neutrosophic numbers with core of truth, indeterminacy and falsity degrees, $l_i = m_i = y_i$, for all i, with left and right spans as in Definition 2.13.

Definition 2.16. Based on definition 2.6, the score function for each $\mu_{\tilde{S}_i}(x)$, $\varphi_{\tilde{S}_i}(x)$ and $v_{\tilde{S}_i}(x)$, and de-neutrosophic value, the deneutrosophication of neutrosophic triangular fuzzy number is defined as:

$$De(\tilde{S}_i) = \frac{4l_i + \triangle_{l_{i2}} - \triangle_{b_{i1}} + 4m_i + \triangle_{m_{i2}} - \triangle_{m_{i1}} + 4y_i + \triangle_{y_{i2}} - \triangle_{y_{i1}}}{12} \quad (2)$$

Neutrosophic Linear Membership Functions: De [42] while solving an EOQ under fuzzy reasoning defined the idea of fuzzy number. Consider left membership function for fuzzy number $\tilde{A}$ with initial at x_0.

$$x \geq x_0, \forall x \in X.$$

For an fuzzy number $\tilde{A}$,define a fuzzy linear or nonlinear membership function $\mu_{\tilde{A}}(x)$ as:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , x \leq x_0 - \delta \\ \frac{x-(x_0-\delta)}{\delta} = 1 - \frac{x_0 - x}{\delta} & , x_0 - \delta \leq x \leq x_0 \\ 1 & , x \geq x_0 \end{cases} \quad (3)$$

where x_0 is the initial value of the fuzzy number $\tilde{A}$ and δ is the maximum tolerance level. The graphical representation of the corresponding fuzzy number is

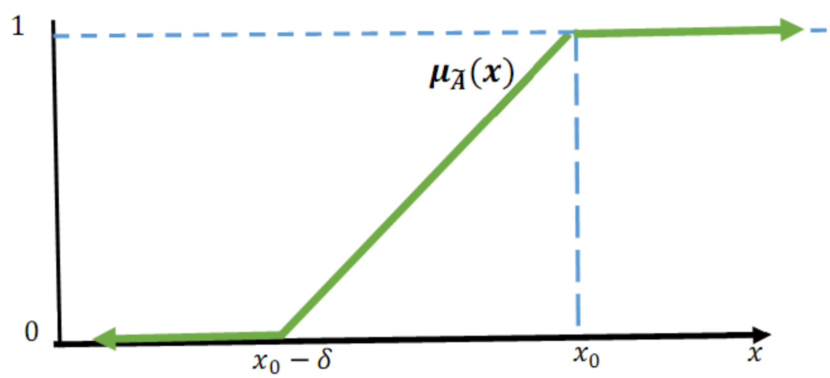

FIGURE 2. Fuzzy linear membership function

Figure 2 defined for at most maximum point at x_0, seeking to find optimal solution in the interval $[x_0 - \delta, x_0]$ with a maximum tolerance δ, implying that there is an optimal solution in between. However, for a defuzzied model, considering tolerance at the optimal value can make the optimal solution more realistic, approximated optimal solution for defining the membership function in order to apply FOT. According to Smarandache [41], any neutrosophic number can be expressed with infimum and supremum, means that for any core value there are left and right tolerable spans.

Determination of tolerance Levels: Establish the tolerance levels for the left (ϵ_l) and right (ϵ_r) sides of the optimal value. These tolerance levels represent acceptable deviations from the optimal value (say f^*). It helps in accommodating slight under-performance without considering the solution completely unacceptable and over-performance, which might be beneficial in some context but could also have associated costs or risks. Define the lower bound (LB) as the optimal value minus left tolerance ($f^* - \epsilon_l$) and the upper bound (UB) as the optimal value plus right tolerance ($f^* + \epsilon_r$). Then define the truth membership degree by how much a value belongs to the interval $[f^* - \epsilon_l, f^* + \epsilon_r]$; the indeterminacy function, which represents the uncertainty or indeterminacy around the interval; and the falsity membership function by how much a value does not belong to the interval.

Definition 2.17. The linear/nonlinear membership function $\mu_{\tilde{B}}(f)$ for fuzzy number $\tilde{B}$ in fuzzy space $E(X)$ with approximate value f^*, left and right tolerances $\epsilon_l > 0$, $\epsilon_r \geq 0$ is defined as:

$$\mu_{\tilde{B}}(x) = \begin{cases} 1 & , f \geq f^* + \epsilon_r \\ \frac{f-(f^*-\epsilon_l)}{\epsilon} = 1 - \frac{f^*-f}{\epsilon_r - \epsilon_l} & , f^* - \epsilon_l \leq f \leq f^* + \epsilon_r \\ 0 & , f \leq f^* - \epsilon_l \end{cases} \tag{4}$$

where $f^* + \epsilon_r$ is the tolerated initial value of the fuzzy number $\tilde{B}$. The graphical representation of the corresponding fuzzy number is described in Figure 3.

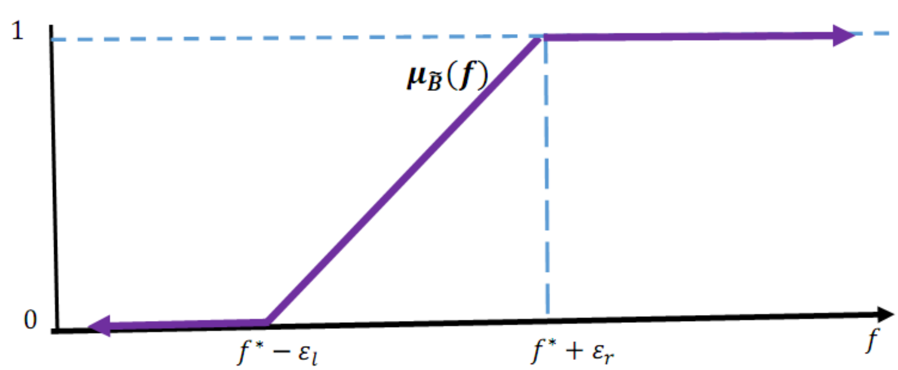

FIGURE 3. Fuzzy linear membership function with maximum tolerance

Lemma 2.18. *Let $T^{min} = f^* - \epsilon_l$ and $T^{max} = f^* + \epsilon_r$ for truth membership. The lower and upper bound of Indeterminacy and non-membership are defined based on the approach of [43]* *as:*

$$\begin{cases} I^{min} = T^{min} + t_1(T^{max} - T^{min}), t_1 \in (0,1), & I^{max} = T^{max} \\ F^{min} = T^{min}, & F^{max} = T^{max} - t_2(T^{max} - T^{min}), t_2 \in [0,1) \end{cases}$$

Remark. Both upper and lower bounds can be modified based on the decision maker's interest in application area.

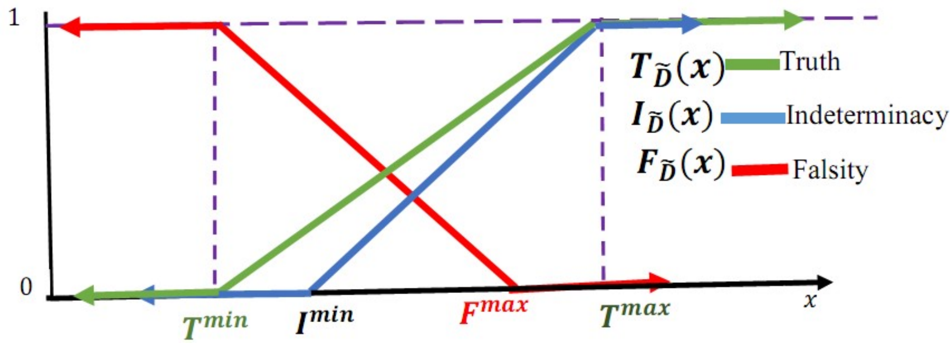

FIGURE 4. Neutrosophic fuzzy linear membership functions

$$T_{\tilde{D}}(x) = \begin{cases} 0 & , x \leq T^{min} \\ \frac{x - T^{min}}{T^{max} - T^{min}} & , T^{min} \leq x \leq T^{max} \\ 1 & , x \geq T^{max} \end{cases},$$

$$I_{\tilde{D}}(x) = \begin{cases} 0 & , x \leq I^{min} \\ \frac{x - I^{min}}{I^{max} - I^{min}} & , I^{min} \leq x \leq I^{max} \\ 1 & , x \geq I^{max} \end{cases}, \quad (5)$$

$$F_{\tilde{D}}(x) = \begin{cases} 1 & , x \leq F^{min} \\ \frac{F^{max} - x}{F^{max} - F^{min}} & , F^{min} \leq x \leq F^{max} \\ 0 & , x \geq F^{max} \end{cases}$$

Optimization problem: Let us consider a crisp optimization problem of the form:

$$\begin{aligned} &\textit{Maximize } G(x), \\ &x \geq 0, \end{aligned} \quad (6)$$

where $x \in R^n$ is decision variable, $G(x)$ is the objective function.

The solution to this crisp optimization problem satisfies $x \geq 0$ exactly. In the analogous fuzzy optimization problem, the degree of satisfaction of the objective is maximized under $x \geq 0$. Again, according to Angelov [44] on IFO technique, when the degree of rejection (non-acceptance) is defined simultaneously with the degree of acceptance and when both these degrees are not complementary to each other, intuitionistic fuzzy set can be used as a more general tool for describing this fuzziness Chakrabortty *et al.* [43] and Panda and Sahoo [19].

Further, when inconstancy exists with neither acceptance nor rejection, indeterminacy occurs and in this case neutrosophic fuzzy (NF) set can be used as the most general tool for describing fuzziness.The corresponding neutrosophic problem is

$$\begin{aligned} &Maximize\ \tilde{G}(x), \\ &x \geq 0, \end{aligned} \tag{7}$$

where $x \in R^n$ is decision variable, $\tilde{G}(x)$ is the neutrosophied objective function.

So, based on Angelov [44] approach, to maximize the degree of acceptance of NF objectives for $x \geq 0$, minimize the degree of rejection, IFOT was developed by Panda and Sahoo [11]. The corresponding de-neutrosophied problem is

$$\begin{aligned} &Maximize\ \hat{G}(x), \\ &x \geq 0, \end{aligned} \tag{8}$$

where $x \in R^n$ is decision variable, $\hat{G}(x)$ is the neutrosophied objective function.

We extend IFOT to NeFOT to minimize indeterminacy in NF objectives with $x \geq 0$.
As in Lemma 2.18, let $\mu(x) = T_D(x), \varphi(x) = I_D(x)$ and $v(x) = F_D(x)$ are a set of degrees of acceptance, indeterminacy, and falsity of the neutrosophic fuzzy decision solution under decision space (D), Bellman and Zadeh [45], studied these concepts and used in many real-life applications of decision-making in a fuzzy environment and Ahmad [46] used such concept in neutrosophic environment for MOO-NLOP.
We have the following equivalent crisp multi-objective revision of the original problem:

$$\begin{aligned} Maximize\ &\mu(x), \\ Minimize\ &\varphi(x), \\ Minimize\ &v(x), \\ \text{S.t}\ &\mu(x) \geq v(x), \\ &\mu(x) \geq \varphi(x), \\ &\mu(x), \varphi(x), v(x) \geq 0, \\ &0 \leq \mu(x) + \varphi(x) + v(x) \leq 3. \\ &x \geq 0. \end{aligned} \tag{9}$$

where $\mu(x)$ denotes the degree of acceptance, $\varphi(x)$ represents the degree of indeterminacy, and $v(x)$ denotes the degree of non-membership of $x \in R^n$.

Definition 2.19. *Neutrosophic Fuzzy Pareto Optimal Solution:* A point $x^* \in R^n$ is said to be neutrosophic Pareto(NF-Pareto) optimal solution to the problem (9) if and only if there does not exist $x \in R^n$ such that $\mu(G(x)) \geq \mu(G(x^*))$, $\varphi(G(x)) \leq \varphi(G(x^*))$, $\nu(G(x)) \leq \nu(G(x^*))$ and $\mu(G(x)) \neq \mu(G(x^*))$, $\varphi(G(x)) \neq \varphi(G(x^*))$, $\nu(G(x)) \neq \nu(G(x^*))$, for at least one objective.

Definition 2.20. *Weak Neutrosophic Fuzzy Pareto Optimal Solution:* A point $x^* \in R^n$ is said to be Weak Neutrosophic Pareto(WNF-Pareto) optimal solution to the problem (9) if and only if there does not exist $x \in R^n$ such that $\mu(G(x)) > \mu(G(x^*))$, $\varphi(G(x)) < \varphi(G(x^*))$, $\nu(G(x)) < \nu(G(x^*))$.

Using Zimmermann [47] approach to maximize the minimum degree of acceptance, minimize the maximum degree of rejection, and to minimize the maximum indeterminacy of NF objectives and constraints, we have the following transformed system of equations:

$$\alpha \leq \mu(x)$$
$$\gamma \geq \varphi(x)$$
$$\beta \geq v(x)$$

where, α is the minimal acceptable degree, γ is the maximal indeterminacy degree, and β is the maximal rejection degree of the objective(s) and constraints. Now the NeFOP can be transformed into the following crisp (non-fuzzy) optimization problem, which can be easily solved numerically or by using standard software.

$$\begin{aligned} &Maximize\ \alpha, \\ &Minimize\ \gamma, \\ &Minimize\ \beta, \\ \text{S.t}\ &\mu(x) \geq \alpha, \\ &\varphi(x) \leq \gamma, \\ &v(x) \leq \beta, \\ &\alpha, \gamma, \beta \geq 0, \\ &0 \leq \alpha + \gamma + \beta \leq 3. \\ &x \geq 0. \end{aligned} \tag{10}$$

2.2.1. *Solution Approach: Algorithm for Neutrosophic Fuzzy Optimization Technique*

In our neutrosophic model, we want to maximize the degree of acceptance and minimize the degrees of rejection and indeterminacy and also obtain optimal solution of the neutrosophic unconstrained objective function.

Algorithm for NeFOT:

Step-I: Solve $\hat{G}(x)$ using the nonlinear differentiable-optimization technique employing gradient method. Let $\left\{x^*, \hat{G}(x^*)\right\}$ be the optimal solution of the de-neutrosophied model (8) of neutrosophic model (7) .

Step-II: Define the lower and upper boundaries of the objective function, $T^{min} = \hat{G}^* - \epsilon_l$ and $T^{max} = \hat{G}^* + \epsilon_r$ with maximum tolerances ϵ_1, ϵ_r on the approximated optimal solution $\hat{G}^*$. Define lower and upper boundaries for indeterminacy $\varphi_G(x)$ and non-membership function $v_G(x)$.

From Lemma 2.18

$$\begin{cases} I^{min} = T^{min} + t_1(T^{max} - T^{min}), & I^{max} = T^{max}, t_1 \in (0,1) \\ F^{min} = T^{min}, & F^{max} = T^{max} - t_2(T^{max} - T^{min}), t_2 \in [0,1) \end{cases}$$

Step-III: The de-neutrosophied (8) objective function of neutrosophic fuzzy model (7) is,

$$\hat{G}(x) \tilde{\geq} T^{max}, \forall x \in X.$$

For the objective function $\hat{G}(x)$, define a neutrosophic fuzzy linear or nonlinear membership function $\mu_{\hat{G}}(\hat{G})$, an indeterminacy function $\varphi_{\hat{G}}(\hat{G})$ and a non-membership $v_{\hat{G}}(\hat{G})$ as:

$$\mu_{\hat{G}}(\hat{G}) = \begin{cases} 0 & , \hat{G}(x) \leq T^{min} \\ \frac{\hat{G}(x) - T^{min}}{T^{max} - T^{min}} & , T^{min} \leq \hat{G}(x) \leq T^{max} \\ 1 & , \hat{G}(x) \geq T^{max} \end{cases}$$

$$\varphi_{\hat{G}}(\hat{G}) = \begin{cases} 0 & , \hat{G}(x) < I^{min} \\ \frac{\hat{G}(x) - I^{min}}{I^{max} - I^{min}} & , I^{min} \leq \hat{G}(x) \leq I^{max} \\ 1 & , \hat{G}(x) > I^{max} \end{cases} \quad (11)$$

$$v_{\hat{G}}(\hat{G}) = \begin{cases} 1 & , \hat{G}(x) \leq F^{min} \\ \frac{F^{max} - \hat{G}(x)}{F^{max} - F^{min}} & , F^{min} \leq \hat{G}(x) \leq F^{max} \\ 0 & , \hat{G}(x) \geq F^{max} \end{cases}$$

The aim is to maximize the degree of acceptance $\mu_{\hat{G}}(\hat{G})$, minimize indeterminacy $\varphi_{\hat{G}}(\hat{G})$, and minimize falsity degree $v_{\hat{G}}(\hat{G})$ of neutrosophic fuzzy decision solution under a single-valued neutrosophic fuzzy decision set.

Step-IV: According to Zimmermann [47] approach, de-neutropsophied version (8) of NeFNLP (7) can be solved by extending Panda and Sahoo [19] IFOT method to NeFOT.

Alternatively we can use Angelov [44] approach of linearization.

$$\max \alpha, \min \gamma, \min \beta \textbf{ or } max(\alpha - \gamma - \beta)$$
$$\text{S.t } \frac{\hat{G}(x) - T^{min}}{T^{max} - T^{min}} \geq \alpha;$$
$$\frac{\hat{G}(x) - I^{min}}{I^{max} - I^{min}} \leq \gamma;$$
$$\frac{\hat{G}(x) - F^{min}}{F^{max} - F^{min}} \leq \beta; \tag{12}$$
$$\alpha > \gamma;$$
$$\alpha > \beta;$$
$$\alpha + \beta + \gamma \leq 3;$$
$$x > 0.$$

α is the minimum aspiration level to be maximized, γ is the maximum indeterminacy level to be minimized, and β is the maximum rejection level to be minimized for the objective function (8).

Step-V: Stop, optimal solution attained.

2.3. *Flowchart for the Proposed Method*

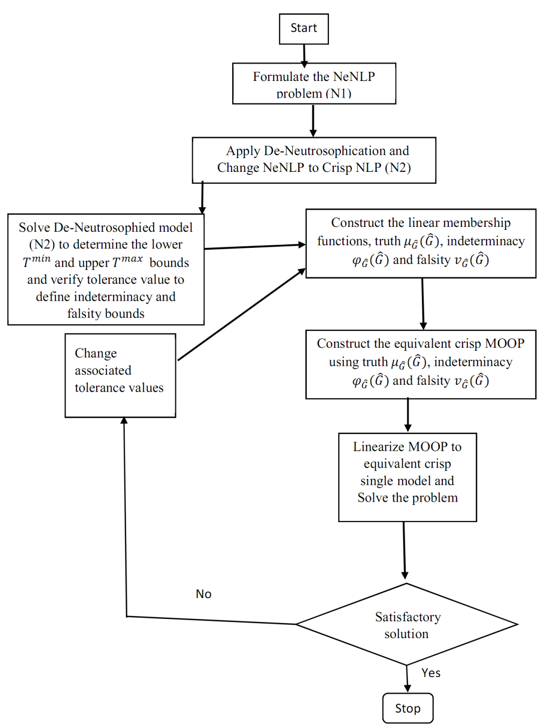

FIGURE 5. Flowchart for the proposed method

3. **Main Results**

In this section, analytical expressions for neutrosophic fuzzy EOQ inventory model for a single perishable item with stock-dependent demand and additional considerations are derived.

To prove the optimality of total net profit function in this work, we transform time variables to order quantity and maximum inventory level variables under shortages by Taylor series approximation and de-neutrosophication is used for the neutrosophic fuzzy parameters in the model to derive necessary and sufficient conditions analytically.

Many scholars have studied preservation technology function $\Theta(c_e)$ in connection with initial rate of deterioration including Shanmugam and Karuppuchamy [10]. But, there is no clear information on the relation between initial deterioration and preservation method, so that generalized preservation approach is used by Mahata and Debnath [4], Gudeta *et al.* [39]. We use this approach during model formulation in neutrosophic environment.

3.1. *Assumptions and Notations*

Assumptions

1. The model is developed for a single perishable item.
2. Supplier and retailer agree to exchange σ% of perfect and $(1-\sigma)$% imperfect items, where $0.88 \leq \sigma \leq 1$ during purchasing at supplier stock due to post harvest loss/or load/unloading, with perish rate $\tilde{\delta}(0 < \delta << 1)$ a constant
3. Price discount for defective items to retailer $\tilde{\pi}_0$ less than cost c and downstream to consumer $\tilde{\pi}$ less than price $\tilde{p}$ per unit item
4. SVT-neutrosophic number demand rate is stock dependent $D(t) = \tilde{a} + \tilde{b}I(t)$, where $I(t)$ is the inventory level at time t, $a > 0$ a constant, $0 < b < 1$, stock influence on demand and $b < 0$ only when shortages occur.
5. Perish start on received item at any time, so that preservation applied with $\Theta = e^{-c_e \xi}$, where SVT-neutrosophic number $c_e > 0$ is preservation cost, $\xi \in [0, 1]$ is control of preservation and generalized preservation technology used on time $[t_1, t_2]$. Θ satisfies the conditions $\frac{\partial \Theta}{\partial c_e} < 0$ and $\frac{\partial^2 \Theta}{\partial c_e^2} > 0$ with $\Theta << \delta < 1$.
6. Holding cost considered for perfect item with price $\tilde{p}$.
7. Shortage allowed and backlogged.
8. Perishable items are not repaired or replaced, and they are removed from the store permanently.
9. All parameters of the model are considered as single valued triangular neutrosophic fuzzy numbers.

Notations

Symbol	Their Representation
Q	: Order quantity/order **(Decision variable)**
M	: Maximum inventory position/order **(Decision variable)**
S	: Defected item due to transport or loading/ unloading and post-harvesting
B	: Backlogged stock-out level /order
N	: Perfect item maximum stock level at time $t = t_1$
t_2	: Stock out time/ maximum life of perishable items
T	: Inventory cycle length
$\tilde{\delta}$	: Neutrosophic deteriorating rate of item with $0 << \delta < 1$.
$\tilde{\Theta}$	: Neutrosophic preservation technology for deterioration rate
$\tilde{c}$	: Neutrosophic purchasing price/ item
$\tilde{c_s}$	: Neutrosophic shortage cost / item
$\tilde{c_h}$	: Neutrosophic holding cost / item / cycle $\tilde{c_h} = i\tilde{p}, i \in (0, 1]$
$\tilde{p}$	: Neutrosophic selling price/ item
r	: Price discount rate for imperfect items,$r_1\%$–supplier and $r_2\%$–retailer
$\tilde{\pi_o}$	: Neutrosophic purchasing cost for defective item $\tilde{\pi_o} = (1 - r_1)\tilde{c}$
$\tilde{\pi}$	: Neutrosophic selling cost for defective item $\tilde{\pi} = (1 - r_2)\tilde{p}$
$\tilde{c_0}$	: Neutrosophic order cost / cycle T
$\tilde{c}_{op}$	: Neutrosophic opportunity(lost sale) cost/cycle T
$\tilde{c_e}$	: Neutrosophic preservation technology cost/ time on $(t_2 - t_1)$
α	: Aspiration level-acceptance level
γ	: Maximum indeterminacy level
β	: Maximum rejection level
$\tilde{G}(Q, M)$	: Neutrosophic total profit

3.2. *Formulation and Mathematical Analysis*

Description: In our model, a two-stage supply chain considers perish from supplier to retailer as well as perish in retailer stock.

The retailer orders Q units with a planned shortage B, when M units of perishable items are available. Therefore, the inventory level at $t = 0$ is M. During purchasing, due to post-harvest loss or loading/unloading or transportation, the inventory level declines with deterioration rate δ and accepting imperfect items as per agreement drops to $N = \sigma M$ at time $t = t_1$. Supplier offers price discount $r_1\%$ and retailer offers a price discount of $r_2\%$ for downstream defective items $(1 - \sigma)M$. The retailer's stock inventory level decreases due to impact of demand (D) and preservation technology $(\Theta \leq \delta)$. For shortage cases, stock is based on actual demand with backlog, and finally, if a partial back-order holds, opportunity cost is considered due to lost sale. For transcendental functions involved in the model, we use

Taylor series approximation which enables us to show the optimality condition analytically in neutrosophic environment. We use de-neutrosophication approach of Mohanta *et al.* [35] Definition 2.16 for neutrosophic parameters, in which the profit function is concave, that is, a unique solution exists in neutrosophic environment. Further, we use a develop NeFOT of Section 2.2 with established solution approaches of Section 2.2.1 which enables us to determine the inherent impreciseness or inconsistent information involved in the developed model as well as optimal solution. The dynamics of the stock level is described in figure 6 with maximum stock and imperfect items I_1 in time $[0, t_1]$, stock level I_2 in time $[t_1, t_2]$ and shortage level I_3 in time $[t_2, T]$ with continuous boundary condition.

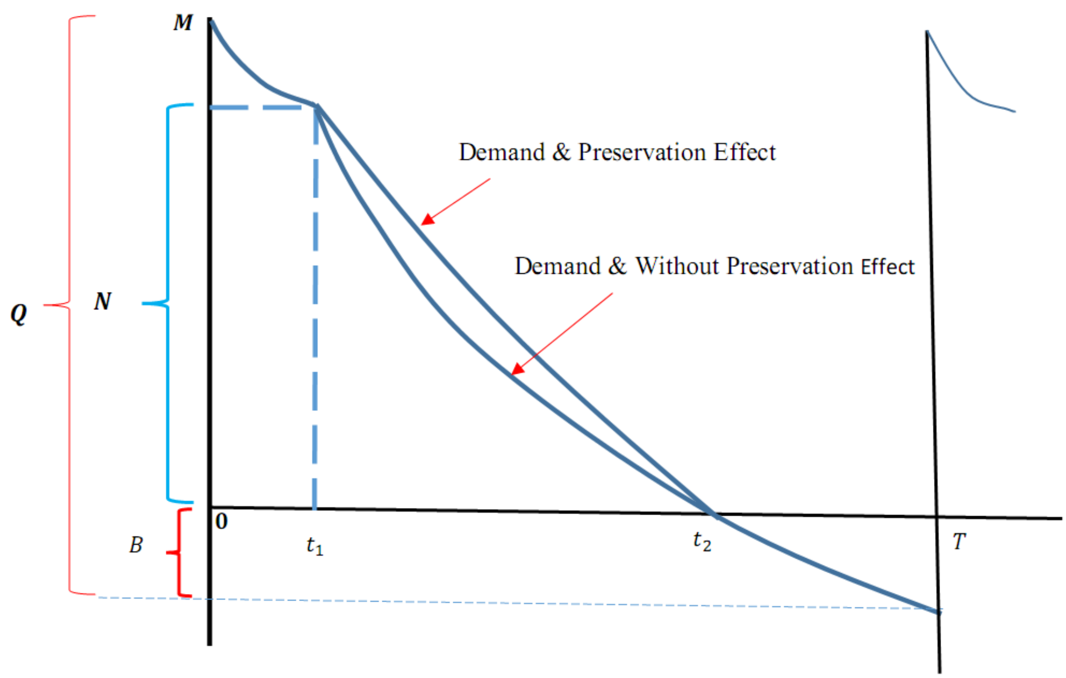

FIGURE 6. Dynamics of inventory level

3.3. *Mathematical Analysis*

Computing inventory levels based on figure 6 and applying boundary condition, we get

$$I_1 = Ne^{\tilde{\delta}(t_1 - t)}, 0 \leq t \leq t_1 \tag{13}$$

$$I_2(t) = \frac{\tilde{a}}{\tilde{b} + \tilde{\Theta}} \left(e^{(\tilde{b}+\tilde{\Theta})(t_2 - t)} - 1 \right), t_1 \leq t \leq t_2 \tag{14}$$

By continuity condition the maximum stock level at time t_1 is N. From (13) and (14), we have

$$N = \frac{\tilde{a}}{\tilde{b} + \tilde{\Theta}} \left(e^{(\tilde{b}+\tilde{\Theta})(t_2 - t_1)} - 1 \right) \tag{15}$$

Solving for $(t_2 - t_1)$ we get

$$t_2 - t_1 = \frac{1}{(\tilde{b}+\tilde{\Theta})} ln \left(1 + \frac{(\tilde{b}+\tilde{\Theta})N}{\tilde{a}}\right) \tag{16}$$

Using Taylor series expansion and neglecting higher powers, we have

$$t_2 - t_1 = \frac{N}{\tilde{a}} \left(1 - \frac{(\tilde{b}+\tilde{\Theta})N}{2\tilde{a}}\right) \tag{17}$$

Holding cost H_c in $[t_1, t_2]$ with $N = \sigma M$ using (14), $\tilde{w} = \tilde{b} + \tilde{\Theta}$ and applying Taylor series expansion and neglecting higher powers more than second degree, H_c is given by

$$H_c = \tilde{c}_h \int_{t_1}^{t_2} I_2(t)dt = \int_{t_1}^{t_2} -\frac{\tilde{a}}{\tilde{w}} + \left(N + \frac{\tilde{a}}{\tilde{w}}\right)\left(e^{-\tilde{w}(t_2-t)} - 1\right) dt$$

$$H_c = \tilde{c}_h \frac{N^2}{2\tilde{a}} \left(1 - \frac{2(\tilde{b}+\tilde{\Theta})N}{3\tilde{a}}\right) \tag{18}$$

Wastage/Deteriorating cost on time $[t_1, t_2]$ using Taylor series approximation and simplifying, we get is

$$W_c = \tilde{p}N - \tilde{p}\,Total\ demand\ during\ time[t_1, t_2] = \tilde{p}N - \tilde{p}\int_{t_1}^{t_2} \tilde{a} + \tilde{b}I_2(t)dt$$

$$= \tilde{p}N - \tilde{p}\int_{t_1}^{t_2} \tilde{a} + \tilde{b}\left(-\frac{\tilde{a}}{\tilde{w}} + \left(N + \frac{\tilde{a}}{\tilde{w}}\right)\left(e^{-\tilde{w}(t_2-t)} - 1\right)\right) dt$$

$$W_c = \tilde{p}\frac{\tilde{\Theta}N^2}{2\tilde{a}} \left(1 - \frac{2(\tilde{b}+\tilde{\Theta})N}{3\tilde{a}}\right) = \tilde{p}\tilde{\Theta}H \tag{19}$$

The shortage stock level $I_3(t)$ in time $[t_2, T]$ is

$$I_3(T) = -\tilde{a}(T - t_2)$$

Shortage cost B_c on time $[t_2, T]$ is

$$B_c = -\tilde{c}_s \int_{t_2}^{T} (-I_3(t))dt = \tilde{c}_s \frac{\tilde{a}}{2}(T - t_2)^2 = \frac{\tilde{c}_s(Q-M)^2}{2\tilde{a}}$$

Purchase cost $P_c = \tilde{c}Q = \tilde{c}((\sigma - 1)M + Q) + \tilde{\pi}_0(1-\sigma)M, \tilde{\pi}_0 = (1 - \tilde{r}_1)\tilde{c}$

Defective cost on time $[0, t_1]$, sold with cost $\tilde{\pi}$ less than price $\tilde{p}$ at retailer level are

$$S_c = \tilde{\pi}(1-\sigma)M$$

Sales revenue of retailer is

$$SR = \tilde{p}\ \textit{Total demand} + \tilde{p}\ \textit{Back-ordered} + \tilde{\pi}\ \textit{Defective item}$$

$$SR = \tilde{p}\int_{t_1}^{t_2} \tilde{a} + \tilde{b}I_2(t)dt + \tilde{p}\int_{t_2}^{T} \tilde{a}dt + \tilde{\pi}(1-\sigma)M$$

$$= \tilde{p}\int_{t_1}^{t_2} \tilde{a} + \tilde{b}\left(-\frac{\tilde{a}}{\tilde{w}} + \left(N + \frac{\tilde{a}}{\tilde{w}}\right)\left(e^{-\tilde{w}(t_2-t)} - 1\right)\right) dt + \tilde{p}\int_{t_2}^{T} \tilde{a}dt + \tilde{\pi}(1-\sigma)M$$

Applying Taylor series approximation and simplifying, we get

$$SR = \tilde{p}N - \tilde{p}W_c + \tilde{p}B + \tilde{\pi}(1-\sigma)M$$

Total preservation cost: $T_e = \tilde{c}_e(t_2 - t_1) = \frac{\tilde{c}_e N}{\tilde{a}}\left(1 - \frac{(\tilde{b}+\tilde{\Theta})N}{2\tilde{a}}\right)$
Net revenue

$$\eta(Q,M) = (\tilde{p}-\tilde{c})\sigma M + (\tilde{p}-\tilde{c})(Q-M) + (\tilde{\pi}-\tilde{\pi}_0)(1-\sigma)M - \tilde{p}\tilde{\Theta}H \tag{20}$$

$G(Q,M) =$ *Net revenue- Holding cost-Back order cost-Technology cost-Order cost*

$$G(Q,M) = (\tilde{p}-\tilde{c})((\sigma-1)M+Q) + (\tilde{\pi}-\tilde{\pi}_0)(1-\sigma)M - (\tilde{p}\tilde{\Theta}+\tilde{c}_h)H - \tilde{c}_s B_c - \tilde{c}_e T_e - \tilde{c}_0 \tag{21}$$

Thus, a nonlinear optimization inventory problem for single perishable item in neutrosophic environment following the notations, assumptions stated in Section 3.1 with full backlogged shortage per cycle can be expressed as:

$$\begin{aligned} Max\ \tilde{G}(Q,M) = \tilde{S}_1((\sigma-1)M+Q) + \tilde{S}_2(1-\sigma)M - \tilde{S}_3\frac{\sigma^2M^2}{2\tilde{a}}\left(1-\frac{2\tilde{w}\sigma M}{3\tilde{a}}\right) \\ -\tilde{S}_4\frac{(Q-M)^2}{2\tilde{a}} - \tilde{S}_6\frac{\sigma M}{\tilde{a}}\left(1-\frac{\tilde{w}\sigma M}{2\tilde{a}}\right) - \tilde{S}_7 \end{aligned} \tag{22}$$

where $\tilde{S}_1 = (\tilde{p}-\tilde{c}) > 0$, $\tilde{S}_2 = \tilde{\pi}-\tilde{\pi}_o > 0$, $\tilde{S}_3 = \tilde{p}\tilde{\Theta}+\tilde{c}_h$, $\tilde{S}_4 = \tilde{c}_s$, $\tilde{S}_5 = 0$, $\tilde{S}_6 = \tilde{c}_e$, $\tilde{S}_7 = \tilde{c}_0$, $\tilde{\pi}-\tilde{\pi}_o < \tilde{p}-\tilde{c}$, $Q, M \geq 0, Q > M$, $\tilde{w} = \tilde{\Theta}+\tilde{b}$, $\tilde{\pi}_o = (1-r_1)\tilde{c}$, $\tilde{\pi} = (1-r_2)\tilde{p}$.

Now from Definition 2.16, $\hat{S}_i = De(\tilde{S}_i)$ is a crisp approximate value of the neutrosophic triangular fuzzy numbers $\tilde{S}_i$. Hence (22) becomes

$$\begin{aligned} Max\ \hat{G}(Q,M) = \hat{S}_1((\sigma-1)M+Q) + \hat{S}_2(1-\sigma)M - \hat{S}_3\frac{\sigma^2M^2}{2\hat{a}}\left(1-\frac{2\hat{w}\sigma M}{3\hat{a}}\right) \\ -\hat{S}_4\frac{(Q-M)^2}{2\hat{a}} - \hat{S}_6\frac{\sigma M}{\hat{a}}\left(1-\frac{\hat{w}\sigma M}{2\hat{a}}\right) - \hat{S}_7 \end{aligned} \tag{23}$$

where $\hat{S}_1 = (\hat{p}-\hat{c}) > 0$, $\hat{S}_2 = \hat{\pi}-\hat{\pi}_o > 0$, $\hat{S}_3 = \hat{p}\hat{\Theta}+\hat{c}_h$, $\hat{S}_4 = \hat{c}_s$, $\hat{S}_5 = 0$, $\hat{S}_6 = \hat{c}_e$, $\hat{S}_7 = \hat{c}_0$, $\hat{\pi} < \hat{c}$, $Q, M \geq 0, Q > M$, $\hat{w} = \hat{\Theta}+\hat{b}$, $\hat{\pi}_o = (1-r_1)\hat{c}$, $\hat{\pi} = (1-r_2)\hat{p}$

Lemma 3.1. *Total profit function $\hat{G}(Q,M)$ is strictly concave if $\hat{G}_{11} < 0$ and $\hat{G}_{11}\hat{G}_{22} - \hat{G}_{12}\hat{G}_{21} > 0$.*

Proof: As principal minor matrix $|H_1| < 0$ and $|H_2| > 0$, the Hessian matrix of $\hat{G}$ is negative definite.

Theorem 3.2. *Total profit function $\hat{G}(Q,M)$ is strictly concave with respect to Q and M, hence $\hat{G}(Q,M)$ attains the global maximum at the point $(Q^*.M^*)$.*

Proof: By Lemma 3.1, the Hessian matrix of $\hat{G}(Q, M)$ is negative definite, so that $\hat{G}(Q, M)$ is a strictly concave function that attains a unique global optimal point (Q^*, M^*) obtained from the necessary condition:

$$\frac{\partial \hat{G}}{\partial Q} = \hat{S}_1 - \frac{\hat{S}_4}{a}(Q - M) = 0$$

Hence neglecting higher powers and positive time value for preservation case up to first degree, we get

$$\frac{\partial \hat{G}}{\partial M} = \hat{S}_1(\sigma - 1) + \hat{S}_2(1 - \sigma) - \hat{S}_3\frac{\sigma^2 M}{a} + \frac{\hat{S}_4}{a}(Q - M) - \hat{S}_6\frac{\sigma}{a} = 0$$

$$\hat{S}_1(\sigma - 1) + \hat{S}_2(1 - \sigma) - \hat{S}_3\frac{\sigma^2 M}{a} + \hat{S}_1 - \hat{S}_6\frac{\sigma}{a} = 0.$$

Hence, (Q^*, M^*) is the unique global maximum point of $\hat{G}$ (23), which is also the unique optimal point of the neutrosophic fuzzy objective function $\tilde{G}$ (22).

Consider the neutrosophic fuzzy unconstrained perishable inventory model (22). Using the deneutrosophication method (2) on neutrosophic parameters, we obtain the de-neutrosophied model (23). We convert crisp values to neutrosophic numbers and utilize the approach of Mullai and Surya [27] to choose the tolerances and construct single valued triangular neutrosophic numbers to suit our numerical example. The general approach of converting crisp values to neutrosophic numbers is illustrated below:

$0 < \triangle_{c_{i1}} < c_i$, $\triangle_{c_{i2}} > 0$; $0 < \eta_{l_{i1}} < l_i$, $\eta_{l_{i2}} > 0$; $0 < \varepsilon_{m_{i1}} < m_i$; $\varepsilon_{m_{i2}} > 0$

Table 1. Crisp to neutrosophic numbers and de-neutrosophication value

Crisp	Neutrosophic	De-Neutrosophication-Value
c_i	$c_i = \langle c_i - \triangle_{c_{i1}}, c_i, c_i + \triangle_{c_{i2}}; l_i - \eta_{l_{i1}}, l_i, l_i + \eta_{l_{i2}}; m_i - \varepsilon_{m_{i1}}, m_i, m_i + \varepsilon_{m_{i2}} \rangle$	$\frac{4c_i + \triangle_{c_{i2}} - \triangle_{c_{i1}} + 4l_i + \triangle_{l_{i2}} - \triangle_{l_{i1}} + 4m_i + \triangle_{m_{i2}} - \triangle_{m_{i1}}}{12}$
a	$a = \langle a - \triangle_{a_{11}}, a_1, a_1 + \triangle_{a_{12}}; a_2 - \eta_{a_{21}}, a_2, a_2 + \eta_{a_{22}}; a_3 - \varepsilon_{a_{31}}, a_3, a_3 + \varepsilon_{a_{32}} \rangle$	$\frac{4a_1 + \triangle_{a_{12}} - \triangle_{a_{11}} + 4a_2 + \triangle_{a_{22}} - \triangle_{a_{21}} + 4a_3 + \triangle_{a_{32}} - \triangle_{a_{31}}}{12}$
b	$b = \langle b - \triangle_{b_{11}}, b_1, b_1 + \triangle_{b_{12}}; b_2 - \eta_{b_{21}}, b_2, b_2 + \eta_{b_{22}}; b_3 - \varepsilon_{b_{31}}, b_3, b_3 + \varepsilon_{b_{32}} \rangle$	$\frac{4b_1 + \triangle_{b_{12}} - \triangle_{b_{11}} + 4b_2 + \triangle_{b_{22}} - \triangle_{b_{21}} + 4b_3 + \triangle_{b_{32}} - \triangle_{b_{31}}}{12}$

$0 < \triangle_{a_{11}} < a_1$, $0 < \eta_{a_{21}} < a_2$, $0 < \varepsilon_{a_{31}} < a_3$; $0 < \triangle_{b_{11}} < b_1, 0 < \eta_{b_{21}} < b_2$, $0 < \varepsilon_{b_{31}} < b_3$;
$\triangle_{a_{12}} > 0$, $\eta_{a_{22}} > 0$ $\varepsilon_{a_{32}} > 0$; $\triangle_{b_{12}} > 0$, $\eta_{b_{22}} > 0$, $\varepsilon_{b_{32}} > 0$.

4. Numerical Example

Example 4.1. For the data presented in numerical example Gudeta *et al.* [39] we use the above mentioned technique to convert crisp values to neutrosophic numbers and use tolerances to convert neutrosophic numbers single valued triangular neutrosophic numbers as presented in Tables 2 and 3

Using proposed Algorithm Section 2.2.1

TABLE 2. Crisp,Neutrosophic Numbers and Deneutrosophication Values

Crisp	Neutrosophic	De-Neutrosophication-Value
$c = \$40/unit$	$\tilde{c}_1 = \langle 30, 40, 50; 25, 40, 56; 20, 40, 61\rangle$	40.167
$p = \$69/unit$	$\tilde{c}_2 = \langle 64, 69, 75; 59, 69, 79; 49, 69, 90\rangle$	69.167
$c_h = \$0.05c_2/unit/cycle$	$\tilde{c}_3 = 0.05\tilde{c}_2 = 0.05\langle 64, 69, 75; 59, 69, 79; 49, 69, 90\rangle$	$0.05 * 69.167 = 3.4584$
$c_s = \$20/unit$	$\tilde{c}_4 = \langle 15, 20, 24; 10, 20, 28; 5, 20, 39\rangle$	20.083
$c_{op} = \$0/unit$	$\tilde{c}_5 = \langle 0000; 000; 000\rangle$	0
$c_e = \$5/unittime$	$\tilde{c}_6 = \langle 3, 5, 7; 2, 5, 8; 1, 5, 10\rangle$	5.083
$c_0 = \$60/order$	$\tilde{c}_7 = \langle 50, 60, 65; 45, 60, 77; 40, 60, 80\rangle$	59.75
$\pi_0 = \$0.8c_1/unit$	$\tilde{c}_8 = 0.8\tilde{c}_1 = 0.8\langle 30, 40, 50; 25, 40, 56; 20, 40, 61\rangle$	$0.8 * 40.167 = 32.134$
$\pi = \$0.9c_2/unit$	$\tilde{c}_9 = 0.9\tilde{c}_2 = 0.9\langle 64, 69, 75; 59, 69, 79; 49, 69, 90\rangle$	$0.9 * 69.167 = 62.25$
$a = 80kg$	$\tilde{a} = \langle 70, 80, 90; 65, 80, 100; 60, 80, 110\rangle$	81.25
$b = 0.5$	$\tilde{b} = \langle 0.45, 0.5, 0.6; 0.3, 0.5, 0.8; 0.2, 0.5, 0.9\rangle$	0.5208
$\delta = 0.3$	$\tilde{\delta} = \langle 0.25, 0.3, 0.35; 0.2, 0.3, 0.45; 0.1, 0.3, 0.55, \rangle$	0.308
$\Theta = 0.0183$	$\tilde{\Theta} = \langle 0.0173, 0.0183, 0.0188; 0.0160, 0.0183, 0.019; 0.0130, 0.0183, 0.02\rangle$	0.01783
$w = b + \Theta = 0.5183$	$\tilde{w} = \langle 0.4675, 0.5183, 0.6188; 0.316, 0.5183, 0.819; 0.213, 0.5183, 0.92\rangle$	0.5386

TABLE 3. Crisp,Neutrosophic Numbers and Deneutrosophication Values

Neutrosophic	De-Neutrosophication-Value
$\tilde{s}_1 = \tilde{c}_2 - \tilde{c}_1 > 0$	29
$\tilde{s}_2 = \tilde{c}_9 - \tilde{c}_8 > 0$	30.12
$\tilde{s}_3 = (\tilde{\Theta} + 0.05)\tilde{c}_2$	4.7064
$\tilde{s}_4 = \tilde{c}_s$; $\tilde{s}_5 = \tilde{c}_{op}$; $\tilde{s}_6 = \tilde{c}_e$; $\tilde{s}_7 = \tilde{c}_0$	

Step-I: Using the parameter values presented in Tables 2 and 3, Equation (23) becomes,

$$Max\ \hat{G}(Q, M) = -59.75 - 0.1236(Q - M)^2 + 29(Q - 0.1M) + 2.9553M - 0.02346M^2 \quad (24)$$

The solution of (24) using Mathematica 11.3 and LINGO 18.0 software gave the following result. The optimal de-neutrosophied value of the objective function is $\hat{G}^*(Q, M) = 10638$.

Step-II: Utilize historical data or past performance data in Gudeta *et al.* [39] value of $G^* = 10465.21$ to estimate reasonable tolerance levels to define lower and upper bounds for approximated optimal value $\hat{G} = 10638$ obtained using deneutrosophication on neutrosophic parameters to infer acceptable tolerance ranges in order to define truth, indeterminacy, and falsity membership functions to apply NeFOT. Let $\epsilon_l = 30$ and $\epsilon_r = 20$, with $\epsilon_r - \epsilon_l \leq |\hat{G} - G^*|$ to make it reasonable. We seek an optimal solution that accommodates the inherent impreciseness, vagueness, and inconsistency of the information that exist in problems.

$T^{min} = \hat{G} - \epsilon_l = 10608$ and $T^{max} = \hat{G} + \epsilon_r = 10658$

$$\begin{cases} I^{min} = T^{min} + t_1(T^{max} - T^{min}) = 10608 + t_1(50), t_1 \in (0,1) \\ I^{max} = T^{max} = 10658 \\ F^{min} = T^{min} = 10608 \\ F^{max} = T^{max} - t_2(T^{max} - T^{min}) = 10658 - t_2(50), t_2 \in [0,1) \end{cases}$$

In particular, let $t_1 = 0.2, t_2 = 0.3$, ensuring that core point is inside the interval

$$\begin{cases} I^{min} = 10608 + t_1(50) = 10608 + 10 = 10618 \\ I^{max} = T^{max} = 10,658 \\ F^{min} = T^{min} = 10608 \\ F^{max} = 10658 - t_2(50) = 10658 - 15 = 10643 \end{cases}$$

Step-III: Objective function (22) is neutrosophic fuzzy whose de-neutrosophied version is (23):

$$\hat{G}(x) \tilde{\geq} T^{max}, \forall x \in X.$$

For the objective function $\hat{G}(x)$, define a neutrosophic fuzzy linear or nonlinear membership function $\mu_{\hat{G}}(\hat{G})$, indeterminacy function $\varphi_{\hat{G}}(\hat{G})$ and non-membership $v_{\hat{G}}(\hat{G})$ are defined as:

$$\mu_{\hat{G}}(\hat{G}) = \begin{cases} 0 & , \hat{G}(x) \leq T^{min} \\ \frac{\hat{G}(x) - T^{min}}{T^{max} - T^{min}} & , T^{min} \leq \hat{G}(x) \leq T^{max} \\ 1 & , \hat{G}(x) \geq T^{max} \end{cases}$$

$$\varphi_{\hat{G}}(\hat{G}) = \begin{cases} 0 & , \hat{G}(x) < I^{min} \\ \frac{\hat{G}(x) - I^{min}}{I^{max} - I^{min}} & , I^{min} \leq \hat{G}(x) \leq I^{max} \\ 1 & , \hat{G}(x) > I^{max} \end{cases} \tag{25}$$

$$v_{\hat{G}}(\hat{G}) = \begin{cases} 1 & , \hat{G}(x) \leq F^{min} \\ \frac{F^{max} - \hat{G}(x)}{F^{max} - F^{min}} & , F^{min} \leq \hat{G}(x) \leq F^{max} \\ 0 & , \hat{G}(x) \geq F^{max} \end{cases}$$

The aim is to maximize the degree of acceptance $\mu_{\hat{G}}(\hat{G})$, minimize indeterminacy $\varphi_{\hat{G}}(\hat{G})$, and minimize falsity degree $v_{\hat{G}}(\hat{G})$ of neutrosophic fuzzy decision solution under a single-valued neutrosophic fuzzy decision set.

Step-IV: According to Zimmermann [47] approach, de-neutrosophied version (23) of NeFNLP (22) can be solved by extending Panda and Sahoo [19] IFOT method to NeFOT. Alternatively we can use Angelov [44] approach of linearization.

$$\begin{aligned}
&\max(\alpha, \min\gamma \min\beta) \ \textbf{or} \ \max(\alpha-\gamma-\beta)\\
\text{S.t}\ &\left(\hat{G}(x)-10608\right)/50 \geq \alpha;\\
&\left(\hat{G}(x)-10618\right)/40 \leq \gamma;\\
&\left(10643-\hat{G}(x)\right)/35 \leq \beta;\\
&\alpha \geq \gamma;\\
&\alpha \geq \beta;\\
&\alpha+\beta+\gamma \leq 3;\\
&\alpha,\gamma,\beta \in [0,1];\\
&x > 0.
\end{aligned} \tag{26}$$

α is the minimum aspiration level to be maximized, γ is the maximum indeterminacy level to be minimized, and β is the maximum rejection level to be minimized for the objective function (23), and also (22).

Step-V: Solve the following and obtain optimal solution $\alpha^*, \gamma^*, \beta^*$ and Q^*, M^* with optimal value G^*

$$\begin{aligned}
&Max(\alpha-\gamma-\beta)\\
S.t\ &(-59.75-0.1236(Q-M)^2+29(Q-0.1M)+2.9553M-0.02346M^2-10608)/50 \geq \alpha;\\
&(-59.75-0.1236(Q-M)^2+29(Q-0.1M)+2.9553M-0.02346M^2-10618)/40 \leq \gamma;\\
&(10643-(-59.75-0.1236(Q-M)^2+29(Q-0.1M)+2.9553M-0.02346M^2))/35 \leq \beta;\\
&\alpha \geq \gamma; \alpha \geq \beta;\\
&\alpha \leq 1; \gamma \leq 1; \beta \leq 1;\\
&\alpha+\gamma+\beta \geq 0;\\
&\alpha+\gamma+\beta \leq 3;\\
&Q \geq 0; M \geq 0;\\
&Q \geq M.
\end{aligned} \tag{27}$$

The following optimal results are obtained
$Max(\alpha-\gamma-\beta) = -0.04072$, $\alpha^* = 0.6018$, $\gamma^* = 0.5023$, $\beta^* = 0.1403$, $M^* = 619.269$, $Q^* = 736.595$, $B^* = 117$, $S^* = 61.93$ and objective function optimal value: $\hat{G}^*(Q,M) = 10638$. Graphical illustration of numerical example for the developed model is given below. The objective function value is plotted against the decision variables order quantity and maximum stock level.

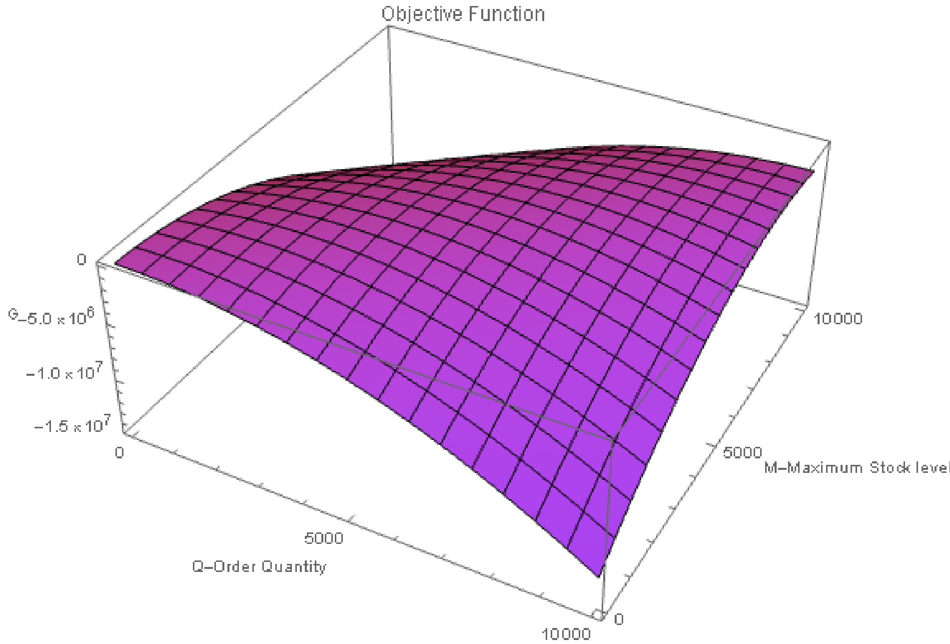

FIGURE 7. Concavity of total profit $\hat{G}$ verses (Q, M)

5. **Discussion**

In any real-life model, impreciseness/vagueness and inconsistency inherent cannot be ignored during model formulation, so that by providing an appropriate tolerance level for crisp parameters or optimal values, corresponding neutrosophic numbers are defined in order to apply the neutrosophic optimization method, which provides us with the inherent impreciseness/vagueness and inconsistency involved in the model as well as a unique optimal solution. But the study by Gudeta *et al.* [39] does not consider uncertainty in the model of single perishable item. Taylor series approximation is used to transform the transcendental function, and as a result, the profit function is transformed. Moreover, in addition to the verified theoretical optimality condition, the concavity of the total profit function and the existence of a unique global optimal solution for the de-neutrosophied model using the Hessian matrix are established. NeFOT method is developed to solve EOQ model and to estimate the maximum acceptance level α^*, minimum indeterminacy level γ^*, and minimum falsity level β^* for the optimal solution, which is a novel development not achieved by any other technique available in the literature. Also from graphical illustration of numerical example using LINGO 18.0 and Mathematica 11.3, we observed that the de-neutrosophied model confirms the concavity property of the objective function with respect to the decision variables and NeFOT also provide us optimal values of G^*, Q^*, M^*, B^*, S^*, t_2^*, and T^*.

6. Conclusion

In this article, we have developed a neutrosophic economic order quantity (EOQ) inventory model for single perishable item. To handle uncertainty due to imprecise, inadequate, and inconsistent information, stock-based demand, retail cost, ordering cost, stocking cost, wastage cost, imperfect item cost, and cost for preservation technology are considered as single valued triangular neutrosophic numbers to maximize total profit per cycle in neutrosophic environment. The retailer and supplier both adopt a price discount policy for imperfect items, and preservation cost is also taken into account. Taylor series approximation is used to transform the transcendental function. A theoretical optimality condition is established for the de-neutrosophied model using Hessian matrix. Our main objective is to find the maximum value of the retailer's profit by making a decision on the order quantity and maximum stock level and to determine the inherent impreciseness, vagueness, and inconsistency involved in the model. De-neutrosophication on SVTNN and NeFOT, which is an extension of IFOT is utilized to determine the minimum of the maximum acceptance level, the maximum of the minimum indeterminacy level, and the maximum of the minimum rejection level and optimal value of the compromise solution. The study is verified with a numerical example and graph.

As future research directions, multi-item inventories for perishable items in neutrosophic environment with and without constraint(s) are under our consideration. Further, any interested researcher can consider advance payment, credit approaches, and other types of fuzziness of parameters and neutrosophic numbers. This model can also be extended to perishable items of fixed life with advanced payment or credit payment.

Acknowledgments The authors would like to acknowledge the Editors of the Journal and anonymous reviewers for their encouragement and constructive comments in revising the article. The authors would like to acknowledge the ArbaMinch University for facilitating and supporting this research work.
Funding For this study no fund was received at all.
Conflict of Interest The authors declare that there is no conflict of interest concerning the publication of this article.

References

1. Harris, F. W. How many parts to make at once,1913.
2. Nahmias, S. Perishable inventory theory: A review. Operations research, 1982, 30(4):680–708.
3. Giri, BC; Pal, S. G. A. and Chaudhuri, K. An inventory model for deteriorating items with stock-dependent demand rate. European Journal of Operational Research, 1996, 95(3):604–610.

4. Mahata, S. and Debnath, B. K. A profit maximization single item inventory problem considering deterioration during carrying for price dependent demand and preservation technology investment. RAIRO: Operations Research,2022, 56(4).
5. Dey, B. K., Bhuniya, S., and Sarkar, B. Involvement of controllable lead time and variable demand for a smart manufacturing system under a supply chain management. Expert Systems with Applications, 2021, 184:115464.
6. Momena, A. F.and Haque, R. R. M., Salahshour, S., and Mondal, S. P. The existence and uniqueness conditions for solving neutrosophic differential equations and its consequence on optimal order quantity strategy. Logistics, 2024, 8(1):18.
7. Asma, A F. and Amirtharaj, E H. Solving multi objective inventory model of deteriorating items with two constraints using fuzzy optimization. Intern. J. Fuzzy Mathematical Archive, 2016(a), Vol.10(1),41–48.
8. Poswal, P., Chauhan, A., Boadh, R., Rajoria, Y. K., Kumar, A., and Khatak, N. Investigation and analysis of fuzzy eoq model for price sensitive and stock dependent demand under shortages. Materials Today: Proceedings, 2022, 56:542–548.
9. Mondal, S. K. and Khara, B. Imperfect production inventory system considering effects of production reliability. In Fuzzy Optimization, Decision-making and Operations Research: Theory and Applications,Springer, 2023:587–617.
10. Shunmugam, H. and Karuppuchamy, A. Inventory model with preservation technology and exponential holding cost in fuzzy scenario. Ratio Mathematica,2023, 48.
11. Mahapatra, A. S., Sarkar, B., Mahapatra, M. S., Soni, H. N., and Mazumder, S. K. Development of a fuzzy economic order quantity model of deteriorating items with promotional effort and learning in fuzziness with a finite time horizon. Inventions,2019, 4(3):36.
12. Barman, H., Pervin, M., Roy, S. K., and Weber, G.-W. Back-ordered inventory model with inflation in a cloudy-fuzzy environment. Journal of Industrial and Management Optimization, 2021, 17(4):1913–1941.
13. Chaudhary, R. and Mittal, M. and Jayaswal, M. K. A sustainable inventory model for defective items under fuzzy environment. Decision Analytics Journal,Elsevier,2023, Vol.7:100-207.
14. Kar, C., Mondal, B., and Roy, T. K. An inventory model under space constraint in neutrosophic environment: a neutrosophic geometric programming approach. NSS: An International Book Series in Information Science and Engineering, 2018, 21:201893.
15. Karthick, B. and Uthayakumar, R. Optimizing an imperfect production model with varying setup cost, price discount, and lead time under fuzzy demand. Process Integration and Optimization for Sustainability,2021, 5:13–29.
16. Asma, A. F. and Amrirtharj, E. H. Solution of single objective inventory model of deteriorating items with fuzzy cost components as triangular fuzzy numbers,2016b.
17. Pawar, S., Patel, P., and Mirajkar, A. Intuitionistic fuzzy approach in multi-objective optimization for krbmc irrigation system, india. ISH Journal of Hydraulic Engineering,2022, 28(sup1):463–470.
18. Tsegaye, H., Thillaigovindan, N., and Alemayehu, G. An efficient method for solving intuitionistic fuzzy multi-objective optimization problems. Punjab University Journal of Mathematics,2021, 53(9).
19. Panda, M. and Sahoo, A. Novel methods for solving multi-objective non-linear inventory model of highly deteriorating items. In Operations Research Forum,Springer,2023, vol. 4:56.
20. Wang, H., Smarandache, F., Zhang, Y., and Sunderraman, R. Single valued neutrosophic sets. Infinite study, 2010, 12.
21. Mohamed, Z., M.Ismail,M. and Abd El-Gawad,A.F. Sustainable Supplier Selection using Neutrosophic Multi-Criterian Decision Making Methodology. SMIJ 2023,Vol. 2:1-9, doi:10.61185/SMIJ.2023.33102

22. Christianto, V. and Smarandache, F.The Enduring Power of Giving and Caring from Cavemen to Kind Hearts: A Neutrosophic Multi-Criteria Decision Making Approach. Syst. Assess. Eng. Manage.2024 Vol. 1:26–33, https://doi.org/10.61356/j.saem.2024.1252
23. Mohamed, M., Salam,A. 1 and Ye, J. Selection of Sustainable Material for the Construction of Drone Aerodynamic Wing using Neutrosophic RAWEC.Syst. Assess. Eng. Manage. 2024 Vol. 1: 54–72, https://doi.org/10. 61356/j.saem.2024.1295
24. Abdel-Basset, M., Saleh, M., Gamal, A., and Smarandache, F. An approach of topsis technique for developing supplier selection with group decision making under type-2 neutrosophic number. Applied Soft Computing, 2019, 77:438–452.
25. Smarandache, F., Abdel-Basset, M., and Broumi, S. Neutrosophic Sets and Systems. NSS,2022, vol. 49(1):38.
26. Mullai, M. and Broumi, S. Neutrosophic inventory model without shortages. Asian Journal of Mathematics and Computer Research,2018, 23:214–219.
27. Mullai, M. and Surya, R. Neutrosophic eoq model with price break. NSS, 2018, 19:24–29.
28. Mondal, B., Kar, C., Garai, A., and Roy, T. K. Optimization of EOQ model with limited storage capacity by neutrosophic geometric programming. Infinite Study 2020.
29. Mullai, M. and Surya, R. Neutrosophic inventory backorder problem using triangular neutrosophic numbers. NSS, 2020, 31:148–155.
30. Pal, Sh. and Chakraborty, A. and others. Triangular neutrosophic-based eoq model for noninstantaneous deteriorating item under shortages. Am J Bus Oper Res,2020, 1(1):28–35.
31. Mondal, B.and Garai, A.and Mukhopadhyay, A. and Majumder, S. K. Inventory policies for seasonal items with logistic-growth demand rate under fully permissible delay in payment: a neutrosophic optimization approach, Soft Computing,Springer,2021, Vol.25:3725–3750.
32. Kar, C., Roy, T. K., and Maiti, M. Eoq model with price, marketing, service and green dependent neutrosophic demand under uncertain resource constraint: A geometric programming approach. NSS, 2022, 51:797–823.
33. Bhavani, G. D., Georgise, F. B., Mahapatra, G., and Maneckshaw, B. Neutrosophic cost pattern of inventory system with novel demand incorporating deterioration and discount on defective items using particle swarm algorithm. Computational Intelligence and Neuroscience, 2022.
34. Bhavani, G. D. and Mahapatra, G. Inventory system with generalized triangular neutrosophic cost pattern incorporating maximum life-time-based deterioration and novel demand through pso. Soft Computing, 2023, 27(5):2385–2402.
35. Mohanta,K., Jha, A.K., Dey, A. and Pal, A. An application of neutrosophic logic on an inventory model with two-level partial trade credit policy for time-dependent perishable products. Soft Computing, 2023a 27:4795–4822, doi.org/10.1007/s00500-022-07619-2
36. Mohanta, K. S., S., Dey, A., De, D., and Pal, A. Cost optimization of an inventory control problem in neutrosophic environment for smart manufacturing,2023b.
37. Deb, S. C. and Islam, S. Application of neutrosophic interval valued goal programming to a supply chain inventory model for deteriorating items with time dependent demand. NSS,2023, 53(1):35
38. Surya, R., Mullai, M., and Vetrivel, G. Neutrosophic inventory system for decaying items with price dependent demand. NSS, 2024, 66:55–75.
39. Gudeta, G. A., Netesan, Th., and Wole, G. A. Inventory model for a single perishable item with stock-dependent demand, preservation, shortages, and a two-stage price discount for imperfect items. (Submitted).
40. Benson, H. Multicriteria optimization,matthias ehrgott,springer (2005). JORS , 2007, 491:p-323.
41. Smarandache, F. Neutrosophic set–a generalization of the intuitionistic fuzzy set. JoDRM, 2010, 1(1):107–116.

42. De, S. K. Solving an eoq model under fuzzy reasoning. Applied Soft Computing, 2021, 99:106892.
43. S. Chakrabortty, S., and M. P. and Nayak, P. K. Intuitionistic fuzzy optimization technique for pareto optimal solution of manufacturing inventory models with shortages. EJOR,2013, 228:381–387.
44. Angelov, P. P. Optimization in an intuitionistic fuzzy environment. FSS, 1997, 86(3):299–306.
45. Bellman, R. E. and Zadeh, L. A. Decision-making in a fuzzy environment. MS, 1970, 17(4):B–141.
46. Ahmad, F., Adhami, A. Y., and Smarandache, F. Single valued neutrosophic hesitant fuzzy computational algorithm for multiobjective nonlinear optimization problem. NSS, 2018, 22:76–86.
47. Zimmermann, H.-J. Fuzzy programming and linear programming with several objective functions. FSS, 1978,1(1):45–55.

Received: Aug 5, 2024. Accepted: Nov 6, 2024