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Enhancing Business English Teaching Quality: A Single-Valued Neutrosophic Framework for Multi-Attribute Decision-Making

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Abstract : The evaluation of teaching effectiveness in college business English courses focuses on enhancing students' language skills, applying business knowledge, cross-cultural communication abilities, and career readiness. Evaluation methods include student feedback, exam results, class participation, and project presentations. A multidimensional evaluation system ensures course content aligns with real-world business needs, improving students' competitiveness and adaptability in a global business environment. The business English teaching quality evaluation is useful for MADM. The single-valued neutrosophic sets (SVNSs) is a useful approach to administrate fuzzy information during the business English teaching quality evaluation. Currently, the Aczel-Alsina operations and power geometric (PG) approach was administrated to put forward the multiple-attribute decision-making (MADM). In this study, the single-valued neutrosophic number Aczel-Alsina weighted power geometric (SVNNAAWPG) approach is administrated in light with Aczel-Alsina operations and PG approach and SVNSs. The SVNNAAWPG approach is administrated for MADM. Finally, numerical example for business English teaching quality evaluation is administrated to conduct the SVNNAAWPG approach. The major contributions of this work are tackled: (1) Aczel-Alsina operations and PG approach are enhanced under SVNSs; (2) SVNNAAWPG approach is administrated in light with Aczel-Alsina operations and PG approach; (3) SVNNAAWPG approach is tackled for MADM with SVNSs; (4) Finally, numerical example for business English teaching quality evaluation is administrated to conduct the SVNNAAWPG approach.

Keywords-multiple-attribute decision-making; single-valued neutrosophic sets (SVNSs); power geometric (PG); business English teaching quality evaluation

1. Introduction

University English teaching quality evaluation is a comprehensive process aimed at ensuring that teaching effectiveness and student learning outcomes meet expected standards. The evaluation typically covers several aspects, including course design, teaching methods, teacher qualifications, student feedback, and learning achievements. Firstly, course design must be up-to-date and align with students' needs and societal trends. Secondly, diverse and innovative teaching methods such as interactive and project-based learning are crucial for improving teaching quality. The professional competence of teachers significantly impacts teaching quality, making teacher development and training an essential part of the evaluation. Additionally, student feedback provides vital insights, gathered through surveys and interviews to capture opinions and suggestions about courses and teaching. Finally, learning achievements are usually assessed through exam scores, skill tests, and comprehensive evaluations. Overall, university English teaching quality evaluation assesses teaching effectiveness and serves as an essential tool for educational improvement and innovation, helping institutions continuously enhance teaching standards. Over the past decade, research on college English teaching quality has deepened, covering various teaching models and evaluation systems. In 2014, Yue [1] studied the quality assurance system for college English in distance open education, emphasizing construction from school management, teaching, and quality supervision to ensure the effectiveness of adult education. In 2015, Li [2] analyzed the current state of intensive reading, highlighting its importance for enhancing students' comprehensive abilities, and proposed specific improvement measures to enhance teaching quality. In 2016, Zhao [3] explored the application of micro-lessons in college English teaching, suggesting that they improve teaching efficiency and promote deeper understanding. Feng [4], in 2018, through surveys of several universities, proposed diversified evaluation methods suitable for applied colleges, providing a reference for reasonable evaluation. In 2019, Zhang [5]discussed the construction of a college English teaching quality assurance system through survey analysis, suggesting strategies from the perspectives of teaching process monitoring and evaluation systems. In the same year, Cai and Ouyang [6] used fuzzy comprehensive evaluation to build a scientific teaching quality evaluation system, improving existing systems. Liu [7], in 2019, proposed that teachers should update professional knowledge and use modern educational technology to improve teaching quality in the context of a new era. In 2020, Ru [8] explored the impact of the modern apprenticeship model on improving college English teaching quality, emphasizing the importance of school-enterprise cooperation and integration of education with industry. In 2021, Li and Gao [9] analyzed teaching reforms under the background of ideological and political education, proposing diversified teaching subjects and heuristic teaching to enhance quality. In the same year, Wang [10] proposed an evaluation index system for online classroom teaching quality to improve effectiveness. Zhao [11], in 2021, studied the hybrid teaching quality evaluation under the SPOC model, constructing a corresponding quality evaluation system. In 2022, Xu [12] emphasized the importance of establishing an internal assurance system and proposed strategies for adjustment based on actual teaching situations. In 2023, Song [13] constructed a classroom teaching quality evaluation system based on feedback from teachers and students, as well as expert opinions, to improve quality. Finally, in 2024, Wang, Zhao and Zuo [14] analyzed CET-4 pass rate data, optimized tiered teaching plans, and implemented various measures to improve teaching quality, achieving preliminary success. These studies offer diverse paths and strategies for college English teaching, promoting continuous improvement in teaching quality.

MADM is a decision-making method used to select the optimal option from multiple alternatives, especially when each option involves multiple attributes or criteria[15, 16]. Each attribute typically reflects different aspects of the decision problem, such as cost, benefits, risks, etc., and these attributes may have varying levels of importance or weights [17, 18]. The core idea of MADM is to comprehensively evaluate and rank the alternatives based on the consideration of all attributes, thereby providing decision-makers with a rational basis for selection[19, 20]. In practice, MADM is widely applied in complex decision-making scenarios in fields such as business, economics, engineering, and management[21]. For example, when selecting a supplier, the decision-maker may need to consider factors such as quality, price, delivery time, and service. To achieve a scientific decision, commonly used MADM methods include the Weighted Sum Method, TOPSIS, and AHP (Analytic Hierarchy Process). These methods assign different weights to each attribute and use various mathematical models to help decision-makers choose the best option among multiple alternatives. The advantage of MADM lies in its ability to systematically account for the combined effects of multiple factors, which helps reduce subjective biases in the decision-making process, thus enhancing the scientific and rational nature of decisions[22, 23].

The evaluation of business English teaching quality is a valuable application of MADM. SVNSs [24] are an effective approach for handling fuzzy information in such evaluations. Recently, the Aczel-Alsina operations [25, 26] and the power geometric (PG) approach [27, 28] have been utilized to enhance MADM. Many methods have employed both the Aczel-Alsina operations [25, 26] and PG approach [27, 28] independently for MADM. However, there has been little to no exploration of combining Aczel-Alsina operations and the PG approach within the framework of SVNSs. In this study, the integration of Single-Valued Neutrosophic MADM with Aczel-Alsina operations and the PG method presents several advantages, making this approach particularly useful for evaluating business English teaching quality. Thus, the SVNNAAWPG method is employed, leveraging the Aczel-Alsina operations, PG approach, and SVNSs. The SVNNAAWPG approach is applied to MADM, and a numerical example is provided to demonstrate its effectiveness in business English teaching quality evaluation.

The key contributions of this work are outlined: (1) The Aczel-Alsina operations and PG approach are extended under the SVNS framework; (2) The SVNNAAWPG approach is applied, utilizing both Aczel-Alsina operations and the PG method; (3) The SVNNAAWPG approach is implemented for MADM with SVNSs; (4) A numerical example is provided to validate the SVNNAAWPG approach for evaluating business English teaching quality.

The structure of this research is outlined. Section 2 introduces the concept of SVNSs. In Section 3, the SVNNAAWPG approach is applied using Aczel-Alsina operations and the PG method. Section 4 presents the application of the SVNNAAWPG approach to MADM problems with SVNSs. In Section 5, a numerical example is provided to evaluate the quality of business English teaching through comparative analysis. Finally, Section 6 offers concluding remarks.

2. Preliminaries

Wang et al. [24] resolved the SVNSs

Definition 1 [24]. The SVNSs is resolved:

$$TA=\left\{\left(\theta,TT_A(\theta),TI_A(\theta),TF_A(\theta)\right)\middle|\theta\in X\right\} \quad (1)$$

where $TT_A(\theta),TI_A(\theta),TF_A(\theta)$ is truth-membership (TM), indeterminacy-membership (IM) and falsity-membership (FM), $TT_A(\theta),TI_A(\theta),TF_A(\theta)\in[0,1]$, $0\le TT_A(\theta)+TI_A(\theta)+TF_A(\theta)\le 3$.

The SVNN is structured as $TA=(TT_A,TI_A,TF_A)$, where $TT_A\in[0,1],TI_A\in[0,1],TF_A\in[0,1]$, and $0\le TT_A+TI_A+TF_A\le 3$.

Definition 2 [29]. Let $TA=(TT_A,TI_A,TF_A)$ and $TB=(TT_B,TI_B,TF_B)$, the score value is resolved:

$$TSV(TA)=\frac{(2+TT_A-TI_A-TF_A)}{3}, \qquad TSV(TA)\in[0,1]. \quad (2)$$

$$TSV(TB)=\frac{(2+TT_B-TI_B-TF_B)}{3}, \qquad TSV(TA)\in[0,1]. \quad (3)$$

Definition 3[29]. Let $TA=(TT_A,TI_A,TF_A)$ and $TB=(TT_B,TI_B,TF_B)$, the accuracy value is resolved:

$$TAV(TA)=TT_A-TF_A,\ TAV(TA)\in[-1,1]\ . \quad (4)$$

$$TAV(TA)=TT_A-TF_A,\ RAV(TA)\in[-1,1]\ . \quad (5)$$

Peng et al.[29] resolved the order issues.

Definition 4[29]. Let $TA=(TT_A,TI_A,TF_A)$ and $TB=(TT_B,TI_B,TF_B)$, let $TSV(TA)=\frac{(2+TT_A-TI_A-TF_A)}{3}$ and $TSV(TB)=\frac{(2+TT_B-TI_B-TF_B)}{3}$, and let $TAV(TA)=TT_A-TF_A$ and $TAV(TA)=TT_A-TF_A$, if $TSV(TA)<TSV(TB)$, $TA<TB$; if $TSV(TA)=TSV(TB)$, (1)if $TAV(TA)=TAV(TB)$, $TA=TB$; (2) if $TAV(TA)>TAV(TB)$, $TA<TB$.

Definition 5[24, 30]. Let $TA=\left(TT_A,TI_A,TF_A\right)$ and $TB=\left(TT_B,TI_B,TF_B\right)$, $r\theta\geq1, r\lambda>0$, the Aczel-Alsina operations among with SVNNs are administrated:

(1) $$TA\oplus TB=\left(\begin{array}{l}1-e^{-\left(\left(-\ln\left(1-TT_A\right)\right)^{t\theta}+\left(-\ln\left(1-TT_B\right)\right)^{t\theta}\right)^{1/t\theta}},\\ e^{-\left(\left(-\ln\left(TI_A\right)\right)^{t\theta}+\left(-\ln\left(TI_B\right)\right)^{t\theta}\right)^{1/t\theta}},e^{-\left(\left(-\ln\left(TF_A\right)\right)^{t\theta}+\left(-\ln\left(TF_B\right)\right)^{t\theta}\right)^{1/t\theta}}\end{array}\right);$$

(2) $$TA\otimes TB=\left(\begin{array}{l}e^{-\left(\left(-\ln\left(TT_A\right)\right)^{t\theta}+\left(-\ln\left(TT_B\right)\right)^{t\theta}\right)^{1/t\theta}},\\ 1-e^{-\left(\left(-\ln\left(1-TI_A\right)\right)^{t\theta}+\left(-\ln\left(1-TI_B\right)\right)^{t\theta}\right)^{1/t\theta}},1-e^{-\left(\left(-\ln\left(1-TF_A\right)\right)^{t\theta}+\left(-\ln\left(1-TF_B\right)\right)^{t\theta}\right)^{1/t\theta}}\end{array}\right)$$

(3) $$t\lambda TA=\left(1-e^{-\left(t\lambda\left(-\ln\left(1-TT_A\right)\right)^{t\theta}\right)^{1/t\theta}},e^{-\left(t\lambda\left(-\ln\left(TI_A\right)\right)^{t\theta}\right)^{1/t\theta}},e^{-\left(t\lambda\left(-\ln\left(TF_A\right)\right)^{t\theta}\right)^{1/t\theta}}\right)$$

(4) $$\left(TA\right)^{r\lambda}=\left(e^{-\left(t\lambda\left(-\ln\left(TT_A\right)\right)^{t\theta}\right)^{1/t\theta}},1-e^{-\left(t\lambda\left(-\ln\left(1-TI_A\right)\right)^{t\theta}\right)^{1/t\theta}},1-e^{-\left(t\lambda\left(-\ln\left(1-TF_A\right)\right)^{t\theta}\right)^{1/t\theta}}\right).$$

Definition 6[31]. Let $TA=\left(TT_A,TI_A,TF_A\right)$ and $TB=\left(TT_B,TI_B,TF_B\right)$, then SVNN Hamming distance (SVNNHD) and SVNN Euclidean distance (SVNNED) are interpreted:

$$SVNNHD\left(TA,TB\right)=\frac{\left|TT_A-TT_B\right|+\left|TI_A-TI_B\right|+\left|TF_A-TF_B\right|}{3}\tag{4}$$

$$SVNNED\left(TA,TB\right)=\sqrt{\frac{\left|TT_A-TT_B\right|^2+\left|TI_A-TI_B\right|^2+\left|TF_A-TF_B\right|^2}{3}}\tag{5}$$

3. SNNAAWPG operator

Yong et al. [32] and Ashraf et al. [30] administrated the SVNNAAWG operator.

Definition 6. Let $TA_i=(TT_i,TI_i,TF_i)$ be SVNNs with weight $tw_i=(tw_1,tw_2,\ldots,tw_n)^T$, $\sum_{i=1}^{n}tw_i=1$, $t\theta\geq1$. If

$$\begin{aligned} &\text{SVNNAAWG}_{tw}\left(TA_1,TA_2,\cdots,TA_n\right)=\underset{i=1}{\overset{n}{\otimes}}\left(TA_i\right)^{tw_i} \\ &=\left(\begin{aligned} &e^{-\left(\sum_{i=1}^{n}tw_i\left(-\ln\left(1-TT_j\right)\right)^{t\theta}\right)^{1/t\theta}}, \\ &1-e^{-\left(\sum_{i=1}^{n}tw_i\left(-\ln\left(TI_j\right)\right)^{t\theta}\right)^{1/t\theta}},1-e^{-\left(\sum_{i=1}^{n}tw_i\left(-\ln\left(TF_j\right)\right)^{t\theta}\right)^{1/t\theta}} \end{aligned}\right) \end{aligned} \tag{6}$$

The SVNNAAWG has three properties.

Property 1. (idempotency). If $TA_i = TA = \left(TT, TI, TF\right)$,

$$\text{SVNNAAWG}_{tw}\left(TA_1,TA_2,\cdots,TA_n\right)=TA \tag{7}$$

Property2. (Monotonicity). Let $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$, $TB_i = (TT_{B_i}, TI_{B_i}, TF_{B_i})$. If $TT_{A_i} \le TT_{B_i}, TI_{A_i} \ge TI_{B_i}$, $TF_{A_i} \ge TF_{B_i}$ holds for all i, then

$$\begin{aligned} &\text{SVNNAAWG}_{tw}\left(TA_1,TA_2,\cdots,TA_n\right) \\ &\le \text{SVNNAAWG}_{tw}\left(TB_1,TB_2,\cdots,TB_n\right) \end{aligned} \tag{8}$$

Property 3 (Boundedness). Let $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$. If

$TA^+ = (\max_i(TT_i), \min_i(TI_i), \min_i(TF_i))$, $TA^- = (\min_i(TT_i), \max_i(TI_i), \max_i(TF_i))$ then

$$TA^- \le \text{SVNNAAWG}_{tw}\left(TA_1,TA_2,\cdots,TA_n\right) \le TA^+ \tag{9}$$

Then, the SVNNAAWPG operator is resolved on SVNNAAWG technique and PG technique [27, 28].

Definition 7. Let $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$ be SVNNs, $t\theta \ge 1$. If

$$\text{SVNNAAWPG}_{tw}\left(TA_1,TA_2,\cdots,TA_n\right)=\underset{i=1}{\overset{n}{\otimes}}\left(TA_i\right)^{tw_i\left(t+TT\left(TA_i\right)\right)\Big/\sum_{i=1}^{n}tw_i\left(t+TT\left(TA_i\right)\right)} \tag{10}$$

where $t \in R, t \ge 0$, $TT\left(TA_a\right)=\sum_{\substack{j=1\\ a\ne j}}^{m} Sup\left(TA_a, TA_j\right)$,

$Sup\left(TA_a, TA_j\right) = 1 - SVNNED\left(TA_a, TA_j\right)$ is support for TA_a from TA_j, with serval conditions: (1) $Sup\left(TA_a, TA_b\right) \in \left[0,1\right]$; (2) $Sup\left(TA_b, TA_a\right) = Sup\left(TA_a, TA_b\right)$; (3) $Sup\left(TA_a, TA_b\right) \ge Sup\left(TA_s, TA_t\right)$, if $SVNNED\left(TA_a, TA_b\right) \le SVNNED\left(TA_s, TA_t\right)$ and $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$ with weight $tw_i = (tw_1, tw_2, \ldots, tw_n)^T$, $\sum_{i=1}^{n} tw_i = 1$.

$tw_i = (tw_1, tw_2, \ldots, tw_n)^T$ is obtained through entropy approach,

$$tw_i = \frac{1 + \frac{1}{\ln m}\sum_{i=1}^{m}\frac{TSV(TA_i)}{\sum_{i=1}^{m} TSV(TA_i)}\ln\frac{TSV(TA_i)}{\sum_{i=1}^{m} TSV(TA_i)}}{\sum_{j=1}^{n}\left(1 + 1 + \frac{1}{\ln m}\sum_{i=1}^{m}\frac{TSV(TA_i)}{\sum_{i=1}^{m} TSV(TA_i)}\ln\frac{TSV(TA_i)}{\sum_{i=1}^{m} TSV(TA_i)}\right)}.$$

The Theorem 2 is resolved.

Theorem 2. Let $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$ be the SVNNs, $t\theta \geq 1$. If

$$\begin{aligned}
&\text{SVNNAAWPG}_{tw}(TA_1, TA_2, \cdots, TA_n) \\
&= \otimes_{i=1}^{n} (TA_i)^{tw_i(t+TT(TA_i)) \Big/ \sum_{i=1}^{n} tw_i(t+TT(TA_i))} \\
&= \left(\begin{array}{l}
e^{-\left(\sum_{i=1}^{n}\frac{tw_i(t+TT(TA_i))}{\sum_{i=1}^{n} tw_i(t+TT(TA_i))}\left(-\ln(1-TT_j)\right)^{t\theta}\right)^{1/t\theta}}, \\
1 - e^{-\left(\sum_{i=1}^{n}\frac{tw_i(t+TT(TA_i))}{\sum_{i=1}^{n} tw_i(t+TT(TA_i))}\left(-\ln(TI_j)\right)^{t\theta}\right)^{1/t\theta}}, 1 - e^{-\left(\sum_{i=1}^{n}\frac{tw_i(t+TT(TA_i))}{\sum_{i=1}^{n} tw_i(t+TT(TA_i))}\left(-\ln(TF_j)\right)^{t\theta}\right)^{1/t\theta}}
\end{array} \right)
\end{aligned} \tag{11}$$

where $t \in R, t \geq 0$, $TT(TA_a) = \sum_{\substack{j=1 \\ a \neq j}}^{m} Sup(TA_a, TA_j)$, $Sup(TA_a, TA_j) = 1 - SVNNED(TA_a, TA_j)$ is support for TA_a from TA_j, with serval conditions: (1) $Sup(TA_a, TA_b) \in [0,1]$; (2) $Sup(TA_b, TA_a) = Sup(TA_a, TA_b)$; (3) $Sup(TA_a, TA_b) \geq Sup(TA_s, TA_t)$, if $SVNNED(TA_a, TA_b) \leq SVNNED(TA_s, TA_t)$ and $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$ with weight $tw_i = (tw_1, tw_2, \ldots, tw_n)^T$, $\sum_{i=1}^{n} tw_i = 1$.

$tw_i = (tw_1, tw_2, \ldots, tw_n)^T$ is obtained through entropy approach,

$$tw_i = \frac{1+\frac{1}{\ln m}\sum_{i=1}^{m}\frac{TSV(TA_i)}{\sum_{i=1}^{m}TSV(TA_i)}\ln\frac{TSV(TA_i)}{\sum_{i=1}^{m}TSV(TA_i)}}{\sum_{j=1}^{n}\left(1+1+\frac{1}{\ln m}\sum_{i=1}^{m}\frac{TSV(TA_i)}{\sum_{i=1}^{m}TSV(TA_i)}\ln\frac{TSV(TA_i)}{\sum_{i=1}^{m}TSV(TA_i)}\right)}.$$

Proof:

(a) Let $i=2$, we have:

$$\mathrm{SVNNAAWPG}_{tw}(TA_1, TA_2) = \left(\begin{array}{l} (TA_1)^{tw_1(t+TT(TA_1))\Big/\sum_{i=1}^{2}tw_i(t+TT(RA_i))} \\ \otimes (TA_2)^{tw_2(t+TT(RA_2))\Big/\sum_{i=1}^{2}tw_i(t+TT(RA_i))} \end{array} \right)$$

$$= \left(\begin{array}{l} e^{-\left(\frac{tw_1(t+TT(TA_1))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(1-TT_1)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\frac{tw_1(t+TT(TA_1))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(1-TI_1)\right)^{t\theta}\right)^{1/r\theta}}, \\ 1-e^{-\left(\frac{tw_1(t+TT(TA_1))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(1-TT_1)\right)^{t\theta}\left(-\ln(TF_1)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right) \otimes$$

$$\left(\begin{array}{l} e^{-\left(\frac{tw_2(t+TT(RA_2))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(1-TT_2)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\frac{tw_2(t+TT(RA_2))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(TI_2)\right)^{t\theta}\right)^{1/t\theta}}, \\ 1-e^{-\left(\frac{tw_2(t+TT(RA_2))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(TF_2)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right)$$

$$= \left(\begin{array}{l} e^{-\left(\sum_{i=1}^{2}\frac{tw_i(t+TT(RA_i))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(1-TT_j)\right)^{t\theta}\right)^{1/t\theta}}, \\ 1-e^{-\left(\sum_{i=1}^{2}\frac{tw_i(t+TT(RA_i))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(TI_j)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\sum_{i=1}^{2}\frac{tw_i(t+TT(RA_i))}{\sum_{i=1}^{2}tw_i(t+TT(RA_i))}\left(-\ln(TF_j)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right)$$

(b) If Eq. (11) hold for $i=k$, we have:

$$\mathrm{SVNNAAWPG}_{tw}\left(TA_1, TA_2, \cdots, TA_k\right) = \bigotimes_{i=1}^{k}\left(TA_i\right)^{tw_i\left(t+TT\left(TA_i\right)\right) \Big/ \sum_{i=1}^{k} tw_i\left(t+TT\left(RA_i\right)\right)}$$

$$= \left(e^{-\left(\sum_{i=1}^{k}\frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{k}tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(1-TT_j\right)\right)^{t\theta}\right)^{1/t\theta}}, \right.$$
$$\left. 1-e^{-\left(\sum_{i=1}^{k}\frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{k}tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(TI_j\right)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\sum_{i=1}^{k}\frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{k}tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(TF_j\right)\right)^{t\theta}\right)^{1/t\theta}} \right)$$

(c) Let $i = k+1$. From Definition 5 and Eq. (11), we obtain

$$\mathrm{SVNNAAWPG}_{tw}\left(TA_1, TA_2, \cdots, TA_k,\ TA_{k+1}\right)$$
$$= \bigotimes_{i=1}^{k}\left(TA_i\right)^{tw_i\left(t+TT\left(TA_i\right)\right) \Big/ \sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)} \otimes \left(TA_{k+1}\right)^{\left(t+TT\left(TA_{k+1}\right)\right) \Big/ \sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}$$

$$= \left(\begin{array}{l} e^{-\left(\sum_{i=1}^{k} \frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(1-TT_j\right)\right)^{t\theta}\right)^{1/t\theta}}, \\ 1-e^{-\left(\sum_{i=1}^{k} \frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(TI_j\right)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\sum_{i=1}^{k} \frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(TF_j\right)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right) \otimes$$

$$\left(\begin{array}{l} e^{-\left(\frac{\left(t+TT\left(TA_{k+1}\right)\right)}{\sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(1-TT_{k+1}\right)\right)^{t\theta}\right)^{1/t\theta}}, \\ 1-e^{-\left(\frac{\left(t+TT\left(TA_{k+1}\right)\right)}{\sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(OI_{k+1}\right)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\frac{\left(t+TT\left(TA_{k+1}\right)\right)}{\sum_{i=1}^{k+1} tw_i\left(t+TT\left(RA_i\right)\right)}\left(-\ln\left(TF_{k+1}\right)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right)$$

$$= \left(\begin{array}{l} e^{-\left(\sum_{i=1}^{n} \frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{n} tw_i\left(t+TT\left(TA_i\right)\right)}\left(-\ln\left(1-TT_j\right)\right)^{t\theta}\right)^{1/t\theta}}, \\ 1-e^{-\left(\sum_{i=1}^{n} \frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{n} tw_i\left(t+TT\left(TA_i\right)\right)}\left(-\ln\left(TI_j\right)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\sum_{i=1}^{n} \frac{tw_i\left(t+TT\left(TA_i\right)\right)}{\sum_{i=1}^{n} tw_i\left(t+TT\left(TA_i\right)\right)}\left(-\ln\left(TF_j\right)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right).$$

In light with (a), (b), and (c), it could be known that Eq. (11) holds any *i*.

The SVNNAAWPG has serval properties.

Property 4 (idempotency). If $TA_i = TA = \left(TT, TI, TF\right)$,

$$\mathrm{SVNNAAWPG}_{tw}\left(TA_1, TA_2, \cdots, TA_n\right) = TA \tag{11}$$

Property 5. (Monotonicity). Let $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$, $TB_i = (TT_{B_i}, TI_{B_i}, TF_{B_i})$. If $TT_{A_i} \le TT_{B_i}, TI_{A_i} \ge TI_{B_i}, TF_{A_i} \ge TF_{B_i}$ holds for all i, then

$$\begin{aligned} &\mathrm{SVNNAAWPG}_{tw}\left(TA_1, TA_2, \cdots, TA_n\right) \\ &\le \mathrm{SVNNAAWPG}_{tw}\left(TB_1, TB_2, \cdots, TB_n\right) \end{aligned} \tag{12}$$

Property 3 (Boundedness). Let $TA_i = (TT_{A_i}, TI_{A_i}, TF_{A_i})$. If
$TA^{+} = (\max_i(TT_i), \min_i(TI_i), \min_i(TF_i))$, $TA^{-} = (\min_i(TT_i), \max_i(TI_i), \max_i(TF_i))$ then

$$TA^{-} \le \text{SVNNAAWPG}_{tw}\left(TA_1, TA_2, \cdots, TA_n\right) \le TA^{+} \tag{13}$$

4. MADM technique

Then, the SVNNAAWPG technique is resolved for MADM with SVNSs. Let $TZ = \{TZ_1, TZ_2, \ldots, TZ_n\}$ be attributes. Let $TP = \{TP_1, TP_2, \ldots, TP_m\}$ be alternatives. $TQ = \left(TQ_{ij}\right)_{m\times n} = \left(TT_{ij}, TI_{ij}, TF_{ij}\right)_{m\times n}$ is the SVNN-matrix. The SVNNAAWPG approach is resolved for MADM.

Step 1. Put forward the SVNN-matrix $TQ = \left(TQ_{ij}\right)_{m\times n} = \left(TT_{ij}, TI_{ij}, TF_{ij}\right)_{m\times n}$.

$$TQ = \left[TQ_{ij}\right]_{m\times n} = \begin{bmatrix} TQ_{11} & TQ_{12} & \ldots & TQ_{1n} \\ TQ_{21} & TQ_{22} & \ldots & TQ_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ TQ_{m1} & TQ_{m2} & \ldots & TQ_{mn} \end{bmatrix} \tag{14}$$

$$TQ = \left[TQ_{ij}\right]_{m\times n} = \left(TT_{ij}, TI_{ij}, TF_{ij}\right)_{m\times n}$$
$$= \begin{bmatrix} \left(TT_{11}, TI_{11}, TF_{11}\right) & \left(TT_{12}, TI_{1}, TF_{12}\right) & \ldots & \left(TT_{1n}, TI_{1n}, TF_{1n}\right) \\ \left(TT_{21}, TI_{21}, TF_{21}\right) & \left(TT_{22}, TI_{22}, TF_{22}\right) & \ldots & \left(TT_{2n}, TI_{2n}, TF_{2n}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(TT_{m1}, TI_{m1}, TF_{m1}\right) & \left(TT_{m2}, TI_{m2}, TF_{m2}\right) & \ldots & \left(TT_{mn}, TI_{mn}, TF_{mn}\right) \end{bmatrix} \tag{15}$$

Step 2. Normalize the $TQ = \left(TQ_{ij}\right)_{m\times n} = \left(TT_{ij}, TI_{ij}, TF_{ij}\right)_{m\times n}$ to $NTQ = \left[NTQ_{ij}\right]_{m\times n} = \left(NTT_{ij}, NTI_{ij}, NTF_{ij}\right)_{m\times n}$.

$$NTQ_{ij} = \left(NTT_{ij}, NTI_{ij}, NTF_{ij}\right) = \begin{cases} \left(TT_{ij}, TI_{ij}, TF_{ij}\right), & TZ_j \text{ is benefit attribute} \\ \left(TF_{ij}, 1 - TI_{ij}, TT_{ij}\right), & TZ_j \text{ is cost attribute} \end{cases} \tag{16}$$

Step 3. Compute the attributes weight.

Entropy [33] is utilized to produce the weight. The SVNN-matrix TT_{ij} is constructed:

$$TT_{ij} = \frac{TSV\left(NTT_{ij}, NTI_{ij}, NTF_{ij}\right)+1}{\sum_{i=1}^{m}\left(TSV\left(NTT_{ij}, NTI_{ij}, NTF_{ij}\right)+1\right)}, \quad (17)$$

The fuzzy Shannon information entropy $FSIE = \left(FSIE_1, FSIE_2, \cdots, FSIE_n\right)$ is constructed:

$$FSIE_j = -\frac{1}{\ln m}\sum_{i=1}^{m} TT_{ij} \ln TT_{ij} \quad (18)$$

and $TT_{ij} \ln TT_{ij} = 0$ if $TT_{ij} = 0$.

The weights are derived: $tw_j = \dfrac{1-FSIE_j}{\sum_{j=1}^{n}\left(1-FSIE_j\right)}$, $j = 1, 2, \cdots, n.$ (19)

Step 4. In light with $NTQ = \left[NTQ_{ij}\right]_{m\times n} = \left(NTT_{ij}, NTI_{ij}, NTF_{ij}\right)_{m\times n}$, $NTQ = \left[NTQ_{ij}\right]_{m\times n} = \left(NTT_{ij}, NTI_{ij}, NTF_{ij}\right)_{m\times n}$ with SVNNAAWPG to get the SVNNs $NTQ_i = \left(NTT_i, NTI_i, NTF_i\right)\left(i = 1, 2, \cdots, m\right)$:

$$NTQ_i = \text{SVNNAAWPG}(NTQ_{i1}, NTQ_{i2}, \cdots, NTQ_{in})$$

$$= \otimes_{j=1}^{n}\left(NTQ_{ij}\right)^{tw_j\left(t+TT\left(NTQ_{ij}\right)\right)\Big/\sum_{j=1}^{n} tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}$$

$$= \left(\begin{array}{c} e^{-\left(\sum_{i=1}^{n}\frac{tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}{\sum_{j=1}^{n} tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}\left(-\ln\left(1-NTT_{ij}\right)\right)^{t\theta}\right)^{1/t\theta}}, \\ 1-e^{-\left(\sum_{j=1}^{n}\frac{tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}{\sum_{j=1}^{n} tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}\left(-\ln\left(NTI_{ij}\right)\right)^{t\theta}\right)^{1/t\theta}}, 1-e^{-\left(\sum_{j=1}^{n}\frac{tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}{\sum_{j=1}^{n} tw_j\left(t+TT\left(NTQ_{ij}\right)\right)}\left(-\ln\left(NTF_{ij}\right)\right)^{t\theta}\right)^{1/t\theta}} \end{array} \right)$$

(20)

Step 5. Construct the $TSV\left(NTQ_i\right)$, $TAV\left(NTQ_i\right)\left(i = 1, 2, \cdots, m\right)$.

$$TSV\left(NTQ_i\right) = \frac{\left(2+NTT_i-NTI_i-NTF_i\right)}{3}, \quad TAV\left(NTQ_i\right) = NTT_i - NTF_i \quad (21)$$

Step 6. Rank the alternatives through $RSV\left(NRQ_i\right)$, $RAV\left(NRQ_i\right)$.

5. Numerical example and comparative analysis

5.1. Numerical example for business English teaching quality evaluation

The evaluation of teaching effectiveness in university Business English courses is a crucial component in assessing the quality of the curriculum and student learning outcomes. It not only helps instructors understand the extent to which learning objectives have been achieved but also provides a basis for improving and refining the course. An effective evaluation system should comprehensively and holistically reflect the results of teacher-student interactions during the learning process, as well as the enhancement of students' knowledge, skills, and overall competence. First, the evaluation of teaching effectiveness should focus on students' learning experiences and the knowledge they have gained. Business English courses are not just about developing language skills but also about improving students' ability to apply what they have learned in practical business contexts. Therefore, the assessment should examine whether students can effectively use the Business English knowledge learned in class in real-world situations. By incorporating reflective learning, case analysis, and role-playing activities, students can experience realistic business scenarios in the course and gradually improve their language application and business thinking skills. Second, the evaluation should consider students' level of participation and enthusiasm for learning. Active student participation is essential in Business English courses. Classroom discussions, group collaboration, presentations, and simulated negotiations can stimulate students' interest in learning while fostering teamwork and communication skills. Therefore, the effectiveness of teaching is not only measured by students' academic performance but also by their classroom engagement, participation, and enthusiasm for learning. Additionally, the teaching methods and course design adopted by instructors play a direct role in the effectiveness of teaching. Business English instruction should not only impart language knowledge but also guide students in understanding cultural differences and business practices in international business environments. Instructors should employ diverse teaching methods, such as multimedia instruction, case studies, and situational simulations, to help students better understand and master the course content. Through the evaluation of teaching effectiveness, instructors can reflect on the efficacy of their teaching strategies and continuously adjust and improve based on student feedback. Finally, the evaluation of teaching effectiveness should emphasize the cultivation of students' comprehensive abilities. Business English courses require students not only to possess strong language skills but also to develop cross-cultural communication abilities, critical thinking, and problem-solving skills. Therefore, the evaluation should fully reflect students' growth and progress in these areas, ensuring that they are well-prepared to succeed in various roles within the international business environment. In conclusion, the evaluation of teaching effectiveness in university Business English courses should not only focus on students' learning outcomes but also consider their real-world application of knowledge, learning attitudes, and the development of comprehensive skills. This approach will contribute to improving educational quality and ensuring the continuous improvement of the curriculum. The business English teaching quality evaluation is useful MADM. Five business English colleges are evaluated with four attributes: TZ_1 is **Improvement in Language Proficiency:** This indicator assesses students' progress in the four essential language skills: listening, speaking, reading, and writing, with a focus on their ability to use English effectively in business contexts. Tests, presentations, and case studies can be used to evaluate whether students are able to communicate accurately and fluently in Business English. TZ_2 is **Cross-Cultural Communication Skills:** This evaluates students' understanding and adaptability in handling international business situations across different cultures. Through simulated cross-cultural business negotiations, discussions, and project collaborations, the indicator examines whether students possess the skills to communicate smoothly in a global environment. TZ_3 is **Mastery of Business and Professional Knowledge:** This indicator measures students' comprehension of core business concepts, terminology, and the international business landscape. Tasks such as case analysis and business plan writing help assess whether students can integrate language skills with professional knowledge to understand and solve real-world business problems.TZ_4 is **Learning Initiative and Innovation Ability:** This evaluates students' level of participation, proactivity, and innovative thinking throughout the learning process. Classroom discussions, group collaboration, and project presentations are used to assess whether students actively engage, offer unique insights, and propose creative solutions in complex business scenarios. These four indicators provide a comprehensive assessment of the teaching effectiveness of university Business English courses, helping to cultivate well-rounded professionals with language proficiency, cross-cultural communication skills, and business acumen. Then, the SVNNAAWPG approach is put forward for business English teaching quality evaluation under SVNNs.

Step 1. Implement the SVNN-matrix $TQ = \left(TQ_{ij}\right)_{5\times4}$ as in Table 1 in light with statistical analysis.

Table 1. SVNN-matrix

	C_1	C_2	C_3	C_4
A_1	(0.6341, 0.1843, 0.1812)	(0.7124, 0.2157, 0.0728)	(0.5419, 0.2241, 0.1347)	(0.8126, 0.1029, 0.0853)
A_2	(0.5232, 0.2748, 0.1427)	(0.6781, 0.1985, 0.1229)	(0.4653, 0.2562, 0.1097)	(0.6893, 0.1347, 0.1769)
A_3	(0.7565, 0.1287, 0.1154)	(0.5983, 0.2473, 0.1538)	(0.6739, 0.1869, 0.1405)	(0.7342, 0.1937, 0.0726)
A_4	(0.8129, 0.1421, 0.0459)	(0.4875, 0.2989, 0.2132)	(0.5127, 0.2369, 0.2517)	(0.6458, 0.1782, 0.1754)
A_5	(0.6917, 0.1639, 0.1458)	(0.7341, 0.1536, 0.1124)	(0.7684, 0.1891, 0.0427)	(0.5897, 0.2049, 0.2062)

Step 2. Normalize $TQ = \left(TQ_{ij}\right)_{5\times4}$ to $NTQ = \left[NTQ_{ij}\right]_{5\times4}$ (See Table 2).

Table 2. The NTQ matrix

	C_1	C_2	C_3	C_4
A_1	(0.6341, 0.1843, 0.1812)	(0.7124, 0.2157, 0.0728)	(0.5419, 0.2241, 0.1347)	(0.8126, 0.1029, 0.0853)
A_2	(0.5232, 0.2748, 0.1427)	(0.6781, 0.1985, 0.1229)	(0.4653, 0.2562, 0.1097)	(0.6893, 0.1347, 0.1769)
A_3	(0.7565, 0.1287, 0.1154)	(0.5983, 0.2473, 0.1538)	(0.6739, 0.1869, 0.1405)	(0.7342, 0.1937, 0.0726)
A_4	(0.8129, 0.1421, 0.0459)	(0.4875, 0.2989, 0.2132)	(0.5127, 0.2369, 0.2517)	(0.6458, 0.1782, 0.1754)

A_5	(0.6917, 0.1639, 0.1458)	(0.7341, 0.1536, 0.1124)	(0.7684, 0.1891, 0.0427)	(0.5897, 0.2049, 0.2062)

Step 3. Compute the weight: $tw_1 = 0.1423, tw_2 = 0.3587, tw_3 = 0.2990, tw_4 = 0.2000$.

Step 4. Put forward the $NTQ_i\ (i = 1,2,3,4,5)$ with SVNNAAWPG approach

Step 4. Obtain $TSV(NTQ_i)\ (i = 1,2,3,4,5)$ (See Figure 1).

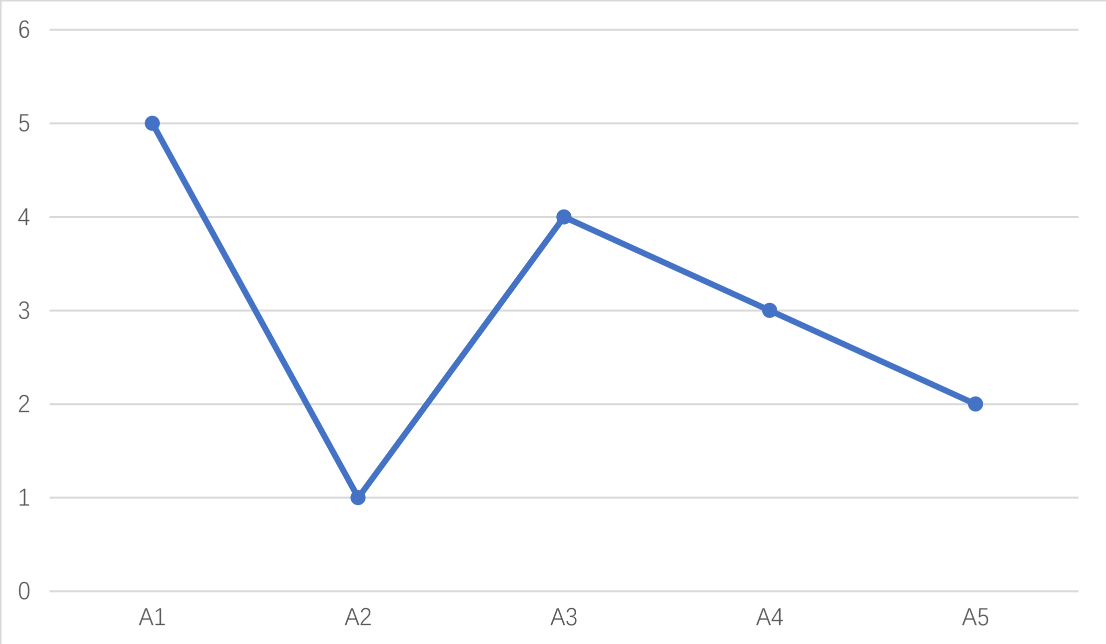

Figure 1. The rank of alternatives.

Step 5. The order is $A_2 > A_5 > A_3 > A_4 > A_1$, and the best business English colleges is A_2.

5.2. Influence analysis

To administrate the effects for final results according to parameters of SVNNAAWPG approach, the results are administrated in Figure 2 and 3.

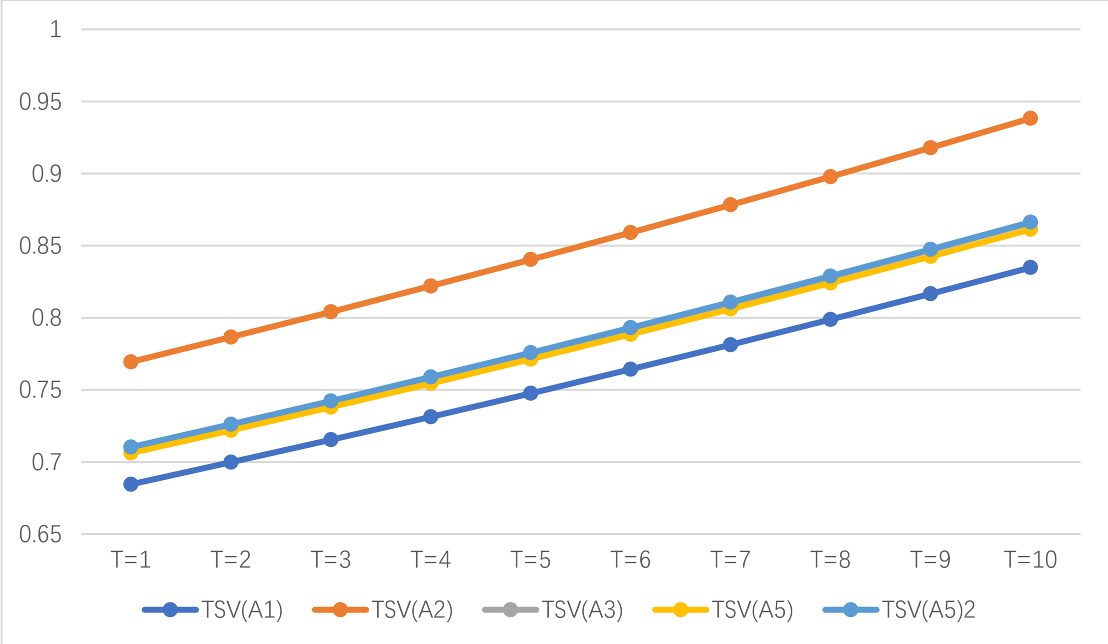

Figure 2. Parameter values for SVNNAAWPG approach.

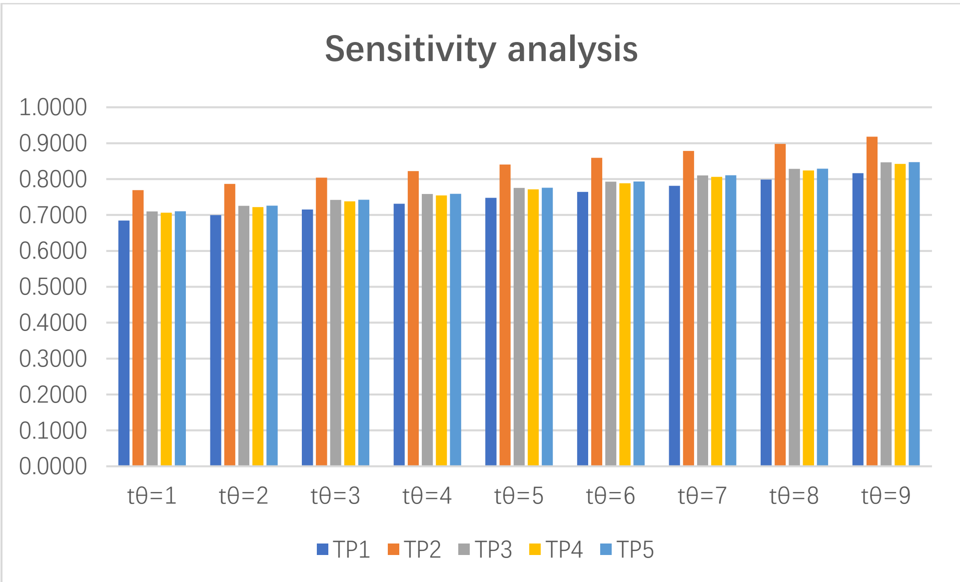

Figure 3. Different parameters for SVNNAAWPG approach

It could be seen when different parameter values are resolved, the order is slightly different.

5.3. Comparative analysis

The SVNNAAWPG approach is compared with SVNNWA approach and SVNNWG approach [29], SVNN-EDAS technique [34] and SVNN-CODAS technique[35] and. The results are resolved in Figure 4. Obtained from Figure 4, it is administrated that the order of these approaches is slightly different, however, the optimal business English college is A_1 and the worst business English college is A_1

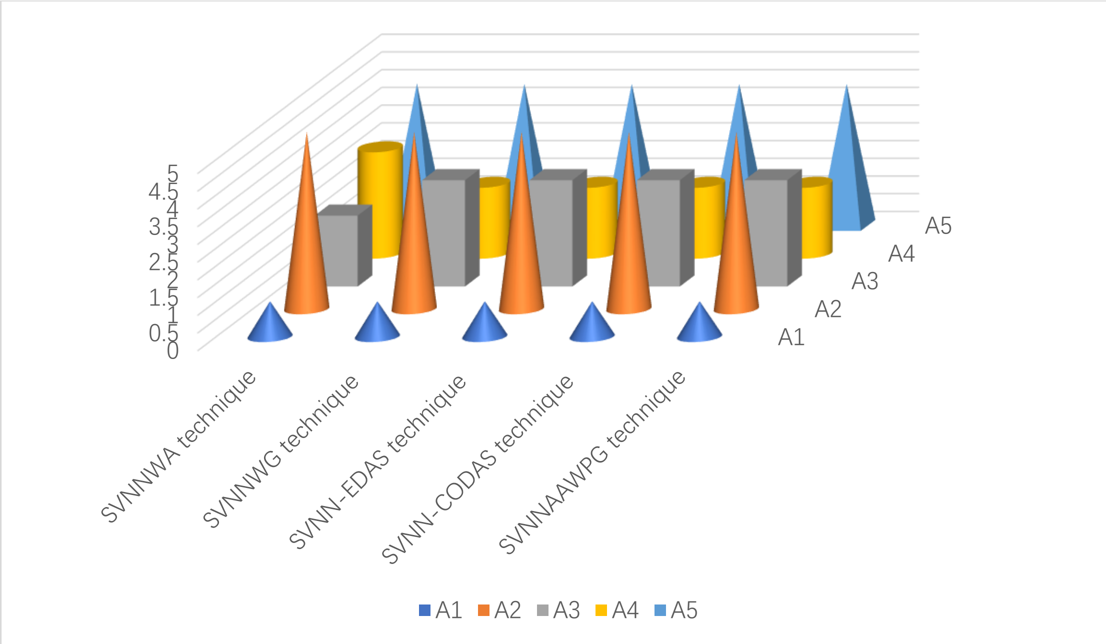

Figure 4. Comparative analysis.

5.4. Discussion analysis

The SVNNAAWPG approach presents distinct advantages and some drawbacks in multi-attribute decision-making, particularly in complex scenarios such as evaluating business English teaching quality. Some **advantages for** SVNNAAWPG approach are outlined: **(1) Effective handling of uncertainty**: The SVNNAAWPG method uses SVNSs to manage complex fuzzy and uncertain information. This three-dimensional representation captures the truth, falsity, and indeterminacy of information, allowing decision-makers to better understand and manage complexity. This capability enhances the accuracy and reliability of evaluations, helping to make more precise judgments, especially in subjective assessments. **(2) High flexibility**: The method incorporates Aczel-Alsina operations, providing the ability to flexibly adjust the weights of various factors. This flexibility allows decision-makers to tailor the evaluation model to specific needs and environmental conditions, adapting to different assessment scenarios. This is particularly important in educational quality assessments where conditions and priorities may change over time. **(3) Comprehensiveness and inclusiveness**: The introduction of the power geometric method allows SVNNAAWPG to perform weighted averaging of multiple attributes, ensuring that all relevant factors are reasonably considered in the decision-making process. This comprehensiveness not only ensures fairness but also balances different factors, making the evaluation results more objective and credible.

At the same time, some disadvantages for SVNNAAWPG approach are outlined: **(1) Mathematical complexity**: The mathematical complexity of the SVNNAAWPG method may pose challenges in practical applications. For users without a technical background, such as some educators or administrators, understanding and applying this method can be difficult. This complexity may limit its adoption in non-technical fields, affecting its effectiveness in practical scenarios. **(2) Data Requirements and Challenges**: The method relies on large amounts of accurate data to ensure reliable and effective evaluations. The process of data collection and processing can be time-consuming and challenging, especially when data is incomplete or inaccurate. This requirement may increase the cost and difficulty of evaluation, impacting its applicability in resource-limited environments.

In summary, while the SVNNAAWPG method demonstrates significant advantages in handling complex decision-making problems, it requires overcoming certain limitations and challenges in application. This necessitates appropriate adjustments and optimizations based on specific circumstances to maximize its potential.

5.5. Managerial implications

In this study, the SVNNAAWPG approach is applied, leveraging Aczel-Alsina operations and the PG approach within the framework of SVNSs. This method is specifically designed for MADM applications, aiming to enhance the accuracy and reliability of the evaluation process. A numerical example is provided to illustrate the application of the SVNNAAWPG approach in evaluating business English teaching quality. Based on the research content and methods, the managerial implications of this study include: **(1) Enhancing evaluation accuracy**: The SVNNAAWPG method significantly improves the accuracy of business English teaching quality assessments. By handling uncertainty and fuzziness, managers can obtain more reliable data to make informed decisions and optimize teaching strategies. **(2) Flexibly adjusting educational strategies**: The method's flexibility allows for dynamic adjustments according to different teaching environments and needs. Managers can adapt course design and teaching methods based on evaluation results to meet rapidly changing market and student demands. **(3) Comprehensive consideration of multiple factors**: The SVNNAAWPG approach enables weighted averaging of multiple attributes in decision-making, ensuring all factors are reasonably considered. This comprehensiveness helps in thoroughly evaluating teaching quality, ensuring fairness and objectivity in decisions, and enhancing overall educational management.

6. Conclusion

Business English plays a crucial role in international business communication, fostering versatile professionals with global perspectives and strong professional competencies. As globalization accelerates, Chinese universities have placed increasing emphasis on business English education. However, traditional teaching methods often fall short in addressing the diverse demands of the international business landscape, making it challenging to cultivate professionals who can thrive in a global environment and possess essential cross-cultural communication skills. In today's business world, Business English is more than just a communication tool; it is a convergence of business acumen, cultural understanding, and specialized knowledge. As a result, there is a pressing need to explore new teaching models that can inspire students' initiative and creativity while fostering the development of their comprehensive skill sets in business English education. The evaluation of business English teaching quality is a crucial aspect of MADM. In this study, the SVNNAAWPG approach is applied, leveraging

Aczel-Alsina operations and the PG approach within the framework of SVNSs. This method is specifically designed for MADM applications, aiming to enhance the accuracy and reliability of the evaluation process. A numerical example is provided to illustrate the application of the SVNNAAWPG approach in evaluating business English teaching quality. The major contributions of this work are highlighted as follows: (1) **Enhancement of Aczel-Alsina operations and PG approach under SVNSs**: The study enhances the Aczel-Alsina operations and the PG approach within the context of SVNSs. These operations and methods are particularly effective in handling uncertainty and vagueness, which are inherent in decision-making processes. By applying these enhanced operations, the study aims to improve the precision and robustness of the evaluation. (2) **Administration of the SVNNAAWPG approach**: The SVNNAAWPG approach is systematically applied, integrating Aczel-Alsina operations and the PG approach. This integration ensures a comprehensive evaluation framework that can address the complexities associated with business English teaching quality assessment. (3) **Application of SVNNAAWPG for MADM with SVNSs**: The SVNNAAWPG approach is tailored for MADM scenarios involving SVNSs. SVNSs are adept at representing uncertain and imprecise information, making them suitable for evaluating qualitative aspects of teaching quality. This application demonstrates the method's capability to handle complex decision-making environments effectively. (4) **Validation through numerical example**: A numerical example is employed to validate the SVNNAAWPG approach in the context of business English teaching quality evaluation. This example showcases the practical application and potential benefits of the method, highlighting its effectiveness in real-world scenarios. The example provides a step-by-step implementation, demonstrating how the SVNNAAWPG approach can be utilized to derive meaningful insights and support decision-making.

In conclusion, the application of the SVNNAAWPG approach in evaluating business English teaching quality offers significant improvements in the accuracy and reliability of the evaluation process. By enhancing Aczel-Alsina operations and the PG approach within SVNSs, the study provides a robust framework for MADM. The numerical example further validates the method's practical applicability, making it a valuable tool for educators and decision-makers in the field of business English education. This approach not only addresses the inherent uncertainties in teaching quality evaluation but also offers a systematic method for making informed decisions.

In light with above analysis, this study faces a few challenges. First, the complexity of the SVNNAAWPG method might make it hard for educators and administrators to use, especially if they don’t have a strong math background. This could limit how widely the method is adopted in evaluating teaching quality. Additionally, the research might rely on a limited data set, which could affect its applicability across diverse educational contexts. Another issue is that the model might not be dynamic enough to keep up with the rapid changes in global trade and language demands, potentially making it less effective over time. Looking ahead, there are several promising directions for further research. One important step is to simplify the model, making it more user-friendly and practical for a broader audience. Developing intuitive software tools could help non-technical users apply the method more easily. It’s also crucial to validate the method internationally by testing it in different cultural and educational settings. This would ensure its broader applicability and effectiveness. Lastly, enhancing the model’s adaptability to changes in international trade policies and technological advancements would be beneficial. By regularly updating the model and incorporating machine learning, it could adjust to new trends and requirements in real-time. These efforts could significantly improve the evaluation of business English teaching quality.

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