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A Comprehensive Discussion on Fuzzy Hypersoft Expert, Superhypersoft, and IndetermSoft Graphs

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Abstract. Graph theory, a branch of mathematics, studies relationships among entities through vertices and edges. To capture the inherent uncertainties in real-world networks, Uncertain Graph Theory has evolved within this field. Soft Expert Graphs combine conventional graph theory with expert assessments, using fuzzy sets for vertices and edges, while allowing expert opinions to shape the uncertainties and relationships within the graph. Hypersoft Graphs extend this concept further by incorporating multi-attribute nodes that represent multiple distinct attribute values, enabling the modeling of more complex, multi-dimensional relationships.

In this paper, we define the Hypersoft Expert Graph and explore its connections to other classes of graphs. We also consider the SuperHypersoft Graph, TreeSoft Graph, and IndetermSoft Graph.

Keywords: Hypersoft Expert Graph, Fuzzy graph, Soft Graph, Hypersoft Graph, Soft Set

1. Introduction

1.1. *Uncertain Graph Classes*

Graph theory, a foundational area of mathematics, models relationships within network structures through vertices (or nodes) and edges. Graphs serve as versatile tools for representing connections and interactions between various elements, often referred to as concepts or sets. These foundational concepts were first introduced in the 1700s, and since then, they have been extensively studied and developed up to the present day [26, 30, 44, 119, 129, 181].

Mathematical concepts capable of handling real-world uncertainties, such as Fuzzy Sets [179] and Neutrosophic Sets [152, 153], have been proposed to address various ambiguous scenarios. This paper investigates multiple models of uncertain graphs, which expand classical graph theory by adding layers of uncertainty, enhancing the representation of complex, ambiguous relationships. These uncertain graph models have proven highly applicable across real-world domains, leading to the development of numerous related graph classes [47–60, 62, 63, 63–72]. These uncertain graphs are especially useful in decision-making applications [3, 10, 33–35, 107, 108, 150, 158, 180].

Given the extensive research and applications in this field, uncertain graphs have become a vital area of study. For additional context and recent advancements, readers may refer to recent survey papers [61, 65, 66].

1.2. *Soft Expert Graphs and Hypersoft Graphs*

The concept of Soft Expert Sets extends into the domain of uncertain graphs, exploring their connections with other graph classes. A Soft Expert Graph integrates classical graph theory with expert evaluations, associating fuzzy sets with vertices and edges while using expert opinions to define uncertainties and relationships. This approach aligns with the principles of Soft Set Theory [15, 16, 110, 113, 177] and Soft Graph Theory [8, 88].

The framework includes models like Fuzzy Soft Expert Graphs [146], Intuitionistic Fuzzy Soft Expert Graphs [171], and Neutrosophic Soft Expert Graphs [173], each of which has proven effective in multi-criteria decision-making [21, 45, 136, 167, 170]. Extensive research has already been conducted on Soft Expert Sets and their applications [13, 14, 17, 18, 130, 145].

A Hypersoft Graph further extends traditional graphs by incorporating multi-attribute nodes, allowing each node to represent several distinct attribute values, which supports more complex, multi-dimensional relationships [132, 140, 142–144]. In essence, it can be viewed as the graph-based concept of Hypersoft Sets [1, 39, 111, 116, 117, 133, 149, 154, 178]. Additionally, Hypersoft Expert Sets have been introduced as extensions of Hypersoft Sets [2, 95–102].

1.3. *Our Contribution in This Paper*

As noted above, while research on Soft Expert Graphs and related areas is progressing, studies on Hypersoft Expert Graphs remain limited. In this paper, we define the Hypersoft Expert Graph and analyze its connections with other graph classes. Additionally, in the concluding section, we examine the SuperHypersoft Graph, TreeSoft Graph, and IndetermSoft Graph. These are graph concepts based on extensions of the Soft Set framework, specifically the SuperHypersoft Set [75, 112, 159, 163], TreeSoft Set [41, 122, 160, 161, 164], and IndetermSoft Set [160, 161].

2. **Preliminaries and Definitions**

This section offers an overview of the fundamental definitions and notations used throughout the paper. Additionally, some foundational concepts from set theory are applied in parts of this work. For further details, please consult relevant references as needed [46, 87, 90, 104, 109].

2.1. *Basic Graph Concepts*

This section provides a concise overview of essential concepts in graph theory. For a more detailed study and additional notational conventions, see [42–44, 82, 175].

Definition 2.1 (Graph)**.** [44] A *graph* G is a mathematical structure used to model pairwise relations between objects. It is composed of a set of vertices $V(G)$ and a set of edges $E(G)$, where each edge represents a connection between two vertices. Formally, a graph is denoted by $G = (V, E)$, with V as the set of vertices and E as the set of edges.

Definition 2.2 (Vertex Degree)**.** [44] For a graph $G = (V, E)$, the *degree* of a vertex $v \in V$, denoted $\deg(v)$, is defined as the number of edges incident to v. For an undirected graph, this is expressed as:

$$\deg(v) = |\{e \in E \mid v \in e\}|.$$

This measure reflects the connectivity of v within the graph.

2.2. *Uncertain Graph*

Approximately half a century has passed since the introduction of the Fuzzy Set, leading to the development of various graph concepts designed to handle uncertainty. Here, we provide definitions for frameworks including Fuzzy, Intuitionistic Fuzzy, Neutrosophic, Vague, and Single-Valued Pentapartitioned Neutrosophic.

Definition 2.3 (Unified Uncertain Graphs Framework)**.** (cf. [65]) Let $G = (V, E)$ be a classical graph with a set of vertices V and a set of edges E. Depending on the type of graph, each vertex $v \in V$ and edge $e \in E$ is assigned membership values to represent various degrees of truth, indeterminacy, falsity, and other nuanced measures of uncertainty.

(1) *Fuzzy Graph [25, 73, 76, 106, 114, 120, 137, 138, 166, 174]:*
  - Each vertex $v \in V$ is assigned a membership degree $\sigma(v) \in [0, 1]$.
  - Each edge $e = (u, v) \in E$ is assigned a membership degree $\mu(u, v) \in [0, 1]$.

(2) *Intuitionistic Fuzzy Graph (IFG) [4, 24, 37, 103, 115, 168, 172, 182]:*
  - Each vertex $v \in V$ is assigned two values: $\mu_A(v) \in [0, 1]$ (degree of membership) and $\nu_A(v) \in [0, 1]$ (degree of non-membership), such that $\mu_A(v) + \nu_A(v) \leq 1$.
  - Each edge $e = (u, v) \in E$ is assigned two values: $\mu_B(u, v) \in [0, 1]$ and $\nu_B(u, v) \in [0, 1]$, with $\mu_B(u, v) + \nu_B(u, v) \leq 1$.

(3) *Neutrosophic Graph [7, 11, 32, 61, 67, 83, 91, 105, 147, 156, 165]:*
  - Each vertex $v \in V$ is assigned a triplet $\sigma(v) = (\sigma_T(v), \sigma_I(v), \sigma_F(v))$, where $\sigma_T(v), \sigma_I(v), \sigma_F(v) \in [0, 1]$ and $\sigma_T(v) + \sigma_I(v) + \sigma_F(v) \leq 3$.
  - Each edge $e = (u, v) \in E$ is assigned a triplet $\mu(e) = (\mu_T(e), \mu_I(e), \mu_F(e))$.

(4) *Vague Graph [5, 6, 27–29, 134, 135, 148]:*
- Each vertex $v \in V$ is assigned a pair $(\tau(v), \phi(v))$, where $\tau(v) \in [0,1]$ is the degree of truth-membership and $\phi(v) \in [0,1]$ is the degree of false-membership, with $\tau(v) + \phi(v) \leq 1$.
- The grade of membership is characterized by the interval $[\tau(v), 1 - \phi(v)]$.
- Each edge $e = (u, v) \in E$ is assigned a pair $(\tau(e), \phi(e))$, satisfying:

$$\tau(e) \leq \min\{\tau(u), \tau(v)\}, \quad \phi(e) \geq \max\{\phi(u), \phi(v)\}.$$

(5) *Hesitant Fuzzy Graph [23, 77, 121, 123, 176]:*
- Each vertex $v \in V$ is assigned a hesitant fuzzy set $\sigma(v)$, represented by a finite subset of $[0,1]$, denoted $\sigma(v) \subseteq [0,1]$.
- Each edge $e = (u, v) \in E$ is assigned a hesitant fuzzy set $\mu(e) \subseteq [0,1]$.
- Operations on hesitant fuzzy sets (e.g., intersection, union) are defined to handle the hesitation in membership degrees.

(6) *Single-Valued Pentapartitioned Neutrosophic Graph [38, 93, 94, 131]:*
- Each vertex $v \in V$ is assigned a quintuple $\sigma(v) = (T(v), C(v), R(v), U(v), F(v))$, where:
  - $T(v) \in [0,1]$ is the truth-membership degree.
  - $C(v) \in [0,1]$ is the contradiction-membership degree.
  - $R(v) \in [0,1]$ is the ignorance-membership degree.
  - $U(v) \in [0,1]$ is the unknown-membership degree.
  - $F(v) \in [0,1]$ is the false-membership degree.
  - $T(v) + C(v) + R(v) + U(v) + F(v) \leq 5$.
- Each edge $e = (u, v) \in E$ is assigned a quintuple $\mu(e) = (T(e), C(e), R(e), U(e), F(e))$, satisfying:

$$\begin{cases} T(e) \leq \min\{T(u), T(v)\}, \\ C(e) \leq \min\{C(u), C(v)\}, \\ R(e) \geq \max\{R(u), R(v)\}, \\ U(e) \geq \max\{U(u), U(v)\}, \\ F(e) \geq \max\{F(u), F(v)\}. \end{cases}$$

2.3. *Hypersoft Graph*

A HyperSoft Graph represents multi-attribute nodes where each node can hold distinct attribute values, enabling complex, multi-dimensional relationships. The definition of a Hypersoft Graph is provided as follows [132, 140, 142–144].

Definition 2.4 (Hypersoft Set)**.** [154] Let X be a non-empty finite universe, and let $T_1, T_2, \ldots, T_n$ be n-distinct attributes with corresponding disjoint sets $J_1, J_2, \ldots, J_n$. A pair (F, J) is called a *hypersoft set* over the universal set X, where F is a mapping defined by

$$F : J \to \mathcal{P}(X),$$

with $J = J_1 \times J_2 \times \cdots \times J_n$.

Definition 2.5 (Hypersoft Graph)**.** Let $G = (V, E)$ be a simple connected graph, where V is the set of vertices and E is the set of edges. Consider $J = J_1 \times J_2 \times \cdots \times J_n$, where each $J_i \subseteq V$ and $J_i \cap J_j = \emptyset$ for $i \neq j$. A *Hypersoft Graph* (HS-Graph) of G is defined as a hypersoft set (F, J) over V such that for each $x \in J$, $F(x)$ induces a connected subgraph of G. The set of all HS-Graphs of G is denoted by $\mathrm{HsG}(G)$.

2.4. *Fuzzy Soft Graph and Fuzzy Hypersoft Graph*

A Fuzzy Soft Graph assigns fuzzy membership values to vertices and edges based on parameters, representing uncertainty in relationships [9, 12, 19, 20, 22, 34, 92, 151]. And a Fuzzy Hypersoft Graph extends fuzzy soft graphs by using multi-parameter combinations to assign fuzzy membership, capturing complex multi-attribute relationships. The definitions of Fuzzy Soft Graph and Fuzzy Hypersoft Graph are presented as follows.

Definition 2.6 (Fuzzy Soft Graph)**.** [9, 12] Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be a non-empty set, and let $A \subseteq E$, where E is a set of parameters. A *fuzzy soft graph* is defined by two mappings:

- $\rho : A \to F(X)$, where $F(X)$ is the collection of all fuzzy subsets in X. For each parameter $e \in A$,
$$\rho(e) : X \to [0, 1]$$
assigns a membership degree $\rho_e(x_i)$ to each element $x_i \in X$. The pair (A, ρ) represents the fuzzy soft vertices.
- $\beta : A \to F(X \times X)$, where $F(X \times X)$ is the collection of all fuzzy subsets in $X \times X$. For each parameter $e \in A$,
$$\beta(e) : X \times X \to [0, 1]$$
assigns a membership degree $\beta_e(x_i, x_j)$ to each pair $(x_i, x_j) \in X \times X$. The pair (A, β) represents the fuzzy soft edges.

The structure $((A, \rho), (A, \beta))$ is called a *fuzzy soft graph* if

$$\beta_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$$

for all $e \in A$ and for all $i, j = 1, 2, \ldots, n$. This graph is denoted as $\hat{G}_{A,X}$.

Definition 2.7 (Fuzzy Hypersoft Graph). [139,141] Let $X = \{x_1, x_2, x_3, \ldots, x_n\}$ be a universe of discourse, and let E be a set of parameters with subsets $K_i \subseteq E$ for $i = 1, 2, \ldots, n$. Define $M = K_1 \times K_2 \times \cdots \times K_n$, representing combinations of parameter values. A *fuzzy hypersoft graph* is defined by:

- $\rho : M \to F(X)$, where $F(X)$ is the collection of all fuzzy subsets in X. For each $e = (k_1, k_2, \ldots, k_n) \in M$,
$$\rho(e) : X \to [0, 1]$$
assigns a membership degree $\rho_e(x_i)$ to each element $x_i \in X$. The pair (M, ρ) represents the fuzzy hypersoft vertices.
- $\beta : M \to F(X \times X)$, where $F(X \times X)$ is the collection of all fuzzy subsets in $X \times X$. For each $e = (k_1, k_2, \ldots, k_n) \in M$,
$$\beta(e) : X \times X \to [0, 1]$$
assigns a membership degree $\beta_e(x_i, x_j)$ to each pair $(x_i, x_j) \in X \times X$. The pair (M, β) represents the fuzzy hypersoft edges.

The structure $((M, \rho), (M, \beta))$ is called a *fuzzy hypersoft graph* if

$$\beta_e(x_i, x_j) \leq \rho_e(x_i) \wedge \rho_e(x_j)$$

for all $e \in M$ and for all $i, j = 1, 2, \ldots, n$. This graph is denoted as $\hat{G}_{M,X}$.

2.5. *Fuzzy Soft Expert Graph*

A Fuzzy Soft Expert Graph combines fuzzy memberships for vertices and edges with expert opinions, defining relationships with parameters and expert evaluations. The definition of a Fuzzy Soft Expert Graph is provided as follows.

Definition 2.8 (Fuzzy Soft Expert Graph). [146] Let V be a universe of discourse, Y a set of parameters, X a set of experts (agents), and $S = \{1, 0\}$ a set of opinions, where 1 represents agreement and 0 represents disagreement. Define $M = Y \times X \times S$ as the space of all parameter-expert-opinion combinations, and let $E \subseteq M \times M$ represent the set of edges in the graph.

A *fuzzy soft expert graph* (FSEG) is defined as a 4-tuple $G = (G^*, Y, \rho, \beta)$, where:

- $G^* = (V, E)$ is a simple graph.
- $\rho : Y \to F(V)$ is a fuzzy soft expert vertex function, where for each $y \in Y$,
$$\rho(y) = \langle x, \rho_y(x) \rangle : x \in V$$
defines the membership degree $\rho_y(x)$ for each vertex $x \in V$.

- $\beta : Y \to F(V \times V)$ is a fuzzy soft expert edge function, where for each $y \in Y$ and edge $(x, x') \in V \times V$,

$$\beta(y) = \langle (x, x'), \beta_y(x, x') \rangle : (x, x') \in V \times V$$

assigns a membership degree $\beta_y(x, x')$ for each edge.

The conditions required for G to be a fuzzy soft expert graph are:

$$\beta_y(x, x') \leq \min\{\rho_y(x), \rho_y(x')\},$$

for all $(x, x') \in V \times V$ and $y \in Y$.

We can also represent a FSEG by $G = (G^*, Y, \rho, \beta) = \{G(y) : y \in Y\}$, where each $G(y)$ is a parameterized fuzzy graph in V.

2.6. *Fuzzy Hypersoft Expert Graph*

The definition of the proposed Fuzzy Hypersoft Expert Graph in this paper is presented below. It extends the concept of the Fuzzy Hypersoft Expert Set into a graph framework and, as will be shown in the following theorem, is also an extension of the Fuzzy Soft Expert Graph.

Definition 2.9 (Fuzzy Hypersoft Expert Set)**.** [95] Let $A = \{A_1, A_2, \ldots, A_n\}$ be a collection of non-overlapping subsets of parameters corresponding to different attributes a_i for $i = 1, 2, \ldots, n$. Define the fuzzy parameterized fuzzy hypersoft expert set (FPFHSE-set) over a universal set U as a pair (f, S), where:

- $Q = A_1 \times A_2 \times \cdots \times A_n$ is the Cartesian product of parameter sets.
- $P = Q \times I \times U$ where I is a set of experts (agents), and $U = \{0, 1\}$ denotes the set of opinions, with 1 representing agreement and 0 disagreement.
- $S \subseteq H = \{(\zeta(q)/q, x, u) \mid (q, x, u) \in P, \zeta(q) \in [0, 1]\}$ is the fuzzy hypersoft expert subset of possible evaluations.
- $f : S \to \mathrm{FP}(U)$ is an approximate function mapping elements of S to fuzzy subsets of U.

Thus, the FPFHSE-set (f, S) can be represented as:

$$(f, S) = \{((\zeta(q)/q, x, u), f((\zeta(q)/q, x, u))) \mid (\zeta(q)/q, x, u) \in S\}.$$

Definition 2.10 (Fuzzy Hypersoft Expert Graph)**.** Let V be a finite set of vertices, and E a set of edges such that $E \subseteq V \times V$. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a collection of non-overlapping parameter subsets corresponding to different attributes a_i, where each A_i is associated with attribute a_i for $i = 1, 2, \ldots, n$. Define the parameter space $Q = A_1 \times A_2 \times \cdots \times A_n$.

Let X be a set of experts, and let $U = \{0, 1\}$ be a set of opinions, where 1 represents agreement and 0 represents disagreement. Define the set $M = Q \times X \times U$, representing all combinations of parameters, experts, and opinions.

Define two mappings:

- The *fuzzy hypersoft expert vertex function* $\rho : M \to F(V)$, where $F(V)$ denotes the set of all fuzzy subsets of V. For each $m = (q, x, u) \in M$, the function $\rho_m : V \to [0, 1]$ assigns a membership degree $\rho_m(v)$ to each vertex $v \in V$.
- The *fuzzy hypersoft expert edge function* $\beta : M \to F(V \times V)$, where $F(V \times V)$ denotes the set of all fuzzy subsets of $V \times V$. For each $m = (q, x, u) \in M$, the function $\beta_m : V \times V \to [0, 1]$ assigns a membership degree $\beta_m(u, v)$ to each edge $(u, v) \in V \times V$.

The structure $G = ((M, \rho), (M, \beta))$ is called a *fuzzy hypersoft expert graph* if for all $m \in M$ and for all $u, v \in V$, the following condition holds:

$$\beta_m(u, v) \leq \rho_m(u) \wedge \rho_m(v),$$

where $\wedge$ denotes the minimum operator.

3. Result:Fuzzy Hypersoft Expert Graph

The relationship between the Fuzzy Hypersoft Expert Graph and other classes of graphs is described as follows.

Theorem 3.1. *The fuzzy hypersoft expert graph $G = ((M, \rho), (M, \beta))$ can be transformed into a fuzzy soft expert graph, a fuzzy hypersoft graph, and a fuzzy graph by appropriate reductions of parameters, experts, and opinions.*

Proof. We will show the transformations step by step.
Transformation to Fuzzy Soft Expert Graph. Let us consider the fuzzy hypersoft expert graph $G = ((M, \rho), (M, \beta))$. To obtain a fuzzy soft expert graph, we proceed as follows:

- Fix the parameters by selecting a specific tuple $q \in Q$.
- The set M reduces to $M' = \{(q, x, u) \mid x \in X, u \in U\}$.
- The mappings ρ and β become functions over M'.
- The fuzzy hypersoft expert graph reduces to $G' = ((M', \rho), (M', \beta))$, which corresponds to a fuzzy soft expert graph where the parameters are fixed.

Transformation to Fuzzy Hypersoft Graph. To transform G into a fuzzy hypersoft graph, we proceed by aggregating over experts and opinions:

- Consider the set $M'' = Q$, eliminating the expert set X and opinion set U.
- Define aggregated vertex and edge functions $\rho' : Q \to F(V)$ and $\beta' : Q \to F(V \times V)$ by combining the contributions from all experts and opinions.
- For each $q \in Q$, define:

$$\rho'_q(v) = \max_{(x,u) \in X \times U} \rho_{(q,x,u)}(v),$$

$$\beta'_q(u, v) = \max_{(x,u') \in X \times U} \beta_{(q,x,u')}(u, v).$$

- The structure $G'' = ((Q, \rho'), (Q, \beta'))$ is a fuzzy hypersoft graph.

Transformation to Fuzzy Graph. To reduce G to a fuzzy graph, we fix both the parameters and the experts, and consider the agreed opinions:

- Fix a specific parameter tuple $q \in Q$ and a specific expert $x \in X$.
- Consider the set $M''' = \{(q, x, 1)\}$, where $u = 1$ represents agreement.
- The vertex and edge functions become $\rho_{(q,x,1)}$ and $\beta_{(q,x,1)}$.
- The fuzzy graph is then $G''' = (\rho_{(q,x,1)}, \beta_{(q,x,1)})$.

This proof is completed. □

4. **Future direction of this research**

In this section, we briefly discuss the future direction of this research.

4.1. *Future tasks: Hypergraphs and Superhypergraphs*

As a future direction for this research, we are considering extending Soft Expert Graphs and Hypersoft Expert Graphs to hypergraphs [31,78–80,80,128] (creating Soft Expert Hypergraphs and Hypersoft Expert Hypergraphs) and superhypergraphs [74, 84–86, 155–158, 162] (leading to Soft Expert Superhypergraphs and Hypersoft Expert Superhypergraphs).

In the future, we hope to extend the concepts of Soft Expert Graphs and Hypersoft Expert Graphs by incorporating the framework of Rough Graphs. Note that a Rough Set [81,124–127] (or Rough Graph [36, 40, 89, 118, 169]) is a mathematical model that approximates uncertain or imprecise data using lower and upper approximations to capture data boundaries. We also intend to explore extensions of Hypersoft Expert Graphs into Neutrosophic Graphs and similar frameworks.

We also plan to explore real-world applications and mathematical properties of Soft Expert Graphs and Hypersoft Expert Graphs in greater detail in the future.

4.2. *Discussion: IndetermSoft Graphs, superhypersoft Graphs, and TreeSoft Graphs*

Various derived forms of soft sets have been proposed, considering their potential applications and mathematical significance. We aim to investigate the superhypersoft graph as an extension of superhypersoft sets [75, 112, 159, 163]. Superhypersoft sets generalize the concepts of both hypersoft sets and soft sets. Additionally, we will explore IndetermSoft Sets [160, 161], IndetermHyperSoft Sets [160, 161], and TreeSoft Sets [41, 122, 160, 161, 164].

The IndetermSoft Set is a Soft Set in which attribute values or subsets may contain indeterminate or uncertain elements within their mappings. The IndetermHyperSoft Set is a

HyperSoft Set that allows indeterminacy in attributes, attribute values, or subset mappings, thus accommodating uncertainty. The TreeSoft Set is a hierarchical Soft Set with multi-level attributes organized in a tree structure, facilitating mappings of complex attribute relationships.

As an example, we consider the IndetermSoft Graph and TreeSoft Graph, which are graphical representations of IndetermSoft Sets and TreeSoft Sets, respectively. Although these concepts are still in the preliminary stages, we present their definitions here and plan to explore their potential applications and mathematical properties in the future. Additionally, we aim to define concepts that incorporate the ideas of Soft Expert Graphs and HyperSoft Expert Graphs. We intend to investigate their applications, mathematical properties, and related algorithms.

Definition 4.1 (IndetermSoft Set)**.** [161] Let U be a universe of discourse, H a non-empty subset of U, and $P(H)$ the powerset of H. Let A be a set of attribute values associated with a specific attribute a. A function $F : A \to P(H)$ is defined as an *IndetermSoft Set* if any of the following conditions hold:

(1) The set A possesses some level of indeterminacy.
(2) The powerset $P(H)$ exhibits some indeterminacy.
(3) There exists at least one attribute value $v \in A$ such that $F(v)$ is indeterminate, meaning unclear, uncertain, or not unique.
(4) Any combination of the above conditions holds.

The IndetermSoft Set (F, H) has a certain degree of indeterminacy, and thus represents a specific case of a NeutroFunction, which allows for a mix of determinate, indeterminate, and false components.

Definition 4.2 (IndetermSoft Graph)**.** Let U be a universe of discourse, and let H be a non-empty subset of U, representing the possible vertices of a graph. Let $P(H)$ denote the power set of H. Let A be a set of attribute values associated with a specific attribute a.

An *IndetermSoft Graph* is defined as a pair $G = ((F_V, H), (F_E, H \times H))$, where:

(1) **IndetermSoft Vertex Function** $F_V : A \to P(H)$:
  - F_V maps each attribute value $v \in A$ to a subset $F_V(v) \subseteq H$ of vertices.
  - The function F_V may exhibit indeterminacy according to the definition of an IndetermSoft Set; that is, for some $v \in A$, $F_V(v)$ may be indeterminate (unclear, uncertain, or not unique).

(2) **IndetermSoft Edge Function** $F_E : A \to P(H \times H)$:
  - F_E maps each attribute value $v \in A$ to a subset $F_E(v) \subseteq H \times H$ of edges.
  - Similar to F_V, the function F_E may also exhibit indeterminacy.

The IndetermSoft Graph $G = ((F_V, H), (F_E, H \times H))$ incorporates indeterminacy in both its vertex and edge sets, allowing for the representation of uncertain or ambiguous relationships within the graph.

Definition 4.3 (IndetermHyperSoft Set)**.** [161] Let U be a universe of discourse, H a non-empty subset of U, and $P(H)$ the power set of H.

Let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$ for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$.

Then, the pair $(F, A_1 \times A_2 \times \cdots \times A_n)$, where $F : A_1 \times A_2 \times \cdots \times A_n \to P(H)$, is called an *IndetermHyperSoft Set* over U if at least one of the following conditions occurs:

(1) At least one of the sets $A_1, A_2, \ldots, A_n$ possesses some indeterminacy.
(2) The set H or the power set $P(H)$ possesses some indeterminacy.
(3) There exists at least one n-tuple $(e_1, e_2, \ldots, e_n) \in A_1 \times A_2 \times \cdots \times A_n$ such that $F(e_1, e_2, \ldots, e_n)$ is indeterminate (unclear, uncertain, conflicting, or not unique).

Thus, an IndetermHyperSoft Set is an extension of the HyperSoft Set when there is indeterminate data, or indeterminate functions, or indeterminate sets.

Definition 4.4 (IndetermHyperSoft Graph)**.** Let $G = (V, E)$ be a graph, where V is the set of vertices and E is the set of edges.

Let U be a universe of discourse, and let H be a non-empty subset of U, where $H \subseteq V \cup E$, representing possible vertices and edges.

Let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attribute values are respectively the sets $A_1, A_2, \ldots, A_n$, with $A_i \cap A_j = \emptyset$ for $i \neq j$, and $i, j \in \{1, 2, \ldots, n\}$.

Then, the pair $(F, A_1 \times A_2 \times \cdots \times A_n)$, where $F : A_1 \times A_2 \times \cdots \times A_n \to P(H)$, is called an *IndetermHyperSoft Graph* over G if at least one of the following conditions occurs:

(1) At least one of the sets $A_1, A_2, \ldots, A_n$ possesses some indeterminacy.
(2) The set H or the power set $P(H)$ possesses some indeterminacy.
(3) There exists at least one n-tuple $(e_1, e_2, \ldots, e_n) \in A_1 \times A_2 \times \cdots \times A_n$ such that $F(e_1, e_2, \ldots, e_n)$ is indeterminate (unclear, uncertain, conflicting, or not unique).

In this context, F maps combinations of attribute values to subsets of H (which could be vertices or edges), incorporating indeterminacy in the attributes, the attribute values, the mapping F, or the set H.

Theorem 4.5. *Every IndetermHyperSoft Graph can be transformed into an IndetermSoft Graph.*

Proof. Let $G = (V, E)$ be a graph, and let $(F, A_1 \times A_2 \times \cdots \times A_n)$ be an IndetermHyperSoft Graph over G, where $F : A_1 \times A_2 \times \cdots \times A_n \to P(H)$, and $H \subseteq V \cup E$.

We can transform the IndetermHyperSoft Graph into an IndetermSoft Graph by combining the multiple attributes into a single composite attribute.

Define a new attribute set A as the Cartesian product:

$$A = A_1 \times A_2 \times \cdots \times A_n.$$

Consider the function F as a mapping $F' : A \to P(H)$, where for each $a = (e_1, e_2, \ldots, e_n) \in A$, we have $F'(a) = F(e_1, e_2, \ldots, e_n)$.

Now, (F', A) can be viewed as an IndetermSoft Graph, where:

- A is the set of composite attribute values (each n-tuple is treated as a single attribute value).
- F' maps attribute values $a \in A$ to subsets of H (vertices or edges), possibly with indeterminacy.

Since the conditions of indeterminacy in the IndetermHyperSoft Graph (indeterminacy in A_i, H, or F) are preserved in the IndetermSoft Graph (indeterminacy in A, H, or F'), the transformation is valid.

Thus, every IndetermHyperSoft Graph can be transformed into an IndetermSoft Graph. □

Definition 4.6 (TreeSoft Set)**.** [161] Let U be a universe of discourse, and H a non-empty subset of U, with $P(H)$ representing the power set of H. Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of attributes, where $n \geq 1$ is an integer and each A_i represents a first-level attribute. Each attribute A_i can be further subdivided as follows:

$$A_1 = \{A_{1,1}, A_{1,2}, \ldots\}$$
$$A_2 = \{A_{2,1}, A_{2,2}, \ldots\}$$
$$\vdots$$
$$A_n = \{A_{n,1}, A_{n,2}, \ldots\}$$

where each $A_{i,j}$ represents a second-level sub-attribute. This hierarchical structure can continue further, forming sub-sub-attributes, such as $A_{i,j,k}$, and so forth, up to the m-th level, denoted as $A_{i_1,i_2,\ldots,i_m}$. This structure forms a graph-tree, denoted as $\text{Tree}(A)$, rooted at A (considered as level zero) and extending to nodes at levels 1 through m.

A TreeSoft Set is a mapping defined as:

$$F : P(\text{Tree}(A)) \to P(H)$$

where $\text{Tree}(A)$ is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and $P(\text{Tree}(A))$ is the power set of $\text{Tree}(A)$.

The node sets of the TreeSoft Set of level m are:

$$\text{Tree}(A) = \{A_{i_1} \mid i_1 = 1, 2, \ldots\}$$

If the graph-tree has only two levels (i.e., $m = 2$), then the TreeSoft Set reduces to a MultiSoft Set.

Definition 4.7 (TreeSoft Graph)**.** Let U be a universe of discourse, and let H be a non-empty subset of U, representing the possible vertices of a graph. Let $P(H)$ denote the power set of H.

Let $A = \{A_1, A_2, \ldots, A_n\}$ be a set of attributes, where each A_i is a first-level attribute. Each attribute A_i can be further subdivided into sub-attributes, forming a hierarchical structure:

$$A_i = \{A_{i,1}, A_{i,2}, \ldots\} \quad \text{for } i = 1, 2, \ldots, n$$

Each sub-attribute $A_{i,j}$ can be further divided into sub-sub-attributes $A_{i,j,k}$, and so on, up to level m. This hierarchical structure forms a graph-tree, denoted as $\text{Tree}(A)$, rooted at A and extending to nodes at levels 1 through m.

A *TreeSoft Graph* is defined via two functions:

(1) **Vertex Function** $F_V : P(\text{Tree}(A)) \to P(H)$:
   - F_V maps subsets of the attribute tree $\text{Tree}(A)$ to subsets of H (vertices).

(2) **Edge Function** $F_E : P(\text{Tree}(A)) \to P(H \times H)$:
   - F_E maps subsets of the attribute tree $\text{Tree}(A)$ to subsets of $H \times H$ (edges).

Thus, the TreeSoft Graph $G = (F_V, F_E)$ represents a graph whose vertices and edges are defined based on hierarchical attributes, capturing complex relationships in a structured manner.

Definition 4.8 (SuperHyperSoft Set)**.** [159] Let U be a universe of discourse, and let $P(U)$ denote the power set of U. Let $a_1, a_2, \ldots, a_n$ be n distinct attributes, where $n \geq 1$. Each attribute a_i has a corresponding set of attribute values A_i, with the property that $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Let $P(A_i)$ denote the power set of A_i for each $i = 1, 2, \ldots, n$.

Then, the pair $(F, P(A_1) \times P(A_2) \times \cdots \times P(A_n))$, where

$$F : P(A_1) \times P(A_2) \times \cdots \times P(A_n) \to P(U),$$

is called a *SuperHyperSoft Set* over U.

Definition 4.9 (SuperHyperSoft Graph)**.** Let $G = (V, E)$ be a graph, where V is the set of vertices and E is the set of edges. Let $U = V \cup E$, and let $P(U)$ denote the power set of U.

Let $a_1, a_2, \ldots, a_n$ be n distinct attributes, each with a corresponding set of attribute values A_i, such that $A_i \cap A_j = \emptyset$ for all $i \neq j$.

Let $P(A_i)$ denote the power set of A_i for each $i = 1, 2, \ldots, n$.

Define two functions:

(1) **Vertex Function**:

$$F_V : P(A_1) \times P(A_2) \times \cdots \times P(A_n) \to P(V),$$

which maps combinations of attribute value subsets to subsets of vertices.

(2) **Edge Function**:

$$F_E : P(A_1) \times P(A_2) \times \cdots \times P(A_n) \to P(E),$$

which maps combinations of attribute value subsets to subsets of edges.

Then, the pair (F_V, F_E) is called a *SuperHyperSoft Graph* over G.

Theorem 4.10. *The SuperHyperSoft Graph generalizes both the HyperSoft Graph and the general Graph. Specifically:*

(1) *Every HyperSoft Graph is a special case of a SuperHyperSoft Graph.*
(2) *Every general Graph can be represented as a SuperHyperSoft Graph.*

Proof. We will prove the two aspects stated in the theorem in sequence.

1. SuperHyperSoft Graph generalizes the HyperSoft Graph:

Recall that in a HyperSoft Graph, the attribute functions are defined over the Cartesian product of attribute value sets $A_1 \times A_2 \times \cdots \times A_n$.

In the SuperHyperSoft Graph, the domain of the functions F_V and F_E is extended to the Cartesian product of the power sets $P(A_1) \times P(A_2) \times \cdots \times P(A_n)$.

Since each attribute value set A_i is a subset of its power set $P(A_i)$ (specifically, $A_i \subseteq P(A_i)$), any function defined on $A_1 \times A_2 \times \cdots \times A_n$ can be considered as a function defined on $P(A_1) \times P(A_2) \times P(A_n)$ by restricting the domain to singleton subsets.

Therefore, every HyperSoft Graph is a SuperHyperSoft Graph where the functions F_V and F_E are defined only on singleton subsets of attribute values.

2. SuperHyperSoft Graph generalizes the general Graph:

Any graph $G = (V, E)$ can be represented as a SuperHyperSoft Graph by defining trivial attributes and functions.

Let us consider the simplest case with a single attribute a_1 and a corresponding attribute value set $A_1 = \{*\}$. Then, the power set $P(A_1) = \{\emptyset, \{*\}\}$.

Define the functions:

- $F_V : P(A_1) \to P(V)$ by:

  - $F_V(\emptyset) = \emptyset$,
  - $F_V(\{*\}) = V$.
- $F_E : P(A_1) \to P(E)$ by:
  - $F_E(\emptyset) = \emptyset$,
  - $F_E(\{*\}) = E$.

Thus, the SuperHyperSoft Graph (F_V, F_E) represents the entire graph G. □

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Ethical Approval

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Conflicts of Interest

The authors declare no conflicts of interest related to the publication of this paper.

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